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Extreme QCD Matter with Machine Learning

Kai Zhou (CUHK-Shenzhen)

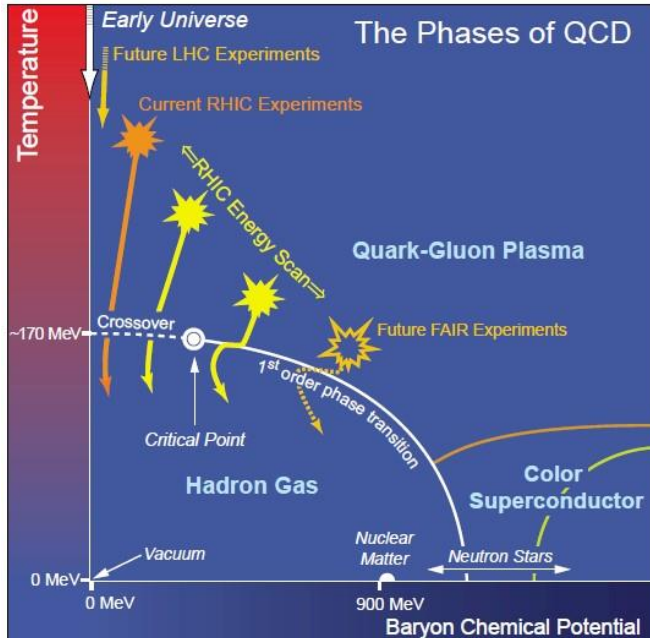
AI in Fundamental Physics - EuCAIFCon 2024 (Amsterdam)

Overview : QCD matter in extreme

- **Phases of matter** : solid, liquid, gas, plasma
- Matter in extreme conditions reveals its **constituents** : nuclear matter → quark matter

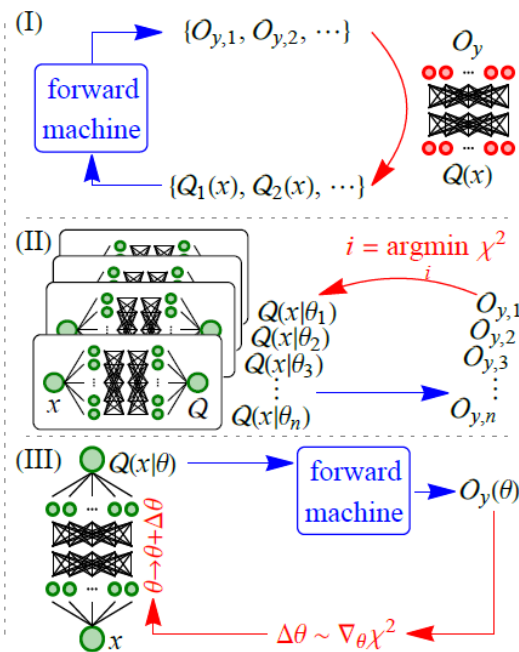
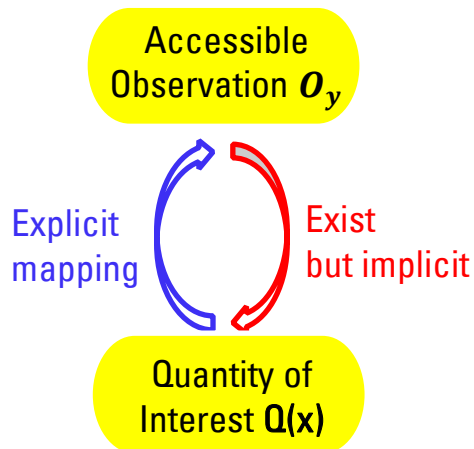


“It would be intriguing to explore new phenomena by distributing high energy or high nuclear matter density over a relatively large volume.” - - T.D. Lee (1974)



To study QCD matter under extreme conditions :

- **Nuclear Collisions** : heat & compress matter
- **Lattice Field Theory** : numerically solve partition function
- **Neutron Star** : dense matter, astronomy constraints



