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ON HIGHER-ORDER SEMICLASSICAL CORRECTIONS TO HIGH-ENERGY CROSS SECTIONS IN THE ONE-INSTANTON SECTOR*

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The relationship between classical euclidean field configurations and particle production in the electroweak theory in the presence of an instanton is examined. A new calculation of the W-boson propagator corrections to the $(E/M_{sp})^2$ contribution to $\ln\sigma$ is given. It is shown explicitly how these corrections are related to minimum action configurations of well-separated instanton–anti-instanton pairs.

1. Introduction

In this paper we examine the relationship between particle production in the presence of an instanton [1–3] and classical euclidean field configurations which minimize the action of an instanton–anti-instanton pair at a fixed euclidean separation [4–9]. If one writes the cross section for $W(p_1) + W(p_2) \to g$ antiquarks + many W-bosons as [10–12] $\sigma\alpha\, e^{(-4\pi/\alpha)F(E/E_0)}$ with E the center-of-mass energy of two incident W-bosons and with $E_0 = \sqrt{6}\,\pi M_W/\alpha$, then we explicitly treat the "low-energy" terms in F of size $(E/E_0)^{4/3}$ and $(E/E_0)^2$. The main purpose of the paper is to compare the method developed in ref. [10] with the valley method [6–8]. Through order $(E/E_0)^2$ we demonstrate that these methods are equivalent. As a by-product we perform an independent calculation of the W-boson propagator contribution to the $(E/E_0)^2$ term, a term around which there has been some controversy [8, 13, 14], and confirm the result of Khoze and Ringwald [8]. Our final answer to this order is

$$F(E/E_0) = 1 - \tfrac{9}{8}(E/E_0)^{4/3} + \tfrac{9}{16}(E/E_0)^2.$$

The $\frac{9}{8}(E/E_0)^{4/3}$ is the term found in ref. [10]. The $\frac{9}{16}(E/E_0)^2$ term is the same as the result of Khoze and Ringwald and consists of parts which have been previously

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discussed. The first part is Higgs production. (Higgs propagator corrections are not important at the level of approximation considered here.) A second part is the finite W-boson mass correction to the leading-order semiclassical result. The third part is the W-boson propagator correction at $M_W^2 \ll k^2 \ll 1/\rho^2$ with k a typical momentum of a produced W-boson and ρ the instanton size. At this level the zero-mode problem for the W-propagator does not arise. Khoze and Ringwald have not included W-boson mass corrections, the 6th term on the right-hand side of eq. (26), though they are included implicitly in their Higgs term. This mass correction was first given in ref. [13]. Higgs production, the 5th term on the right-hand side of eq. (26), is well known [1–3]. The result of ref. [8] appears different in the Higgs part because the W-boson mass term is included there. The W-boson propagator correction, the final term on the right-hand side of eq. (26), has been previously calculated in refs. [8, 13, 14]. Our calculation, given in sect. 2, agrees with the answer given in refs. [8, 14] but disagrees with the answer given in ref. [13]. We feel that this disagreement is not of a profound nature.

In sect. 2, we briefly review the main results of ref. [10] giving the total cross section for the semiclassical production of particles in the one-instanton sector. We then carry out a calculation of the W-boson propagator corrections at the level of $(E/E_0)^2$ corrections to $F(E/E_0)$.

In sect. 3, we give a formula for minimizing the classical euclidean action for an instanton–anti-instanton pair at fixed euclidean separation. In general, one must deal with the zero-mode problem, however, there are no essential difficulties which arise when calculating F to order $(E/E_0)^2$. We then show how the formalism of ref. [10] is reproduced by taking the imaginary part of the exponential of this classical action when continued to Minkowski space in x^2 with x the instanton–anti-instanton separation. We then show that zero-mode difficulties only arise at order-$(E/E_0)^{8/3}$ terms in F.

Of course, the semiclassical expansion in powers of $(E/E_0)^{2/3}$ is not guaranteed to be useful for the baryon number violation problem. While we expect the valley method to give the semiclassical corrections correctly so long as the instanton–anti-instanton separation is large compared to the instanton size, the interesting region of E/E_0 on the order of 1 probably involves instanton–anti-instanton separations comparable to the instanton size. In addition, there are high-energy corrections to the initial particles involved in the scattering which may not be semiclassical [15, 16].

2. Expansion of the cross section for $E/E_{\rm sph} \ll 1$

In this section we shall briefly review the calculation of the leading semiclassical production cross section in the presence of a single instanton [10]. As is well known this leading semiclassical cross section is due to W-production with the Higgs field only serving to cut off the large instanton sizes. [As usual we assume $\sin^2\theta_{\rm w} = 0$ so

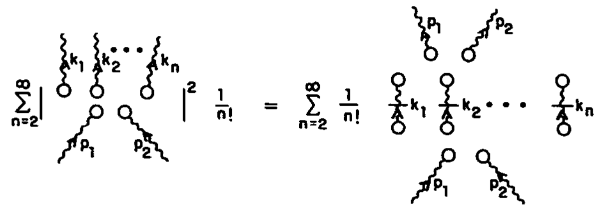

Fig. 1. The total cross section for 2 W's $\to n$ W's. The circle with the wiggly line represents a classical (instanton) solution continued to Minkowski space and evaluated on-shell after extracting $-i/k_i^2$.

that only the SU(2) part of the usual SU(2) ⊗ U(1) electroweak theory is effective. For simplicity we also suppose that the Higgs coupling, λ, and α_{w} are comparable so that the masses of the Higgs particle and the W-mesons are not too different.] Next, we shall calculate the next-to-leading low-energy corrections. Here both W-bosons and Higgs particles are important, though only the propagator corrections of the W-bosons present any technical difficulties.

