

# Accelerating the search for mass bumps using the Data-Directed Paradigm

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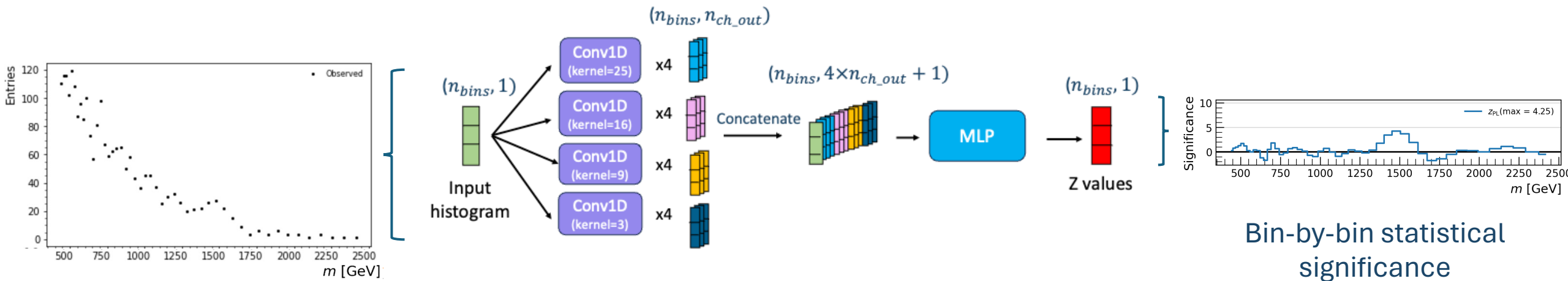
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# What ?

- We want to maximize our chances to **find new physics** in collider data
- Train a neural network to **identify mass bumps in real data** without the need of simulation or analytical fit to estimate the background



Invariant mass histogram

Bin-by-bin statistical significance

# Why ?

- Exploit the **discovery potential of the data**
  - Impossible to check all final states with a traditional analysis
  - Many possible resonances in unexplored final states → bumps