- **Direct inverse mapping capturing :** with Supervised Learning
- **Statistical approach to χ^2 fitting :** Bayesian Reconstruction for posterior or Heuristic (Generic) Algorithm to min.
- **Automatic Differentiation :** fuse physical prior into reconstruction via differentiable programming strategy

$$\chi^2 = \sum_y \left(\frac{\mathcal{F}_y[Q_{\text{NN}}(x|\theta)] - O_y}{\Delta O_y} \right)^2$$

$$\frac{1}{2} \nabla_{\theta} \chi^2 = \sum_y \frac{\mathcal{F}_y[Q_{\text{NN}}(x|\theta)] - O_y}{(\Delta O_y)^2} \int dx \frac{\delta \mathcal{F}_y[Q(x)]}{\delta Q(x)} \Big|_{Q(x)=Q_{\text{NN}}(x|\theta)} \nabla_{\theta} Q_{\text{NN}}(x|\theta)$$

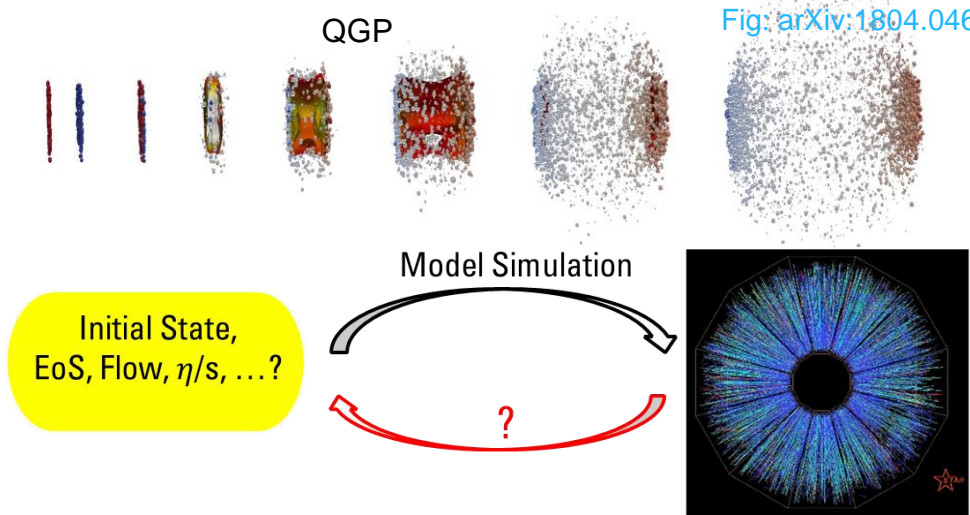


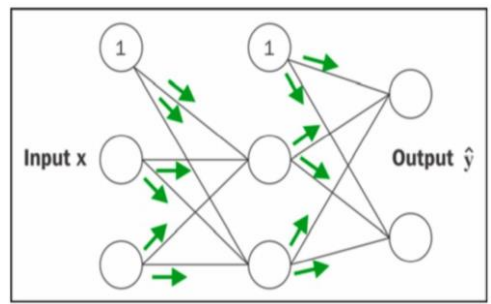
Fig: arXiv:1804.04649

- **Uncertainties** in HIC modeling
- Multiple parameters **entangle** with multiple observables
- How to **disentangle different factors** to reveal fundamental physics from the dynamical environment final state?



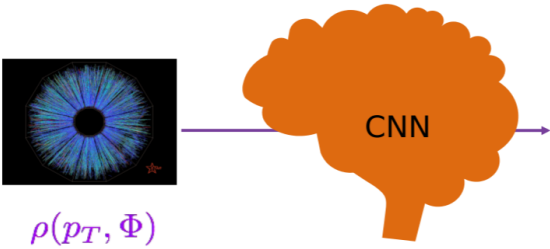
Bayes' Theorem

$$\underbrace{P(\theta | y)}_{\text{Posterior}} \propto \prod_i^N \underbrace{P(y_i | \theta)}_{\text{Data Likelihood}} \underbrace{P(\theta)}_{\text{Prior}}$$