2.1 THE LEADING SEMICLASSICAL APPROXIMATION FOR $E/E_{\mathrm{sph}} \ll 1$ [10]

In the leading semiclassical approximation, illustrated in fig. 1,

$$\sigma \alpha \int \mathrm{d}\rho^2 \,\mathrm{d}\rho'^2 \exp\left[-\frac{4\pi}{\alpha} - \pi^2 v^2(\rho^2+\rho'^2)\right] \mathrm{d}U(\xi)\,\mathrm{d}U(\xi') \sum_{n=0}^{\infty} \frac{1}{n!} \prod_{i=1}^{n} \int \frac{\mathrm{d}^3 k_i}{(2\pi)^3 2E_i} (2\pi)^4$$

$$\times \delta\left(p_1+p_2-\sum_{i=1}^{n} k_i\right) \prod_{i=1}^{n} R_\mu^{a*}(\xi', k_i, \rho'^2) R_\mu^a(\xi, k_i, \rho^2)(-1), \tag{1}$$

where the two incoming W's have momentum p_1 and p_2. ρ and ρ' are the scale sizes of the instantons in the amplitude and in the complex conjugate amplitude, respectively. Let $A_\mu^a(\xi, x, \rho^2)$ be the instanton field continued to Minkowski space,

$$A_\mu^a(\xi, x, \rho^2) = \sum_{a'} U_{aa'}(\xi) \frac{2\rho^2 \bar{\eta}^{(-)}_{a'\mu\nu} x_\nu}{g(x^2-i\epsilon)(x^2-\rho^2-i\epsilon)}, \tag{2}$$

where ξ specifies the orientation in weak isospin space and where $\bar{\eta}^{(-)}_{aij} = \epsilon_{aij}$,

$\bar{\eta}^{(-)}_{ai0} = i\delta_{ai}$ and $\bar{\eta}^{(-)}_{a\mu\nu} = -\bar{\eta}^{(-)}_{a\nu\mu}$. Then defining

$$A^a_\mu(\xi, k, \rho^2) = i\int d^4x\, e^{-ikx} A^a_\mu(\xi, x, \rho^2) \tag{3}$$

one has

$$A^2_\mu(\xi, -k, \rho^2) \underset{k^2\to 0}{\longrightarrow} \frac{-i}{k^2} R^a_\mu(\xi, k, \rho^2), \tag{4}$$

with

$$R^a_\mu(\xi, k, \rho^2) = \sum_{a'} U_{aa'}(\xi) \frac{4\pi^2\rho^2}{g} \bar{\eta}^{(-)}_{a'\mu\nu} k_\nu, \tag{5}$$

and where $U_{aa'}(\xi)$ gives a rotation in weak isospin space. It is convenient to work in the center-of-mass frame where $p_1 + p_2 = (E, 0, 0, 0)$. Then so long as $E/E_{sp} \ll 1$ we shall see that the produced W's are relativistic so that the massless gauge field approximation used in eq. (2) is justified. The phase space integration in (1) may also be approximated by the phase space for zero mass W's. Replacing the $(2\pi)^4\delta^4(p_1 + p_2 - \Sigma^n_{i=1} k_i)$ by an integral over the position of the instanton in the complex conjugate amplitude relative to the position of the instanton in the $2 \to n$ production amplitude,

$$(2\pi)^4 \delta^4\left(p_1 + p_2 - \sum_{i=1}^n k_1\right) = \int d^4x\, e^{-i(p_1+p_2-k_1-k_2-\ldots-k_n)\cdot x},$$

gives an exponential series in (1). One finds

$$\sigma\alpha\int d\rho^2\, d\rho'^2\, dU(\xi)\, dU(\xi')\, d^4x \exp\left[-iEt - \frac{4\pi^2}{\alpha} - (\rho^2 + \rho'^2)\pi^2 v^2 \right.$$
$$\left. - \int \frac{e^{ik\cdot X}\, d^3k}{(2\pi)^3 2E_k} R^{a*}_\mu(\xi', k, \rho'^2) R^a_\mu(\xi, k, \rho^2)\right], \tag{6}$$

where only the terms explicitly relating to W-production have been kept in σ. Of course σ is a cross section for baryon number violation, however, the factors explicitly relating to baryon number violation only give non-exponential prefactors in eq. (6) and so have been suppressed.

It is straightforward to show that

$$-R^{a*}_\mu(\xi', k, \rho'^2) R^a_\mu(\xi, k, \rho^2) = \frac{32\pi^4}{g^2} \rho^2\rho'^2 \sum_{a,b} (U^{-1}(\xi')U(\xi))_{ab}(\delta_{ab}k^2 - k_a k_b). \tag{7}$$

When g is small and when $E^2/gv^2 \gg 1$ the collective coordinate integrations can be evaluated in the saddle-point approximation. The saddle point is at $x=0$, $\xi=\xi'$, $\rho^2=\rho'^2$. Using

$$\int \frac{d^3k}{(2\pi)^3 2k} k^2 e^{ikt} = \frac{3}{2\pi^2(t+i\epsilon)^4} \tag{8}$$

one finds

$$\sigma \propto \int d\rho^2 \, dt \, e^W, \tag{9}$$

with [10]

$$W = -\frac{4\pi}{\alpha} - 2\pi^2 v^2 \rho^2 - iEt + \frac{96\pi^2\rho^4}{g^2(t+i\epsilon)^4}. \tag{10}$$

The final saddle points in eq. (9) are at

$$t = t_1 = i\left(\frac{24E}{\pi^2 v^4 g^2}\right)^{1/3}, \tag{11a}$$

$$\rho^2 = \rho_1^2 = \left(\frac{3E^4}{8\pi^8 v^{10} g^2}\right)^{1/3}, \tag{11b}$$

giving

$$W = -\frac{4\pi}{\alpha}\left(1 - \tfrac{9}{8}(E/E_0)^{4/3}\right), \tag{12}$$

with $E_0 = \sqrt{6}\,\pi M_W/\alpha$.

2.2. THE NEXT-TO-LEADING SEMICLASSICAL APPROXIMATION

In the next-to-leading semiclassical approximation there are three sources of corrections of size $(4\pi/\alpha^2)(E/E_0)^2$ to the W of (9) and (10). They are: (i) W-boson mass effects where one simply replaces the pole at $k^2=0$ in eq. (4) by a pole at $k^2=M_W^2$ and where one keeps the phase space integral in eq. (6) as $(1/2E_k)d^3k$ rather than using the $(1/2k)d^3k$ approximation [13]; (ii) Higgs production from the Higgs part of the electroweak instanton. Higgs propagator corrections are not important when one uses the saddle point given in eq. (11) [3, 10, 17]; (iii) W-production due to the W-propagator in the background instanton field. (i) and (ii) are well understood and non-controversial. (iii) is more subtle and has been the topic of much discussion recently [8, 9, 13, 14]. The contribution of

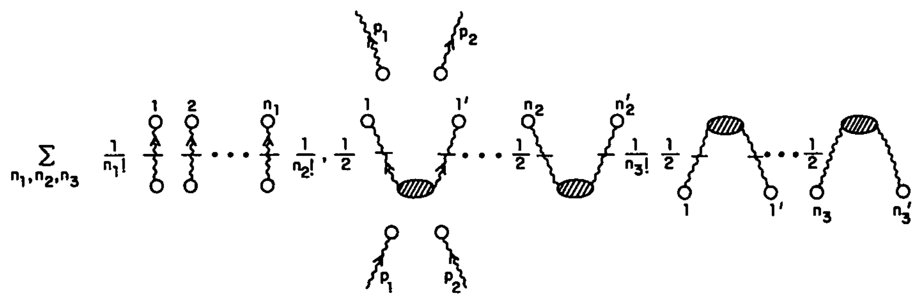

Fig. 2. The 2 W → $(n_1 + n_2 + n_3)$ W cross section including W-boson propagator effects.

