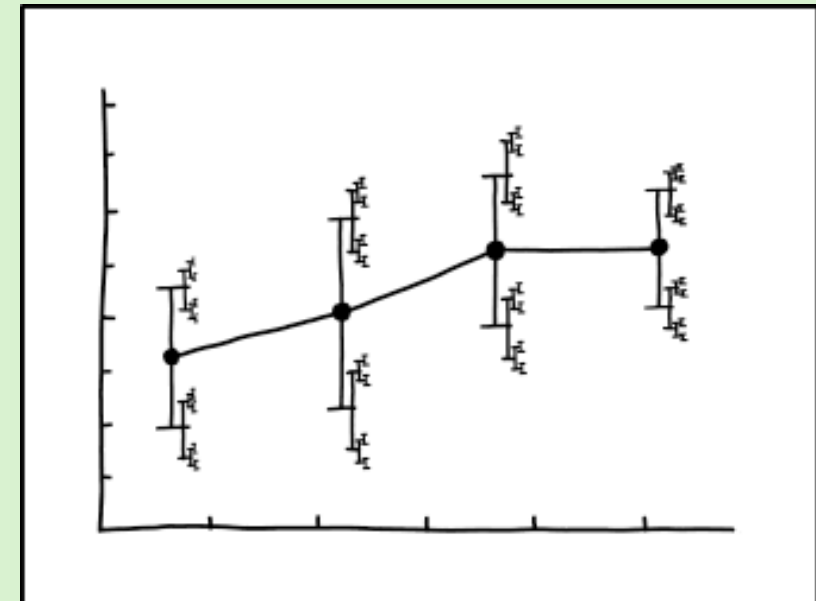
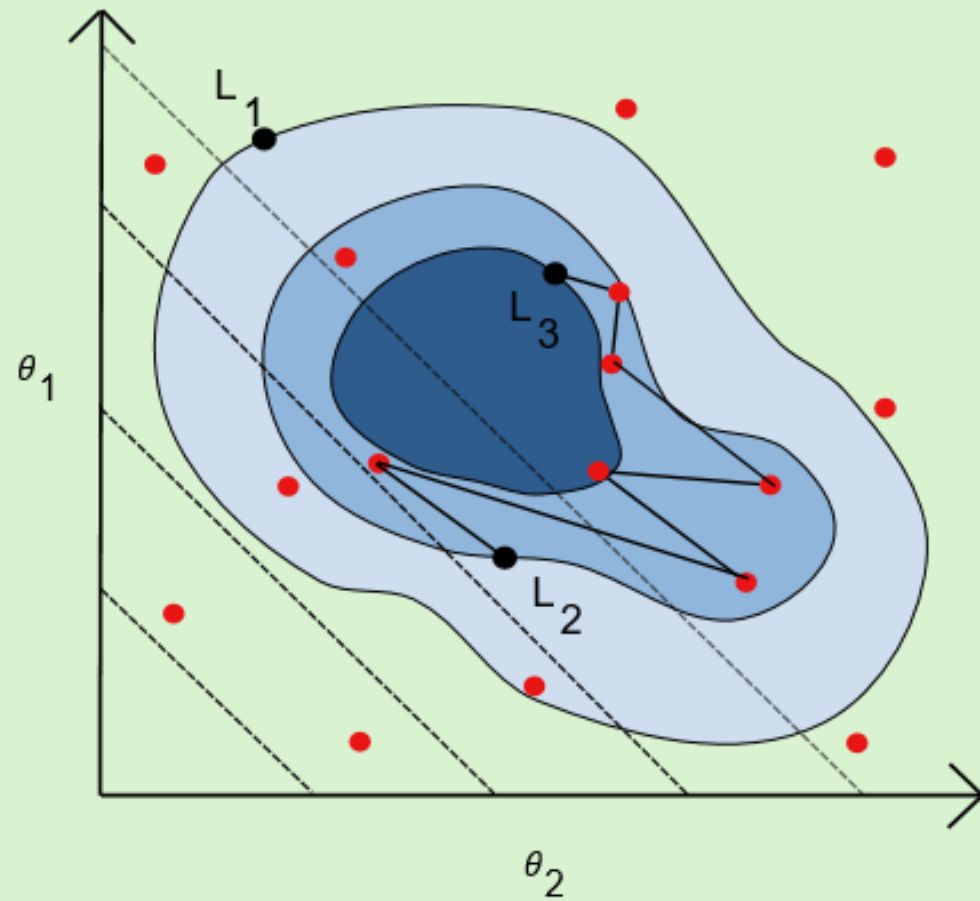


