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QUANTUM CHEMISTRY AND EDMs

NIKHEF WORKSHOP: THEORY MEETS EXPERIMENT

Lukáš F. Pašteka

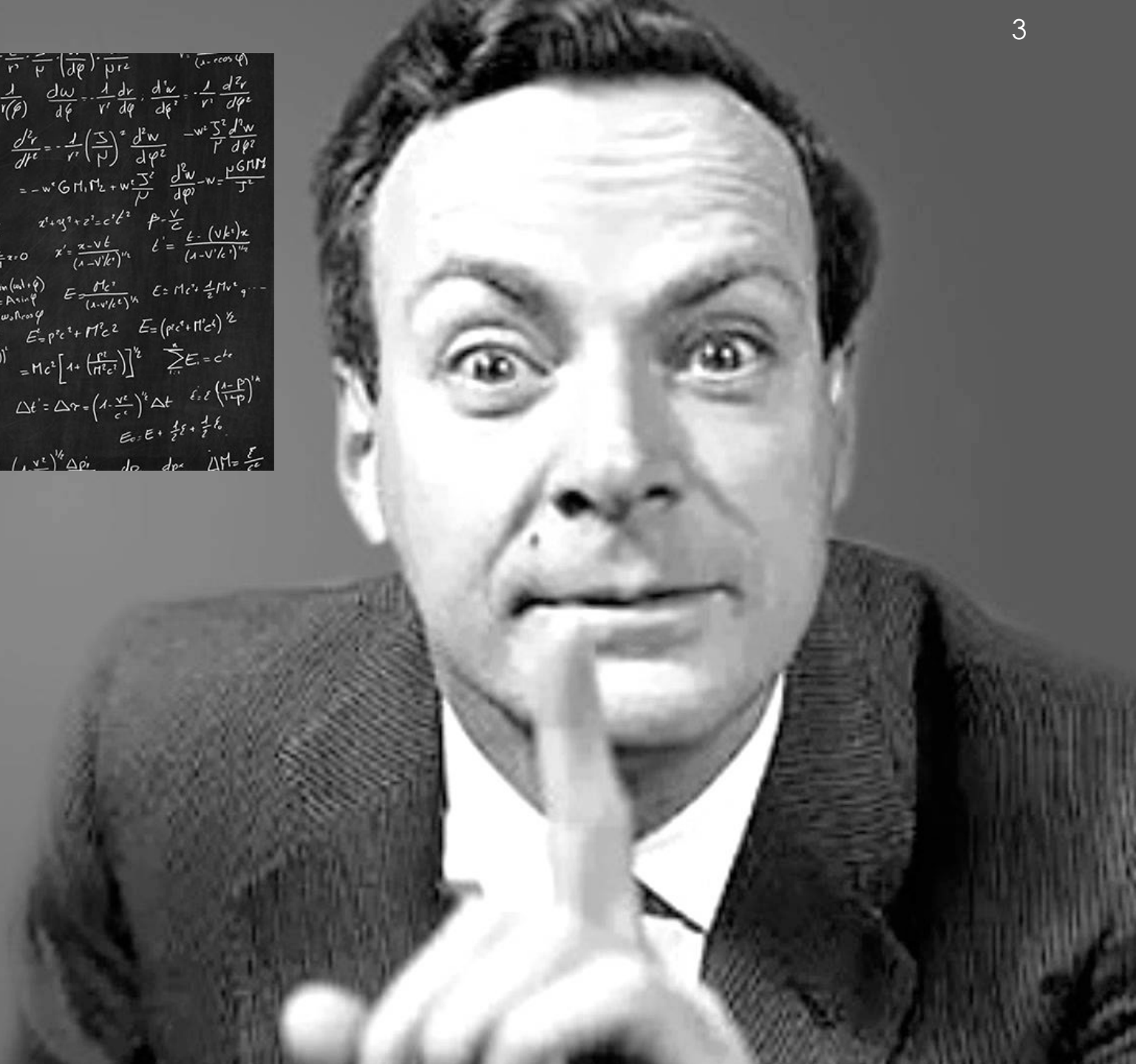
Intro

Why bother with theory?

- experiments are challenging and expensive
- small amounts of unstable, short-lived elements
- unprecedented sensitivity needed to detect the tiny effects of new physics
- special precautions are required (hazardous materials)
- practical considerations do not apply to theory
- alongside specially developed experimental techniques, theoretical support becomes crucial

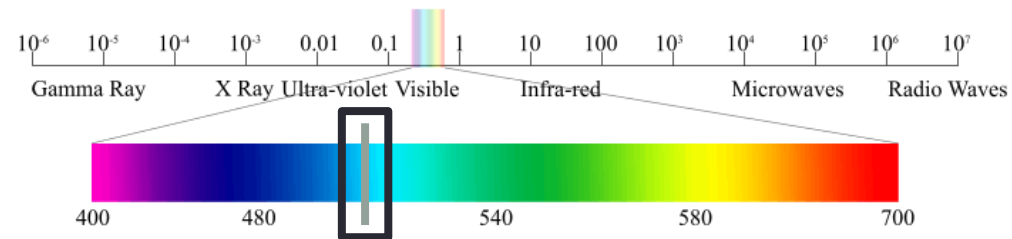
Handwritten mathematical notes covering quantum mechanics and relativity. The notes include:

- Wave function definitions: $\hat{X} = \frac{m\omega}{\hbar} X$, $\hat{P} = \frac{1}{\sqrt{m\omega}} P$
- Hamiltonian: $\hat{H} = \frac{1}{2}(\hat{X} + i\hat{P})^2$
- Eigenstates: $\hat{H}|\psi_n\rangle = E_n|\psi_n\rangle$
- Energy levels: $E_n = \hbar\omega(n + \frac{1}{2})$
- Probability distributions: $|\psi_n(x)|^2$ plots for various states.
- Relativistic energy: $E = \gamma mc^2$, $E = \sqrt{p^2 c^2 + m^2 c^4}$
- Momentum and velocity: $p = \gamma mv$, $v = \frac{pc}{E}$
- Wave number and wavelength: $k = \frac{2\pi}{\lambda}$, $\lambda = \frac{h}{p}$
- Compton shift: $\Delta\lambda = \frac{h}{m_e c} (1 - \cos\theta)$
- De Broglie wavelength: $\lambda = \frac{h}{mv}$
- Bohr radius: $a_0 = \frac{4\pi\epsilon_0 \hbar^2}{m_e e^2}$
- Angular momentum: $L = \hbar\sqrt{l(l+1)}$
- Spin: $S = \hbar\sqrt{s(s+1)}$
- Dirac equation: $(\gamma^\mu \partial_\mu + m)\psi = 0$
- Dirac spinors: $u(p, s)$, $v(p, s)$
- Dirac current: $j^\mu = \bar{\psi} \gamma^\mu \psi$
- Dirac equation in an electromagnetic field: $(\gamma^\mu (\partial_\mu + ieA_\mu) + m)\psi = 0$
- Dirac equation in a scalar field: $(\gamma^\mu \partial_\mu + m + S\phi)\psi = 0$
- Dirac equation in a vector field: $(\gamma^\mu (\partial_\mu + ieA_\mu + gV_\mu) + m)\psi = 0$
- Dirac equation in a tensor field: $(\gamma^\mu \partial_\mu + m + T_{\mu\nu} \sigma^{\mu\nu})\psi = 0$
- Dirac equation in a spinor field: $(\gamma^\mu \partial_\mu + m + \chi\psi)\psi = 0$
- Dirac equation in a vector-spinor field: $(\gamma^\mu \partial_\mu + m + \chi\psi)\psi = 0$
- Dirac equation in a spinor-vector field: $(\gamma^\mu \partial_\mu + m + \chi\psi)\psi = 0$
- Dirac equation in a spinor-tensor field: $(\gamma^\mu \partial_\mu + m + \chi\psi)\psi = 0$
- Dirac equation in a spinor-vector-tensor field: $(\gamma^\mu \partial_\mu + m + \chi\psi)\psi = 0$
- Dirac equation in a spinor-vector-spinor field: $(\gamma^\mu \partial_\mu + m + \chi\psi)\psi = 0$
- Dirac equation in a spinor-vector-spinor-tensor field: $(\gamma^\mu \partial_\mu + m + \chi\psi)\psi = 0$



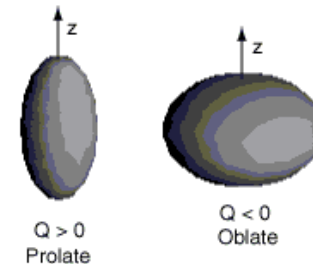
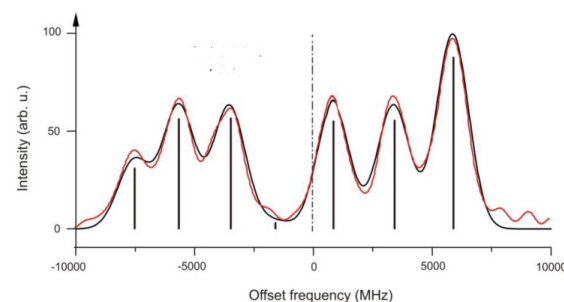
Where does theory fit?

- Identification of promising candidates for precision measurements
 - practical considerations (stability, laser-coolability, etc.)
 - high sensitivity to desired effect
- Reliable predictions for planning the experiment