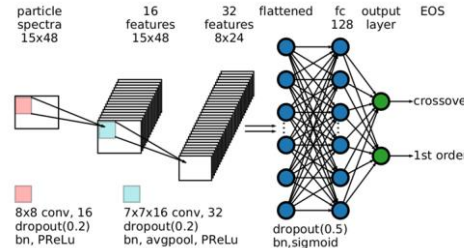
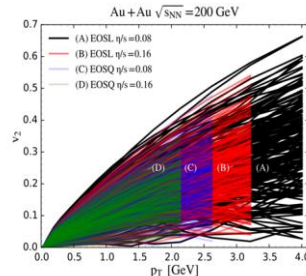
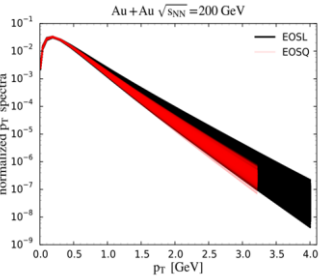
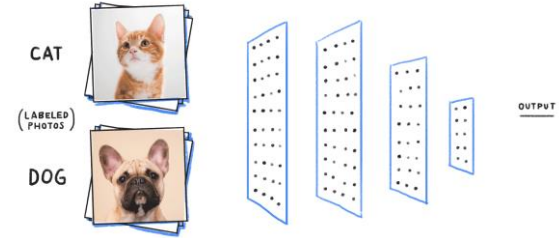
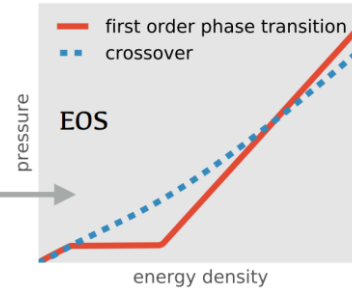
- Universal approximator
- Differentiable programming
- Gradient based optimization

Direct inverse mapping? CNN make the road



Crossover "(0,1)"
?

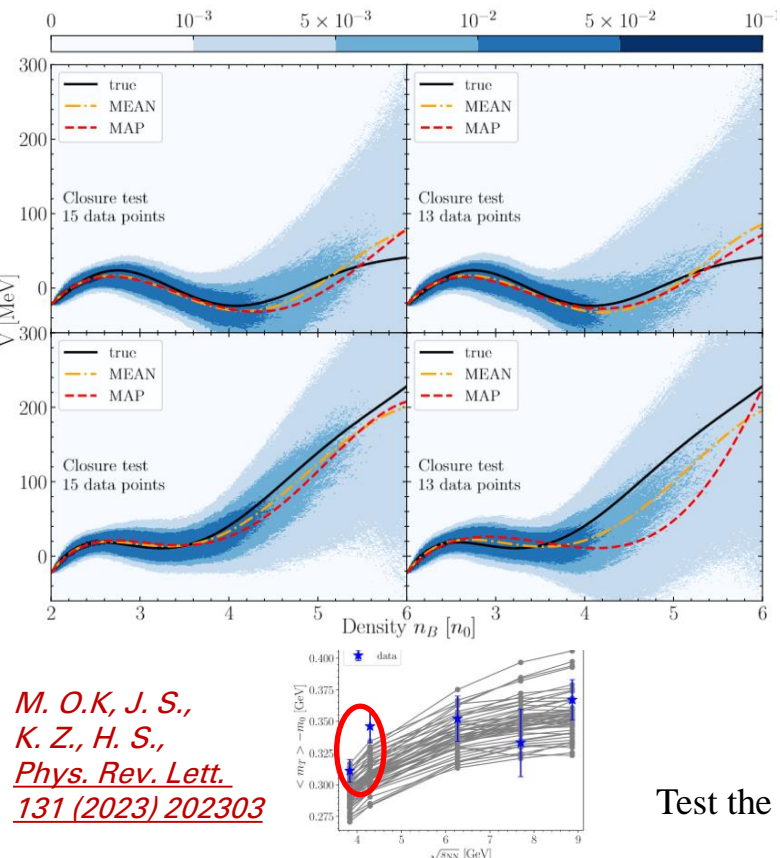
1st Order "(1,0)"