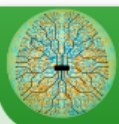
Costless correction of nested sampling parameter estimation

Metha Prathaban



I DON'T KNOW HOW TO PROPAGATE ERROR CORRECTLY, SO I JUST PUT ERROR BARS ON ALL MY ERROR BARS.

[<https://xkcd.com/2110/>]



Phantom points in nested sampling parameter estimation

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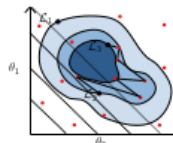
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Nested sampling parameter estimation differs from evidence estimation, in that it incurs an additional source of error. This error affects estimates of parameter means and credible intervals in gravitational wave analyses and beyond, and yet, it is typically not accounted for in standard error estimation methods. We present two novel methods to quantify this error more accurately for any chain-based nested sampler, using the additional likelihood calls made at runtime in producing independent samples. Using injected signals of black hole binary coalescences as an example, we demonstrate how these extra points may be carefully utilised to estimate the true error correctly, and provide a way to check the accuracy of the resulting error bars.



1. Nested sampling and phantom points



Nested sampling (NS) is a popular Bayesian inference tool for parameter estimation and model comparison. A set of live points is drawn from the prior and at each iteration, the live point with the lowest likelihood is deleted. A new point is drawn, with the constraint that its likelihood must be higher than that of the deleted point [1, 2]. In this way, a series of nested iso-likelihood contours are defined [3].

There are many ways to generate a new live point with the hard likelihood constraint $\mathcal{L} > \mathcal{L}_d$. However, many NS implementations use a Markov-Chain based procedure, where new points are continually generated within the likelihood contour until we are satisfied that the new point is independent from the deleted point. This point is then assigned as the new live point, and the points generated in the chain between the deleted and new live point (red) are typically discarded. These 'phantom points', though deemed too correlated to use in evidence estimation, can provide useful information about the parameter space, though this has been largely unexplored.

Errors in evidence vs parameter estimation

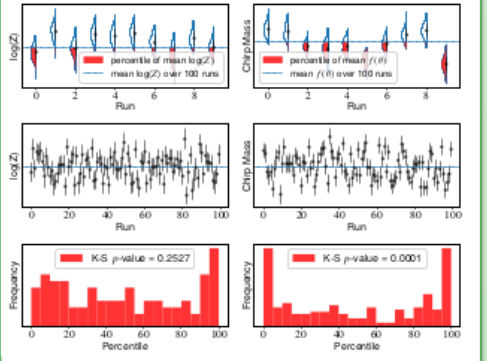
Evidence estimation From Bayes' theorem, $Z = \int \mathcal{L}(\theta) \pi(\theta) d\theta. \quad (1)$	Parameter estimation The expected value of $f(\theta)$ is [4] $E[f(\theta)] = \int f(\theta) \frac{\mathcal{L}(\theta) \pi(\theta)}{Z} d\theta \quad (2)$
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Changing the integration variable to the fraction of prior volume within an iso-likelihood contour, X , and approximating this as a sum over the dead points [3, 5]:

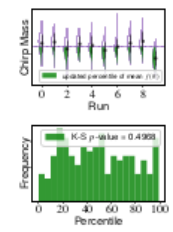
$\int_0^1 \mathcal{L}(X) dX \approx \sum_{\theta \in \text{dead points}} \mathcal{L}_d \Delta X_i \quad (3)$	$\frac{1}{Z} \int f(X) \mathcal{L}(X) dX \approx \frac{1}{N} \sum_{i=1}^N \tilde{f}(X_i) \mathcal{L}_d \Delta X_i \quad (4)$
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By construction, all points along a given contour have the same likelihood value; using the likelihood value of a single dead point, \mathcal{L}_d , as a proxy for $\mathcal{L}(X)$ is an exact substitution. The dominant error is the unknown volumes of the fractional prior volume 'shells' between contours, ΔX .

For a binary black hole, we can apply the 'simulated weights' method [6, 7], suggested by Skilling, to estimate the errors on the evidence and chirp mass per run. If this is sufficient, the percentiles of the 'true evidence' and 'true chirp mass' (estimated over 100 runs) should be uniformly distributed.



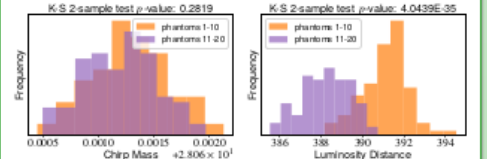
3. Likelihood binning method



- Bin phantom points by their likelihood values, such that each dead point is associated with a set of phantom points from the run which sit very close to the contour defined by it.
- We make the assumption that, though phantom points do not lie exactly on the dead point's iso-likelihood contour, they are still representative of the $f(\theta)$ values along the contour.
- For each dead point in equation 4, resample a new $f(\theta)$ value from the associated bin (which includes the original dead point itself) and a new ΔX .
- Repeat many times to obtain the error as the standard deviation of the resulting distribution of estimates.

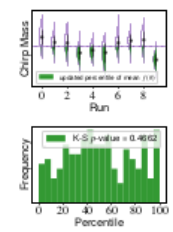
Verifying accuracy of error bars

For certain parameters, the chain length of the sampler may not be long enough to accumulate sufficient uncorrelated phantom points to use these methods. We can check from a single run whether we have the correct error bars by splitting the phantom points in two halves, and applying either method separately to the two sets of points to check for convergent results using the K-S 2-sample test. The default chain length in PolyChord [1, 2] is long enough for these methods to work well on the chirp mass, but not the luminosity distance.



We can apply something similar at runtime to reduce the sampling time by calculating the optimal chain length.

Reconstructed runs method



- All phantom points are perfectly valid, except that they may be too correlated with their associated dead point to use both in the same run.
- We can therefore take the 1st phantom point in every chain in the run and combine these carefully to form an equally valid nested sampling run to the original.
- Repeat with other phantom points to reconstruct multiple valid runs from original.
- Combine the parameter estimates from each of these reconstructed runs, as well as the original run, and compare the corresponding new error bar from this.

References

[1] S. J. G. Murray and S. J. G. Murray, *Bayesian Inference for Gravitational Wave Parameter Estimation*, Cambridge University Press, 2019.

[2] S. J. G. Murray, *Bayesian Inference for Gravitational Wave Parameter Estimation*, Cambridge University Press, 2019.

[3] S. J. G. Murray, *Bayesian Inference for Gravitational Wave Parameter Estimation*, Cambridge University Press, 2019.

[4] S. J. G. Murray, *Bayesian Inference for Gravitational Wave Parameter Estimation*, Cambridge University Press, 2019.

[5] S. J. G. Murray, *Bayesian Inference for Gravitational Wave Parameter Estimation*, Cambridge University Press, 2019.

[6] S. J. G. Murray, *Bayesian Inference for Gravitational Wave Parameter Estimation*, Cambridge University Press, 2019.

[7] S. J. G. Murray, *Bayesian Inference for Gravitational Wave Parameter Estimation*, Cambridge University Press, 2019.

zendo repository with the source code and pdf for this poster.