- Extraction of data from measurements

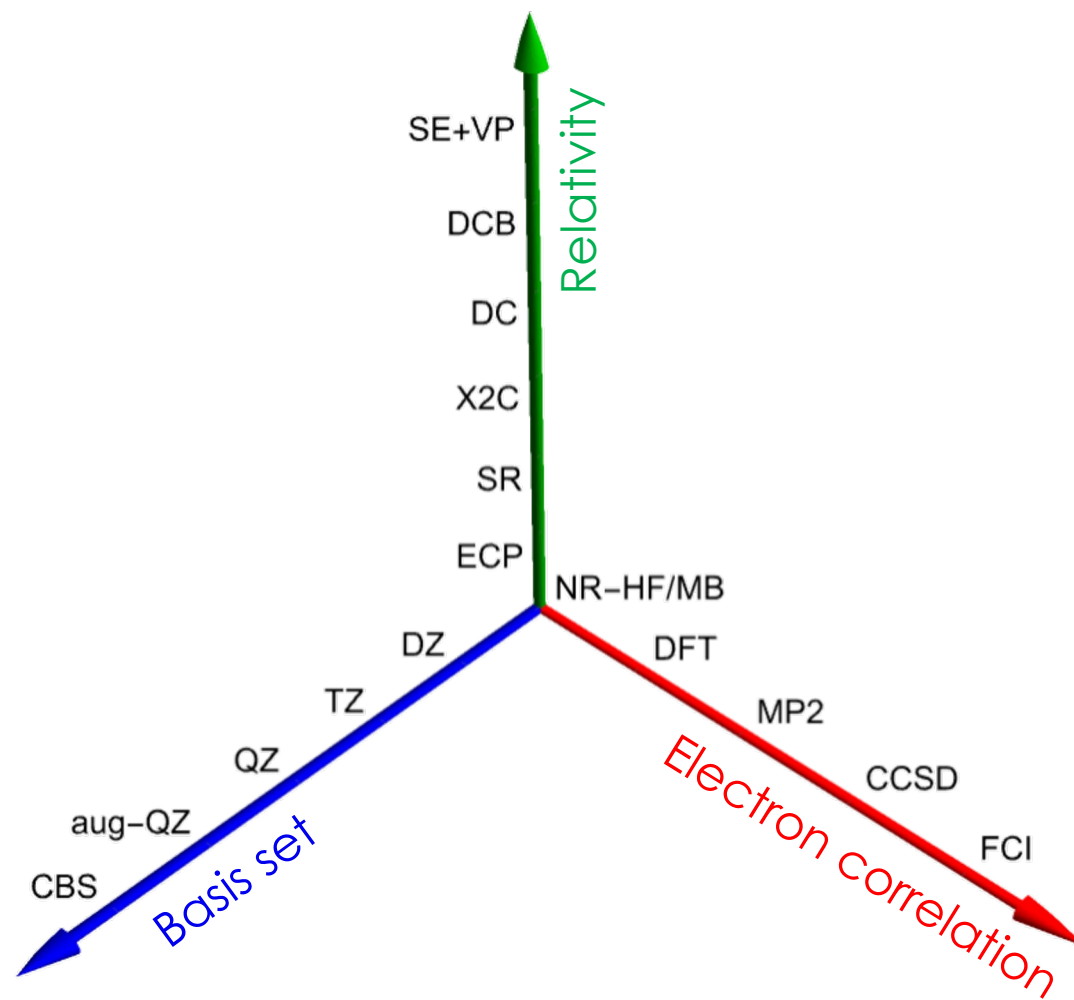
HFS
 $B = eQ_s q$



Computational methods a general idea

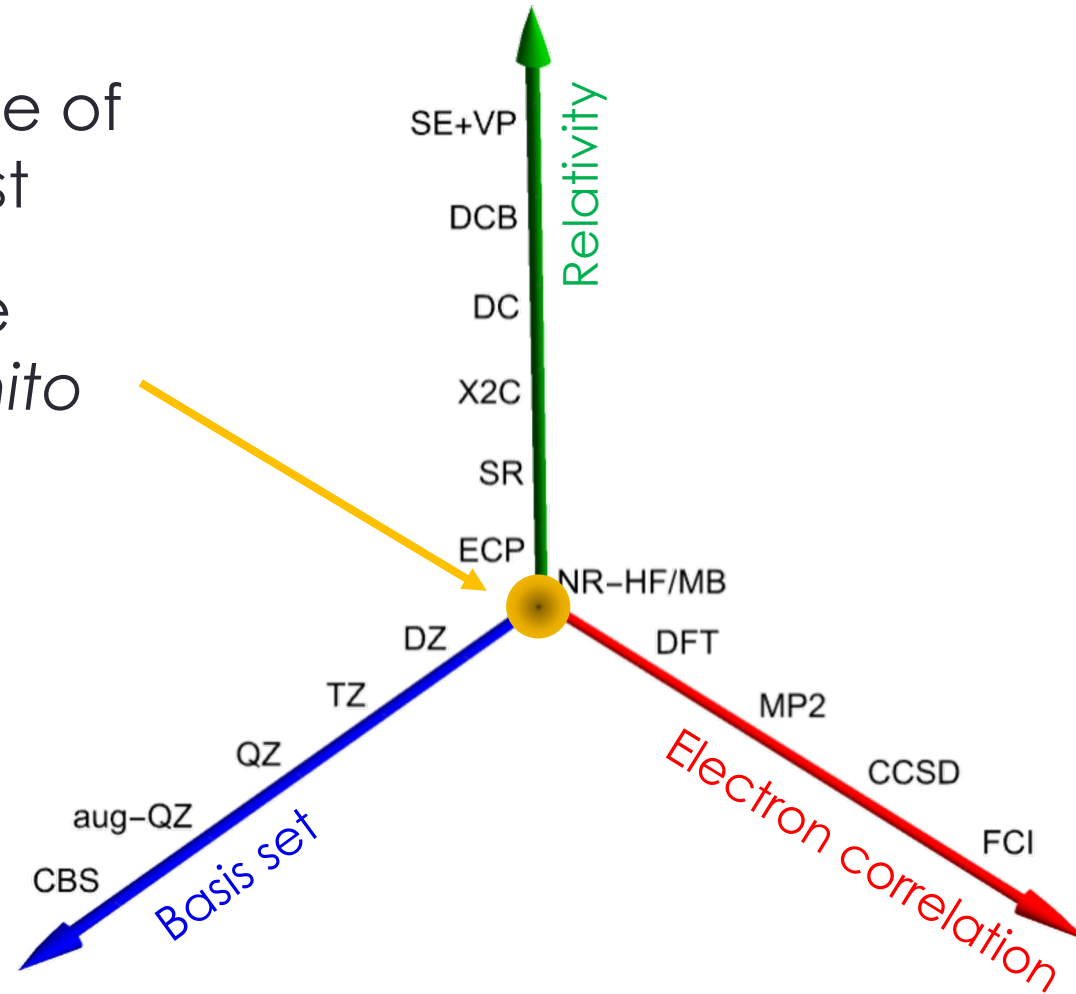


Computational methods



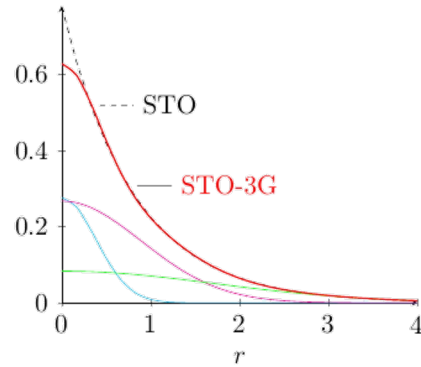
Computational methods

- systematic increase of accuracy and cost
- starting point is the cheapest still *ab initio* method
- computation is a balancing act

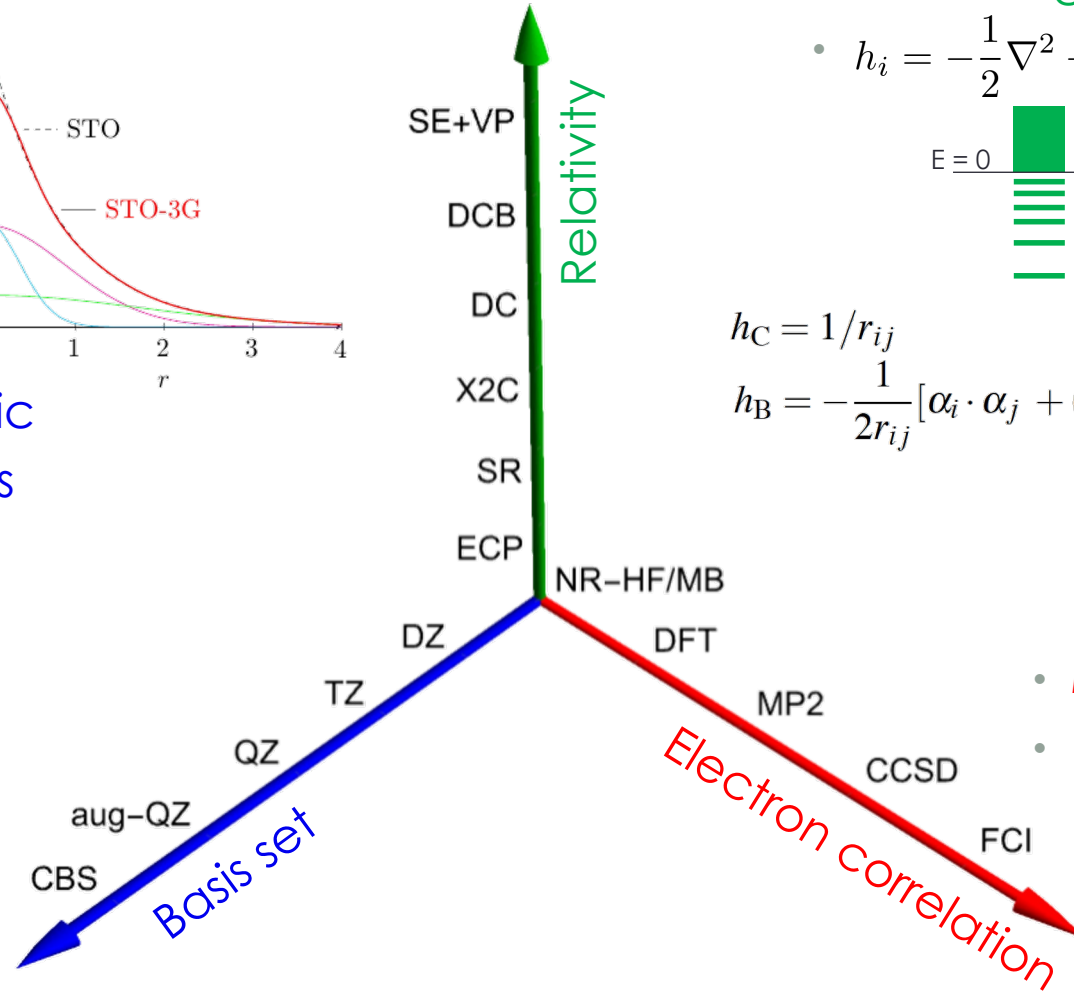
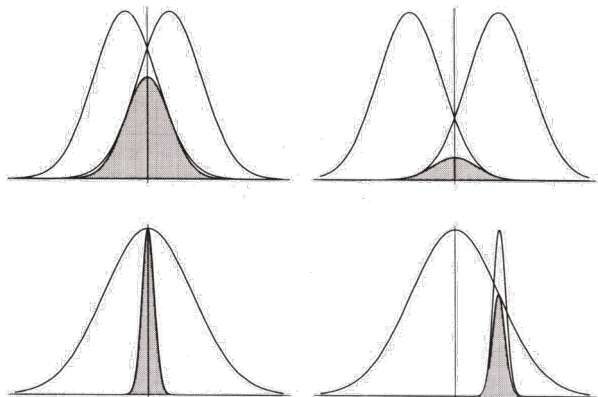


Computational methods

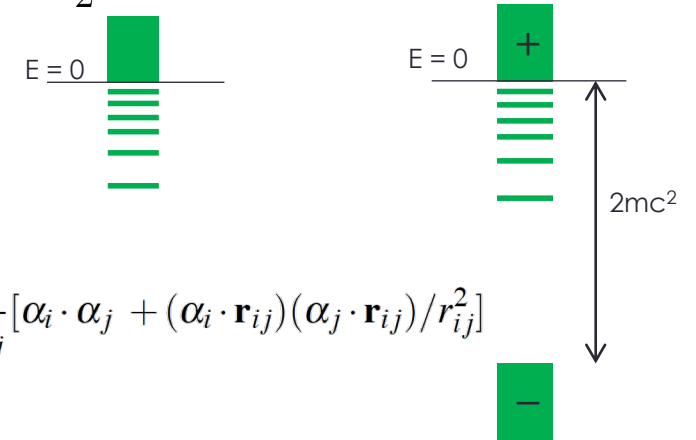
- vector space
- 1-electron expansion
- MO-LCAO
- atom and property specific
- Gaussian (radial) basis sets



$$G_{nlm}(r, \theta, \psi) = N_n \underbrace{r^{n-1} e^{-ar^2}}_{\text{radial part}} \underbrace{Y_l^m(\theta, \psi)}_{\text{angular part}}$$



- i.e. Hamiltonian $H = \sum_i h_i + \sum_{i<j} h_{ij}$
- Schrödinger \rightarrow Dirac \rightarrow beyond
- $h_i = -\frac{1}{2}\nabla^2 + V_n \rightarrow h_i = c\alpha \cdot \mathbf{p} + \beta c^2 + V_n$



$$h_C = 1/r_{ij}$$

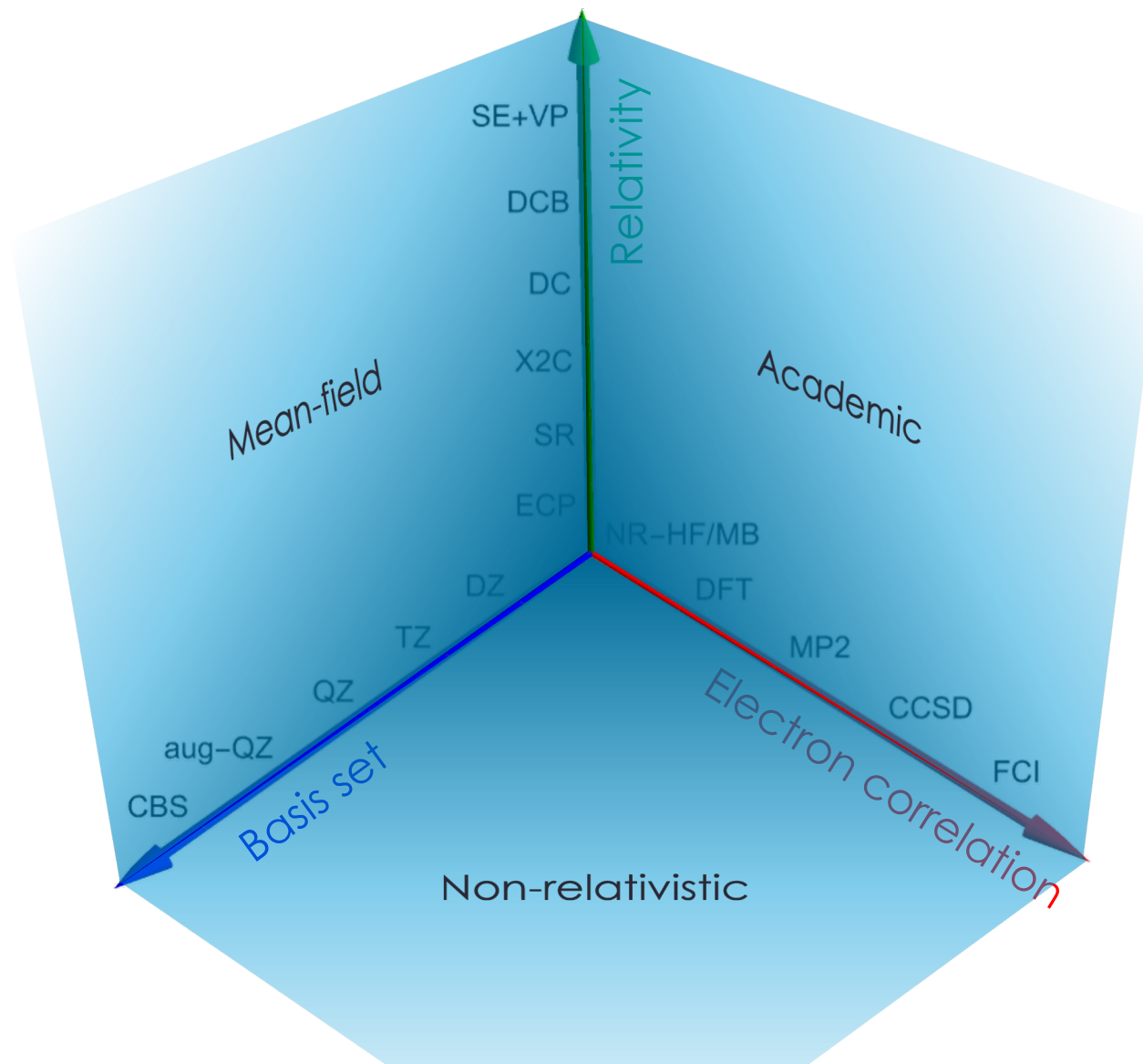
$$h_B = -\frac{1}{2r_{ij}} [\alpha_i \cdot \alpha_j + (\alpha_i \cdot \mathbf{r}_{ij})(\alpha_j \cdot \mathbf{r}_{ij})/r_{ij}^2]$$