W-boson mass effects to W, (i) above, is [13]

$$\delta W^{(\mathrm{i})} = \frac{12\pi^2}{g^2} M_{\mathrm{W}}^2 \frac{\rho^2 \rho'^2}{t^2} \tag{13}$$

while the contribution of Higgs production, (ii) above, is [1–3]

$$\delta W^{(\mathrm{ii})} = -\frac{\pi^2 v^2 \rho^2 \rho'^2}{t^2}. \tag{14}$$

Our purpose in this section is to study the W-boson propagator effects. Pictorially, these contributions are illustrated in fig. 2, where the shaded "blobs" represent the W-boson propagator with the external poles removed. The series in n_1, n_2 and n_3 exponentiate leading to an expression for W illustrated in fig. 3. The first term on the right-hand side of fig. 3 is the term we calculated in subsect 2.1. It is the final term in the exponential on the right-hand side of eq. (6). The second term on the right-hand side of the equation illustrated in fig. 3 is given by [10]

$$\delta W^{(\mathrm{iii})} = \tfrac{1}{2} \int \frac{\mathrm{d}^3 k_1}{(2\pi)^3 2k_1} \frac{\mathrm{d}^3 k_2}{(2\pi)^3 2k_2} \mathrm{e}^{i(k_1+k_2)t} \sum_{a,b} R_\mu^{a*}\left(\xi', k_1, \rho'^2\right)$$
$$\times \Pi_{\mu\nu}^{ab}\left(k_1, k_2, \xi, \rho^2\right) R_\nu^{b*}\left(\xi', k_2, \rho'^2\right). \tag{15}$$

To get the full contribution to the W-boson propagator corrections we should add the third term on the right-hand side of the equation illustrated in fig. 3. That term is given by the complex conjugate of the expression given in eq. (15) along with the

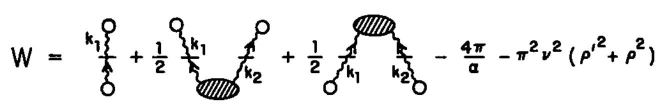

Fig. 3. A pictorial representation of W as used in (9).

changes $\xi \leftrightarrow \xi'$, $\rho \leftrightarrow \rho'$, $t \to -t$. R is given in eq. (5) while $\Pi^{ab}_{\mu\nu}$ is the gluon propagator, with poles $-i/k_1^2$ and $-i/k_2^2$ removed. We may evaluate eq. (15) at $\xi = \xi'$, the saddle-point value found earlier and, indeed, we shall simply take $\xi = \xi' = 0$ as $\delta W^{(\mathrm{iii})}$ can only depend on the relative orientation of the two instantons.

One might try to evaluate $\Pi^{ab}_{\mu\nu}$ by using the expression for the propagator given by Brown et al. [18], however, one is immediately confronted with the problem that the propagator given in ref. [18] has terms with double poles in k_1^2 or k_2^2. The source of the double-pole terms is the requirement that the propagator only represent fluctuations orthogonal to the zero modes of the instanton. A general method of surmounting this double-pole problem has been given in ref. [19]. However, we can avoid the problem in a much easier way which is sufficient for the purposes of evaluating eq. (15). The key point is to realize that the effective values of $|\boldsymbol{k}_1| = k_1$ and $|\boldsymbol{k}_2| = k_2$ in eq. (15) are given by $k_1, k_2 \sim i/t_1$ with t_1 given in eq. (11a). But $i/t_1 \ll 1/\rho_1$ so that $k_1\rho_1$ and $k_2\rho_1$ are very small for the dominant contribution in eq. (15). One can check that zero-mode contributions to $\Pi^{ab}_{\mu\nu}(k_1, k_2)$ are very small when $k_1\rho_1 \ll 1, k_2\rho_1 \ll 1$. Thus, we should be able to solve for $\Pi^{ab}_{\mu\nu}(k_1, k_2)$ when $k_1, k_2 \to 0$ without having to deal with the zero-mode problem at all. (Possible zero-mode contributions to $\Pi^{ab}_{\mu\nu}(k_1, k_2)$ are linear or quadratic, depending on the mode, in k_1 and k_2 as $k_1, k_2 \to 0$.)

Now, formally, $\Pi^{ab}_{\mu\nu}(k_1, k_2)$ obeys the integral equation illustrated in fig. 4. We are looking for a term of size ρ^2 when $k_1, k_2 \to 0$. The first term on the right-hand of the equation illustrated in fig. 4 is such a term. The second term on the right-hand side of that equation is of size ρ^2 only over the region of integration in k for which $k\rho$ is of order one. If $k\rho \gg 1$ the contribution of the, rapidly convergent, integral is small while the region $k\rho \alpha k_1\rho$ contributes like $\rho^2(\rho^2 k_1^2)$ when $k_1^2 \alpha k_2^2 \ll 1/\rho^2$. But in the region $k\rho$ of order one there is no k_1- or k_2-dependence. Indeed, this is a general result. In order to get a contribution of size ρ^2 and not higher order in ρ^2, the integrations over the instanton momentum must be in the region $k\alpha 1/\rho$. This means that there can be no k_1- or k_2-dependence whatsoever in $\Pi^{ab}_{\mu\nu}(k_1, k_2)$ except for the k_1- and k_2-dependence of the first term on the right-hand side of the equation illustrated in fig. 4. Thus we can write

$$\delta w^{(\mathrm{iii})} = R^{a*}_{\mu}(k_1, \rho'^2) R^{b*}_{\nu}(k_2, \rho'^2)\left[\Gamma^{acb}_{\mu\lambda\nu}\frac{-4\pi^2 i\rho^2}{g(k_1+k_2)^2}\bar{\eta}^{(-)}_{c\lambda\rho}(k_1+k_2)_\rho + \Pi^{ab}_{\mathrm{R}\mu\nu}(k_1, k_2)\right] \tag{16}$$

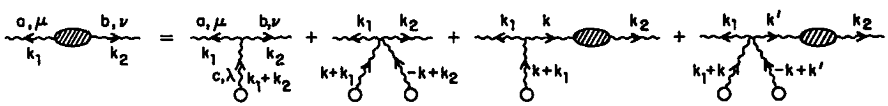

Fig. 4. The integral equation for $\Pi^{ab}_{\mu\nu}(k_1, k_2)$.

