Towards Efficient N3LO Predictions for Color Singlet Processes

Gherardo Vita

Nikhef Theory Seminar - Amsterdam, 11 January 2024

Based on:

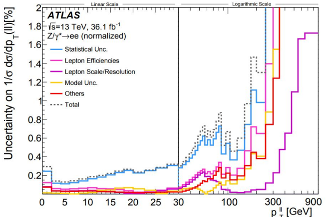
“N3LO Power Corrections for 0-jettiness Subtractions With Fiducial Cuts”

GV

[2401.03017]

Outline

- **Introduction**
  - Why do we need higher order predictions?

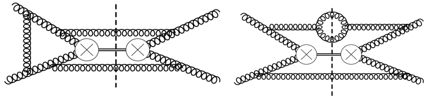


	Q [GeV]	$\delta\sigma^{\mathrm{N^3LO}}$	$\delta\sigma^{\mathrm{NNLO}}$
$gg \to$ Higgs	m_H	3.5%	30%
$b\bar{b} \to$ Higgs	m_H	-2.3%	2.1%
NCDY	30	-4.8%	−0.34%
	100	-2.1%	−2.3%
CCDY(W^+)	30	-4.7%	−0.1%
	150	-2.0%	−0.1%
CCDY(W^-)	30	-5.0%	−0.1%
	150	-2.1%	−0.6%

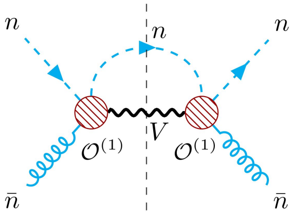
- **Enabling N3LO predictions**

$$\sigma(X) = \underbrace{\int_0^{q_{T\,\mathrm{cut}}} \mathrm{d}q_T \frac{\mathrm{d}\sigma^{\mathrm{sing}}(X)}{\mathrm{d}q_T}}_{\text{Below the cut region}} + \underbrace{\int_{q_{T\,\mathrm{cut}}} \mathrm{d}q_T \frac{\mathrm{d}\sigma(X)}{\mathrm{d}q_T}}_{\text{Above the cut region}} + \underbrace{\Delta\sigma(X, q_{T\,\mathrm{cut}})}_{\text{Residual}}$$

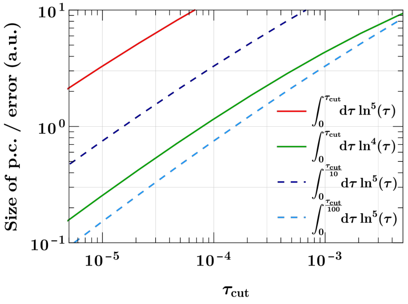
  - Beam Functions for non-local subtractions at N3LO



- **Making N3LO predictions efficient**

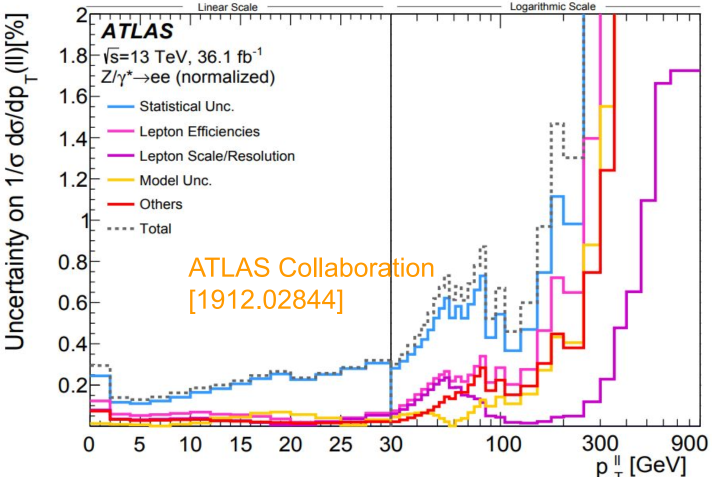


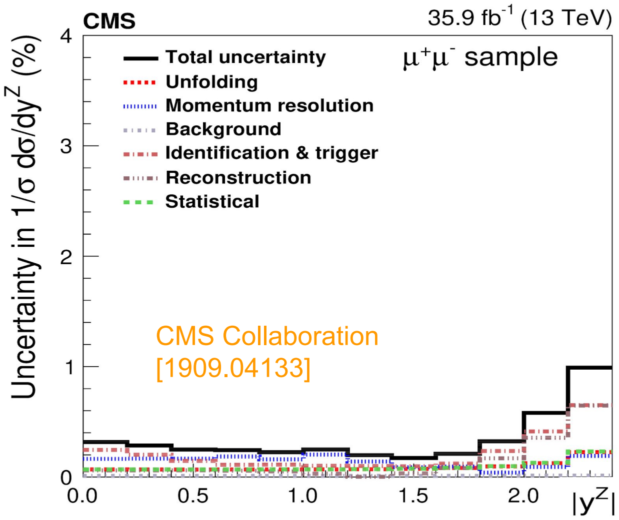
  - N3LO Power corrections for 0-jettiness subtraction

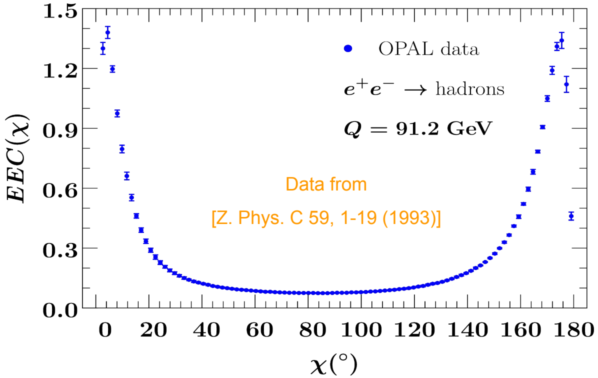


Testing the Standard Model at Colliders

- **Percent** level accurate measurements of several processes key to some of the most pressing questions of contemporary particle physics



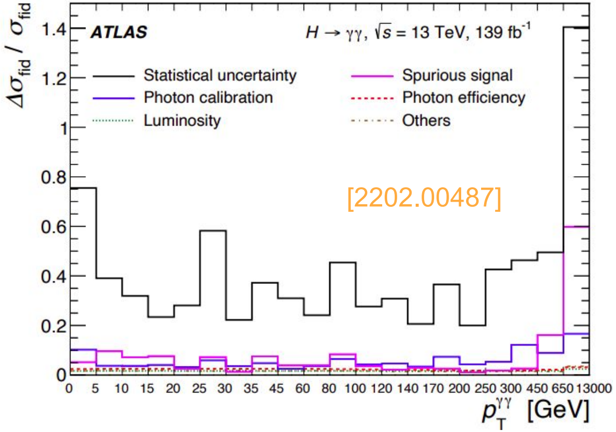




Ability to test the SM at (sub)-percent accuracy!

Testing the Higgs at Colliders

- Measurements of Higgs differential distributions at the moment are **limited by statistics…**

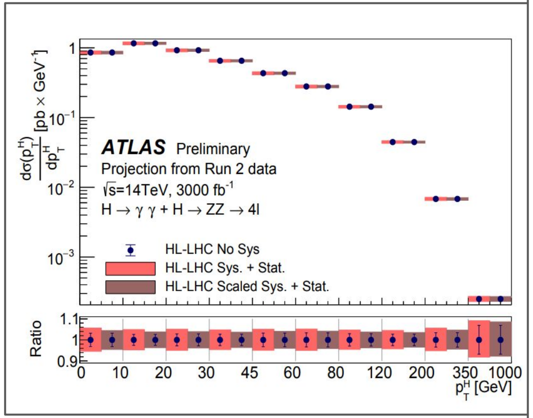


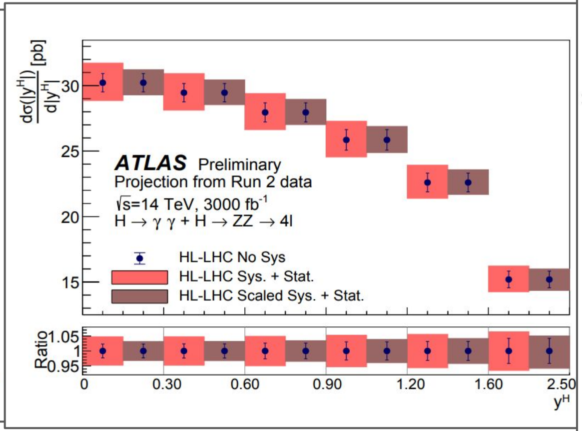
Note:

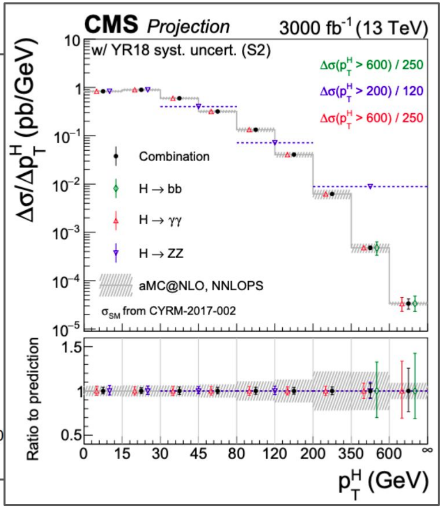
Inclusive cross section:
total number of Higgs produced

Differential distributions: more fine grained questions about the dynamics of Higgs production and decay

…but situation **will improve dramatically** with HL-LHC







Standard Model Phenomenology at percent level

To answer these fundamental questions we need comparable precision from the theory side!

$$\hat{\sigma}_{ab\to X} = \underbrace{\sigma_0}_{\text{LO}} + \underbrace{\alpha_s\sigma_1}_{\text{NLO}} + \underbrace{\alpha_s^2\sigma_2}_{\text{NNLO}} + \underbrace{\alpha_s^3\sigma_3}_{\text{N}^3\text{LO}} + \ldots$$

QCD Perturbation Theory

Improving Theoretical Predictions

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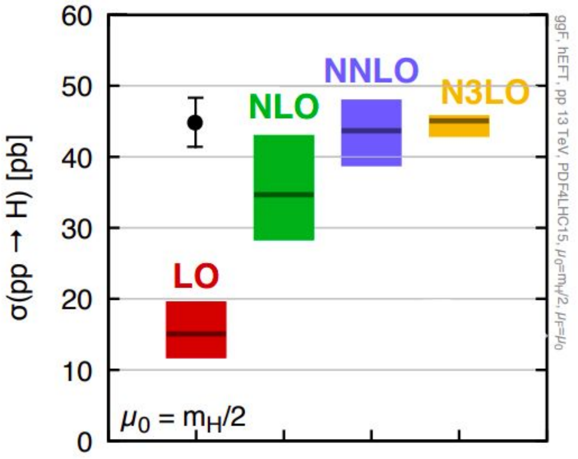

Perturbative QCD at work!