- N -electron expansion (SD / CSF)
- post-HF methods
 - MP perturbation theory
 - configuration interaction $\psi = \sum_I C_I \Phi_I$
 - coupled cluster

$$\psi = \exp(T)\Phi$$

$$T = T_1 + T_2 + T_3 + \dots + T_N$$

Computational methods

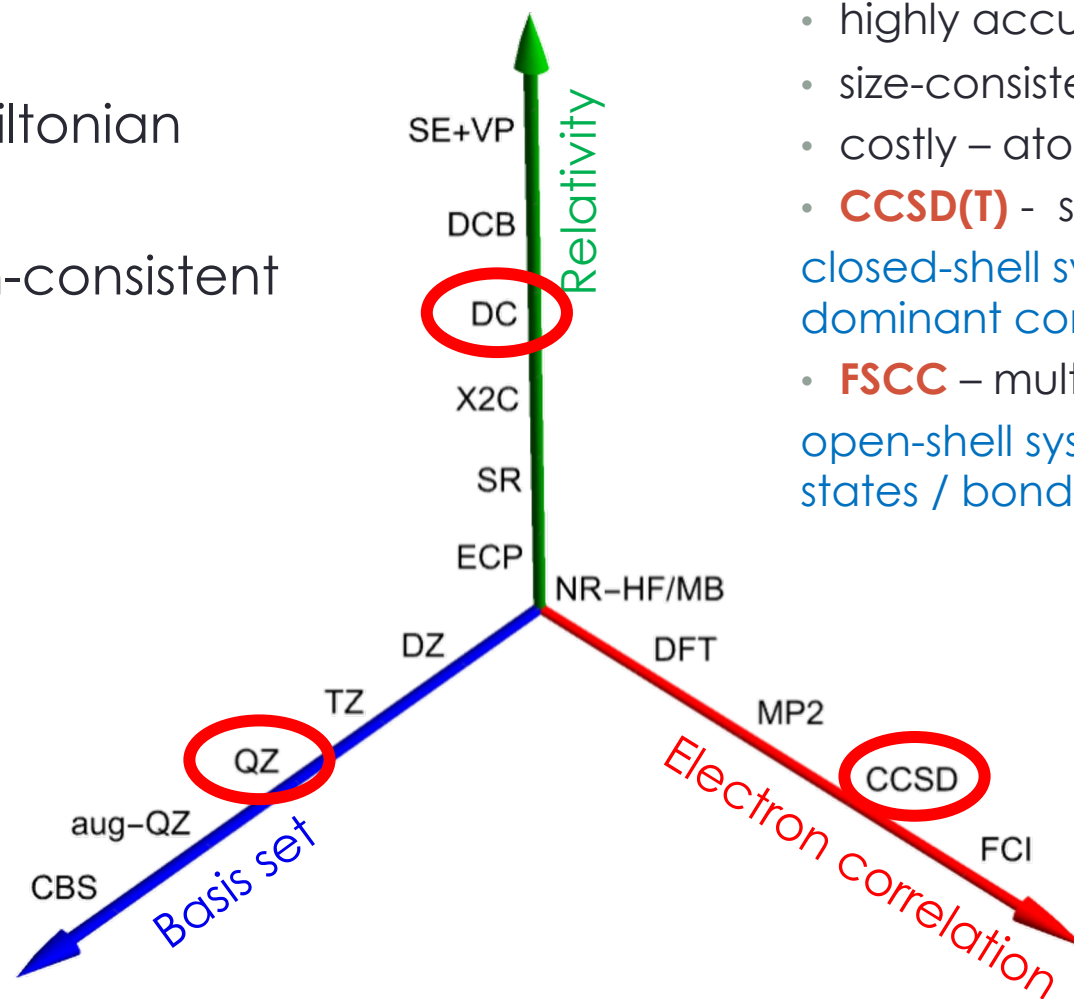


The right tool for the job

- Coupling parameters of $P(T)$ -violating phenomena, variation of fundamental constants, ...
 - relativistic in nature → relativistic methods
- Atomic and molecular parameters needed in experiments
 - heavy systems → relativistic methods
- High accuracy
 - state-of-the-art treatment of correlation + large basis sets
- Uncertainty estimates
 - robust and systematic methodology

Golden standard

- 4c Dirac-Coulomb Hamiltonian
- large flexible correlation-consistent basis sets
 - CBS extrapolation



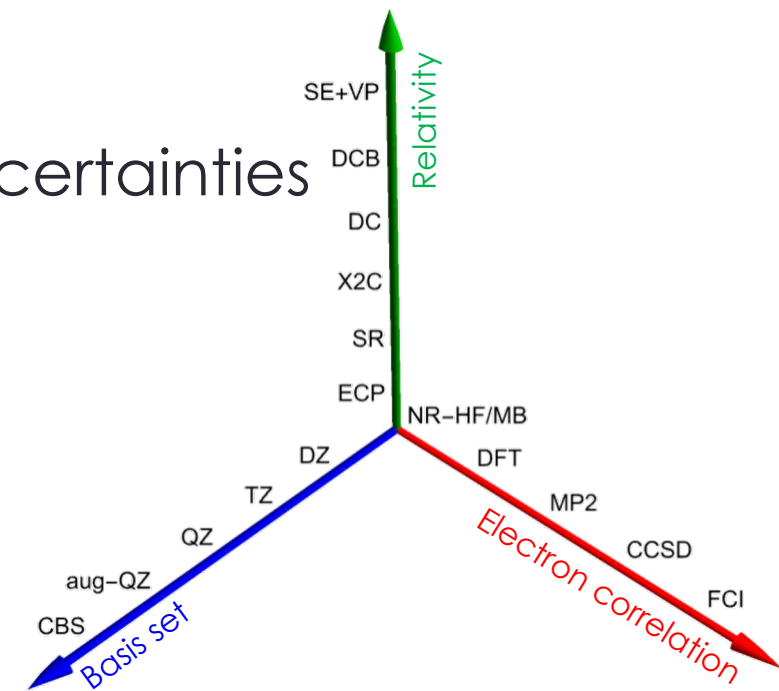
- Coupled-Cluster
 - highly accurate
 - size-consistent
 - costly – atoms & small molecules
 - **CCSD(T)** - single reference CC closed-shell systems / systems with one dominant configuration (e.g. BaF, $X^2\Sigma$)
 - **FSCC** – multireference Fock-space CC open-shell systems / spectrum of excited states / bond dissociation

What can we calculate

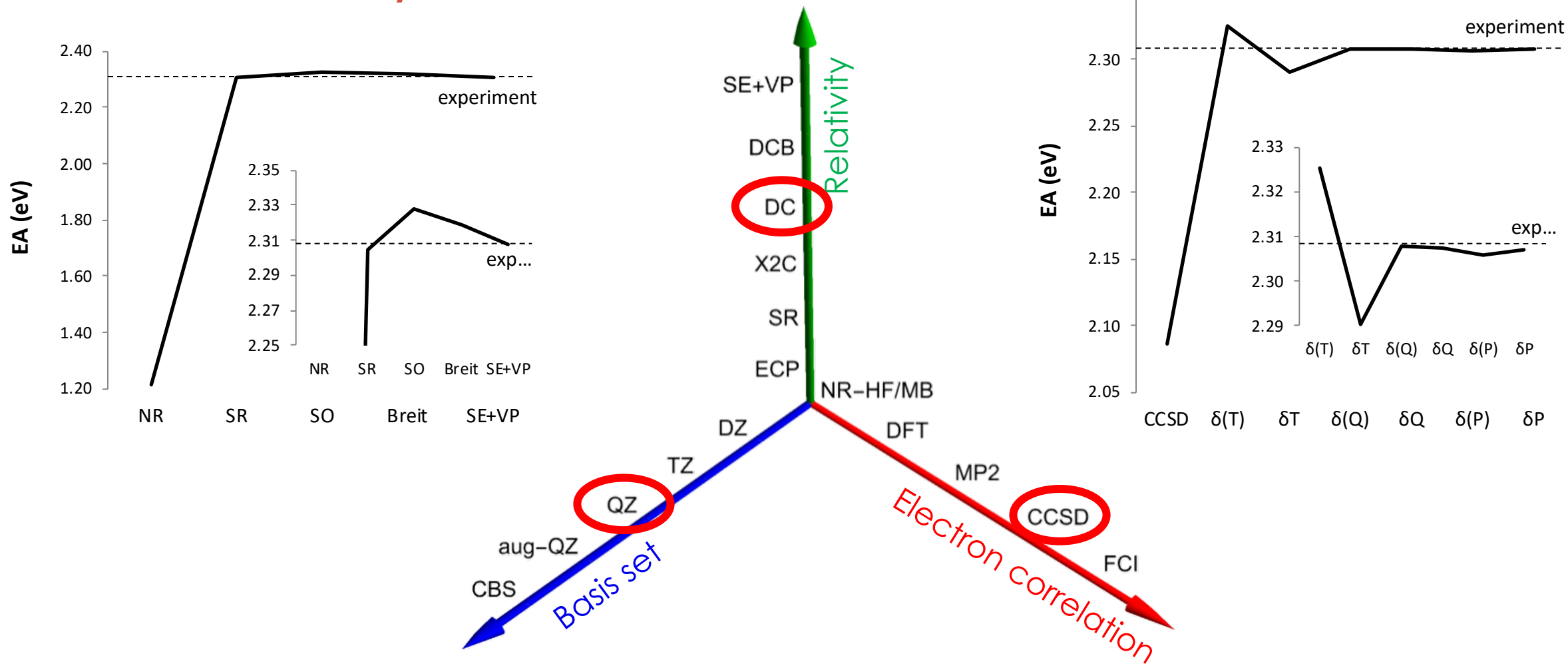
- Atomic properties
 - energies, IPs, EAs, spectra, HFS parameters, polarizabilities, ...
- Molecular properties
 - geometries, spectroscopic constants, laser cooling schemes, ...
- Properties for interpretation of precision measurements:
 - W_d , W_s (eEDM experiments)
 - W_A (NSD-PV, nuclear anapole moments)
 - W_M (NMQ moments)
 - E_{PV} (parity-violating energy shifts)
 - sensitivity to α -variation
- Expected accuracy
 - ~ 10 meV for energies, ~ 5 % for properties
 - can do better
- Systematic improvement and uncertainty evaluation