with

$$\Gamma^{acb}_{\mu\lambda\nu} = ig\epsilon_{acb}\left[g_{\lambda\mu}(2k_1+k_2)_\nu + g_{\mu\nu}(k_2-k_1)_\lambda - g_{\lambda\nu}(2k_2+k_1)_\mu \right], \tag{17}$$

and where $\Pi^{ab}_{\mathrm{R}\mu\nu}$ stands for all the contributions to $\Pi^{ab}_{\mu\nu}$ except the term explicitly given in eq. (16). Then

$$\delta W^{(\mathrm{iii})} = \tfrac{1}{2}\int \frac{\mathrm{d}^3k_1}{(2\pi)^3 2k_1}\frac{\mathrm{d}^3k_2}{(2\pi)^3 2k_2}\,\mathrm{e}^{i(k_1+k_2)t}\,\delta w^{(\mathrm{iii})}. \tag{18}$$

As discussed above, we can assume that $\Pi^{ab}_{\mathrm{R}\mu\nu}(k_1,k_2) = \Pi^{ab}_{\mathrm{R}\mu\nu}(0,0) \equiv \overline{\Pi}^{ab}_{\mu\nu}$. It is straightforward to evaluate (18) and find

$$\delta W^{(\mathrm{iii})} = -\frac{(\rho')^4}{2g^2}\int_0^\infty k_1\,\mathrm{d}k_1\,k_2\,\mathrm{d}k_2\left[16\pi^2\rho^2\left(k_1^2+k_2^2+\tfrac{3}{2}k_1k_2\right) + k_1k_2\overline{\Pi}^{ab}_{ab}\right]\mathrm{e}^{i(k_1+k_2)t}, \tag{19}$$

where a sum over $a, b = 1,2,3$ is assumed in $\overline{\Pi}^{ab}_{ab}$. The final integrals in eq. (19) are easily done to give

$$\delta W^{(\mathrm{iii})} = \frac{2(\rho')^4}{g^2t^6}\left[72\pi^2\rho^2 + \overline{\Pi}^{ab}_{ab}\right]. \tag{20}$$

Now, what is to be done to determine $\overline{\Pi}^{ab}_{ab}$? Surprisingly, the answer is very simple. Refer back to eq. (15). We have directly contracted R^*_μ and R^*_ν with $\Pi^{ab}_{\mu\nu}$. We could also have done the calculation by forming the product

$$R^{a*}_\mu(k_1)g^{\perp}_{\mu\alpha}(k_1)\Pi^{ab}_{\alpha\beta}(k_1,k_2)g^{\perp}_{\beta\nu}(k_2)R^{b*}_\nu(k_2), \tag{21}$$

where

$$g^{\perp}_{\mu\alpha}(k_1) = g_{\mu\alpha} - \frac{\left(k_{1\mu}\overline{k}_{1\alpha} + \overline{k}_{1\mu}k_{1\alpha}\right)}{2k_1^2} \tag{22}$$

with $\overline{k}_{1\mu}k_{1\mu} = 2k_1^2$ and $k_{1\mu}k_{1\mu} = \overline{k}_{1\mu}\overline{k}_{1\mu} = 0$. $g^{\perp}_{\mu\alpha}(k)$ projects onto the transverse polarizations. Of course, we have the *right* to use $g_{\mu\alpha}$ instead of $g^{\perp}_{\mu\alpha}$ since $k_{1\mu}R^{a*}_\mu(k_1) = 0$ means that $R^{a*}_\mu(k_1)$ is a perfectly fine polarization projection for the k_1-line. Indeed, since $R^{a*}_\mu(k_1)k_{1\mu} = 0$ effectively, $g_{\mu\alpha} - g^{\perp}_{\mu\alpha} = \overline{k}_{1\mu}k_{1\alpha}/2k_1^2$, it might seem that requiring $g_{\mu\alpha}$ and $g^{\perp}_{\mu\alpha}$ to give the same result for $\delta W^{(\mathrm{iii})}$ is the same as requiring $k_{1\alpha}\Pi^{ab}_{\alpha\nu}(k_1,k_2) = 0$ but this is not quite right. We could expect $k_{1\alpha}\Pi^{ab}_{\alpha\nu}(k_1,k_2) = 0$ in a covariant gauge quantization after integration of the weak isospin collective coordinate, but in our case $\Pi^{ab}_{\alpha\nu}$ appears in the exponential and the weak isospin collective coordinate is determined by a saddle-point approximation. Thus, our requirement of the equivalence of $g_{\mu\alpha}$ and $g_{\mu\alpha}^{\perp}$ projection is necessary only after the k_1 and k_2 integrals in eq. (15), or eq. (18), have been performed. Let us then make the replacements

$$R_{\mu}^{a*}(k_1) \to R_{\mu'}^{a*}(k_1) g_{\mu'\mu}^{\perp}(k_1), \tag{23a}$$

$$R_{\nu}^{b*}(k_2) \to R_{\nu'}^{b*}(k_2) g_{\nu'\nu}^{\perp}(k_2) \tag{23b}$$

in eq. (16). The requirement that either, or both, of the replacements in (23) not change $\delta W^{(\mathrm{iii})}$ as given by eq. (18) yields the relation

$$\overline{\Pi}_{ab}^{ab} = 24\pi^2\rho^2. \tag{24}$$

Using eq. (24) in eq. (20) gives

$$\delta W^{(\mathrm{iii})} = \frac{192\pi^2\rho^2\rho'^2(\rho^2+\rho'^2)}{g^2 t^6}, \tag{25}$$

where we have added the third term on the right-hand side of the equation illustrated in fig. 3 to get our final result. Eq. (25) is in agreement with the result of ref. [8].

Adding up all the contributions gives

$$W = -\frac{4\pi}{\alpha} - iEt - 2\pi^2 v^2\rho^2 + \frac{96\pi^2\rho^4}{g^2t^4} - \frac{\pi^2 v^2 \rho^4}{t^2} + \frac{12\pi^2}{g^2} M_{\mathrm{W}}^2 \frac{\rho^4}{t^2} + 384\frac{\pi^2\rho^6}{g^2t^6}. \tag{26}$$

Using (26) in (9) and taking the saddle-point approximation gives

$$\sigma \alpha \exp\left\{ -\frac{4\pi}{\alpha}\left[1 - \tfrac{9}{8}(E/E_0)^{4/3} + \tfrac{9}{16}(E/E_0)^2\right]\right\}, \tag{27}$$

with $E_0 = \sqrt{6}\,\pi M_{\mathrm{W}}/\alpha$.