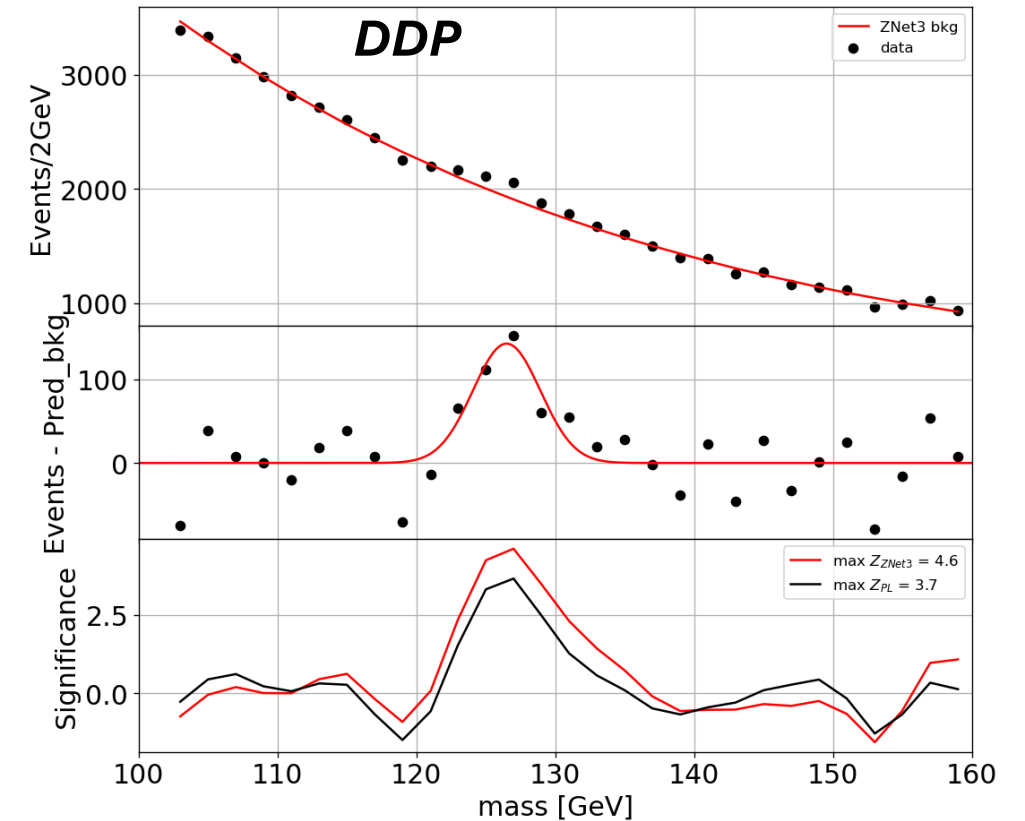
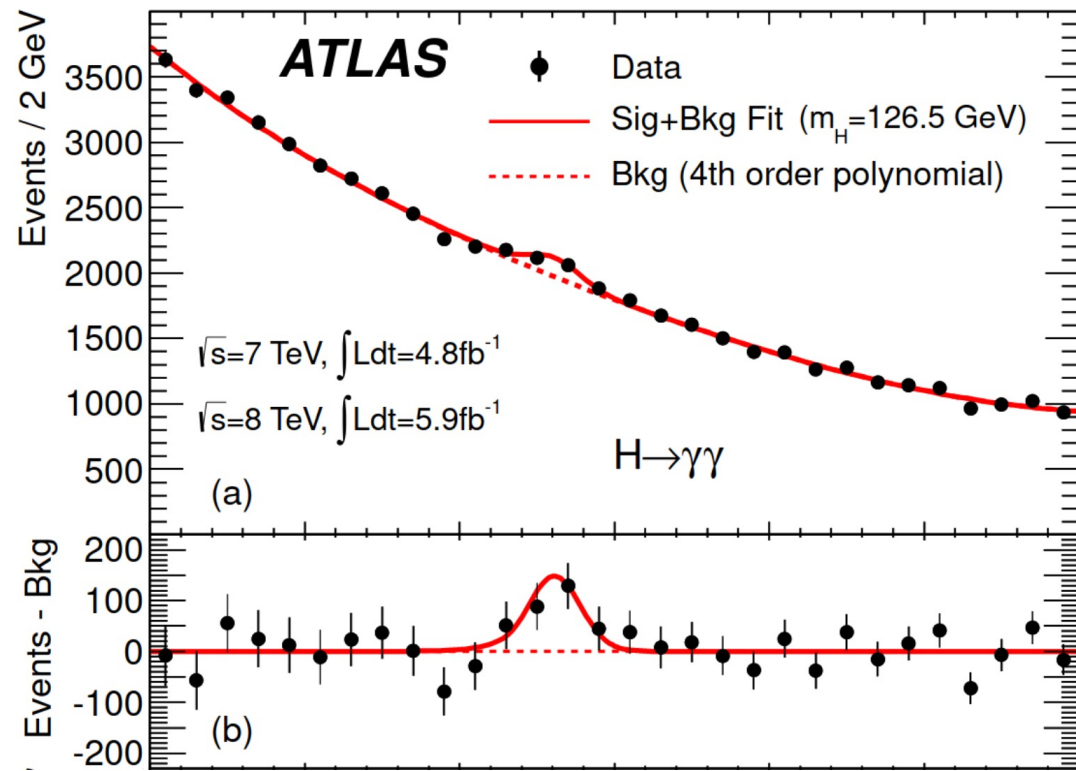
Existing searches for two-body resonances<sup>[1]</sup>