Without higher order predictions for the fundamental mechanism for Higgs boson production at the LHC (gluon fusion) we wouldn't be able to correctly describe collider experiments

Improving Theoretical Predictions

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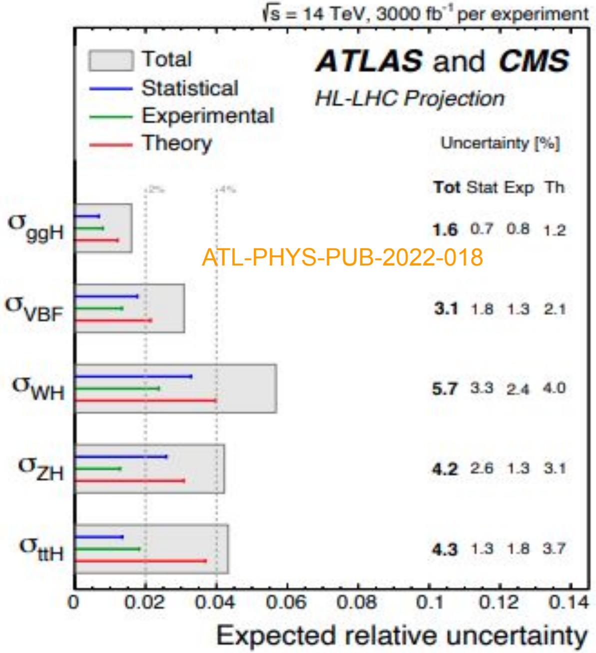
Theory errors are projected to be a **major limiting factor** for Higgs precision program...



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...and the **projected theory error reduction** used in the HL-LHC analysis (a factor of 2 reduction compared to current results) will require **tremendous effort** from the theory community

Improving Theoretical Predictions

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N3loxs [Baglio, Duhr, Mistlberger, Szafron '22]

Note:
N3LO corrections are sizable **not only** for Higgs! They are necessary ingredients for the precision program at LHC and future colliders.

"The Path Forward to N3LO"
Snowmass Whitepaper
[Caola, Chen, Duhr, Liu, Mistlberger, Petriello, GV, Weinzierl]

Predictions for Differential Cross Sections

$$\sigma = f_1 \circ f_2 \int d\Phi \, |M|^2$$

- Cross sections for LHC processes are obtained via **phase space integrals** over **amplitudes** (squared) convoluted with **Parton Distribution Functions (PDFs)**
- Bottlenecks for precision are present for each ingredient. In particular:
  - Efficiently calculate and evaluate multi-loop scattering amplitudes
  - Handling of kinematics limits and phase space singularities
  - Extracting N3LO PDFs

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Main complication for N3LO differential distributions for Higgs, Drell-Yan

Non-local subtractions

- One way to deal with IR singularities for cross sections are EFT-based subtractions:

q_T Subtraction: [Catani, Grazzini '07]

N-Jettiness Subtraction: [Boughezal, Focke, Liu, Petriello '15]
[Gaunt, Stahlhofen, Tackmann, Walsh '15]

$$\sigma(X) = \int_0^{q_{T\,\mathrm{cut}}} \mathrm{d}q_T \frac{\mathrm{d}\sigma^{\mathrm{sing}}(X)}{\mathrm{d}q_T} + \int_{q_{T\,\mathrm{cut}}} \mathrm{d}q_T \frac{\mathrm{d}\sigma(X)}{\mathrm{d}q_T} + \Delta\sigma(X, q_{T\,\mathrm{cut}})$$

Below the cut region:
- Singular distribution
- Contains most complicated cancellation of IR divergences
- Control it analytically via factorization theorems

Above the cut region:
- Resolved extra radiation
- No events in Born configuration
- Lower number of loops
- Calculate numerically and/or with lower order subtraction schemes

Residual:
Non singular terms from below the cut (*power correction*).
Minimized by going to very small values of cut parameter

- Extremely successful program for many color singlet (and top) processes at **NNLO**

[Matrix collaboration]

- With ***N*-Jettiness** ability to tackle also processes with **jets in the final state**

[Boughezal, Focke, Liu, Petriello + Campbell, Ellis, Giele '15, '16] [Campbell, Ellis, Williams '16] [Campbell, Ellis, Seth '19][Mondini, Williams '21]

Singular Region of LHC Observables

- **Singular region** (i.e. below the cut) can be understood at all orders via *Leading power factorization theorems* in Soft and Collinear Effective Theory (**SCET**)

E.g. ***Transverse-Momentum Distributions*** *in pp*

$$\frac{d\sigma}{dQ^2 dY d^2\vec{q}_T} = \sigma_0 \sum_{i,j} H_{ij}(Q^2,\mu) \int d^2\vec{b}_T \, e^{i\vec{q}_T\cdot\vec{b}_T} \tilde{B}_i\left(x_1^B, b_T, \mu, \frac{\nu}{\omega_a}\right) \tilde{B}_j\left(x_2^B, b_T, \mu, \frac{\nu}{\omega_b}\right) \tilde{S}(b_T,\mu,\nu)$$

Hard Function — $H_{ij}(Q^2,\mu)$

q_T Beam Functions — $\tilde{B}_i$, $\tilde{B}_j$

Soft Function — $\tilde{S}(b_T,\mu,\nu)$

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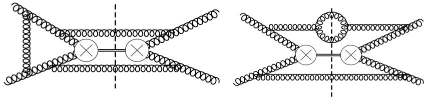
Hard Function — q_T **Beam Functions** — **Soft Function**

- For **N3LO slicing** we need Hard, Beam and Soft functions at N3LO
- For ***H*** and ***S***, necessary ingredients are **constants:** known at N3LO since 2010 (*H*) and 2016 (*S*)
  [Gehrmann, Glover, Huber, Ikizlerli, Studerus '10] *[Li, Zhu '16]*
- For Beam function they are **full functions** (of the collinear splitting variable)

N3LO q_T-Beam Function were the last missing ingredient to extend these methods to N3LO

Beam Functions at N3LO

1 million 3-loop Feynman Diagrams



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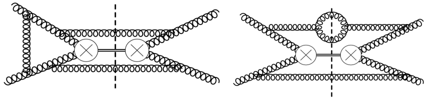

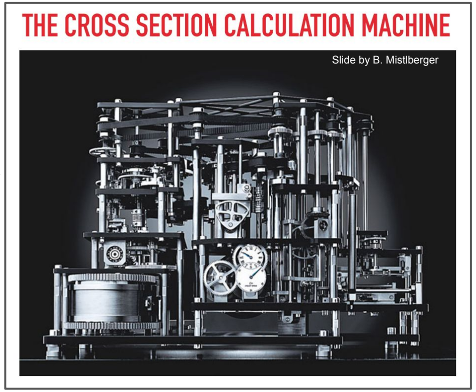

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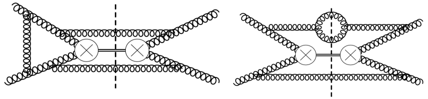
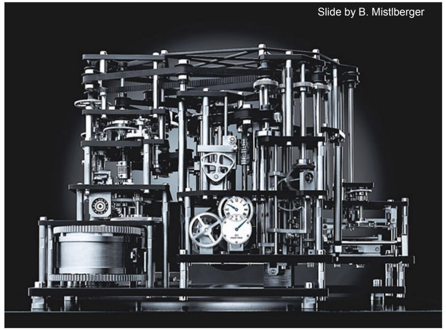




Collinear expansion of the partonic cross section for Drell Yan and Higgs at N3LO **differential in (Q_T, τ, z)**

project to τ: $B_a(t_a, x_1^B, \mu)$

project to q_T: $\tilde{B}_i\left(x_1^B, b_T, \mu, \frac{\nu}{\omega_a}\right)$

"N-Jettiness Beam Functions at N3LO"

M.Ebert, B.Mistlberger, **GV** [2006.03056]

- Quark τ **Beam Functions** (i.e. Quark N-Jettiness BF)
- Gluon τ **Beam Functions** (i.e. Gluon N-Jettiness BF)

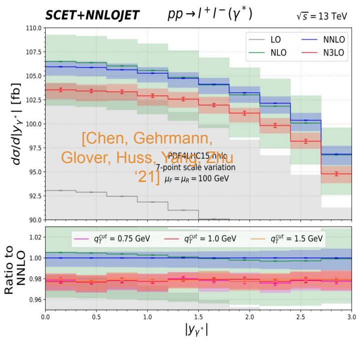
"Transverse Momentum Dependent PDFs at N3LO"