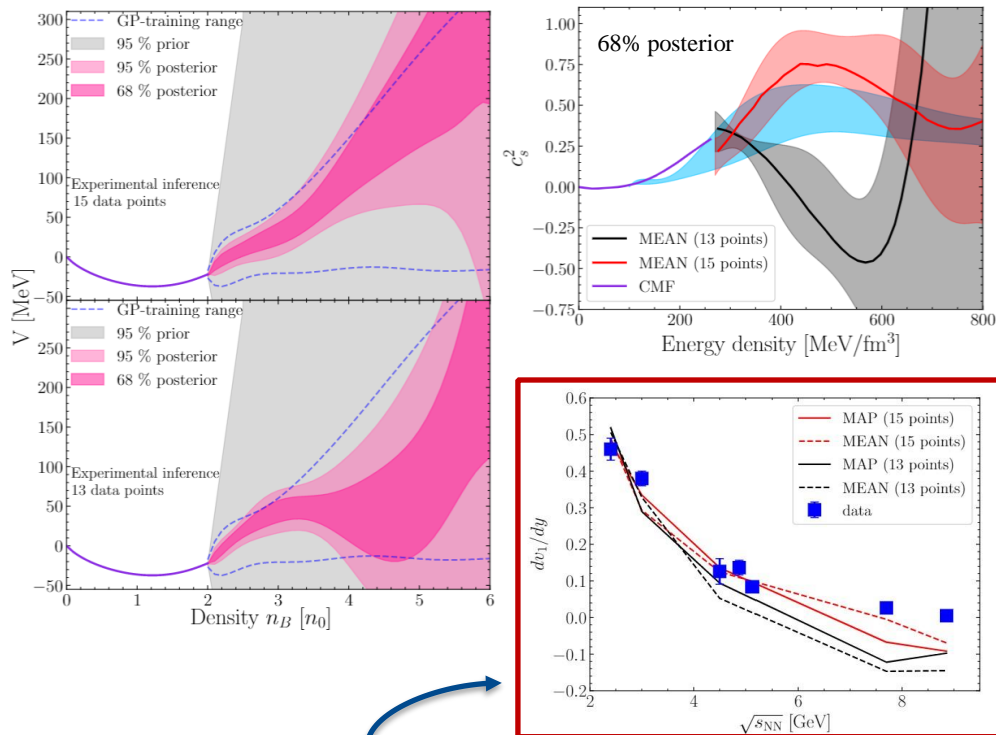
- Conventional obs. hard to distinguish
- Strongly influence from initial fluctuations and other uncertainties
- CNN : 95% event-by-event accuracy!
- Robust to initial conditions, eta/s

Conclusion : Information of early dynamics can **survive** to the end of hydrodynamics and encoded within the final state raw spectra, immune to evolution's uncertainties, **with deep CNN we can decode it back.**

● Posterior \sim Likelihood \times Prior



● With real experimental data



Test the extracted EoS on different observables (not used in Bayesian analysis)

HQ Potential Reconstruction by Inversing Shroedinger Eq.

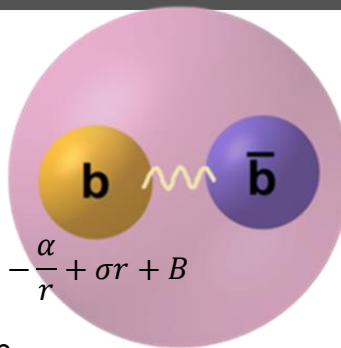
Large mass scale : $m_Q \gg \Lambda_{QCD}, T, p$

- Hard Process production in early stage
- ‘Calibrated’ QCD Force – HQ interaction

Vacuum: NRQCD, Cornell-like $V(r) = -\frac{\alpha}{r} + \sigma r + B$

Medium: Color Screening, Thermal width

Laine, et.al, JHEP(2)