|                                                            | $e$               | $\mu$   | $\tau$  | $q/g$         | $b$         | $t$         | $\gamma$    | $Z/W$       | $H$         | BSM $\rightarrow$ SM <sub>1</sub> $\times$ SM <sub>1</sub> |                   |             |             | BSM $\rightarrow$ SM <sub>1</sub> $\times$ SM <sub>2</sub> |             |             | BSM $\rightarrow$ complex |             |             |     |
|------------------------------------------------------------|-------------------|---------|---------|---------------|-------------|-------------|-------------|-------------|-------------|------------------------------------------------------------|-------------------|-------------|-------------|------------------------------------------------------------|-------------|-------------|---------------------------|-------------|-------------|-----|
|                                                            |                   |         |         |               |             |             |             |             |             | $q/g$                                                      | $\gamma/\pi^0$ 's | $b$         | ...         | $tZ/H$                                                     | $bH$        | ...         | $\tau qq'$                | $eqq'$      | $\mu qq'$   | ... |
| $e$                                                        | [37,38]           | [39,40] | [39]    | $\emptyset$   | $\emptyset$ | $\emptyset$ | [41]        | [42]        | $\emptyset$ | $\emptyset$                                                | $\emptyset$       | $\emptyset$ | $\emptyset$ | $\emptyset$                                                | $\emptyset$ | $\emptyset$ | $\emptyset$               | [43,44]     | $\emptyset$ |     |
| $\mu$                                                      |                   | [37,38] | [39]    | $\emptyset$   | $\emptyset$ | $\emptyset$ | [41]        | [42]        | $\emptyset$ | $\emptyset$                                                | $\emptyset$       | $\emptyset$ | $\emptyset$ | $\emptyset$                                                | $\emptyset$ | $\emptyset$ | $\emptyset$               | $\emptyset$ | [43,44]     |     |
| $\tau$                                                     |                   |         | [45,46] | $\emptyset$   | [47]        | $\emptyset$ | $\emptyset$ | $\emptyset$ | $\emptyset$ | $\emptyset$                                                | $\emptyset$       | $\emptyset$ | $\emptyset$ | $\emptyset$                                                | $\emptyset$ | $\emptyset$ | $\emptyset$               | [48,49]     | $\emptyset$ |     |
| $q/g$                                                      |                   |         |         | [29,30,50,51] | [52]        | $\emptyset$ | [53,54]     | [55]        | $\emptyset$ | $\emptyset$                                                | $\emptyset$       | $\emptyset$ | $\emptyset$ | $\emptyset$                                                | $\emptyset$ | $\emptyset$ | $\emptyset$               | $\emptyset$ | $\emptyset$ |     |
| $b$                                                        |                   |         |         |               | [29,52,56]  | [57]        | [54]        | [58]        | [59]        | $\emptyset$                                                | $\emptyset$       | $\emptyset$ | $\emptyset$ | [60]                                                       | $\emptyset$ | $\emptyset$ | $\emptyset$               | $\emptyset$ | $\emptyset$ |     |
| $t$                                                        |                   |         |         |               |             | [61]        | $\emptyset$ | [62]        | [63]        | $\emptyset$                                                | $\emptyset$       | $\emptyset$ | $\emptyset$ | [64]                                                       | [60]        | $\emptyset$ | $\emptyset$               | $\emptyset$ | $\emptyset$ |     |
| $\gamma$                                                   |                   |         |         |               |             |             | [65,66]     | [67-69]     | [68,70]     | $\emptyset$                                                | $\emptyset$       | $\emptyset$ | $\emptyset$ | $\emptyset$                                                | $\emptyset$ | $\emptyset$ | $\emptyset$               | $\emptyset$ | $\emptyset$ |     |
| $Z/W$                                                      |                   |         |         |               |             |             |             | [71]        | [71]        | $\emptyset$                                                | $\emptyset$       | $\emptyset$ | $\emptyset$ | $\emptyset$                                                | $\emptyset$ | $\emptyset$ | $\emptyset$               | $\emptyset$ | $\emptyset$ |     |
| $H$                                                        |                   |         |         |               |             |             |             |             | [72,73]     | [74]                                                       | $\emptyset$       | $\emptyset$ | $\emptyset$ | $\emptyset$                                                | $\emptyset$ | $\emptyset$ | $\emptyset$               | $\emptyset$ | $\emptyset$ |     |
| BSM $\rightarrow$ SM <sub>1</sub> $\times$ SM <sub>1</sub> | $q/g$             |         |         |               |             |             |             |             |             | $\emptyset$                                                | $\emptyset$       | $\emptyset$ | $\emptyset$ | $\emptyset$                                                | $\emptyset$ | $\emptyset$ | $\emptyset$               | $\emptyset$ | $\emptyset$ |     |
|                                                            | $\gamma/\pi^0$ 's |         |         |               |             |             |             |             |             |                                                            | [75]              | $\emptyset$ | $\emptyset$ | $\emptyset$                                                | $\emptyset$ | $\emptyset$ | $\emptyset$               | $\emptyset$ | $\emptyset$ |     |
|                                                            | $b$               |         |         |               |             |             |             |             |             |                                                            |                   | [76,77]     | $\emptyset$ | $\emptyset$                                                | $\emptyset$ | $\emptyset$ | $\emptyset$               | $\emptyset$ | $\emptyset$ |     |
|                                                            | $\vdots$          |         |         |               |             |             |             |             |             |                                                            |                   |             |             | $\emptyset$                                                | $\emptyset$ | $\emptyset$ | $\emptyset$               | $\emptyset$ | $\emptyset$ |     |
| $\vdots$                                                   |                   |         |         |               |             |             |             |             |             |                                                            |                   |             |             |                                                            |             |             |                           |             |             |     |