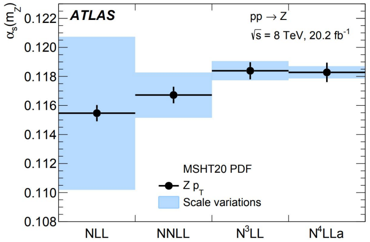
M.Ebert, B.Mistlberger, **GV** [2006.05329]

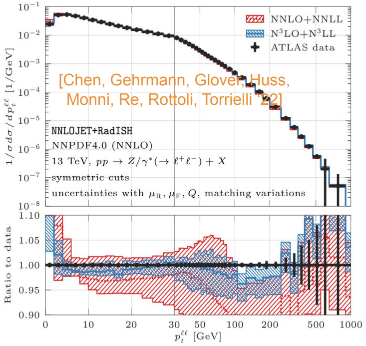
- Quark **TMDPDF** (Quark q_T Beam Function)
- Unpolarized **Gluon TMDPDF** (Gluon q_T Beam Function)

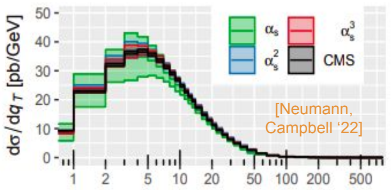
Precision Standard Model Phenomenology at N3LO

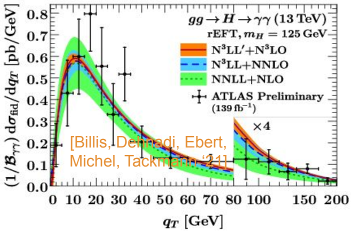
- **N3LO TMDPDF** were **last missing ingredient** for q_T slicing at N3LO
- Enabled N3LO predictions for differential and fiducial Drell-Yan and Higgs production
- Marked the advent of a new level of accuracy for the precision program at the LHC











And many more:
[Ju, Schönherr '21]
[Camarda, Cieri, Ferrera '21]
[Re, Rottoli, Torrielli '21]
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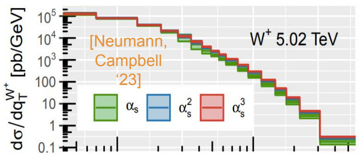

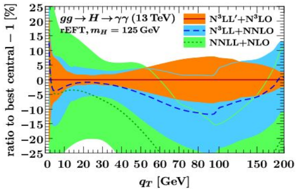

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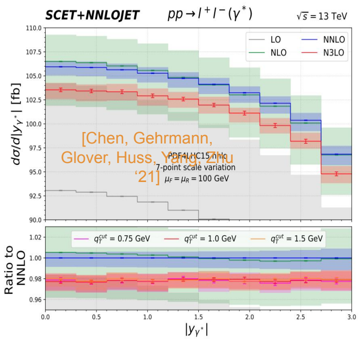

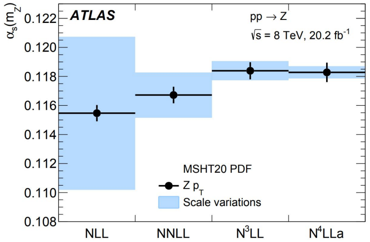

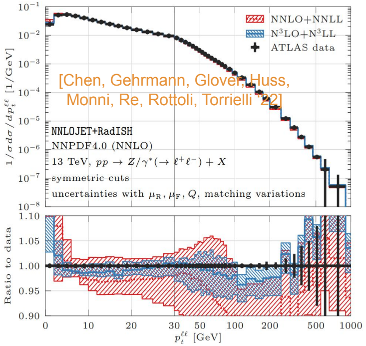

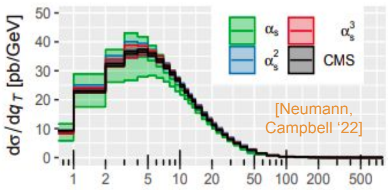

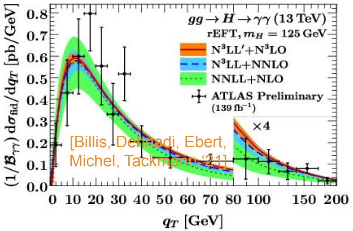

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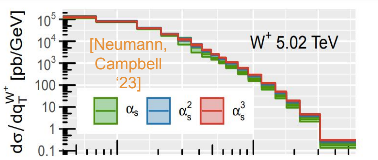
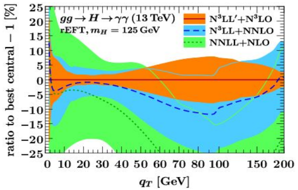




However...

- Numerical (slicing) error of these methods very difficult to control at this order
- Extreme push of NNLO+j predictions well into the IR needed (NNLOjet pushed to q_T = 0.5 GeV)
- Calculations take **O(10 million) CPU hours**
- Almost any change will require to run everything from scratch
- Other results use O(100k) CPU hours and stop at 5 GeV… this requires very delicate extrapolation to 0 to obtain finite results.
- Going forward, these facts pose issues for the practical usability of these predictions

In short, starting to think about how to move from

making N3LO predictions **possible**,

to

making N3LO predictions (more) **efficient**, **stable**, and **usable**

(at least for some color singlet processes…which may also turn out to be a necessary stepping stone to make other processes possible at N3LO)

A deeper look into non-local subtractions

$$\sigma = \int_0^{\tau_{\rm cut}} \mathrm{d}\tau \frac{\mathrm{d}\sigma}{\mathrm{d}\tau} + \int_{\tau_{\rm cut}}^{\tau_{\rm max}} \mathrm{d}\tau \frac{\mathrm{d}\sigma}{\mathrm{d}\tau}$$

$$= \int_0^{\tau_{\rm cut}} \mathrm{d}\tau \left[\frac{\mathrm{d}\sigma^{(0)}}{\mathrm{d}\tau} + \sum_{i>0} \frac{\mathrm{d}\sigma^{(i)}}{\mathrm{d}\tau}\right] + \int_{\tau_{\rm cut}}^{\tau_{\rm max}} \mathrm{d}\tau \frac{\mathrm{d}\sigma}{\mathrm{d}\tau}$$

$$= \sigma_{\rm sub}(\tau_{\rm cut}) + \Delta\sigma(\tau_{\rm cut}) + \int_{\tau_{\rm cut}}^{\tau_{\rm max}} \mathrm{d}\tau \frac{\mathrm{d}\sigma}{\mathrm{d}\tau}\,.$$

Above the cut result

- Resolved extra radiation
- No events in Born configuration
- Lower number of loops
- Calculate numerically with lower order subtraction schemes

Below the cut region:

- Singular distribution
- Contains most complicated cancellation of IR div.
- Control it analytically via factorization theorems

Residual/slicing error:

- Non singular terms from below the cut
- Reducing this requires pushing cut parameter to very small values
- Can be improved analytically by calculation next to leading power distribution

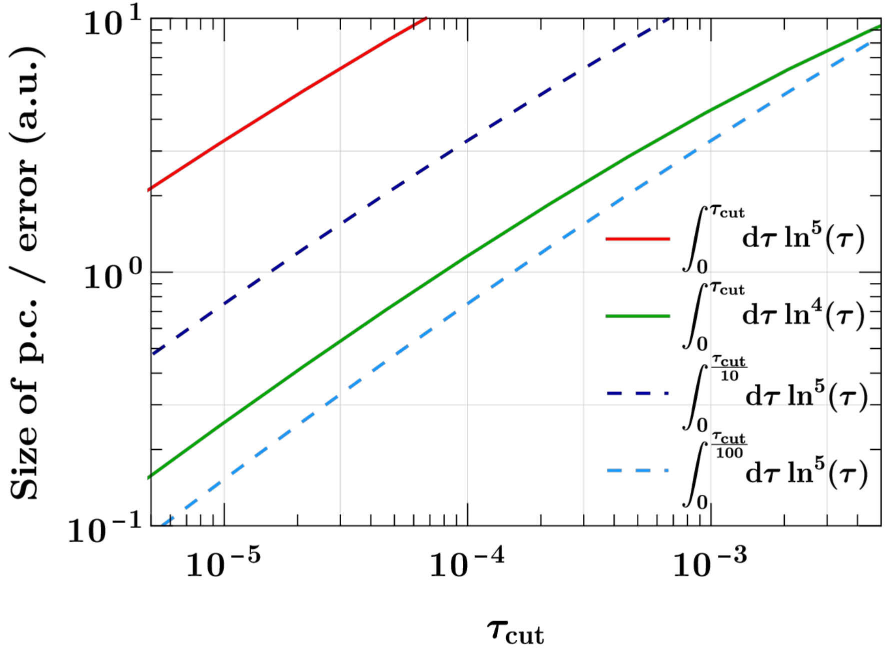
$$\sigma_{\rm sub}(\tau_{\rm cut}) = \int_0^{\tau_{\rm cut}} \mathrm{d}\tau \frac{\mathrm{d}\sigma^{\rm sub}}{\mathrm{d}\tau}\,,$$

$$\Delta\sigma(\tau_{\rm cut}) = \int_0^{\tau_{\rm cut}} \mathrm{d}\tau \left[\frac{\mathrm{d}\sigma}{\mathrm{d}\tau} - \frac{\mathrm{d}\sigma^{\rm sub}}{\mathrm{d}\tau}\right]$$