Uncertainty estimation

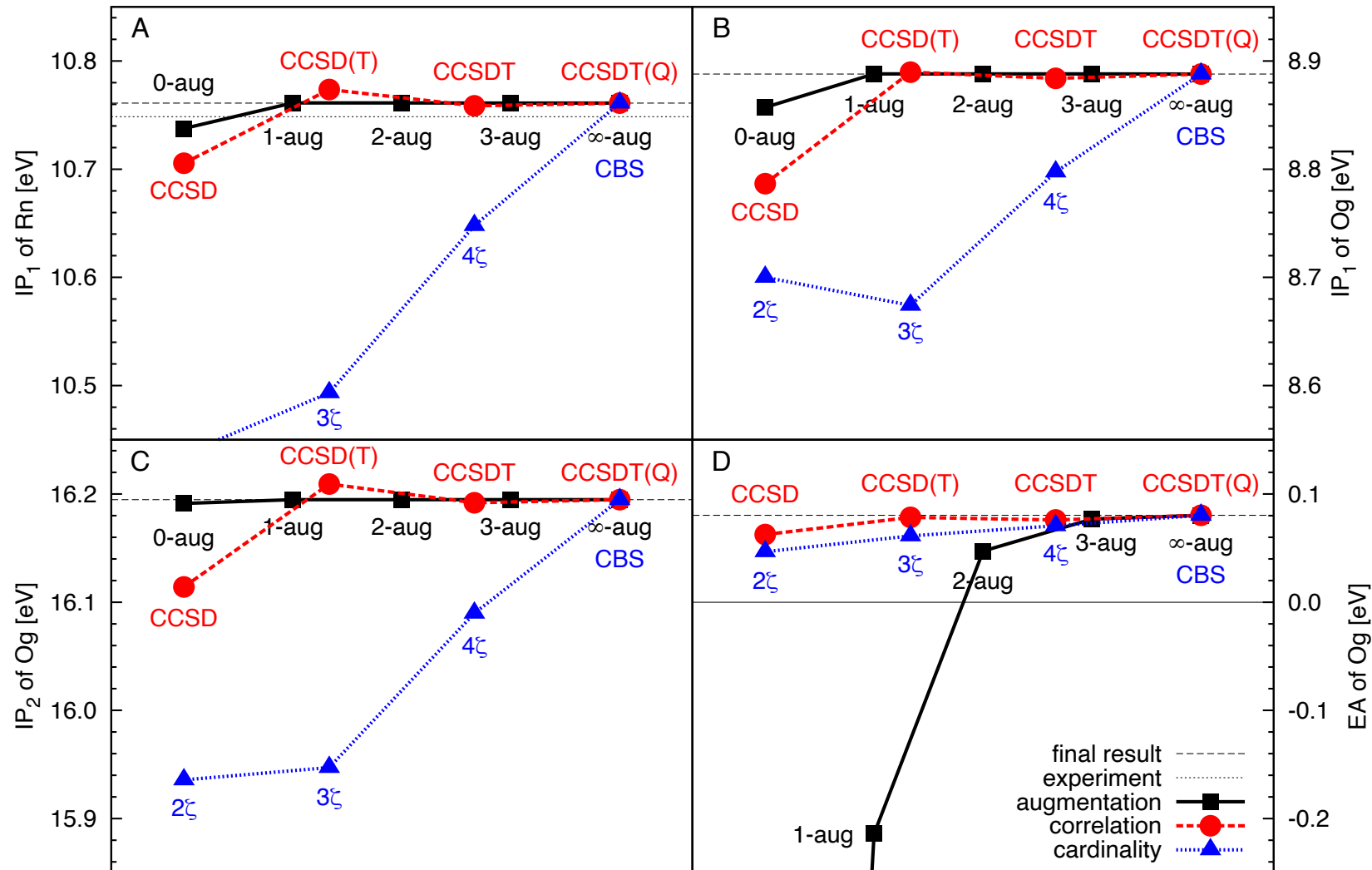
- Calculate the same property in a lighter homologue (and compare to experiment)
- Calculate a different property in the same atom/molecule (and compare to experiment)
- Perform a computational investigation to assign uncertainties based on method incompleteness



Uncertainty estimation



Uncertainty estimation – IP/EA of Rn/Og

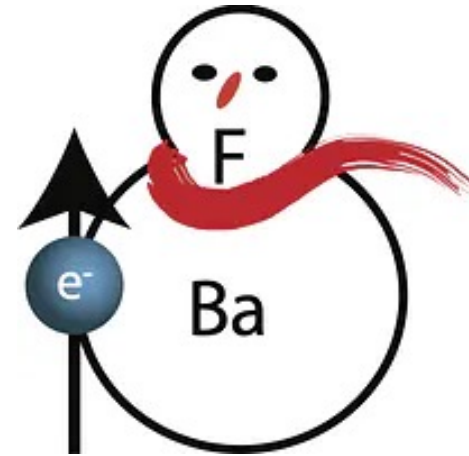


In service to NL-eEDM



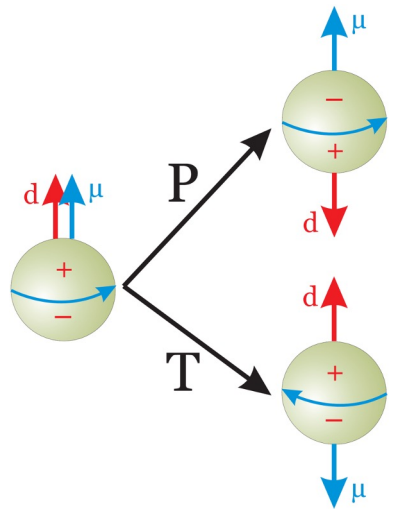
Selected applications

- BaF (and friends)
 - coupling constants
 - laser cooling
 - IPs
- Polyatomic molecules
 - BaCH₃
 - YbCH₃
 - BaOH



Anastasia Borschevsky

P,T-odd properties



$$H_{sr}^{\mathcal{P},\mathcal{T}\text{-odd}} = (W_s k_s + W_d d_e) \Omega$$

Total electronic angular momentum projection

$$\Omega = \mathbf{J}_e \cdot \mathbf{n}$$

Electron electric dipole moment
(eEDM)

\mathcal{P}, \mathcal{T} -odd scalar pseudoscalar (S-PS) neutral-current
electron-nucleon interaction constant

Kriplovich, Nauka (1981), English trans. G&B (1991)

Kozlov, Zh. Eksp. Teor. Fiz. 89, 1933 (1985)

Dimitriev et al, Phys. Lett. A 167, 280 (1992)

W_d/W_s parameters

$$H_{sr}^{\mathcal{P}, \mathcal{T}\text{-odd}} = (W_s k_s + W_d d_e) \Omega$$

$$W_s = \frac{1}{\Omega k_s} \langle \Psi^{(0)} | H^{S-PS} | \Psi^{(0)} \rangle$$

$$W_d = \frac{1}{\Omega d_e} \langle \Psi^{(0)} | H^{eEDM} | \Psi^{(0)} \rangle$$

$$H^{S-PS} = \frac{iG_F k_s}{e\sqrt{2}} \sum_{i,k} \rho_k(\mathbf{r}_i) \gamma^0 \gamma^5$$

$$H^{eEDM} = -d_e \sum_i \gamma^0 \boldsymbol{\Sigma} \cdot \mathbf{E}(\mathbf{r}_i)$$

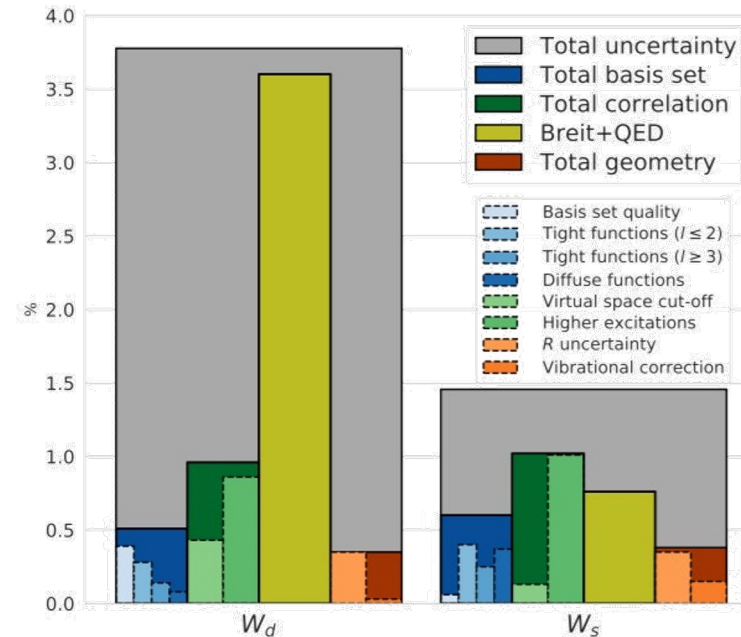
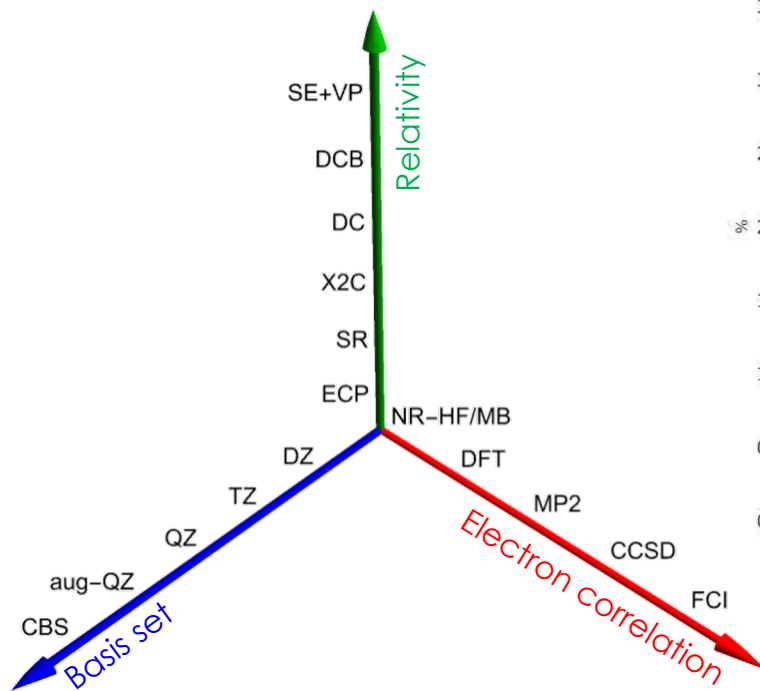
$$W_{s(d)} = \frac{1}{\Omega} \left. \frac{dE(\lambda_{k_s(d_e)})}{d\lambda_{k_s(d_e)}} \right|_{\lambda_{k_s(d_e)}=0}$$

$$E(\lambda_p) = \langle 0 | H_0 + \lambda_p H_p | 0 \rangle$$

- Use relativistic CC to calculate W_d and W_s .
- Systematically improve the calculation up to convergence
- Perform an extensive computational study to estimate uncertainties

BaF: W_d/W_s parameters

- Use relativistic CCSD(T) to calculate W_d and W_s in BaF
- Systematically improve the calculation up to convergence
- Estimate uncertainties



| Source | Estimation scheme | δW_d | δW_s |
|-------------------------|------------------------------|--------------|--------------|
| Basis set | | | |
| Quality | $(vqz - vtz)/2$ | 0.012 | 0.005 |
| Diffuse funct. | s-aug-vqz - vqz | 0.002 | 0.031 |
| Tight funct. $l \leq 2$ | (s, p, d) | 0.009 | 0.033 |
| Tight funct. $l \geq 3$ | aeqz - cvqz | 0.004 | 0.021 |
| Correlation | | | |
| Virtual space cut-off | 6000 a.u - 2000 a.u. | 0.014 | 0.011 |
| Higher excitations | $(CCSD-T - CCSD+T) \cdot 2$ | 0.027 | 0.084 |
| Relativity | | | |
| Breit+QED | $(DC+\Delta G - DC) \cdot 2$ | 0.113 | 0.064 |
| Geometry | | | |
| R uncertainty | $R - (R - \delta R)$ (aetz) | 0.011 | 0.029 |
| Vibrational effects | $R_e - v_0$ (aetz) | 0.001 | 0.013 |
| Total | | | |
| sum | $\sqrt{\sum_i \delta_i^2}$ | 0.119 | 0.122 |
| % | | 3.79 | 1.47 |