3. The relationship between the cross section and classical euclidean field configurations

In this section we shall show how the discussion of sect. 2, in terms of produced particles in Minkowski space, can be recast in terms of classical euclidean field configurations. We proceed by first deriving an expression for the minimum action field configuration between a widely separated instanton and anti-instanton. We shall work at a level of approximation where zero modes, or approximate zero modes, of the field configuration do not cause serious problems. At this level of approximation our procedure is clearly equivalent to the valley method [6–9]. Finally, we show explicitly that minimizing the classical action leads to eq. (26).

3.1. MINIMIZING THE EUCLIDEAN ACTION OF A WELL-SEPARATED INSTANTON AND ANTI-INSTANTON

Let us write the W-boson part of the classical euclidean action of the electroweak theory as

$$S=\int \mathrm{d}^4x\left\{\frac{1}{2}A_\mu^a\Gamma_{\mu\nu}^{(2)}A_\nu^b+\frac{1}{3!}\Gamma_{\mu\nu\rho}^{(3)abc}A_\mu^aA_\nu^bA_\rho^c+\frac{1}{4!}\Gamma_{\mu\nu\rho\sigma}^{(4)abcd}A_\mu^aA_\nu^bA_\rho^cA_\sigma^d\right\}. \tag{28}$$

$\Gamma^{(2)}$ and $\Gamma^{(3)}$ involve derivatives in the usual way. We suppose the gauge $\partial_\mu A_\mu^a=0$ is chosen though this is not necessary for the discussion that follows. We suppose that the Γ's are symmetric under interchange of the indices (a,μ), (b,ν), (c,ρ) and (d,σ). Let $A_{1\mu}^a$ be an instanton centered at x_1,

$$A_{1\mu}^a(x)=\frac{2\rho^2\eta_{a\mu\nu}^{(-)}(x-x_1)_\nu}{g(x-x_1)^2\left[(x-x_1)^2+\rho^2\right]}, \tag{29}$$

and let $A_{2\mu}^a(x)$ be an anti-instanton centered at x_2,

$$A_{2\mu}^a(x)=\frac{2\rho'^2\eta_{a\mu\nu}^{(+)}(x-x_2)_\nu}{g(x-x_2)^2\left[(x-x_2)^2+\rho'^2\right]}. \tag{30}$$

When $(x_1-x_2)^2\gg\rho^2,\rho'^2$ the sum $A_{1\mu}^a+A_{2\mu}^a$ is an approximate solution to the euclidean field equations. For fixed x_1,x_2,ρ^2,ρ'^2 write $A_\mu^a=A_{1\mu}^a+A_{2\mu}^a+a_\mu^a$ and expand $S(A_1+A_2+a)$ through second order in a. One has

$$S(A)=S(A_1+A_2)+\int \mathrm{d}^4x\,\frac{\delta S(A_1+A_2)}{\delta A_\mu^a(x)}a_\mu^a(x)$$

$$+\tfrac{1}{2}\int \mathrm{d}^4x\,\mathrm{d}^4y\,a_\mu^a(x)\,\delta^2\frac{S(A_1+A_2)}{\delta A_\mu^a(x)\delta A_\nu^b(y)}a_\nu^b(y)+\dots. \tag{31}$$

Now, ignoring for the moment any zero-mode complications, let us minimize (31) with respect to small fluctuations a_μ^a. One finds

$$\int \mathrm{d}^4y\,\frac{\delta^2S(A_1+A_2)}{\delta A_\mu^a(x)\delta A_\nu^b(y)}a_\nu^b(y)=-\frac{\delta S(A_1+A_2)}{\delta A_\mu^a(x)}. \tag{32}$$

Thus, one has

$$S(A)=S(A_1+A_2)+\tfrac{1}{2}\int \mathrm{d}^4x\,\frac{\delta S(A_1+A_2)}{\delta A_\mu^a(x)}a_\mu^a(x) \tag{33}$$

with a_μ^a determined by eq. (32).

In terms of the Γ's introduced in eq. (28),

$$\begin{aligned}\frac{\delta S(A_1+A_2)}{\delta A_\mu^a(x)}&=\Gamma_{\mu\nu}^{(2)ab}(A_{1\nu}^b+A_{2\nu}^b)+\frac{1}{2}\Gamma_{\mu\nu\rho}^{(3)abc}(A_{1\nu}^b+A_{2\nu}^b)(A_{1\rho}^c+A_{2\rho}^c)\\&\quad+\frac{1}{3!}\Gamma_{\mu\nu\rho\sigma}^{(4)abcd}(A_{1\nu}^b+A_{2\nu}^b)(A_{1\rho}^c+A_{2\rho}^c)(A_{1\sigma}^d+A_{2\sigma}^d).\end{aligned} \tag{34}$$

Using the classical equations

$$\Gamma_{\mu\nu}^{(2)ab}A_{1\nu}^b+\frac{1}{2}\Gamma_{\mu\nu\rho}^{(3)abc}A_{1\nu}^bA_{1\rho}^c+\frac{1}{3!}\Gamma_{\mu\nu\rho\sigma}^{(4)abcd}A_{1\nu}^bA_{1\rho}^cA_{1\sigma}^d=0 \tag{35}$$

along with a similar equation for A_2 one finds

$$\frac{\delta S(A_1+A_2)}{\delta A_\mu^a}=\Gamma_{\mu\nu\rho}^{(3)abc}A_{1\nu}^bA_{2\rho}^c+\frac{1}{2}\Gamma_{\mu\nu\rho\sigma}^{(4)abcd}A_{1\nu}^bA_{2\rho}^c(A_{1\sigma}^d+A_{2\sigma}^d). \tag{36}$$

Similarly, one can show, using the classical field equations (35),

$$\begin{aligned}S(A_1+A_2)&=S(A_1)+S(A_2)-\int \mathrm{d}^4x A_{1\mu}^a\Gamma_{\mu\nu}^{(2)ab}A_\nu^b\\&\quad+\tfrac{1}{4}\int \mathrm{d}^4x\,\Gamma_{\mu\nu\rho\sigma}^{(4)abcd}A_{1\mu}^aA_{1\nu}^bA_{2\rho}^cA_{2\sigma}^d.\end{aligned} \tag{37}$$

Eq. (33) along with eqs. (32), (36) and (37) then give the minimum action for a well-separated instanton and anti-instanton.