Improving non-local subtraction methods: Power corrections

$$\Delta\sigma(\tau_{\text{cut}}) = \int_0^{\tau_{\text{cut}}} \mathrm{d}\tau \left[\frac{\mathrm{d}\sigma}{\mathrm{d}\tau} - \frac{\mathrm{d}\sigma^{\text{sub}}}{\mathrm{d}\tau}\right]$$

- At N3LO power corrections start with **5th power of log**
- Taking τ_{cut} small reduces single power, but increases size of log => very slow convergence
- Each order in the log equivalent to ~ a 50 fold reduction in τ_{cut}



$$\Delta\sigma^{N3LO}(\tau_{\text{cut}}) \sim \left(\frac{\alpha_s}{4\pi}\right)^3 \int_0^{\tau_{\text{cut}}} \mathrm{d}\tau \left(c_{3,5}^{\text{NLP}} \ln^5\tau + c_{3,4}^{\text{NLP}} \ln^4\tau + c_{3,3}^{\text{NLP}} \ln^3\tau + \ldots\right)$$

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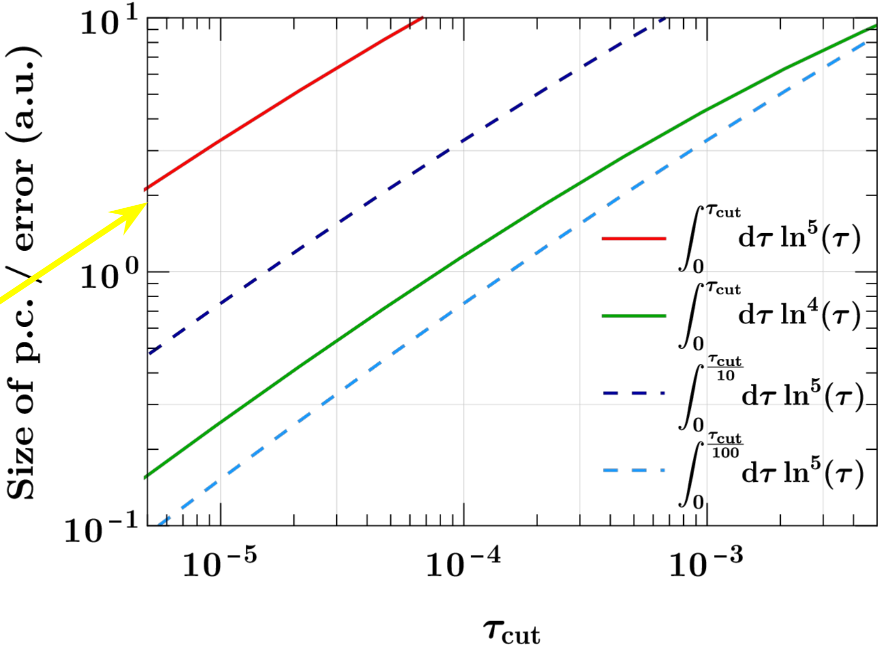
- At … h
  po…
- Tak… t
  incr… ence
- Each … to ~ a 50 fold
  reduct…

Very straightforward way of improving slicing:

Obtain the leading logarithmic term at NLP analytically



$$\Delta\sigma^{N3LO}(\tau_{\text{cut}}) \sim \left(\frac{\alpha_s}{4\pi}\right)^3 \int_0^{\tau_{\text{cut}}} \mathrm{d}\tau \left(c_{3,5}^{\text{NLP}} \ln^5\tau + c_{3,4}^{\text{NLP}} \ln^4\tau + c_{3,3}^{\text{NLP}} \ln^3\tau + \ldots\right)$$

0-Jettiness Power Corrections at N3LO

$$\Delta\sigma^{N3LO}(\tau_{\rm cut}) \sim \left(\frac{\alpha_s}{4\pi}\right)^3 \int_0^{\tau_{\rm cut}} d\tau \left(c_{3,5}^{\rm NLP} \ln^5 \tau + c_{3,4}^{\rm NLP} \ln^4 \tau + c_{3,3}^{\rm NLP} \ln^3 \tau + \ldots \right)$$

- For 0-jettiness, use consistency relations to relate full LL to RVV correction in collinear limit.

[Moult, Rothen, Stewart, Tackmann, Zhu '16] [Moult, Stewart, **GV**, Zhu '19]

- Focus on Drell-Yan and Higgs production. Single collinear emission fully differential in rapidity:

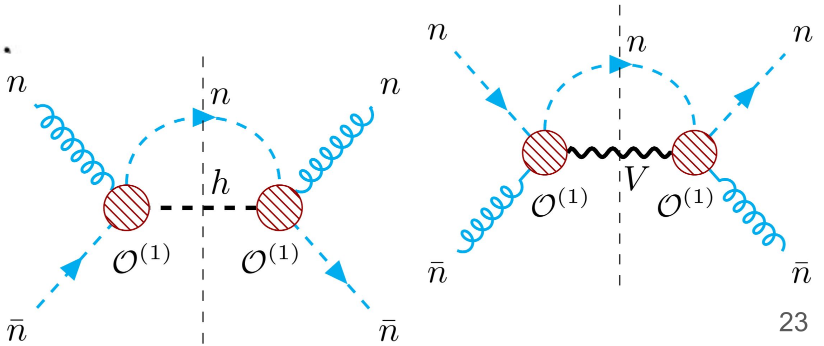
LP Matrix Element — NLP Phase Space

$$\frac{d\sigma_n^{\rm NLP}}{dQ^2 dY d\mathcal{T}} \sim \int_{x_a}^1 \frac{dz_a}{z_a} \frac{(Q^2\tau)^{-\epsilon}}{(1-z_a)^\epsilon} \left\{ \tau A^{(0)}(\tau, z_a, \epsilon) \left[-f_a\left(\frac{x_a}{z_a}\right) f_b(x_b) + f_a\left(\frac{x_a}{z_a}\right) x_b f_b'(x_b) \right] + f_a\left(\frac{x_a}{z_a}\right) f_b(x_b) A^{(2)}(\tau, z_a, \epsilon) \right\}.$$

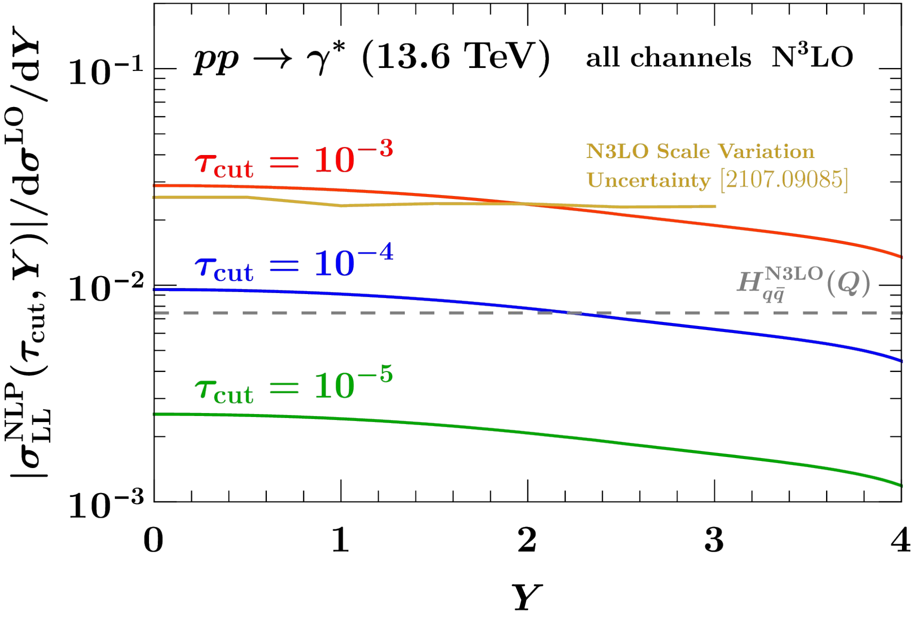
LP Phase Space — NLP Matrix Element

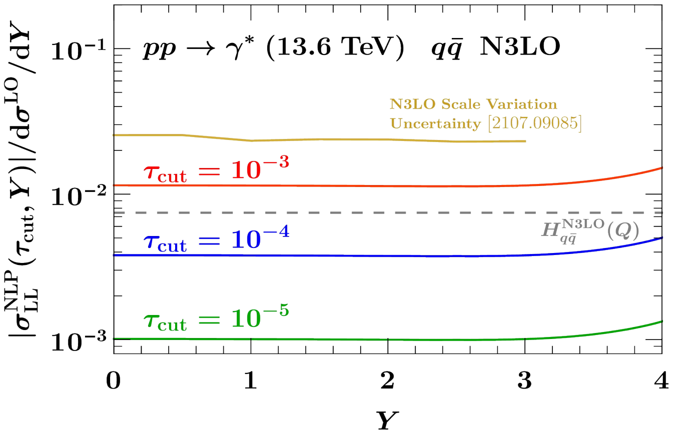
[Ebert, Moult, Stewart, Tackmann, **GV**, Zhu '18]

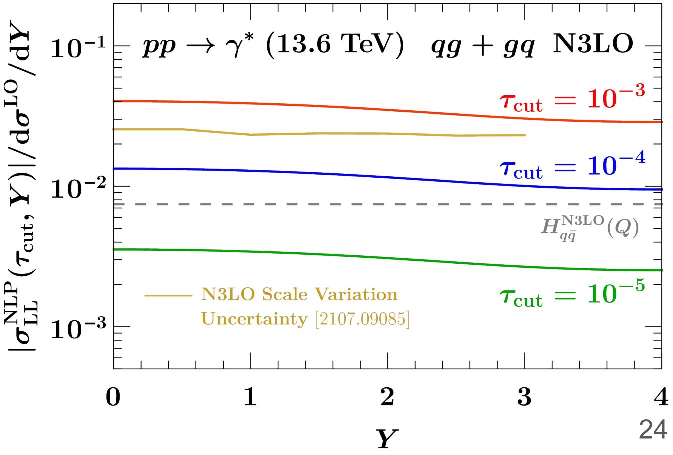
- LL contributions also from off-diagonal qg + gq channels via subleading power hard scattering operators and Lagrangian insertions