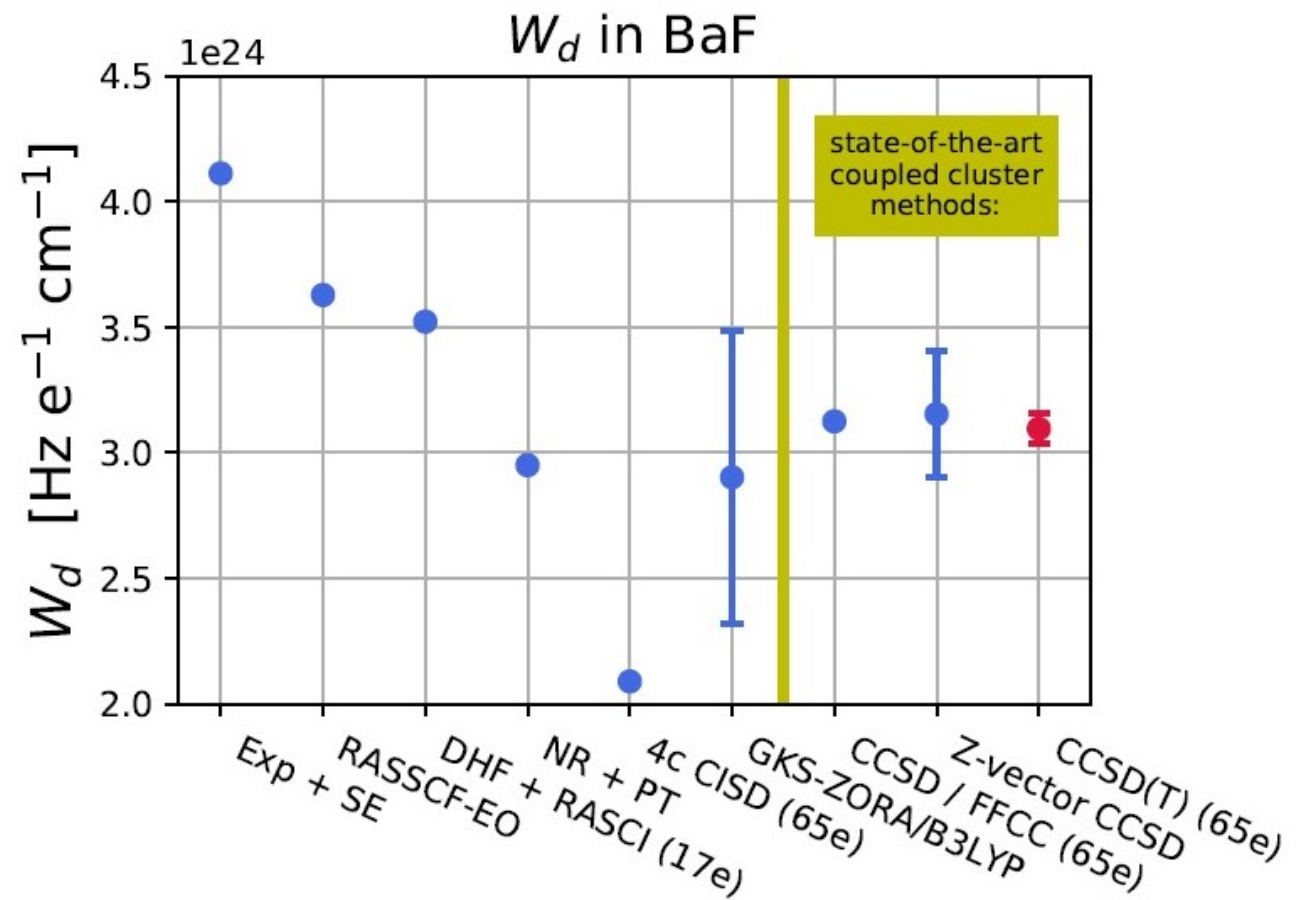
BaF: W_d/W_s parameters

- Final recommended values

| BaF | $W_d^* [\frac{10^{24}\text{Hz}}{\text{e cm}}]$ | $W_s [\text{Hz}]$ |
|------------|------------------------------------------------|-------------------|
| DC CCSD(T) | 3.13 ± 0.24 | 8.29 ± 0.12 |

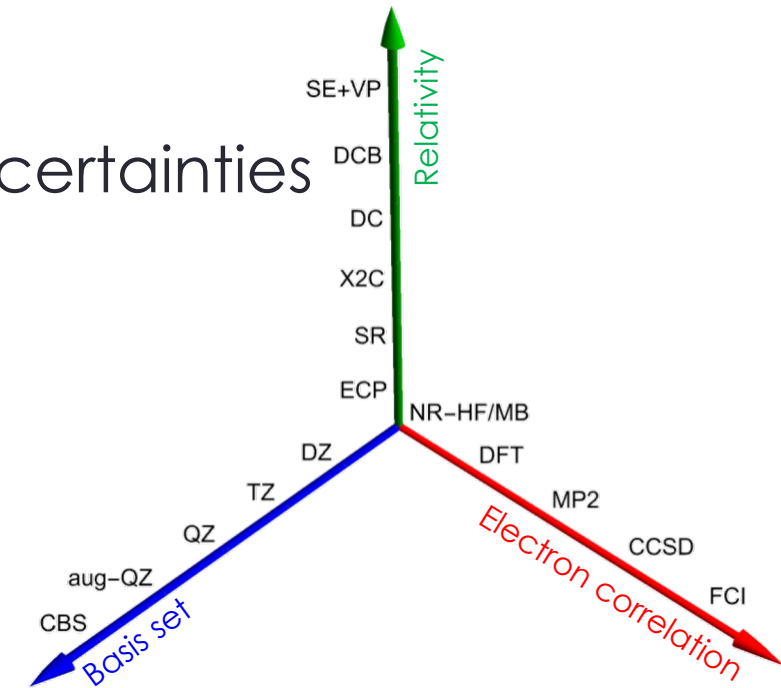


Pi Haase



Uncertainty estimation

- Calculate the same property in a lighter homologue (and compare to experiment)
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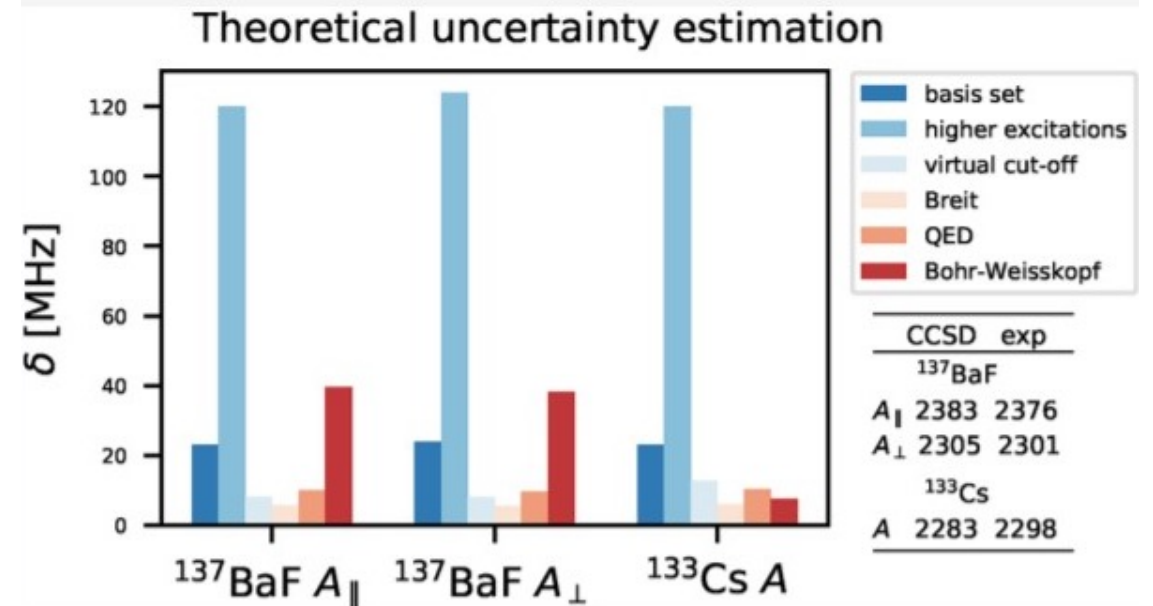
BaF: Magnetic HFS constants

- A proxy property for W_d/W_s parameters

| BaF | $W_d^* [\frac{10^{24}\text{Hz}}{e\text{ cm}}]$ | $W_s [\text{Hz}]$ |
|------------|------------------------------------------------|-------------------|
| DC CCSD(T) | 3.13 ± 0.24 | 8.29 ± 0.12 |

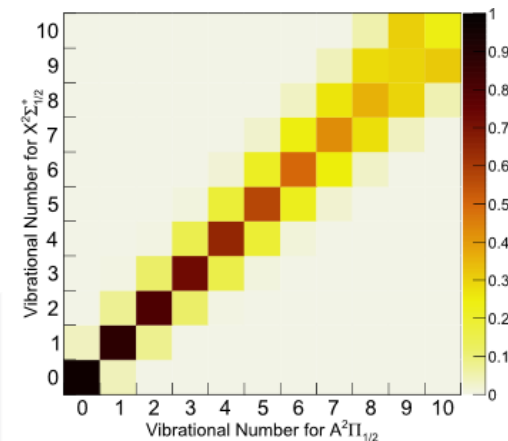
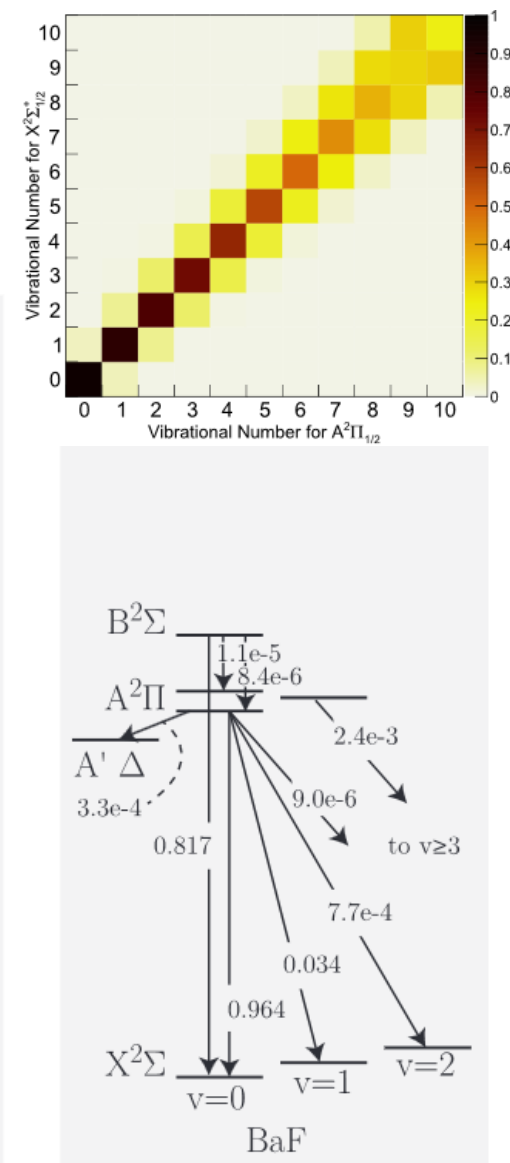
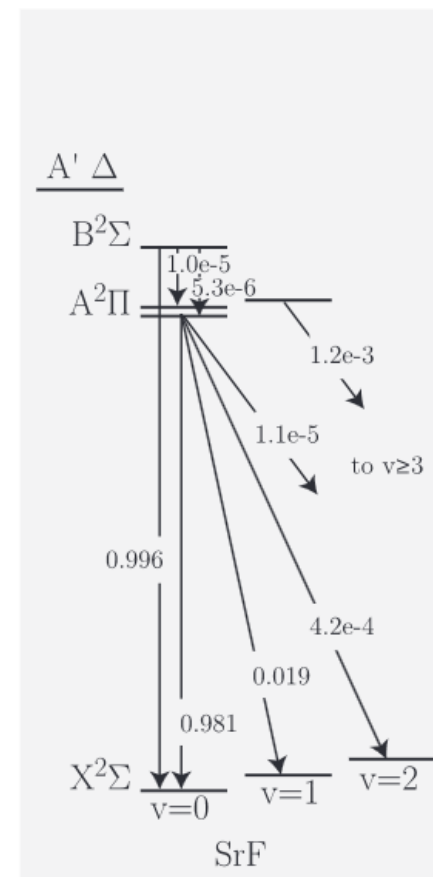
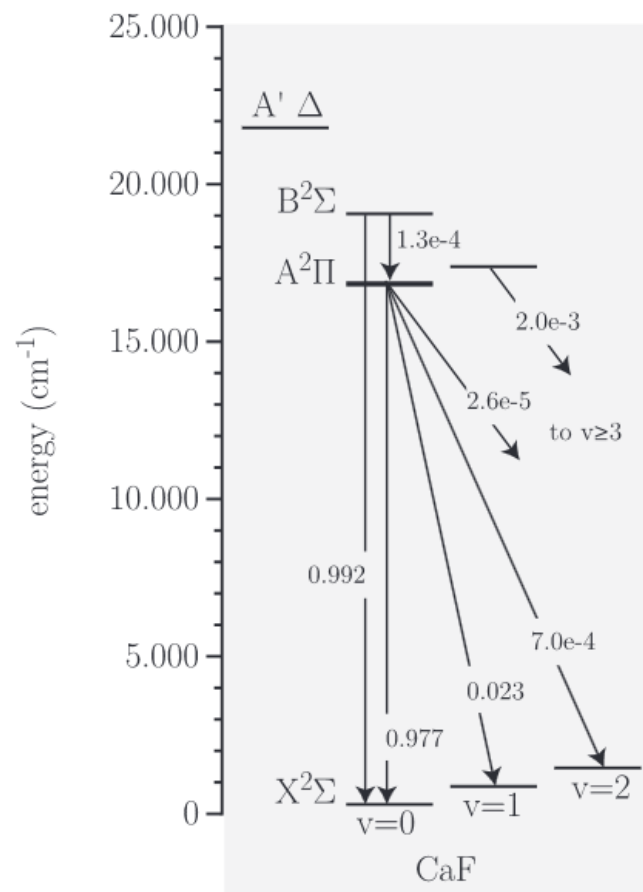
Table 8. A_{\parallel} and A_{\perp} of ^{137}Ba in BaF (MHz)

| method | ^{137}BaF | | | |
|-------------------------------|--------------------|--------|-------------|---------|
| | A_{\parallel} | %(exp) | A_{\perp} | %(exp) |
| GRECP SCF-EO ⁹⁰ | 2264 | -4.71 | 2186 | -5.00 |
| GRECP RASSCF-EO ⁹⁰ | 2272 | -4.38 | 2200 | -4.39 |
| DF RASCI ⁹¹ | 2240 | -5.72 | 2144 | -6.82 |
| DF MBPT ⁹¹ | 2314 | -2.61 | 2254 | -2.04 |
| DC CCSD (this work) | 2383(129) | 0.29 | 2305(132) | 0.17 |
| exp ⁷⁷ | | | 2376(12) | 2301(9) |