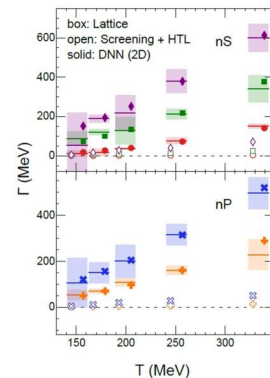
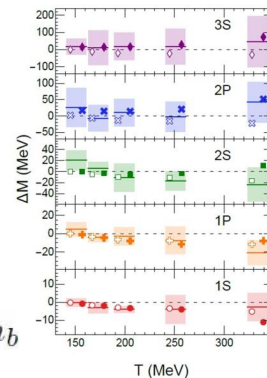
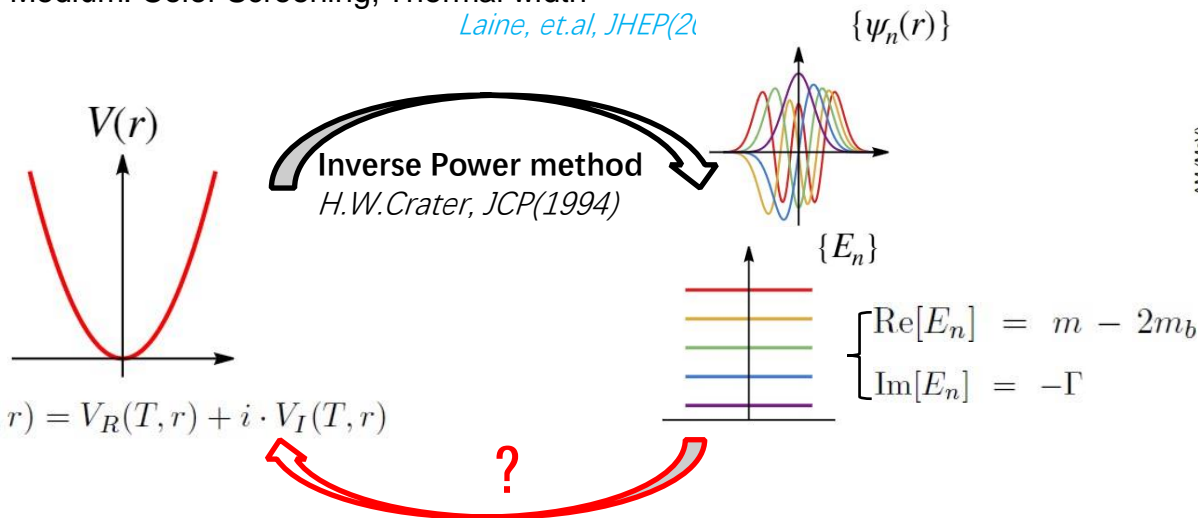


Large mass scale :

$$\hat{H}\psi_n = -\frac{\nabla^2}{2m_\mu}\psi_n + V(r)\psi_n = E_n\psi_n$$