Eq. (33) can be cast into a somewhat more useful form for determining the dependence of S on the instanton–anti-instanton separation. To solve for a_μ^a we formally invert eq. (32) to obtain

$$a_\mu^a(x)=-\int \mathrm{d}^4y\left[\frac{\delta^2S(A_1+A_2)}{\delta A_\mu^a(x)\delta A_\nu^b(y)}\right]^{-1}\frac{\delta S(A_1+A_2)}{\delta A_\nu^b(y)}. \tag{38}$$

[We shall later comment on the zero-mode problem which occurs in writing the inverse operator in (38).] When $(x_1-x_2)^2\gg\rho^2,\rho'^2$ one can write a, as given by

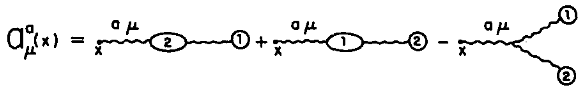

Fig. 5. A pictorial representation of eq. (39).

eq. (38) in terms of the amputated vector propagator in the background field A_1, $\Pi^{ab}_{1\mu\nu}$, and the amputated vector propagator in the background field A_2, $\Pi^{ab}_{2\mu\nu}$. Explicitly,

$$\Gamma^{(2)ab}_{\mu\nu}a^b_\nu(x) = \int \mathrm{d}^4y \left[\Pi^{ab}_{1\mu\nu}(x,y)A^b_{2\nu}(y) + \Pi^{ab}_{2\mu\nu}(x,y)A^b_{1\nu}(y) + \Gamma^{(3)abc}_{\mu\nu\rho}(x,y)A^b_{1\nu}(y)A^c_{2\rho}(y)\right]. \tag{39}$$

Pictorially, (39) is given in fig. 5, the minus sign in the third term in that figure corresponding to the fact that the plus sign in front of the $\Gamma^{(3)}$ term in eq. (39) has a sign opposite to the sign for the Feynman rules in euclidean field theory. Using (36) and (39) in (33) along with (37) one finds

$$\begin{aligned}
S(A) = \frac{4\pi}{\alpha} &- \int \mathrm{d}^4x \Big[A^a_{1\mu}(x)\Gamma^{(2)ab}_{\mu\nu}A^b_{2\nu}(x) + \tfrac{1}{2}\Gamma^{(3)abc}_{\mu\nu\rho}A^a_{1\mu}(x)A^b_{2\nu}(x)\\
&\times\left(A^c_{1\rho}(x) + A^c_{2\rho}(x)\right) + \tfrac{1}{4}\Gamma^{(4)abcd}_{\mu\nu\rho\sigma}A^a_{1\mu}(x)A^b_{1\nu}(x)A^c_{2\rho}(x)A^d_{2\sigma}(x)\Big]\\
&- \tfrac{1}{2}\int \mathrm{d}^4x\,\mathrm{d}^4y \Big[A^a_{1\mu}(x)\Pi^{ab}_{2\mu\nu}(x,y)A^b_{1\nu}(y) + A^a_{2\mu}(x)\Pi^{(ab)}_{1\mu\nu}(x,y)A^b_{2\nu}(y)\\
&- \left(\Gamma^{(3)ace}_{\mu\rho\lambda}A^a_{1\mu}(x)A^c_{1\rho}(x)\right)\left(\Gamma^{(2)^{-1}}\right)^{ef}_{\lambda\gamma}(x,y)\left(\Gamma^{(3)bdf}_{\nu\sigma\gamma}A^b_{1\nu}(x)A^d_{2\sigma}(y)\right)\Big] + R,
\end{aligned} \tag{40}$$

where $R = R_1 + R_2$ with

$$\begin{aligned}
R_1 = \tfrac{1}{4}\int \mathrm{d}^4x\,\mathrm{d}^4y &\Big\{\Gamma^{(4)abcd}_{\mu\nu\rho\sigma}A^a_{1\mu}(x)A^c_{2\rho}(x)A^d_{2\sigma}(x)\\
&\times\Big[\left(\Gamma^{(2)^{-1}}\Pi\right)^{be}_{\nu\gamma}(x,y)A^e_{2\gamma}(y) + \left(\Gamma^{(2)^{-1}}\right)^{be}_{\nu\gamma}(x,y)\Gamma^{(3)efg}_{\gamma\eta\delta}A^f_{1\eta}(y)A^g_{2\delta}(y)\Big]\Big\},
\end{aligned} \tag{41}$$

and where R_2 is obtained from R_1 by exchanging A_1 and A_2. We shall shortly see

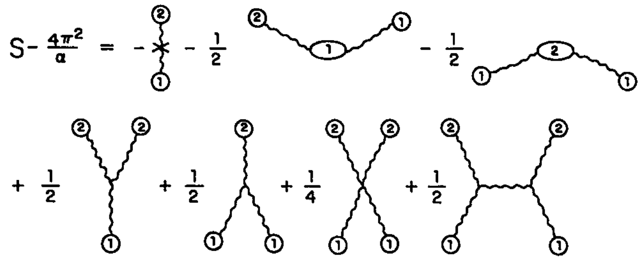

Fig. 6. A pictorial representation of eq. (40).

that R will not come in at the level of approximation we are concerned with. Eq. (40), but without R, is illustrated in fig. 6. In general, eqs. (40) and (41) only make sense once one specifies a procedure for dealing with the zero-mode problem. However, as we shall see in subsect. 3.2 the zero-mode problem does not arise at the level of approximation with which we are concerned.

3.2. THE RELATIONSHIP BETWEEN THE CLASSICAL ACTION AND PARTICLE PRODUCTION

The action given in (40) depends on $\sqrt{(x_1 - x_2)^2}$, the distance between the instanton and anti-instanton, on ρ^2, on ρ'^2 and on the relative orientation between the instanton and anti-instanton. (We have not explicitly put in the orientation dependence in eqs. (40), having anticipated the orientation given in eqs. (29) and (30) as leading to the minimum action.) Call $x = x_1 - x_2$. Then we may continue S from euclidean values of x^2 to Minkowski values. (We emphasize, however, that the minimization of the action is carried out in euclidean space, then the result is continued, in x^2, to Minkowski space [8].) Consider

$$I = i\int \mathrm{d}\rho^2\, \mathrm{d}\rho'^2\, \mathrm{d}^4x \exp\left[-iP\cdot x - \pi^2 v^2(\rho^2 + \rho'^2) - S(x^2 - i\epsilon, \rho^2, \rho'^2)\right], \quad (42)$$

where we have inserted the Higgs term $\pi^2 v^2(\rho^2 + \rho'^2)$ in the action but where we continue to suppress the rest of the Higgs contributions to S. We are now going to show that $\mathrm{Im}\, I$ explicitly generates the contributions shown in fig. 2 so that an alternative way of obtaining eq. (26) is to first minimize the classical euclidean action and then carry out the collective coordinate integrals as indicated in (42) [8].