[1] J. H. Kim et al., version 1, 10.48550/ARXIV.1907.06659 (2019), <https://arxiv.org/abs/1907.06659>

# Promising result

- **Finding the Higgs bump**
  - Predicted significance matches the ATLAS significance within error [2]



# Please visit our poster!

More promising results!

More on the training data!



## Accelerating the search for mass bumps using the Data-Directed Paradigm

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### Motivation

What? Train a neural network to **identify mass bumps in real data** without the need of simulation or analytical fit to estimate the background

Why? Exploit the **discovery potential of the data**.

- Impossible to check all final states with the traditional analysis
- Many possible resonances in unexplored final states → bumps

### Data Directed Paradigm for bump searches

The **Data-Directed Paradigm (DDP)** is a search strategy to efficiently identify regions of interest in the data. It requires two ingredients:

- Property of the Standard Model (SM) on which deviations can be searched for
- Tool to scan the observable-space in search for deviations

Smoothly falling invariant mass

NN mapping invariant mass to statistical significance for bumps

### Neural network architecture

Use of 1D convolution layers followed by a dense layer

Intuitive and agnostic to the number of bins in the histogram

Learns from the neighborhoods of different size from the raw input

Mixes all representations for each bin independently

### Histogram processing and calibration

- Using the **Dark Machines dataset** [2]
  - Designed to test anomaly detection techniques
  - Dataset equivalent to 10 fb<sup>-1</sup> with highest cross section processes at the LHC
- Mass histograms with **all possible combinations** of the following objects:
  - Electron
  - Photon
  - Reconstructed leptonic Z
  - Boosted top
  - Muon
  - Jet
  - Boosted hadronic W/Z
  - High mass jet (m > 200 GeV)
- Additional kinematics cuts on missing energy (MET) and transverse momentum (p<sub>T</sub>) of leading objects
- Split the data according to jet multiplicity to improve S/B ratio and reduce the look-elsewhere effect
- Total of 30 000 mass histograms
- Rebinning that reflects the detector resolution**, using  $p_T \approx m/2$ 
  - Resolution is higher for m(4j) than m(3j), and for smaller masses
  - Binning reflects this with larger bin width when resolution is smaller

### Performance and finding Beyond the Standard Model (BSM) signals

- Accurately predicts maximum significance with no bias and a variance of ±0.64
- Excellent discriminating performance with an AUC of 0.900

### Training data

- Synthetic data generated by injecting a Gaussian signal on two types of backgrounds:
  - Analytical functions
  - Fits to simulation data (e.g. Dark Machines sample)