M. Strickland, et.at., PRC(2015) PRD(2018), PLB(2020)

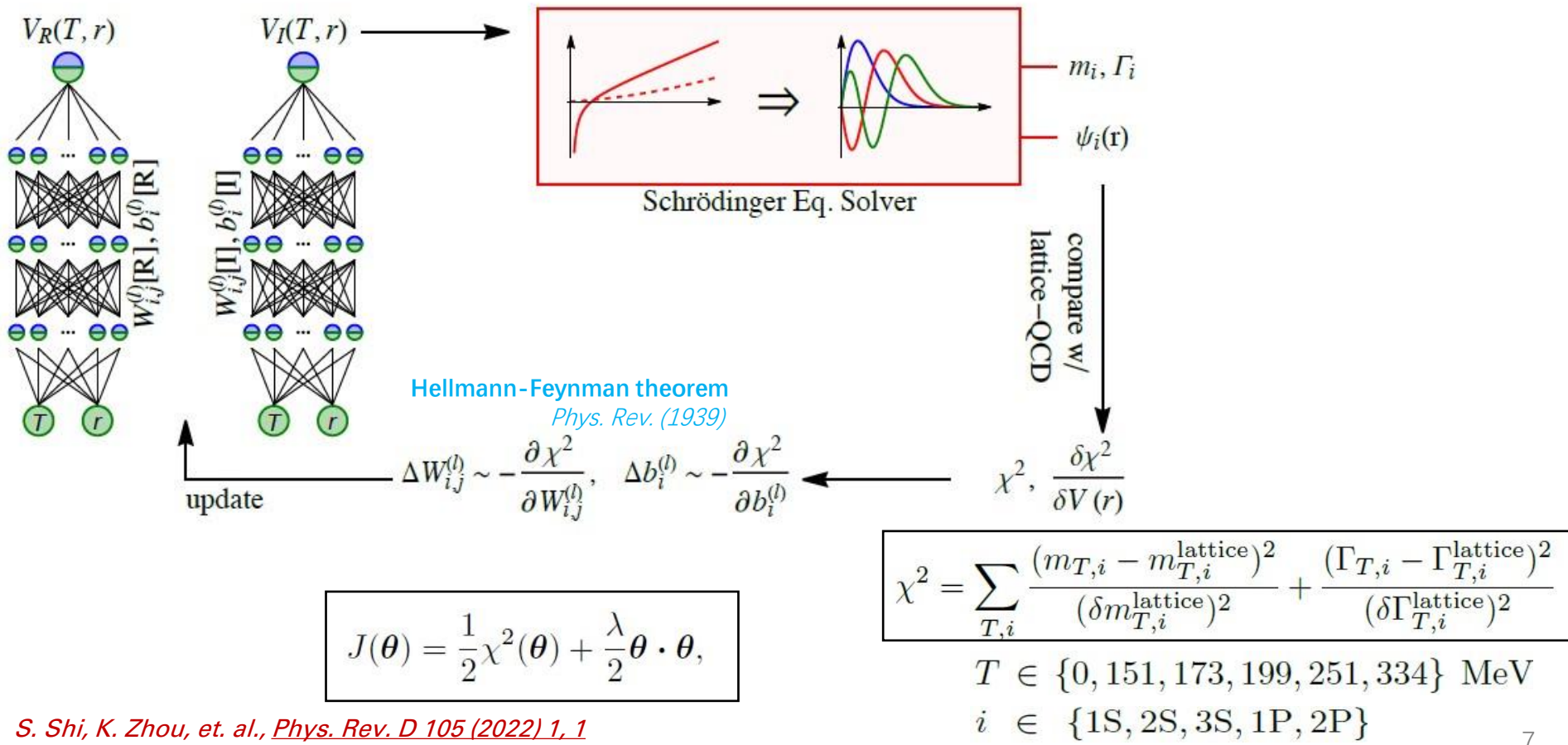
New IQCD results cannot be explained by Perturbative HTL-inspired potentials !



