Estimating classical mutual information for spin systems and scalar field theories using generative neural networks

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Location: 44

2.2 Generative models & Simulation of physical systems

ML enhanced Monte Carlo simulations

Mutual information

Mutual information quantifies the "amount of information" obtained about one random variable by observing the other random variable. The bipartite partitioning into A, B allows defining the Shannon mutual information as

$$I = \sum_{\mathrm{a,b}} p(\mathrm{a,b}) \log \frac{p(\mathrm{a,b})}{p(\mathrm{a})p(\mathrm{b})}$$

Ancestral sampling using autoregressive neural networks

We factorize the probability into a product of conditional probabilities

$$p(\mathrm{s}) \;=\; p(s^1)\prod_{i=2}^{N} p(s^i|s^1,\ldots,s^{i-1}) \;\approx\; q_\theta(s^1)\prod_{i=2}^{N} q_\theta(s^i|s^1,\ldots,s^{i-1}).$$

Using reweighting from p to q_θ provides us with access to the full partition function $Z(\beta)$ as well as $Z(a,\beta)$ and $Z(b,\beta)$ and write MI as

$$I(\beta) = \log\langle \hat{w}(\mathrm{a,b})\rangle_{q_\theta} - \beta\langle w(\mathrm{a,b})E(\mathrm{a,b})\rangle_{q_\theta} - \langle w(\mathrm{a,b})\log Z(\mathrm{a})\rangle_{q_\theta} - \langle w(\mathrm{a,b})\log Z(\mathrm{b})\rangle_{q_\theta}.$$

ML enhanced Monte Carlo simulations

Applications

Our group has applied this approach to various models:

- quantum entanglement in the Ising chain
  $\Rightarrow$ Dawid Zapolski *Calculating entanglement entropy with generative neural networks*, Location 41
- hierarchical algorithm for the three-dimensional Ising model
  $\Rightarrow$ Mateusz Winiarski *Applying hierarchical autoregressive neural networks for three-dimensional Ising model*, Location 74
- higher-dimensional Ising model and Z_2 gauge model
  $\Rightarrow$ Vaibhav Chahar *Simulation of Z_2 model using Variational Autoregressive Network (VAN)*, Location 1
- Potts model