0-Jettiness Power Corrections at N3LO: Results for DY

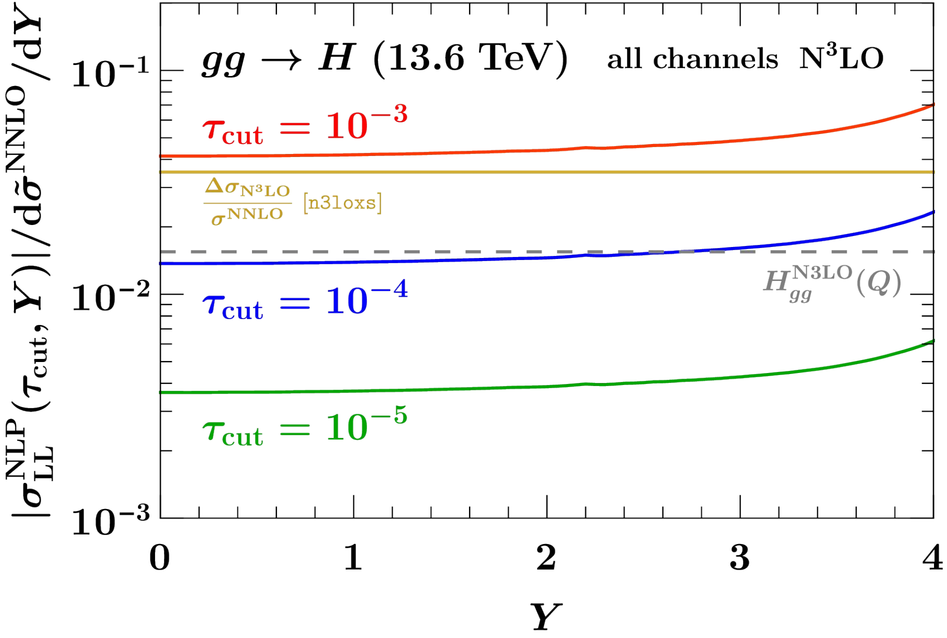


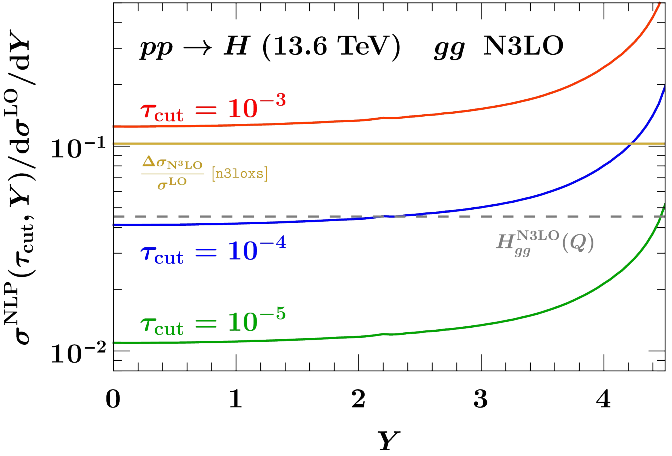


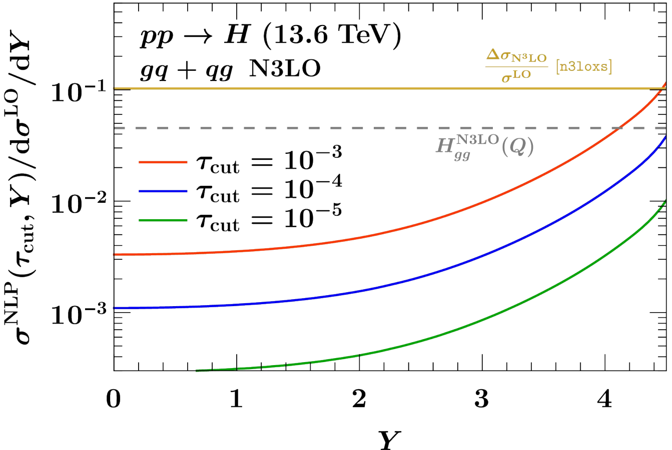


- By the size of LL NLP: 0-jettiness with standard setup (only LP in subtraction term) would require $\tau_{\rm cut} \sim 10^{-5}$ or even smaller.
- Off-diagonal channel has large power corrections (in line with empirical observation in q_T slicing at N3LO)

0-Jettiness Power Corrections at N3LO: Results for Higgs







- Similar story for the case of Higgs production in gluon fusion
- Here the off-diagonal channel is negligible, as it is often the case with the Higgs

Ok, but what about fiducial power corrections?

Fiducial vs Dynamical Power Corrections

- Power corrections can have different sources. We have already seen an example: phase space vs amplitude expansion. But so far we have just looked at the *production* of a color singlet…

- Particular source of p.c. are **fiducial and isolation cuts** on the color singlet decay products.

- Reference on the topic is **(Ebert, Tackmann) [1911.08486]** (extended substantially in (Ebert, Michel, Stewart, Tackmann) [2006.11382] in the case of q_T with also resummation)

- In my paper I refer to these p.c. as **"fiducial"** (irrespectively if we are talking about things induced by p_T lepton cuts or photon isolation cuts), and call the other ones **"dynamical"** as they are mainly related to the subleading power dynamics (the latter is not entirely true so suggestions on the naming are welcome).

Fiducial Power Corrections

- These are **purely kinematic effects**, but have very **large impact** on non-local subtractions due to non canonical scaling in the cut parameter.

- In short:

$$\frac{d\sigma^{(\text{cuts})}(X)}{dQ^2 dY dq_T^2} \sim \frac{1}{q_T^2}\frac{q_T}{Q}, \qquad \frac{d\sigma^{(\text{cuts})}(X)}{dQ^2 dY d\mathcal{T}_0} \sim \frac{1}{\mathcal{T}_0}\sqrt{\frac{\mathcal{T}_0}{Q}}.$$

  - Cuts on leptons induce *linear* terms
    For q_T subtraction they can be captured analytically by a boost, but not for 0-jettiness.

  - Photon Isolations induce p.c. with wild and complicated scaling
    No simple boost trick to account for them.

$$\frac{d\sigma^{(\text{smooth})}(X)}{dQ^2 dY dq_T^2} \sim \frac{R^2}{q_T^2}\left(\frac{q_T}{Q}\right)^{1/n}\left(\frac{Q}{E_T^{\text{iso}}}\right)^{1/n}$$

$$\frac{d\sigma^{(\text{smooth})}(X)}{dQ^2 dY d\mathcal{T}_0} \sim \begin{cases} \frac{R^2}{\mathcal{T}_0}\left(\frac{\mathcal{T}_0}{Q}\right)^{1+1/(2n)}\left(\frac{Q}{E_T^{\text{iso}}}\right)^{1/n} \\ \frac{R^2}{\mathcal{T}_0}\left(\frac{\mathcal{T}_0}{Q}\right)^{1/n}\left(\frac{Q}{E_T^{\text{iso}}}\right)^{1/n} \end{cases}$$

- So, although fiducial power corrections are more trivial conceptually, account for them comes first numerically compared to dynamical power corrections.

Projection to Born Improved Slicing

[Cacciari et al. '15]
[Ebert, Tackmann '19]
[GV '24]

In more general cases, cut-induced power corrections can be numerically accounted for by using **"Projection-to-Born Improved Slicing"**

$$\sigma_{h,\,\mathrm{N^3LO}}(\mathcal{O}) = \sigma_{h,\,\mathrm{N^3LO}}(\tilde{\mathcal{O}}) + \sigma_{h+j,\,\mathrm{NNLO}}(\mathcal{O} - \tilde{\mathcal{O}})$$

P2B correction factor

Slicing calculation for Born projected observable

$$= \int_0^{\tau_{\mathrm{cut}}} \mathrm{d}\tau \, \frac{\mathrm{d}\sigma^{\mathrm{sub}}_{h,\,\mathrm{N^3LO}}}{\mathrm{d}\tau}(\tilde{\mathcal{O}}) + \int_{\tau > \tau_{\mathrm{cut}}} \mathrm{d}\sigma^{\mathrm{full}}_{h+j,\,\mathrm{NNLO}}(\tilde{\mathcal{O}})$$

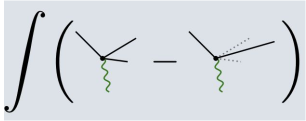
Below the cut term — Above the cut term

$$+ \int_0^{\tau_{\mathrm{cut}}} \mathrm{d}\tau \left[\frac{\mathrm{d}\sigma^{\mathrm{full}}_{h,\,\mathrm{N^3LO}}}{\mathrm{d}\tau} - \frac{\mathrm{d}\sigma^{\mathrm{sub}}_{h,\,\mathrm{N^3LO}}}{\mathrm{d}\tau} \right] (\tilde{\mathcal{O}})$$

Residual Error

$$+ \int \mathrm{d}\sigma^{\mathrm{full}}_{h+j,\,\mathrm{NNLO}}(\mathcal{O} - \tilde{\mathcal{O}})$$