To begin, let's keep only the first two terms on the right-hand side of eq. (40). Then

$$I = i\int \mathrm{d}\rho^2\, \mathrm{d}\rho'^2\, \mathrm{d}^4x \exp\left[-\frac{4\pi}{\alpha} - \pi^2 v^2(\rho^2 + \rho'^2) + \int \mathrm{d}^4y A^a_{2\mu}(y)\Gamma^{(a)ab}_{\mu\nu} A^b_{1\nu}(y) - iP\cdot x\right]. \quad (43)$$

We may write (43) as

$$I=i\int \mathrm{d}^4x\,\mathrm{d}\rho^2\,\mathrm{d}\rho'^2\exp\left[-\frac{4\pi}{\alpha}-\pi^2v^2(\rho^2+\rho'^2)-iP\cdot x\right]$$

$$\times\sum_{n=0}^{\infty}\left[S_0(x^2-i\epsilon,\rho^2,\rho'^2)\right]^n\frac{(-)^n}{n!}\,, \tag{44}$$

where S_0 can be written as

$$S_0=i\int\frac{\mathrm{d}^4k}{(2\pi)^4}\,\mathrm{e}^{ik\cdot x}\,k^2A^a_{2\mu}(k)A^a_{1\mu}(-k)\,. \tag{45}$$

A convenient expression for $A^a_{1\mu}(k)$ is

$$A^a_{1\mu}(k)=-\frac{4\pi^2i\rho^2}{g}\bar{\eta}^{(-)}_{a\mu\lambda}k_\lambda\int_0^\infty \mathrm{d}\mu^2\left[J_0(\mu\rho)+J_2(\mu\rho)\right]\frac{1}{(k^2-\mu^2+i\epsilon)^2} \tag{46}$$

while $A^a_{2\mu}(k)$ is obtained from $A^a_{1\mu}(-k)$ by $\rho^2\to\rho'^2$, $k\to -k$ and $\bar{\eta}^{(-)}_{a\mu\lambda}\to\bar{\eta}^{(+)}_{a\mu\lambda}=\bar{\eta}^{(-)*}_{a\mu\lambda}$. The integral (45) is in Minkowski space with the $i\epsilon$ prescription given in (46) corresponding to the coordinate space $i\epsilon$ prescription given in (44). The d^4x integral in (44) is now trivial and one finds

$$I=i\int \mathrm{d}\rho^2\,\mathrm{d}\rho'^2\exp\left[\frac{4\pi}{\alpha}-\pi^2v^2(\rho^2+\rho'^2)\right]\sum_{n=0}^{\infty}\frac{(-i)^n}{n!}\int\frac{\mathrm{d}^4k_1\ldots\mathrm{d}^4k_n}{(2\pi)^{4n}}$$

$$\times(2\pi)^4\delta^4(P-k_1-k_2-\ldots-k_n)\,J(k_1)J(k_2)\ldots J(k_n)\,, \tag{47}$$

with

$$J(k)=k^2A^a_{2\mu}(k)A^a_{1\mu}(-k)\,. \tag{48}$$

It is easy to show that

$$J(k)=-\frac{16\pi^2\rho^2\rho'^2}{g^2}k^2(k^2+3k_0^2)\int_0^\infty \mathrm{d}\mu^2\,\frac{(J_0(\mu\rho)+J_2(\mu\rho))}{(k^2-\mu^2+i\epsilon)^2}$$

$$\times\int_0^\infty \mathrm{d}\mu^2\,\frac{(J_0(\mu\rho')+J_2(\mu\rho'))}{(k^2-\mu^2+i\epsilon)^2}\,. \tag{49}$$

For a moment keep only the $\rho^2\rho'^2$ contribution to J. Then,

$$J = -\frac{16\pi^4\rho^2\rho'^2}{g^2(k^2+i\epsilon)}\left(3k_0^2+k^2\right) + \mathrm{O}\left(\rho^2\rho'^4\right) + \mathrm{O}\left(\rho^4\rho'^2\right). \tag{50}$$

Now it is a straightforward application of the Cutkosky rules [20] that

$$2\,\mathrm{Im}\, i\int \mathrm{d}^4k_1\ldots\mathrm{d}^4k_n \frac{\delta^4(P-k_1-k_2\ldots k_n)\, i^n}{(k_1^2+i\epsilon)(k_2^2+i\epsilon)\ldots(k_n^2+i\epsilon)}$$
$$= \int \mathrm{d}^4k_1\ldots\mathrm{d}^4k_n\, 2\pi\delta\left(k_1^2\right)2\pi\delta\left(k_2^2\right)\ldots2\pi\delta\left(k_n^2\right)\delta^4(P-k_1-k_2-\ldots-k_n). \tag{51}$$

Thus,

$$2\,\mathrm{Im}\, I = \int \mathrm{d}\rho^2\rho'^2 \exp\left[-\frac{4\pi}{\alpha} - v^2\left(\rho^2+\rho'^2\right)\pi^2\right]\sum_{n=0}^{\infty}\frac{1}{n!}\int\frac{\mathrm{d}^3k_1k_1^2}{(2\pi)^3 2k_1}$$
$$\ldots\frac{\mathrm{d}^3k_nk_n^2}{(2\pi)^3 2k_n}(2\pi)^4\delta^4(P-k_1-k_2-\ldots-k_n)\left(\frac{64\pi^4\rho^2\rho'^2}{g^2}\right)^n. \tag{52}$$

But (52) is exactly the same as (1) when (7) is used and $\xi'=\xi$. Thus, we have shown that the imaginary part of I reproduces the leading semiclassical production of vector particles.

Now let us complete the argument for the rest of the terms in eq. (40) along with the higher-order terms in ρ and ρ' which we neglected. There are really only two essential points. First, expand I in (42) to give

$$I = i\int \mathrm{d}\rho^2\,\mathrm{d}\rho'^2\,\mathrm{d}^4x \exp\left[i(P\cdot x) - \pi^2v^2\left(\rho^2+\rho'^2\right)\right]\sum_{n=0}^{\infty}\frac{(-1)^n}{n!}\left[S\left(x^2-i\epsilon,\rho^2,\rho'^2\right)\right]^n \tag{53}$$

Next, write the fields appearing in eq. (40), and the propagators also, in a Minkowski momentum space representation. Then, the Cutkosky rules immediately tell us that $\mathrm{Im}\, I$ will be given in terms of products of cut graphs, the individual terms coming from those shown in fig. 6. A little more explicitly, write

$$I = i\int \mathrm{d}\rho^2\, \mathrm{d}\rho'^2 \exp\left[-\frac{4\pi}{\alpha} - \pi^2 v^2(\rho^2+\rho'^2)\right] \sum_{n=0}^{\infty} \frac{(-i)^n}{n!} \int \frac{\mathrm{d}^4k_1 \ldots \mathrm{d}^4k_n}{(2\pi)^{4n}} (2\pi)^4$$
$$\times \delta^4(P - k_1 - \ldots - k_n)\, S(k_1) S(k_2) \ldots S(k_n)\,, \tag{54}$$

with

$$S(x^2 - i\epsilon) = \frac{i}{(2\pi)^4} \int \mathrm{d}^4k\, \mathrm{e}^{ikx} S(k)\,. \tag{55}$$