R. Larsen, et.al, PRD(2019), PLB(2020), PRD(2020)

How to extract **effective potential** given **limited spectroscopy** ?

Flow chart for "DNN + Schrödinger Eq."

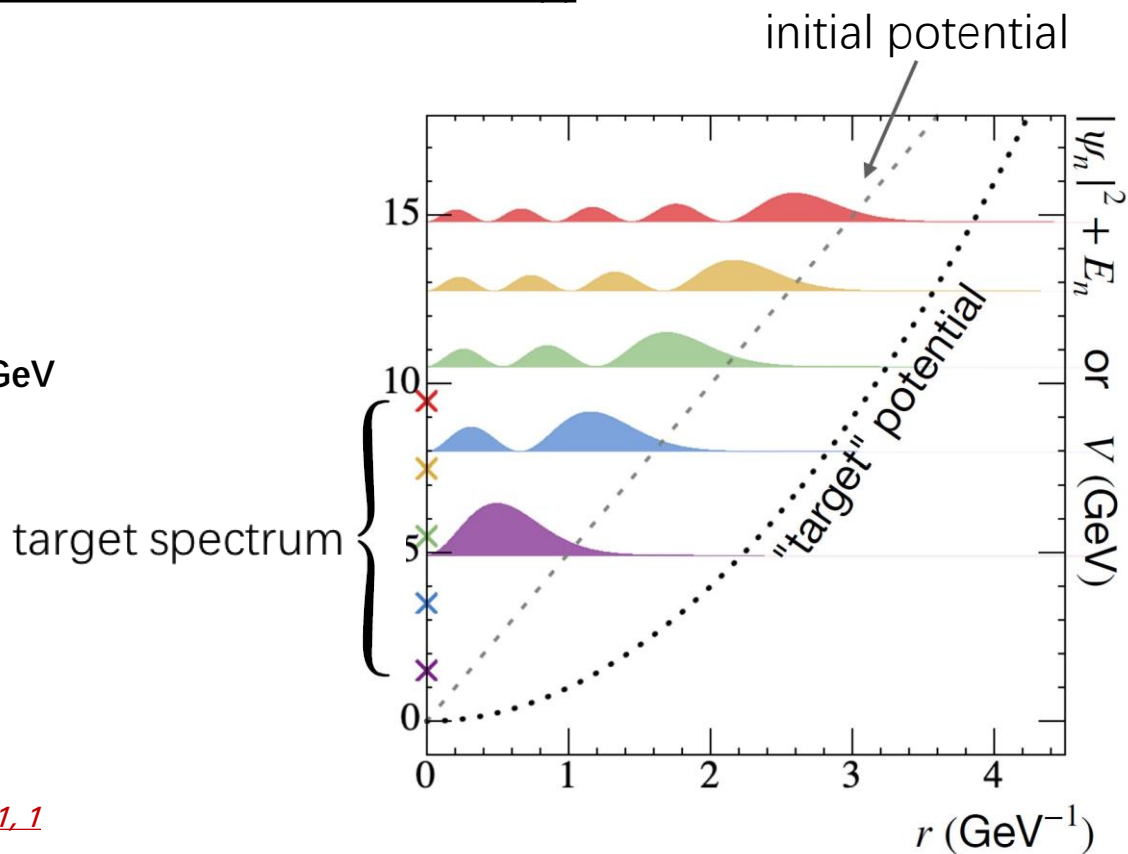


Proof of Concept

limited spectrum $\{ E_n \}$ to continuous interaction $V(r)$?

Learn $V(r)$ from 5 eigenvalues :

$\{ E_n \} = \{ 3/2, 7/2, 11/2, 15/12, 19/2 \}$ GeV



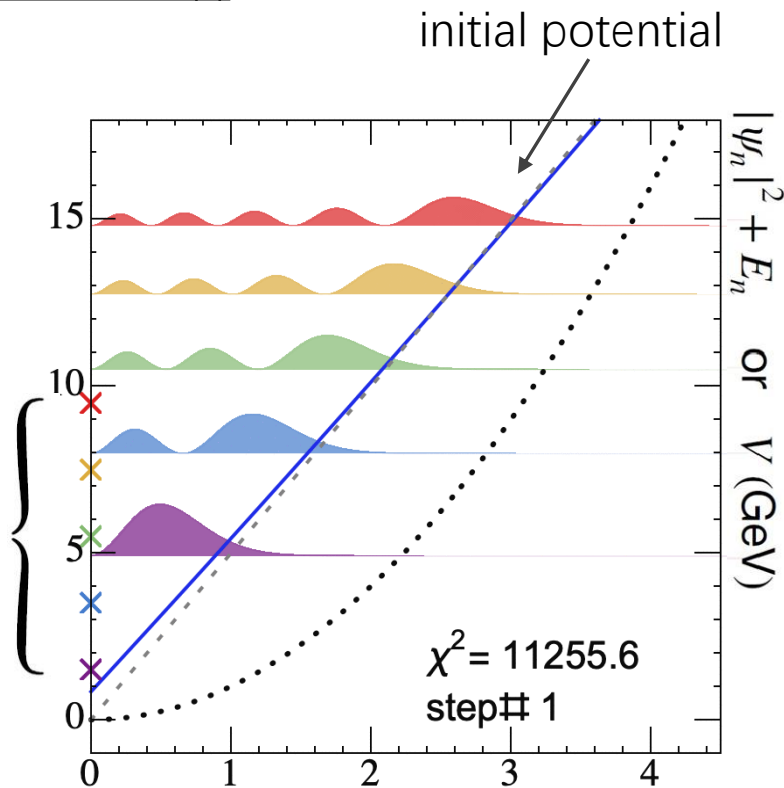
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target spectrum



Proof of Concept

limited spectrum $\{E_n\}$ to continuous interaction $V(r)$?

-- Yes! But to some range decided by the used states.