P2B correction factor



Note:
Because of local cancellation using exact matrix elements, P2B is very efficient numerically.
Sometimes referred as the "*perfect*" subtraction scheme

Projection to Born Improved Slicing

[Cacciari et al. ‘15]
[Ebert, Tackmann ‘19]
[GV ‘24]

In more general cases, cut-induced power corrections can be numerically accounted for by using

This enable us to

- Focus on *analytic* calculation of *dynamical* power corrections
- *Numerically* treat *fiducial* power corrections efficiently with P2B method

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term

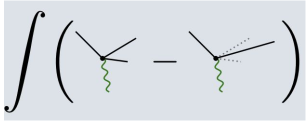
Note:
Because of local cancellation using exact matrix elements, P2B is very efficient numerically.
Sometimes referred as the “*perfect*” subtraction scheme

$$+\int_0^{\tau_{\rm cut}} \mathrm{d}\tau \left[\frac{\mathrm{d}\sigma^{\rm full}_{h,\,\mathrm{N^3LO}}}{\mathrm{d}\tau} - \frac{\mathrm{d}\sigma^{\rm sub}_{h,\,\mathrm{N^3LO}}}{\mathrm{d}\tau}\right](\tilde{\mathcal{O}})$$

Residual Error

$$+\int \mathrm{d}\sigma^{\rm full}_{h+j,\,\mathrm{NNLO}}(\mathcal{O}-\tilde{\mathcal{O}})$$

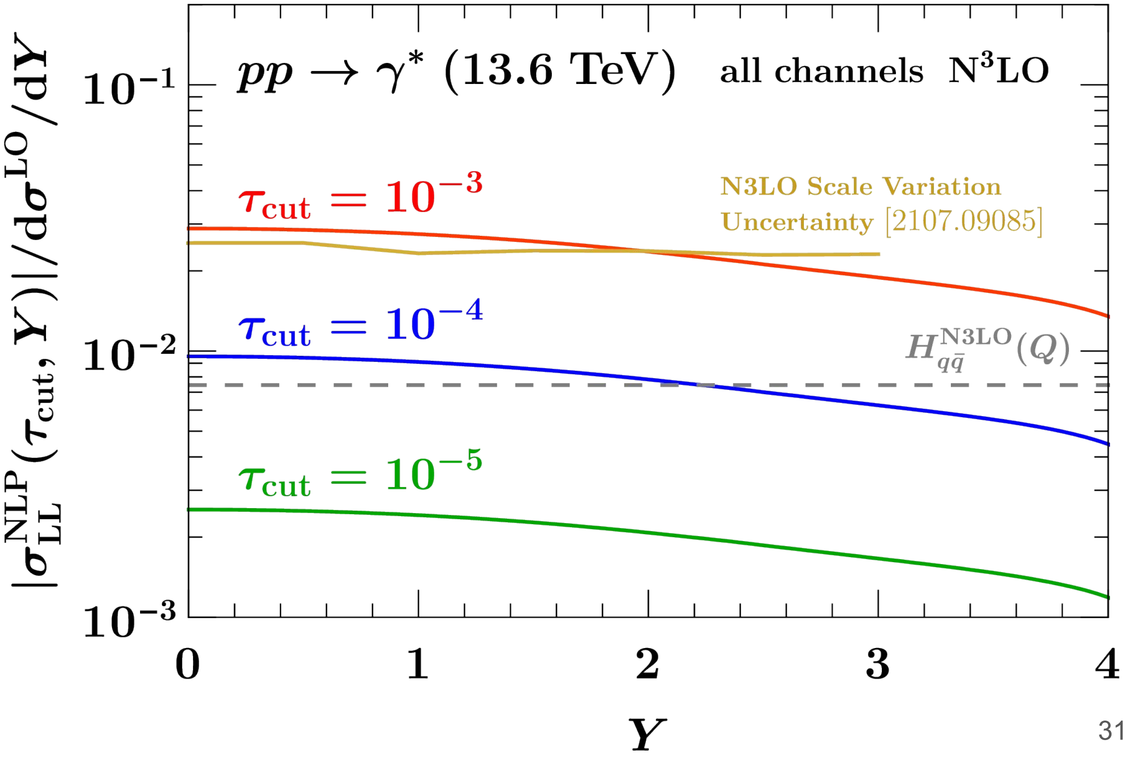
P2B correction factor



0-Jettiness P.C. at N3LO: Improving DY

$$\Delta\sigma^{N3LO}(\tau_{\rm cut}) \sim \left(\frac{\alpha_s}{4\pi}\right)^3 \int_0^{\tau_{\rm cut}} d\tau \left(c_{3,5}^{\rm NLP} \ln^5\tau + c_{3,4}^{\rm NLP} \ln^4\tau + c_{3,3}^{\rm NLP} \ln^3\tau + \ldots\right)$$

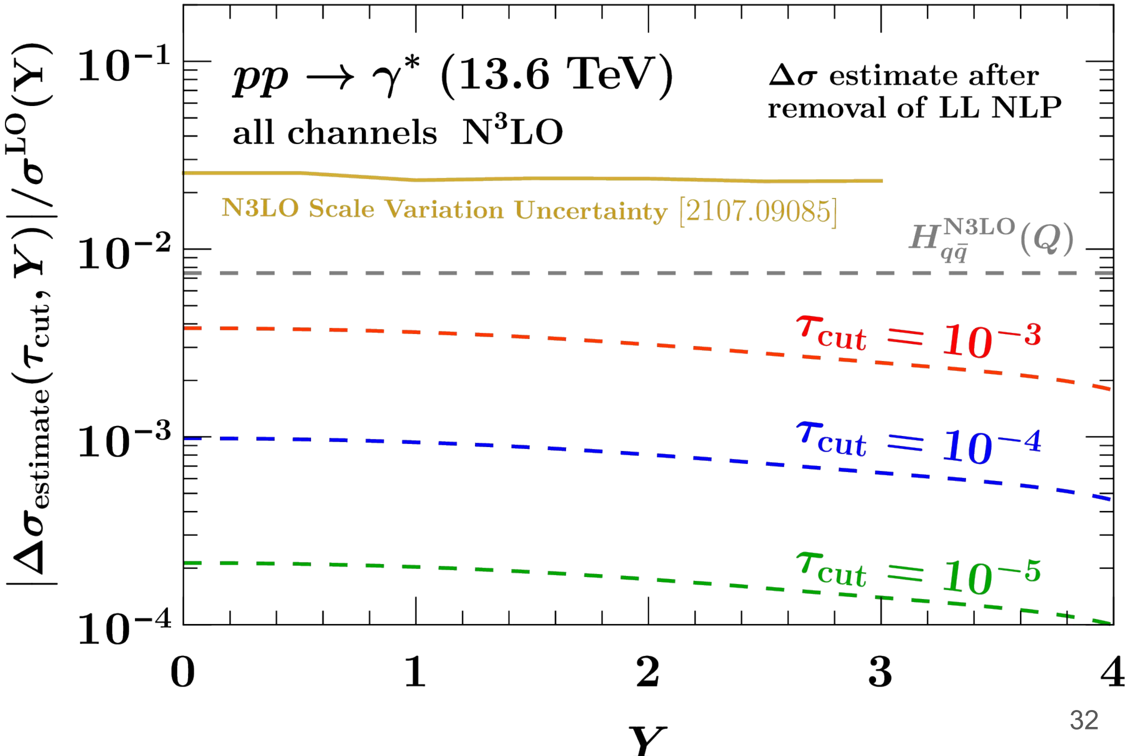
- So now we are back at considering dynamical power corrections
- We have them analytically
- Hence include them in subtraction term below the cut



0-Jettiness P.C. at N3LO: Estimate of residual error for DY

$$\Delta\sigma^{N3LO}(\tau_{\rm cut}) \sim \left(\frac{\alpha_s}{4\pi}\right)^3 \int_0^{\tau_{\rm cut}} d\tau \left(c_{3,5}^{\rm NLP} \ln^5 \tau + c_{3,4}^{\rm NLP} \ln^4 \tau + c_{3,3}^{\rm NLP} \ln^3 \tau + \dots\right)$$

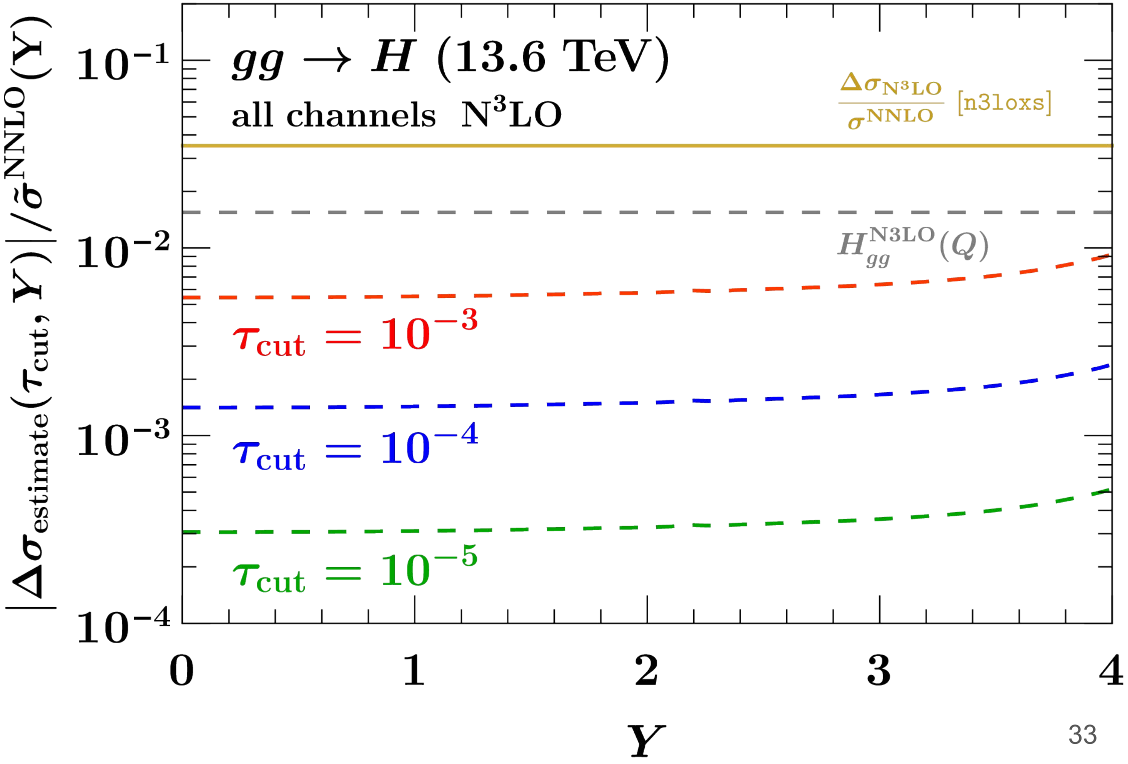
- Estimate residual slicing error removing LL NLP
- Assume same size as LL coefficient (in line with what seen at previous orders) for subleading logs and powers
- Slicing error significantly reduced. O(x50) larger cut allowed.
- May save millions of CPU hours and allow for better convergence studies