MF cooling schemes

- Yongliang Hao



Ionization potentials of MF

- Excellent agreement for CaF and BaF
- For SrF, new IP measurements would be useful



Aleksandra Kuyberis

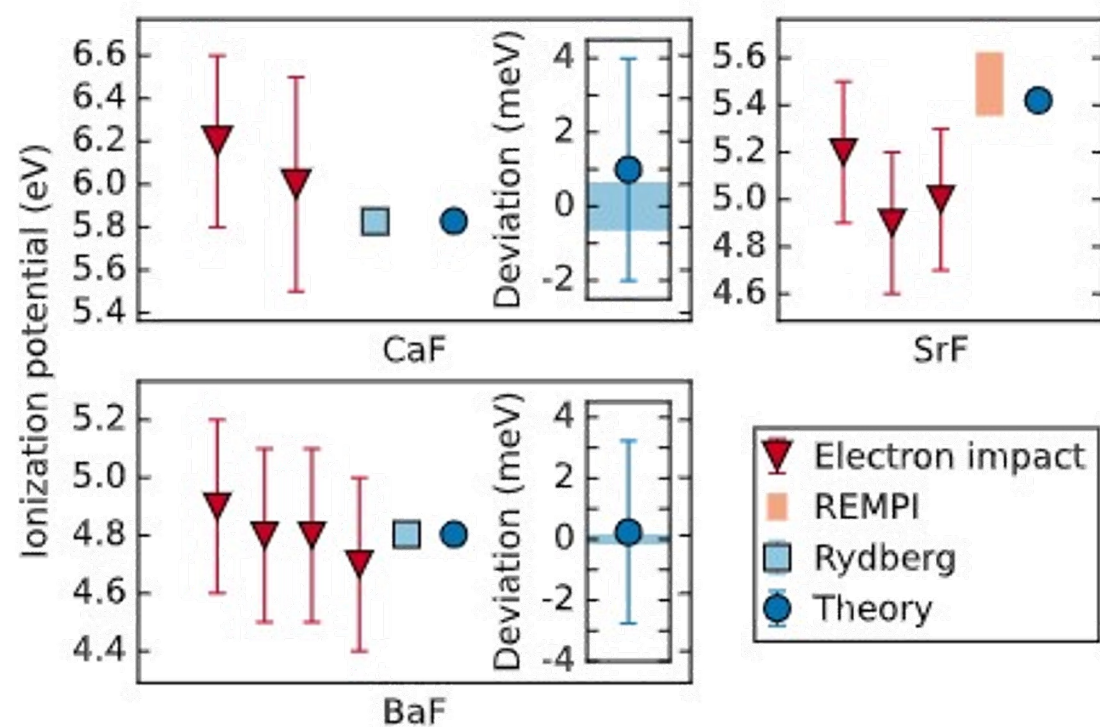


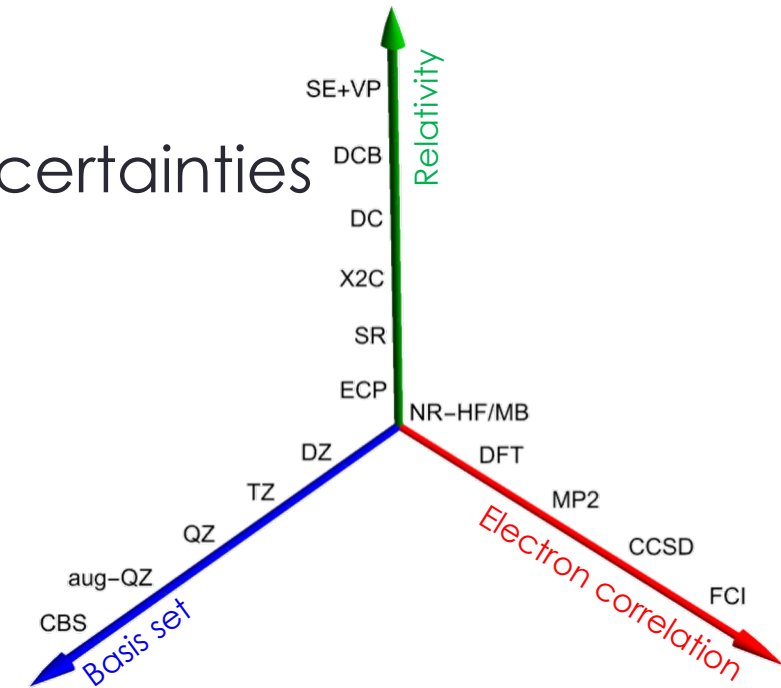
TABLE VI: Recommended theoretical IP of MF (M = Ca, Sr, Ba) [eV] with uncertainties.

| IP | CaF | SrF | BaF |
|-------------------------|-----------|-----------|------------|
| Adiabatic | 5.821(3) | 5.415(1) | 4.800(3) |
| Adiabatic+ZPE | 5.828(3) | 5.420(1) | 4.804(3) |
| Experiment ^a | 5.8270(6) | 5.36–5.62 | 4.80377(1) |

^a-CaF[69], SrF[70] and BaF[39]

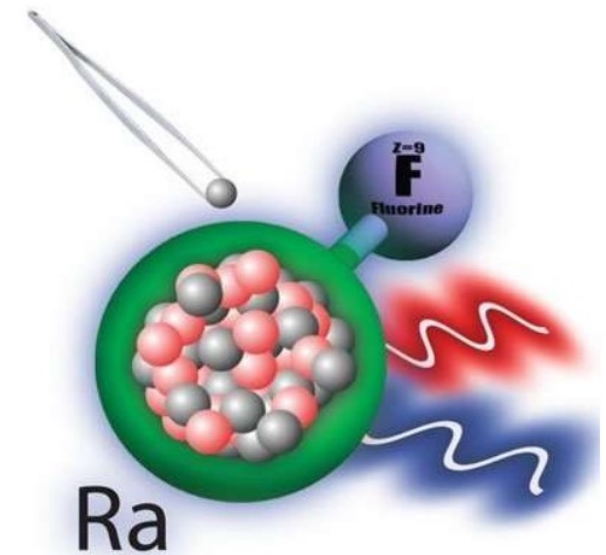
Uncertainty estimation

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Ionization potential of RaF

- a promising candidate for precision measurements
- Many (high-impact) experimental studies
- Spectra, isotope shifts
- New measurements: IP
- Theory before experiment



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Spectroscopy of short-lived radioactive molecules

[R. F. Garcia Ruiz](#) , [R. Berger](#) , [J. Billowes](#), [C. L. Binnersley](#), [M. L. Bissell](#), [A. A. Breier](#), [A. J. Brinson](#), [K. Chrysalidis](#), [T. E. Cocolios](#), [B. S. Cooper](#), [K. T. Flanagan](#), [T. F. Giesen](#), [R. P. de Groote](#), [S. Franchoo](#), [F. P. Gustafsson](#), [T. A. Isaev](#), [Á. Koszorús](#), [G. Neyens](#), [H. A. Perrett](#), [C. M. Ricketts](#), [S. Rothe](#), [L. Schweikhard](#), [A. R. Vernon](#), [K. D. A. Wendt](#), ... [X. F. Yang](#)  Show authors

87. "Precision spectroscopy and laser cooling scheme of a radium-containing molecule".

S.M. Udrescu, S. Wilkins, A. Breier, M. Athanasakis-Kaklamanakis, R.F. Garcia Ruiz et al

[Nature Physics \(2023\)](#).

Featured in Physics

Editors' Suggestion

Open Access

Isotope Shifts of Radium Monofluoride Molecules

S. M. Udrescu *et al.*

Phys. Rev. Lett. **127**, 033001 – Published 14 July 2021

Ionization potential of RaF

- Measurement
 - ionization threshold under multi-step laser excitation
 - both two-step and three-step ionization schemes.
- Theory
 - relativistic CC with higher order corrections

| Method | IP (eV) |
|----------------|----------|
| CBS-DC-CCSD | 4.932 |
| CBS-DC-CCSD(T) | 4.983 |
| +aug+ae.vs.cv | 4.986 |
| + ΔT | 4.987 |
| +Breit | 4.985 |
| +QED | 4.979 |
| Theoretical | 4.979(9) |
| Experimental | 4.972(4) |