Then the Cutkosky rules give

$$2\,\mathrm{Im}\, I = \int \mathrm{d}\rho^2\, \mathrm{d}\rho'^2 \exp\left[-\frac{4\pi}{\alpha} - \pi^2 v^2(\rho^2+\rho'^2)\right] \sum_{n=0}^{\infty} \frac{1}{n!} \int \frac{\mathrm{d}^4k_1 \ldots \mathrm{d}^4k_n}{(2\pi)^{4n}}$$
$$\times (2\pi)^4 \delta^4(P - k_1 - k_2 - \ldots - k_n)\,[2\,\mathrm{Im}\, S(k_1)]$$
$$\times [2\,\mathrm{Im}\, S(k_2)] \ldots [2\,\mathrm{Im}\, S(k_n)]\,. \tag{56}$$

Thus, one can write

$$2\,\mathrm{Im}\, I = \int \mathrm{d}^4x\, \mathrm{d}\rho^2\, \mathrm{d}\rho'^2 \exp\left[-\frac{4\pi}{\alpha} - \pi^2 v^2(\rho^2+\rho'^2) - iP\cdot x\right.$$
$$\left. + \int \frac{\mathrm{d}^4k}{(2\pi)^4} \mathrm{e}^{ikx}\, 2\,\mathrm{Im}\, S(k)\right]. \tag{57}$$

The second essential point is the analysis of $2\,\mathrm{Im}\, S(k)$. To see how this works consider the propagator term in eq. (40),

$$S_2 = -\tfrac{1}{2} \int \mathrm{d}^4y\, \mathrm{d}^4z A^a_{2\mu}(y) \Pi^{ab}_{1\mu\nu}(y, z) A^b_{2\nu}(z)\,. \tag{58}$$

Writing S_1 in momentum space, according to eq. (55),

$$S_2(k) = \frac{i}{2(2\pi)^4} \int \mathrm{d}^4k_1\, A^{\alpha}_{2\mu}(k_1) \Pi^{ab}_{1\mu\nu}(k_1, k - k_1) A^b_{2\nu}(k - k_1)\,, \tag{59}$$

which expression is illustrated in fig. 7 Singularities in k^2, giving an imaginary part

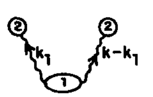

Fig. 7. The W-boson propagator contribution to (40).

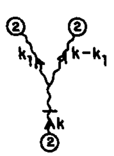

Fig. 8. The 1 W-boson discontinuity to the terms represented in fig. 7.

to S, come from real intermediate states in the k-channel. These on-shell intermediate states can consist of 1 W-boson, 2 W-bosons or higher number of W-bosons. The 1 W-boson intermediate state corresponds to a $\delta(k^2)$ to $\mathrm{Im}\, S(k)$ and comes from the part of (59) illustrated in fig. 8 where the solid line across the k-line indicates the $i/(k^2+i\epsilon)$ is replaced by $2\pi\delta(k^2)$. However, the 1 W-boson part of (59) is exactly cancelled by the 1 W-boson intermediate state of the fourth term on the right-hand side of the equation illustrated in fig. 6. S_1 has 2 W-boson intermediate states coming from k_1^2 and $(k-k_1)^2$ being put on mass-shell and also from two-particle states internal to $\Pi_{1\mu\nu}$. However, the singularities coming from these two particle states internal to $\Pi_{1\mu\nu}$ are cancelled by similar terms coming from the last two terms on the right-hand side of the equation shown in fig. 6. Thus, the only 2 W-boson intermediate states in S_2 that need be kept are given by

$$2\,\mathrm{Im}'\, S_2(k) = \frac{1}{2(2\pi)^4}\int \mathrm{d}^4k_1\, 2\pi\delta\big(k_1^2\big)\, 2\pi\delta\big((k-k_1)^2\big)\, R_{2\mu}^a(k_1)$$

$$\times\, \Pi_{1\mu\nu}^{ab}(k_1, k-k_1)\, R_{2\nu}^b(k-k_1)\,, \tag{60}$$

where the prime indicates that we are taking a particular intermediate state, the 2 W-boson intermediate state. Eq. (60) when used in (57) leads to (15).

Thus, we have identified a 2 W-boson intermediate state of S_2 with the W-boson propagator correction which we have previously discussed in sect. 2. All other two-particle intermediate states cancel between the various contributions given in eq. (40). The final point to be made is that higher than two-particle intermediate state contributions to 2 Im $S(k)$ contribute terms of size $(\rho^2)^a(\rho'^2)^b$ with $a+b \geqslant 4$ and so are small compared to the $\rho^2\rho'^2(\rho^2+\rho'^2)$ contributions to W with which we are concerned here. In particular, R_1 given by (41) has a minimum of three particles in real intermediate states and hence is beyond the scope of our discussion.

Finally, a few comments on the role of zero modes may be in order. Because of the existence of zero modes the vector propagator in the presence of an instanton is ambiguous since to any given propagator one is free to add

$$\delta G_{\mu\nu}^{ab}(k_1, k_2) = \sum_n c_n \psi_\mu^{(n)a}(k_1)\, \psi_\nu^{(n)b}(k_2)\,, \tag{61}$$

where n labels the particular zero mode, ψ is the zero-mode wave function and the c_n are arbitrary. The poles of the zero modes take the form

$$\psi_\mu^{(n)a}(k) = \frac{r_{\mu\nu}^{(n)a} k_\nu}{k^2} \tag{62}$$

for k_μ small. This means that

$$\delta\Pi^{ab}_{\mu\nu}(k_1,k_2) = -\sum_n c_n r^{(n)a}_{\mu\alpha} r_{\nu\beta} k_\alpha k_\beta \tag{63}$$

for small k. If the $\delta\Pi^{ab}_{\mu\nu}$ given in eq. (63) replaces the $\Pi^{ab}_{\mu\nu}$ in eq. (15) it is straightforward to verify that the resulting contribution to W is of size $(\rho/t)^8$ which corresponds to an energy dependence of size $(E/E_0)^{8/3}$ in the exponential as a contribution to σ. When terms of this magnitude are considered it is essential to properly constrain the fluctuations not to be along the zero modes of the instanton. The valley method should be one method of accomplishing this.

I wish to thank Valery Khoze for sending me an early copy of ref. [8] and for telling me of the work of ref. [14]. I also wish to thank Glennys Farrar for giving me a copy of ref. [13].

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