0-Jettiness P.C. at N3LO: Estimate of residual error for Higgs

$$\Delta\sigma^{N3LO}(\tau_{\rm cut}) \sim \left(\frac{\alpha_s}{4\pi}\right)^3 \int_0^{\tau_{\rm cut}} d\tau \left(c_{3,5}^{\rm NLP} \ln^5 \tau + c_{3,4}^{\rm NLP} \ln^4 \tau + c_{3,3}^{\rm NLP} \ln^3 \tau + \ldots \right)$$

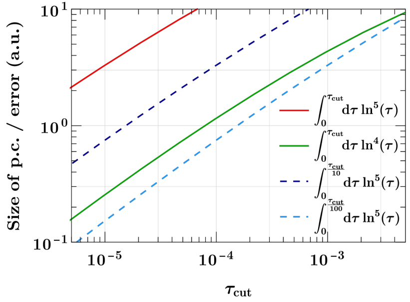
- Play the same game for estimating residual slicing error after the inclusion of LL NLP in the subtraction term:
- Assume same size as LL coefficient (in line with what seen at previous orders) for subleading logs and powers
- Slicing error significantly reduced. O(x50) larger cut allowed.



Conclusion

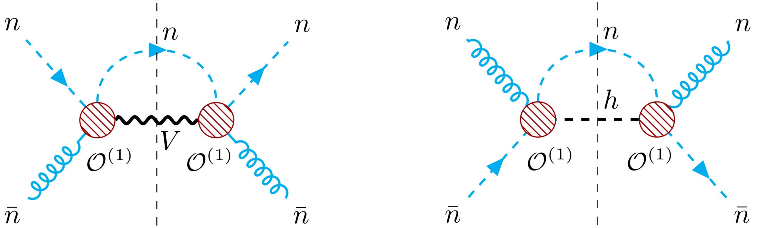
- ➢ Discussed challenges of N3LO calculations and slicing methods

$$\sigma(X) = \underbrace{\int_0^{q_{T\,\mathrm{cut}}} \mathrm{d}q_T \frac{\mathrm{d}\sigma^{\mathrm{sing}}(X)}{\mathrm{d}q_T}}_{\text{Below the cut region}} + \underbrace{\int_{q_{T\,\mathrm{cut}}} \mathrm{d}q_T \frac{\mathrm{d}\sigma(X)}{\mathrm{d}q_T}}_{\text{Above the cut region}} + \underbrace{\Delta\sigma(X, q_{T\,\mathrm{cut}})}_{\text{Residual}}$$

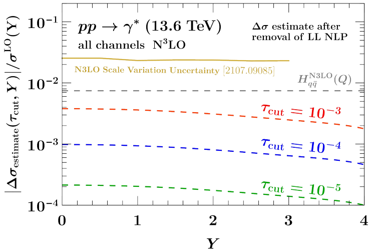


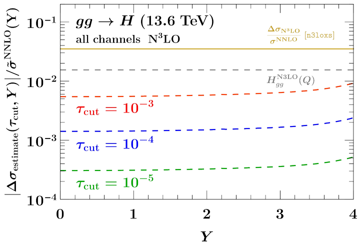
- ➢ Used P2B improved slicing to account for fiducial power corrections

- ➢ Presented the calculation of the LL NLP at N3LO for 0-jettiness



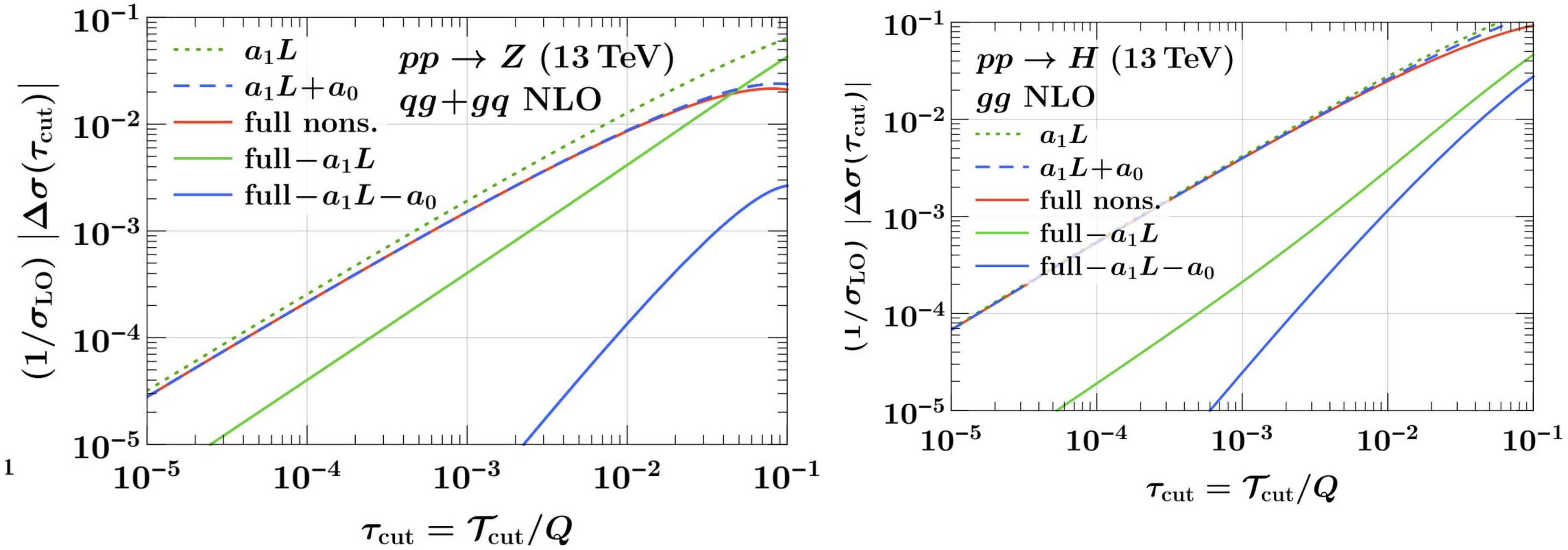
- ➢ Illustrated impact on slicing error for Drell-Yan and Higgs production