$$Ae^{-m^2} + \alpha e^{-\frac{m^2}{\sigma^2}} + \beta \frac{1}{m^2} + \gamma \frac{1}{m^3} + \delta \frac{1}{m^4} + \epsilon \frac{1}{m^5} + \eta \frac{1}{m^6} + \theta \frac{1}{m^7} + \rho \frac{1}{m^8} + \sigma \frac{1}{m^9} + \tau \frac{1}{m^{10}} + \nu \frac{1}{m^{11}} + \xi \frac{1}{m^{12}} + \zeta \frac{1}{m^{13}} + \eta \frac{1}{m^{14}} + \theta \frac{1}{m^{15}} + \rho \frac{1}{m^{16}} + \sigma \frac{1}{m^{17}} + \tau \frac{1}{m^{18}} + \nu \frac{1}{m^{19}} + \xi \frac{1}{m^{20}} + \zeta \frac{1}{m^{21}} + \eta \frac{1}{m^{22}} + \theta \frac{1}{m^{23}} + \rho \frac{1}{m^{24}} + \sigma \frac{1}{m^{25}} + \tau \frac{1}{m^{26}} + \nu \frac{1}{m^{27}} + \xi \frac{1}{m^{28}} + \zeta \frac{1}{m^{29}} + \eta \frac{1}{m^{30}} + \theta \frac{1}{m^{31}} + \rho \frac{1}{m^{32}} + \sigma \frac{1}{m^{33}} + \tau \frac{1}{m^{34}} + \nu \frac{1}{m^{35}} + \xi \frac{1}{m^{36}} + \zeta \frac{1}{m^{37}} + \eta \frac{1}{m^{38}} + \theta \frac{1}{m^{39}} + \rho \frac{1}{m^{40}} + \sigma 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\frac{1}{m^{395}} + \xi \frac{1}{m^{396}} + \zeta \frac{1}{m^{397}} + \eta \frac{1}{m^{398}} + \theta \frac{1}{m^{399}} + \rho \frac{1}{m^{400}} + \sigma \frac{1}{m^{401}} + \tau \frac{1}{m^{402}} + \nu \frac{1}{m^{403}} + \xi \frac{1}{m^{404}} + \zeta \frac{1}{m^{405}} + \eta \frac{1}{m^{406}} + \theta \frac{1}{m^{407}} + \rho \frac{1}{m^{408}} + \sigma \frac{1}{m^{409}} + \tau \frac{1}{m^{410}} + \nu \frac{1}{m^{411}} + \xi \frac{1}{m^{412}} + \zeta \frac{1}{m^{413}} + \eta \frac{1}{m^{414}} + \theta \frac{1}{m^{415}} + \rho \frac{1}{m^{416}} + \sigma \frac{1}{m^{417}} + \tau \frac{1}{m^{418}} + \nu \frac{1}{m^{419}} + \xi \frac{1}{m^{420}} + \zeta \frac{1}{m^{421}} + \eta \frac{1}{m^{422}} + \theta \frac{1}{m^{423}} + \rho \frac{1}{m^{424}} + \sigma \frac{1}{m^{425}} + \tau \frac{1}{m^{426}} + \nu \frac{1}{m^{427}} + \xi \frac{1}{m^{428}} + \zeta \frac{1}{m^{429}} + \eta \frac{1}{m^{430}} + \theta \frac{1}{m^{431}} + \rho \frac{1}{m^{432}} + \sigma \frac{1}{m^{433}} + \tau \frac{1}{m^{434}} + \nu \frac{1}{m^{435}} + \xi \frac{1}{m^{436}} + \zeta \frac{1}{m^{437}} + \eta \frac{1}{m^{438}} + \theta \frac{1}{m^{439}} + \rho \frac{1}{m^{440}} + \sigma \frac{1}{m^{441}} + \tau \frac{1}{m^{442}} + \nu \frac{1}{m^{443}} + \xi \frac{1}{m^{444}} + \zeta \frac{1}{m^{445}} + \eta \frac{1}{m^{446}} + \theta \frac{1}{m^{447}} + \rho \frac{1}{m^{448}} + \sigma \frac{1}{m^{449}} + \tau \frac{1}{m^{450}} + \nu \frac{1}{m^{451}} + \xi \frac{1}{m^{452}} + \zeta \frac{1}{m^{453}} + \eta \frac{1}{m^{454}} + \theta \frac{1}{m^{455}} + \rho \frac{1}{m^{456}} + \sigma \frac{1}{m^{457}} + \tau \frac{1}{m^{458}} + \nu \frac{1}{m^{459}} + \xi \frac{1}{m^{460}} + \zeta \frac{1}{m^{461}} + \eta \frac{1}{m^{462}} + \theta \frac{1}{m^{463}} + \rho \frac{1}{m^{464}} + \sigma \frac{1}{m^{465}} + \tau \frac{1}{m^{466}} + \nu \frac{1}{m^{467}} + \xi \frac{1}{m^{468}} + \zeta \frac{1}{m^{469}} + \eta \frac{1}{m^{470}} + \theta \frac{1}{m^{471}} + \rho \frac{1}{m^{472}} + \sigma \frac{1}{m^{473}} + \tau \frac{1}{m^{474}} + \nu \frac{1}{m^{475}} + \xi \frac{1}{m^{476}} + \zeta \frac{1}{m^{477}} + \eta \frac{1}{m^{478}} + \theta \frac{1}{m^{479}} + \rho \frac{1}{m^{480}} + \sigma \frac{1}{m^{481}} + \tau \frac{1}{m^{482}} + \nu \frac{1}{m^{483}} + \xi \frac{1}{m^{484}} + \zeta \frac{1}{m^{485}} + \eta \frac{1}{m^{486}} + \theta \frac{1}{m^{487}} + \rho \frac{1}{m^{488}} + \sigma \frac{1}{m^{489}} + \tau \frac{1}{m^{490}} + \nu \frac{1}{m^{491}} + \xi \frac{1}{m^{492}} + \zeta \frac{1}{m^{493}} + \eta \frac{1}{m^{494}} + \theta \frac{1}{m^{495}} + \rho \frac{1}{m^{496}} + \sigma \frac{1}{m^{497}} + \tau \frac{1}{m^{498}} + \nu \frac{1}{m^{499}} + \xi \frac{1}{m^{500}} + \zeta \frac{1}{m^{501}} + \eta \frac{1}{m^{502}} + \theta \frac{1}{m^{503}} + \rho \frac{1}{m^{504}} + \sigma \frac{1}{m^{505}} + \tau \frac{1}{m^{506}} + \nu \frac{1}{m^{507}} + \xi \frac{1}{m^{508}} + \zeta \frac{1}{m^{509}} + \eta \frac{1}{m^{510}} + \theta \frac{1}{m^{511}} + \rho \frac{1}{m^{512}} + \sigma \frac{1}{m^{513}} + \tau \frac{1}{m^{514}} + \nu \frac{1}{m^{515}} + \xi \frac{1}{m^{516}} + \zeta \frac{1}{m^{517}} + \eta \frac{1}{m^{518}} + \theta \frac{1}{m^{519}} + \rho \frac{1}{m^{520}} + \sigma \frac{1}{m^{521}} + \tau \frac{1}{m^{522}} + \nu \frac{1}{m^{523}} + \xi \frac{1}{m^{524}} + \zeta \frac{1}{m^{525}} + \eta \frac{1}{m^{526}} + \theta \frac{1}{m^{527}} + \rho \frac{1}{m^{528}} + \sigma \frac{1}{m^{529}} + \tau \frac{1}{m^{530}} + \nu \frac{1}{m^{531}} + \xi \frac{1}{m^{532}} + \zeta \frac{1}{m^{533}} + \eta \frac{1}{m^{534}} + \theta \frac{1}{m^{535}} + \rho \frac{1}{m^{536}} + \sigma \frac{1}{m^{537}} + \tau \frac{1}{m^{538}} + \nu \frac{1}{m^{539}} + \xi \frac{1}{m^{540}} + \zeta \frac{1}{m^{541}} + \eta \frac{1}{m^{542}} + \theta \frac{1}{m^{543}} + \rho \frac{1}{m^{544}} + \sigma \frac{1}{m^{545}} + \tau \frac{1}{m^{546}} + \nu \frac{1}{m^{547}} + \xi \frac{1}{m^{548}} + \zeta \frac{1}{m^{549}} + \eta \frac{1}{m^{550}} + \theta \frac{1}{m^{551}} + \rho \frac{1}{m^{552}} + \sigma \frac{1}{m^{553}} + \tau \frac{1}{m^{554}} + \nu \frac{1}{m^{555}} + \xi \frac{1}{m^{556}} + \zeta \frac{1}{m^{557}} + \eta \frac{1}{m^{558}} + \theta \frac{1}{m^{559}} + \rho \frac{1}{m^{560}} + \sigma \frac{1}{m^{561}} + \tau \frac{1}{m^{562}} + \nu \frac{1}{m^{563}} + \xi \frac{1}{m^{564}} + \zeta \frac{1}{m^{565}} + \eta \frac{1}{m^{566}} + \theta \frac{1}{m^{567}} + \rho \frac{1}{m^{568}} + \sigma \frac{1}{m^{569}} + \tau \frac{1}{m^{570}} + \nu \frac{1}{m^{571}} + \xi \frac{1}{m^{572}} + \zeta \frac{1}{m^{573}} + \eta \frac{1}{m^{574}} + \theta \frac{1}{m^{575}} + \rho \frac{1}{m^{576}} + \sigma \frac{1}{m^{577}} + \tau \frac{1}{m^{578}} + \nu \frac{1}{m^{579}} + \xi \frac{1}{m^{580}} + \zeta \frac{1}{m^{581}} + \eta \frac{1}{m^{582}} + \theta \frac{1}{m^{583}} + \rho \frac{1}{m^{584}} + \sigma \frac{1}{m^{585}} + \tau \frac{1}{m^{586}} + \nu \frac{1}{m^{587}} + \xi \frac{1}{m^{588}} + \zeta \frac{1}{m^{589}} + \eta \frac{1}{m^{590}} + \theta \frac{1}{m^{591}} + \rho \frac{1}{m^{592}} + \sigma \frac{1}{m^{593}} + \tau \frac{1}{m^{594}} + \nu \frac{1}{m^{595}} + \xi \frac{1}{m^{596}} + \zeta \frac{1}{m^{597}} + \eta \frac{1}{m^{598}} + \theta \frac{1}{m^{599}} + \rho \frac{1}{m^{600}} + \sigma \frac{1}{m^{601}} + \tau \frac{1}{m^{602}} + \nu \frac{1}{m^{603}} + \xi \frac{1}{m^{604}} + \zeta \frac{1}{m^{605}} + \eta \frac{1}{m^{606}} + \theta \frac{1}{m^{607}} + \rho \frac{1}{m^{608}} + \sigma \frac{1}{m^{609}} + \tau \frac{1}{m^{610}} + \nu \frac{1}{m^{611}} + \xi \frac{1}{m^{612}} + \zeta \frac{1}{m^{613}} + \eta \frac{1}{m^{614}} + \theta \frac{1}{m^{615}} + \rho \frac{1}{m^{616}} + \sigma \frac{1}{m^{617}} + \tau \frac{1}{m^{618}} + \nu \frac{1}{m^{619}} + \xi \frac{1}{m^{620}} + \zeta \frac{1}{m^{621}} + \eta \frac{1}{m^{622}} + \theta \frac{1}{m^{623}} + \rho \frac{1}{m^{624}} + \sigma \frac{1}{m^{625}} + \tau \frac{1}{m^{626}} + \nu \frac{1}{m^{627}} + \xi \frac{1}{m^{628}} + \zeta \frac{1}{m^{629}} + \eta \frac{1}{m^{630}} + \theta \frac{1}{m^{631}} + \rho \frac{1}{m^{632}} + \sigma \frac{1}{m^{633}} + \tau \frac{1}{m^{634}} + \nu \frac{1}{m^{635}} + \xi \frac{1}{m^{636}} + \zeta \frac{1}{m^{637}} + \eta \frac{1}{m^{638}} + \theta \frac{1}{m^{639}} + \rho \frac{1}{m^{640}} + \sigma \frac{1}{m^{641}} + \tau \frac{1}{m^{642}} + \nu \frac{1}{m^{643}} + \xi \frac{1}{m^{644}} + \zeta \frac{1}{m^{645}} + \eta \frac{1}{m^{646}} + \theta \frac{1}{m^{647}} + \rho \frac{1}{m^{648}} + \sigma \frac{1}{m^{649}} + \tau \frac{1}{m^{650}} + \nu \frac{1}{m^{651}} + \xi \frac{1}{m^{652}} + \zeta \frac{1}{m^{653}} + \eta \frac{1}{m^{654}} + \theta \frac{1}{m^{655}} + \rho \frac{1}{m^{656}} + \sigma \frac{1}{m^{657}} + \tau \frac{1}{m^{658}} + \nu \frac{1}{m^{659}} + \xi \frac{1}{m^{660}} + \zeta \frac{1}{m^{661}} + \eta \frac{1}{m^{662}} + \theta \frac{1}{m^{663}} + \rho \frac{1}{m^{664}} + \sigma \frac{1}{m^{665}} + \tau \frac{1}{m^{666}} + \nu \frac{1}{m^{667}} + \xi \frac{1}{m^{668}} + \zeta \frac{1}{m^{669}} + \eta \frac{1}{m^{670}} + \theta \frac{1}{m^{671}} + \rho \frac{1}{m^{672}} + \sigma \frac{1}{m^{673}} + \tau \frac{1}{m^{674}} + \nu \frac{1}{m^{675}} + \xi \frac{1}{m^{676}} + \zeta \frac{1}{m^{677}} + \eta \frac{1}{m^{678}} + \theta \frac{1}{m^{679}} + \rho \frac{1}{m^{680}} + \sigma \frac{1}{m^{681}} + \tau \frac{1}{m^{682}} + \nu \frac{1}{m^{683}} + \xi \frac{1}{m^{684}} + \zeta \frac{1}{m^{685}} + \eta \frac{1}{m^{686}} + \theta \frac{1}{m^{687}} + \rho \frac{1}{m^{688}} + \sigma \frac{1}{m^{689}} + \tau \frac{1}{m^{690}} + \nu \frac{1}{m^{691}} + \xi \frac{1}{m^{692}} + \zeta \frac{1}{m^{693}} + \eta \frac{1}{m^{694}} + \theta \frac{1}{m^{695}} + \rho \frac{1}{m^{696}} + \sigma \frac{1}{m^{697}} + \tau \frac{1}{m^{698}} + \nu \frac{1}{m^{699}} + \xi \frac{1}{m^{700}} + \zeta \frac{1}{m^{701}} + \eta \frac{1}{m^{702}} + \theta \frac{1}{m^{703}} + \rho \frac{1}{m^{704}} + \sigma \frac{1}{m^{705}} + \tau \frac{1}{m^{706}} + \nu \frac{1}{m^{707}} + \xi \frac{1}{m^{708}} + \zeta \frac{1}{m^{709}} + \eta \frac{1}{m^{710}} + \theta \frac{1}{m^{711}} + \rho \frac{1}{m^{712}} + \sigma \frac{1}{m^{713}} + \tau \frac{1}{m^{714}} + \nu \frac{1}{m^{715}} + \xi \frac{1}{m^{716}} + \zeta \frac{1}{m^{717}} + \eta \frac{1}{m^{718}} + \theta \frac{1}{m^{719}} + \rho \frac{1}{m^{720}} + \sigma \frac{1}{m^{721}} + \tau \frac{1}{m^{722}} + \nu \frac{1}{m^{723}} + \xi \frac{1}{m^{724}} + \zeta \frac{1}{m^{725}} + \eta \frac{1}{m^{726}} + \theta \frac{1}{m^{727}} + \rho \frac{1}{m^{728}} + \sigma \frac{1}{m^{729}} + \tau \frac{1}{m^{730}} + \nu \frac{1}{m^{731}} + \xi \frac{1}{m^{732}} + \zeta \frac{1}{m^{733}} + \eta \frac{1}{m^{734}} + \theta \frac{1}{m^{735}} + \rho \frac{1}{m^{736}} + \sigma \frac{1}{m^{737}} + \tau \frac{1}{m^{738}} + \nu \frac{1}{m^{739}} + \xi \frac{1}{m^{740}} + \zeta \frac{1}{m^{741}} + \eta \frac{1}{m^{742}} + \theta \frac{1}{m^{743}} + \rho \frac{1}{m^{744}} + \sigma \frac{1}{m^{745}} + \tau \frac{1}{m^{746}} + \nu \frac{1}{m^{747}} + \xi \frac{1}{m^{748}} + \zeta \frac{1}{m^{749}} + \eta \frac{1}{m^{750}} + \theta \frac{1}{m^{751}} + \rho \frac{1}{m^{752}} + \sigma \frac{1}{m^{753}} + \tau \frac{1}{m^{754}} + \nu \frac{1}{m^{755}} + \xi \frac{1}{m^{756}} + \zeta \frac{1}{m^{757}} + \eta \frac{1}{m^{758}} + \theta \frac{1}{m^{759}} + \rho \frac{1}{m^{760}} + \sigma \frac{1}{m^{761}} + \tau \frac{1}{m^{762}} + \nu \frac{1}{m^{763}} + \xi \frac{1}{m^{764}} + \zeta \frac{1}{m^{765}} + \eta \frac{1}{m^{766}} + \theta \frac{1}{m^{767}} + \rho \frac{1}{m^{768}} + \sigma \frac{1}{m^{769}} + \tau \frac{1}{m^{770}} + \nu \frac{1}{m^{771}} + \xi \frac{1}{m^{772}} + \zeta \frac{1}{m^{773}} + \eta \frac{1$$