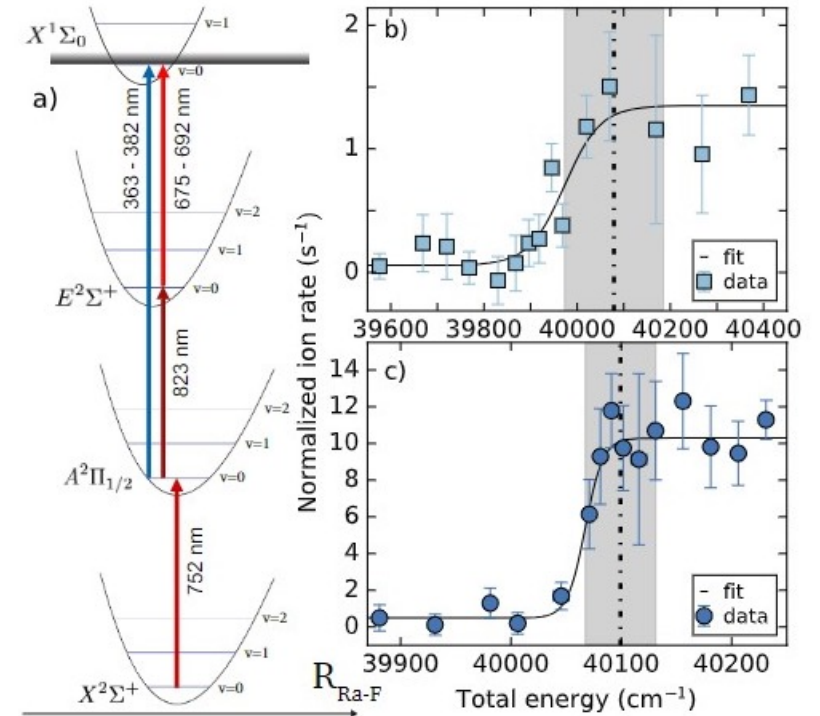
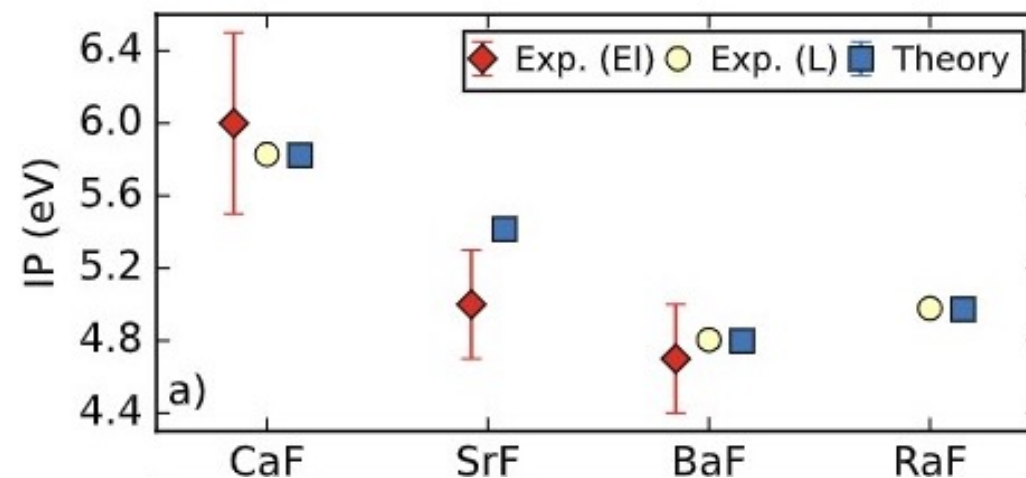
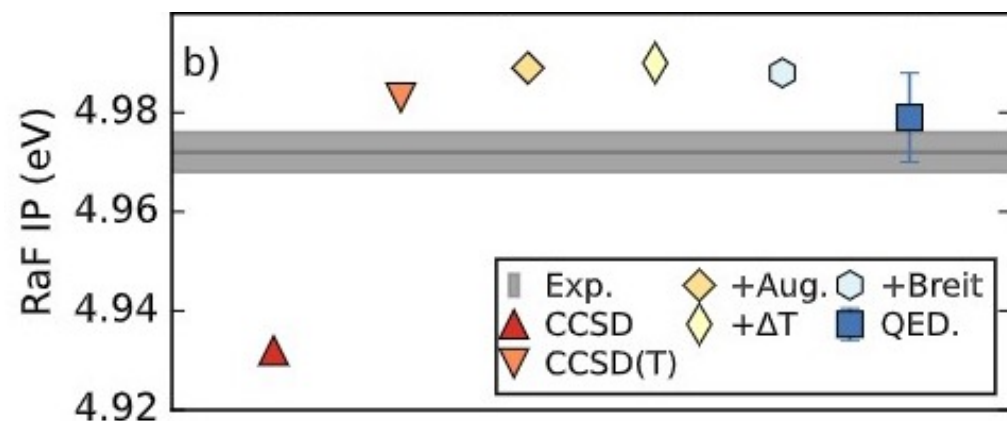
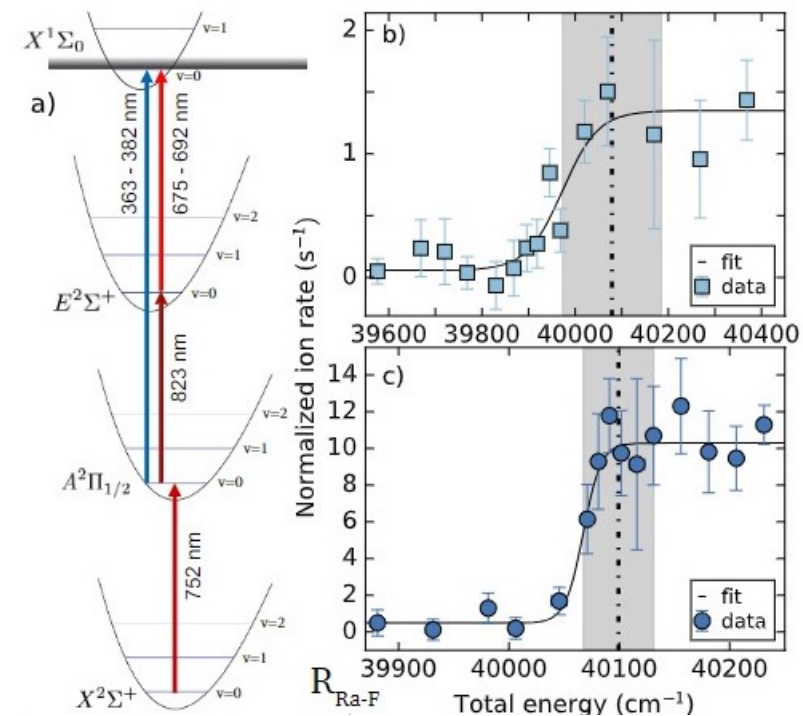


FIG. 1. a) The two-step and three-step ionization schemes used in the experiments. Power-normalized, background-subtracted ion count rate as a function of total photon energy in $^{226}\text{Ra}^{19}\text{F}$ for the b) two-step scheme and c) three-step scheme. The determined IPs are shown as vertical dashed-and-dotted lines with their 1σ uncertainties as gray bands.

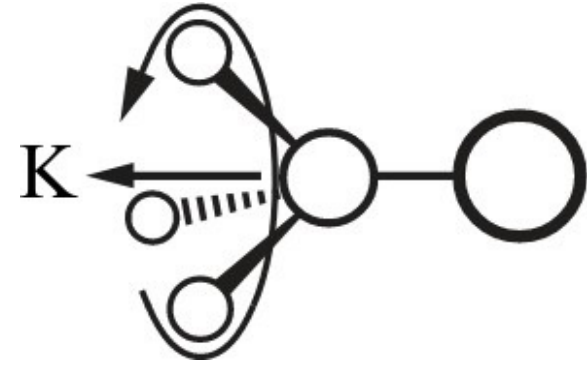
Ionization potential of RaF

- Measurement
 - ionization threshold under multi-step laser excitation
 - both two-step and three-step ionization schemes.
- Theory
 - relativistic CC with higher order corrections



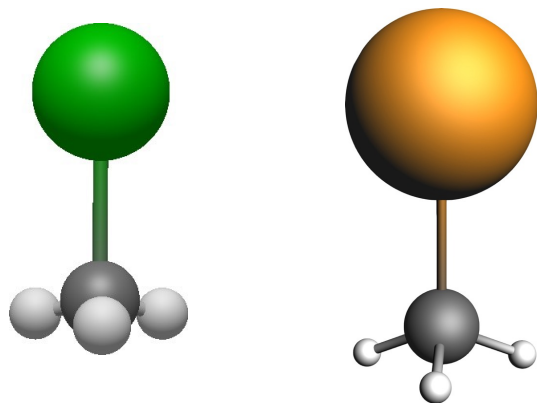
Symmetric top molecules

- BaCH_3 and YbCH_3
- long-lived close-lying opposite parity eigenstates (K-doublets)
- expected to be laser-coolable
- similar W_d/W_s parameters to other Ba/Yb containing molecules



Symmetric top molecules

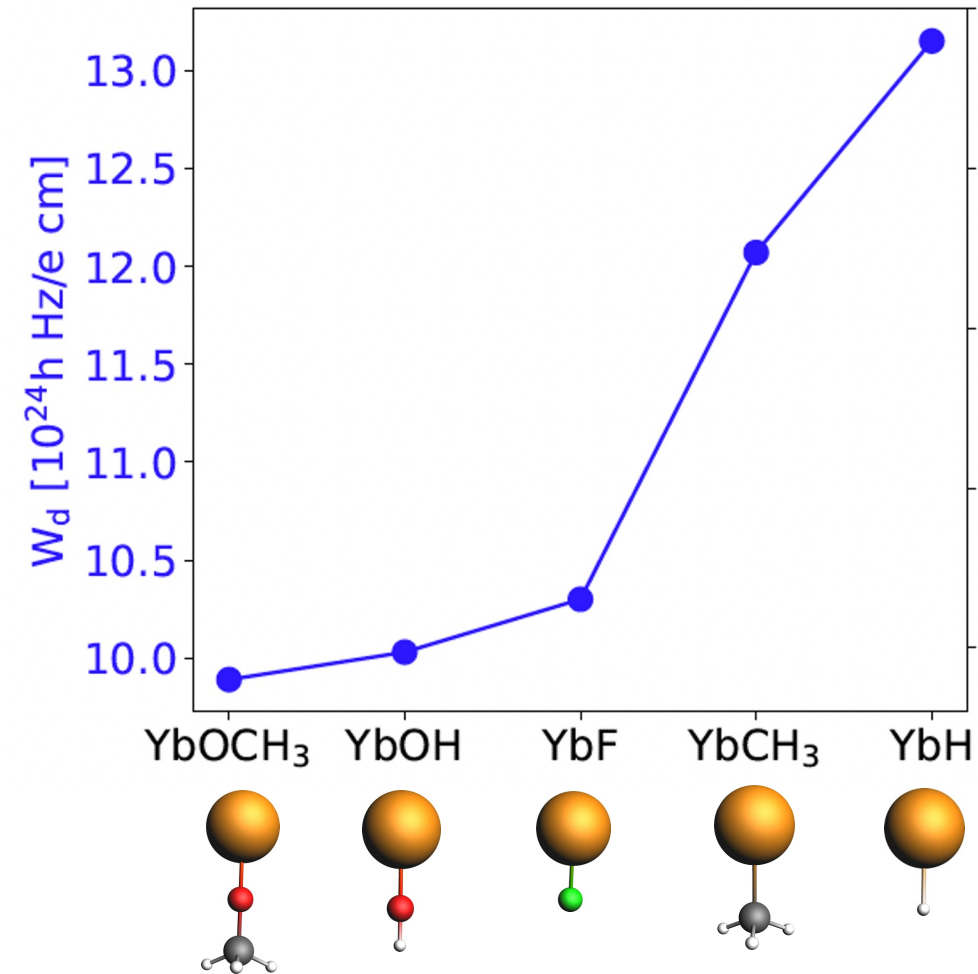
- BaCH_3 and YbCH_3



Yuly Chamorro

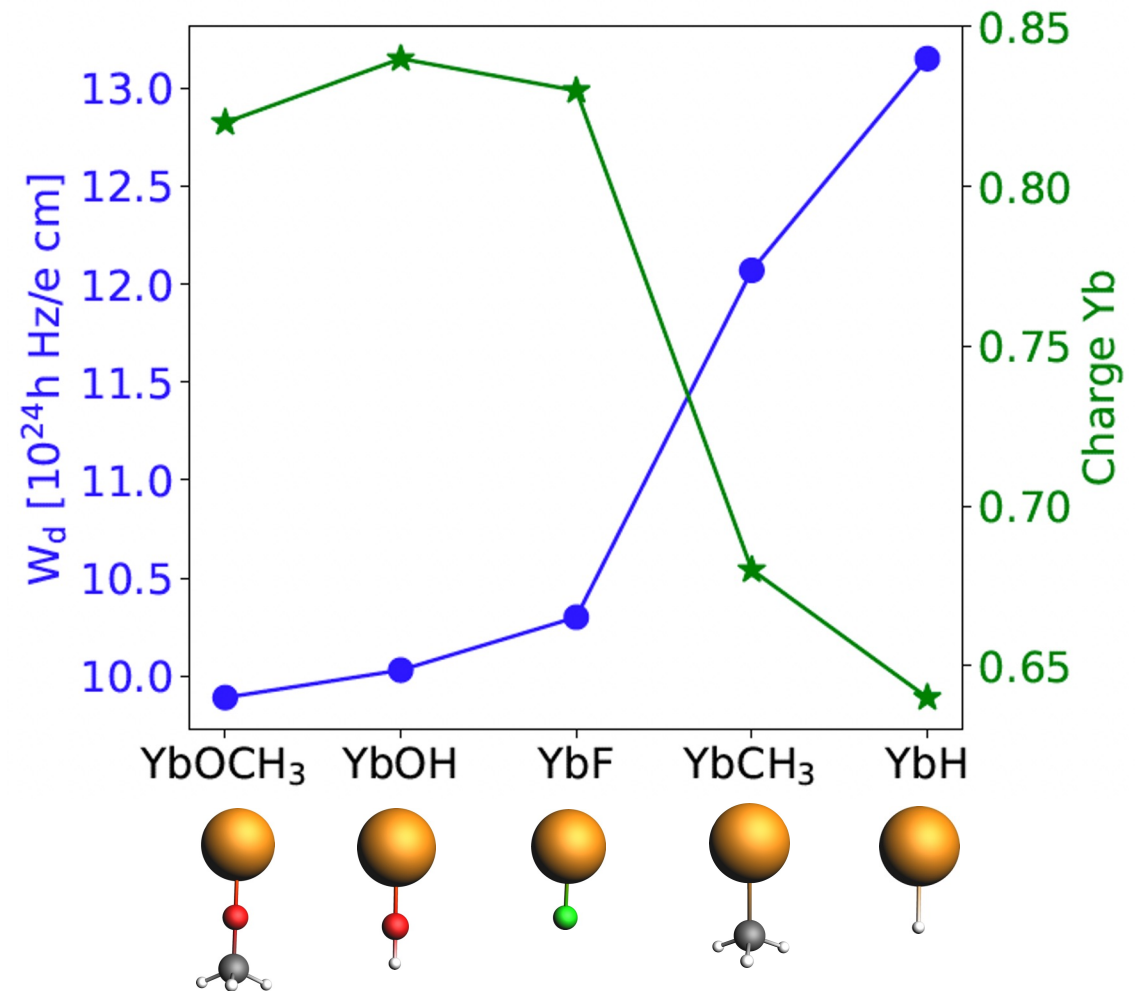
| Molecule | Method | W_d ($10^{24} \frac{\text{hHz}}{\text{ecm}}$) | W_s (hkHz) |
|------------------|--------------|---------------------------------------------------|----------------------|
| BaCH_3 | DC-CCSD(T) | 3.22 ± 0.12 | 8.42 ± 0.29 |
| BaOCH_3 | X2C-CCSD(T) | 3.05 [74] | |
| BaOH | DC-CCSD(T) | 3.10 ± 0.15 [38] | |
| | ZORA-cGHF | 3.32 ± 0.33 [39] | 8.79 ± 0.88 [39] |
| | ZORA-cGKS | 2.98 ± 0.30 [39] | 7.91 ± 0.79 [39] |
| BaF | DC-CCSD(T) | 3.13 ± 0.12 [37] | 8.29 ± 0.12 [37] |
| YbCH_3 | DC-FSCC(0,1) | 13.80 ± 0.35 | 50.16 ± 1.27 |
| YbOCH_3 | X2C-CCSD(T) | 11.60 [74] | |
| YbOH | DC-FSCC(0,1) | 11.30 ± 0.5 [38] | |
| | ZORA-cHFS | 11.40 ± 1.14 [39] | 41.2 ± 4.12 [39] |
| | ZORA-cGKS | 8.54 ± 0.85 [39] | 30.8 ± 3.08 [39] |
| | DC-CCSD | 11.47 ± 0.68 [75] | |
| YbF | DC-FSCC | 11.39 [37] | |