Backup

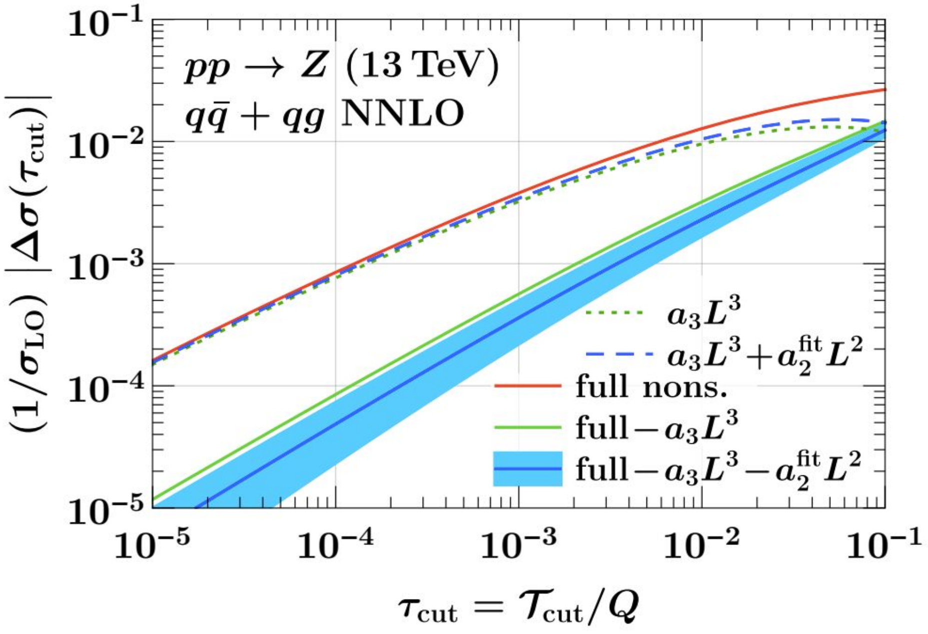
Log behaviour at NLP NLO



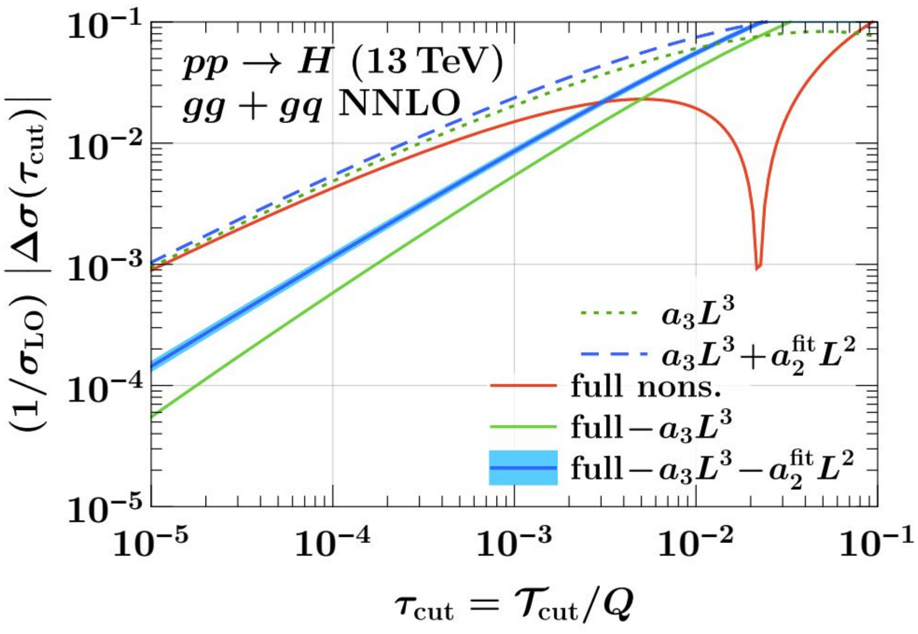
[1807.10764]

Log behaviour at NLP NNLO

[1612.00450]



[1710.03227]



A word on linear vs quadratic power corrections

$$\text{0-jettiness}: \quad \Delta\sigma^{N3LO}(\tau_{\rm cut}) \sim \left(\frac{\alpha_s}{4\pi}\right)^3 \int_0^{\tau_{\rm cut}} \mathrm{d}\tau \left(c_{3,5}^{\rm NLP}\ln^5\tau + c_{3,4}^{\rm NLP}\ln^4\tau + c_{3,3}^{\rm NLP}\ln^3\tau + \dots\right)$$

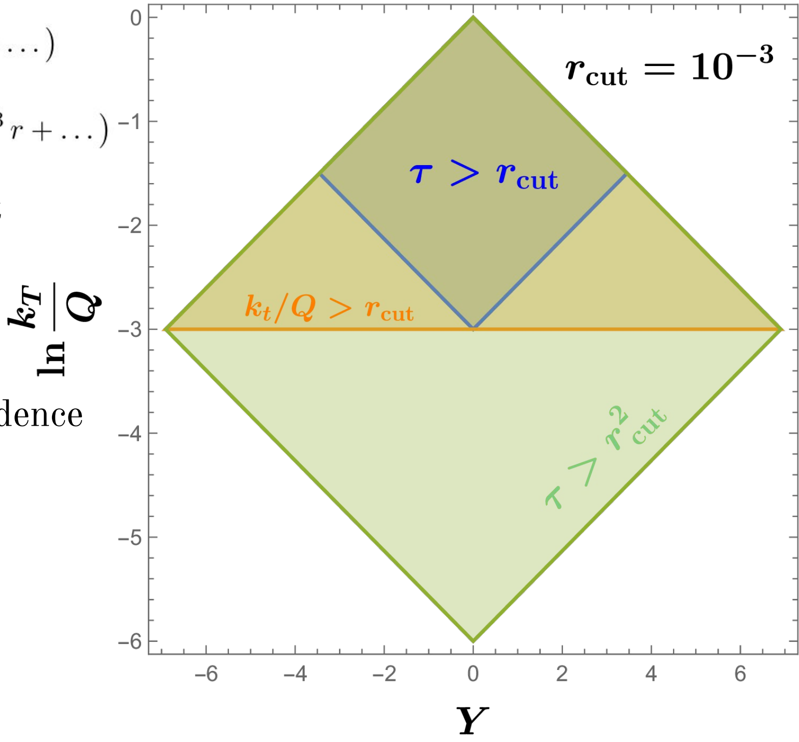
$$q_T: \quad \Delta\sigma^{N3LO}(q_{T\rm cut}) \sim \left(\frac{\alpha_s}{4\pi}\right)^3 \int_0^{q_{T\rm cut}^2/Q^2} \mathrm{d}r \left(d_{3,5}^{\rm NLP}\ln^5 r + d_{3,4}^{\rm NLP}\ln^4 r + d_{3,3}^{\rm NLP}\ln^3 r + \dots\right)$$

- Scaling in q_T of the slicing param. may lead to the impression that q_T subtraction has *quadratic* power corrections, while jettiness has *linear* power corrections.

- But it all comes down to how one decides to treat the angle dependence

$$\tau = \frac{q_T}{Q}e^{-|Y|} \sim \begin{cases} \frac{q_T}{Q} & \text{soft emissions} \\ \frac{q_T^2}{Q^2} & \text{collinear emissions} \end{cases}$$

- In practice, key point is what is more challenging numerically for the above the cut code:
  - 0-jettiness: better suppression of collinear emissions
  - q_T : better suppression of wide angle soft emissions



Note: fiducial p.c. generating *linear* terms in q_T, go as $\sqrt{\tau_{\rm cut}}$ in the case of 0-jettiness

Differential color singlet production at N3LO

- Two methods for differential N3LO predictions for color singlet:

Projection to Born

[Cacciari, Dreyer, Karlberg, Salam, Zanderighi '15]

$$\frac{d\sigma_F^{\mathrm{N}^k\mathrm{LO}}}{d\mathcal{O}} = \underbrace{\left(\frac{d\sigma_{F+\mathrm{jet}}^{\mathrm{N}^{(k-1)}\mathrm{LO}}}{d\mathcal{O}} - \frac{d\sigma_{F+\mathrm{jet}}^{\mathrm{N}^{(k-1)}\mathrm{LO}}}{d\widetilde{\mathcal{O}}}\right)}_{\text{Locally subtracted real emissions}} + \underbrace{\frac{d\sigma_F^{\mathrm{N}^k\mathrm{LO}}}{d\widetilde{\mathcal{O}}}}_{\text{Integrated counterterm}}$$

- **PRO:** Local counterterm is the full Matrix Element => Great numerical efficiency
- **Cons:** Integrated counterterm is very hard to obtain (analytic differential distribution at N3LO in full kinematics)

q_T or **0-jettiness** subtraction

q_T Subtraction: [Catani, Grazzini '07]

N-Jettiness: [Boughezal, Focke, Liu, Petriello '15] [Gaunt, Stahlhofen, Tackmann, Walsh '15]

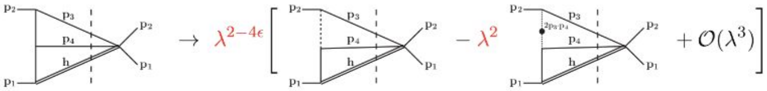
$$\sigma(X) = \underbrace{\int_0^{q_{T\,\mathrm{cut}}} dq_T \frac{d\sigma^{\mathrm{sing}}(X)}{dq_T}}_{\text{Below the cut region}} + \underbrace{\int_{q_{T\,\mathrm{cut}}} dq_T \frac{d\sigma(X)}{dq_T}}_{\text{Above the cut region}} + \underbrace{\Delta\sigma(X, q_{T\,\mathrm{cut}})}_{\text{Residual}}$$

- **PRO:** Analytic control of IR divergences from EFT factorization thm. at Leading Power
- **Cons:** numerically challenging

Beam Functions calculation at N3LO

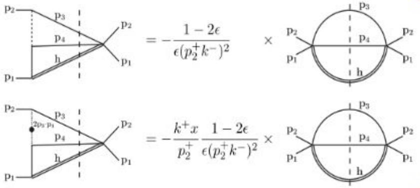
(Ebert, Mistlberger, **GV**) [2006.05329], [2006.03056]

- Calculation of the **collinear expansion of the partonic cross section** for DY and Higgs @N3LO **differential** in **(Q_T, τ, z)**
  - ~ 100k Feynman diagrams
  - Reverse unitarity for phase space integrals
  - Collinear Expansion at the XS level
    "Collinear expansion for color singlet cross sections" [Ebert, Mistlberger, **GV**]

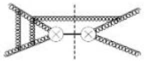


  - Reduction to basis of **Master Integrals** via Integration By Parts (IBPs) using Water

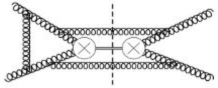
Expanded diagrams admit (simplified) **IBPs** identities



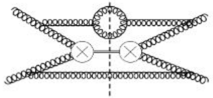
  - RVV: known in full kinematics
    [Duhr, Gehrmann] [Duhr, Gehrmann, Jaquier] [Dulat, Mistlberger]



  - **RRV**: 170 Collinear Master Integrals



  - **RRR**: 320 Collinear Master Integrals



  - Derived system of Differential Equations for the Master Integrals
  - System has 2 non trivial scales with algebraic dependence on the variables (not something solvable algorithmically)
  - Algebraic sectors: constructed dlog integrand basis via calculation of **leading singularities** of candidate integrals on maximal cut surface
  - Boundaries from soft integrals [Anastasiou, Duhr, Dulat, Mistlberger] and constraints on singular behavior