Bonding effects



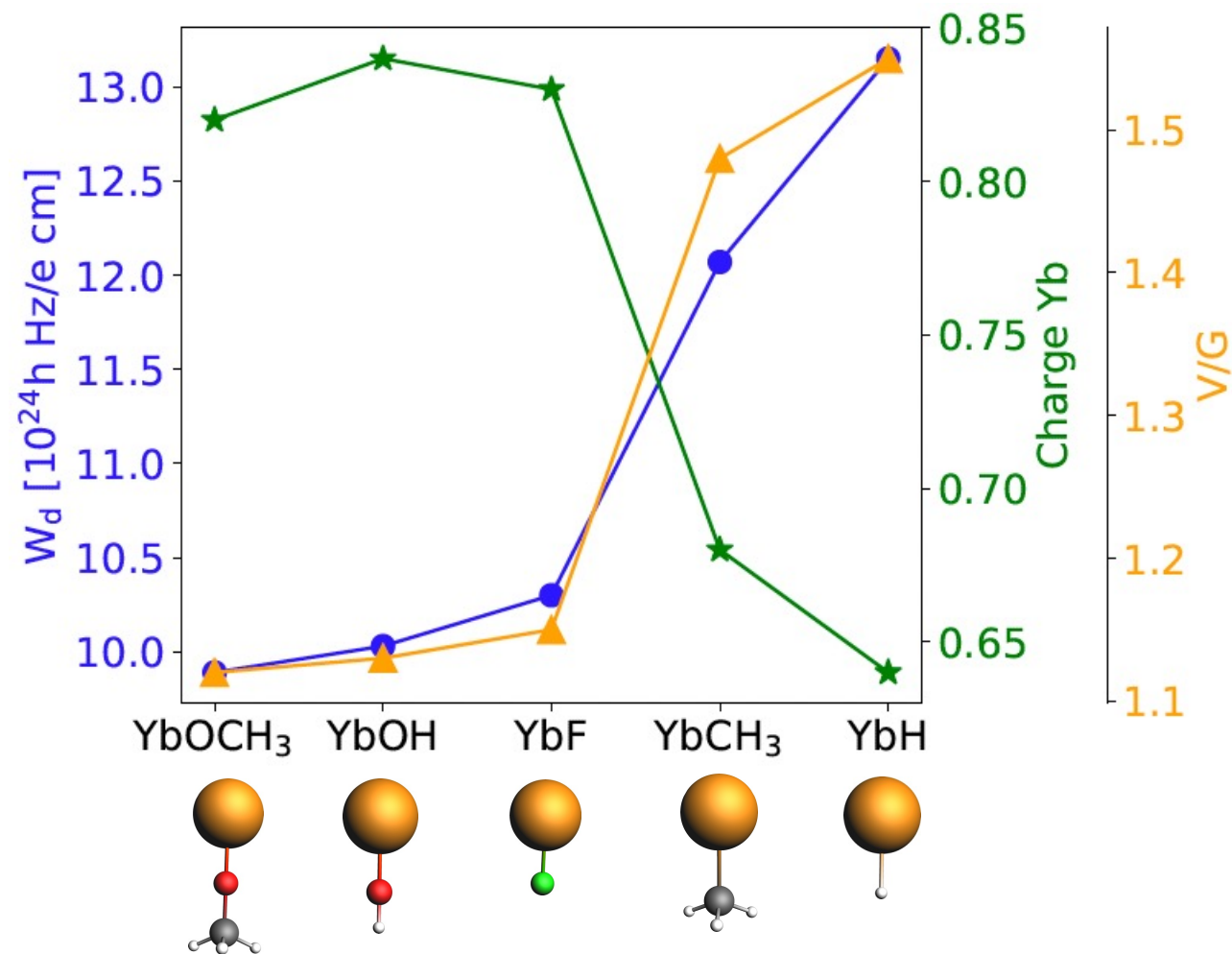
Bonding effects

- smaller charge on Yb
 ↓
 • less polar = more covalent



Bonding effects

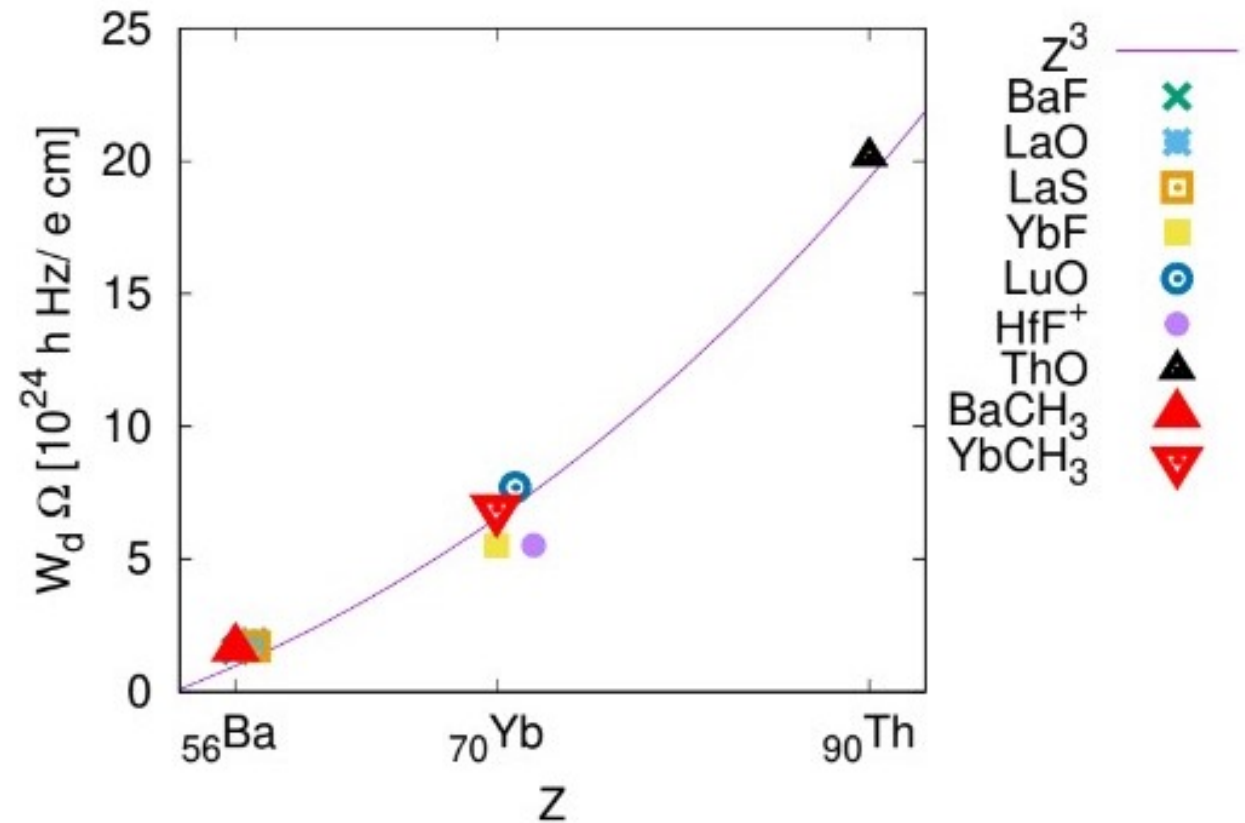
- smaller charge on Yb
 - ↓
- less polar = more covalent
- V/G at BCP
 - measure of covalent character
 - <1 ionic, 1-2 polar, >2 covalent
- downside – laser coolability



Z scaling

- W_d scales with the atomic number Z^3
 - as predicted

Bouchiat, M. A., Bouchiat, C., J. Phys., France 35, 899-927 (1974)



Bouchiat et Bouchiat, J. Phys. France 35, 899 (1974)

BaOH – work in progress

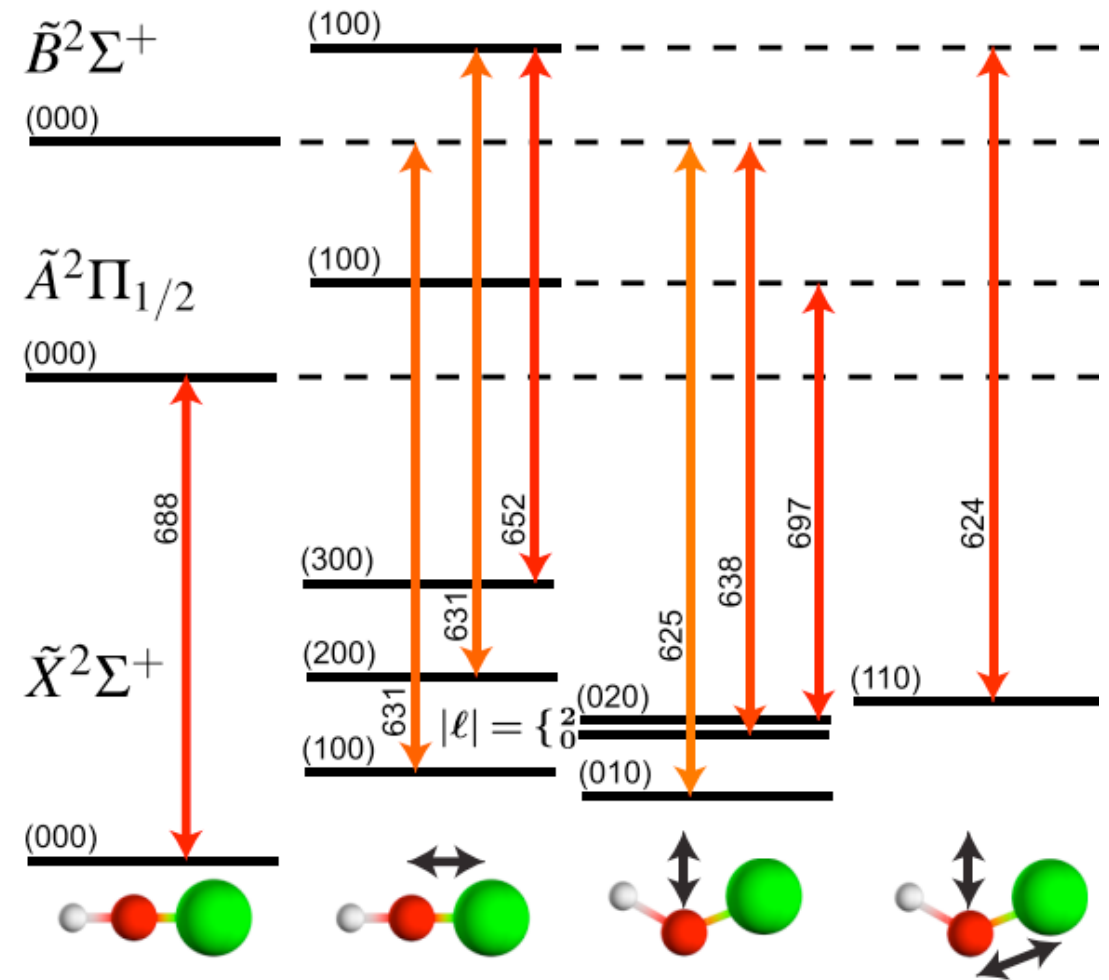
- bending mode lifetime
- cooling scheme

Radoslava Hlavacova



- polarizability and scattering tensors

Eifion Prinsen



(this is fake news – actually SrOH)



Fin