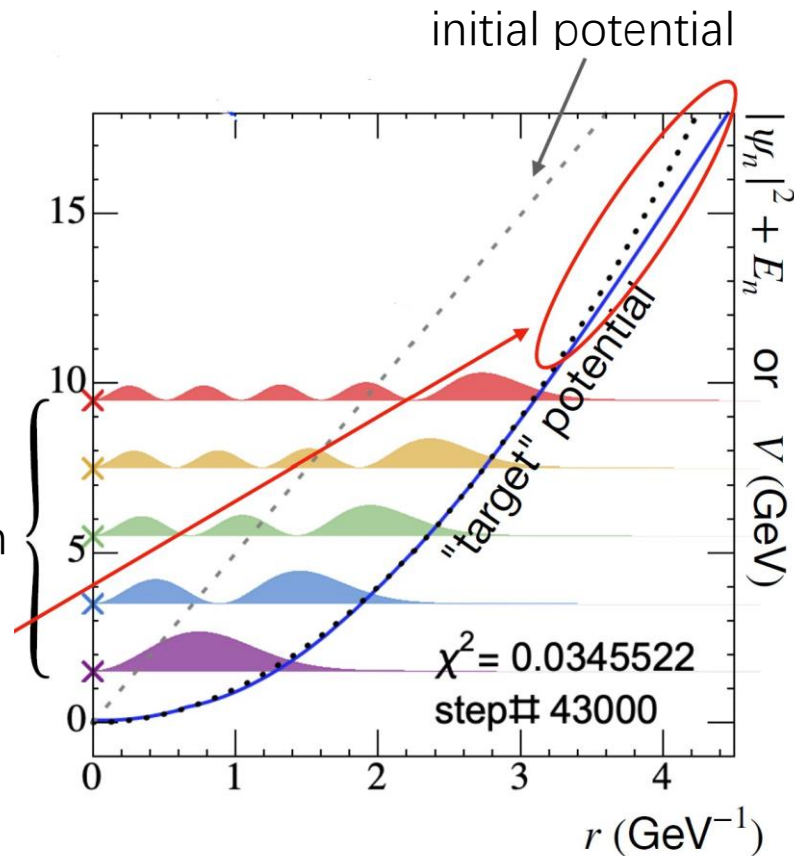
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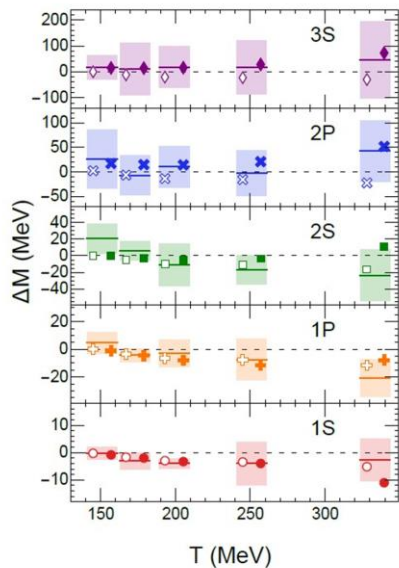
target spectrum

Deviation @ given states' wavefunction vanishes

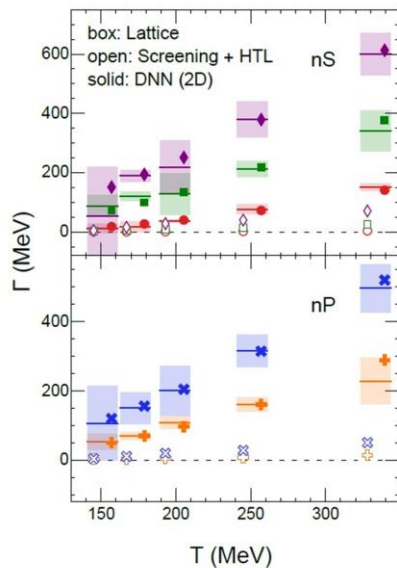
$$\delta E_n = \langle \psi_n | \delta V(r) | \psi_n \rangle$$



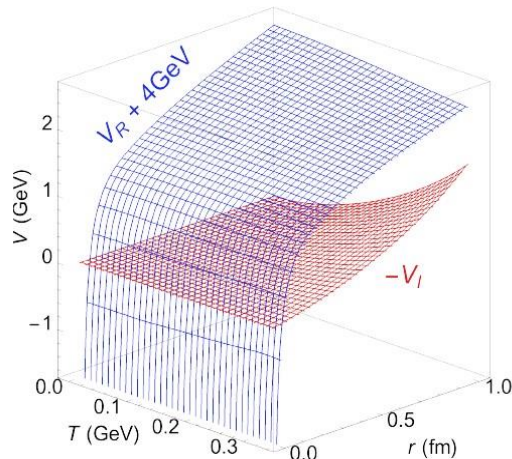
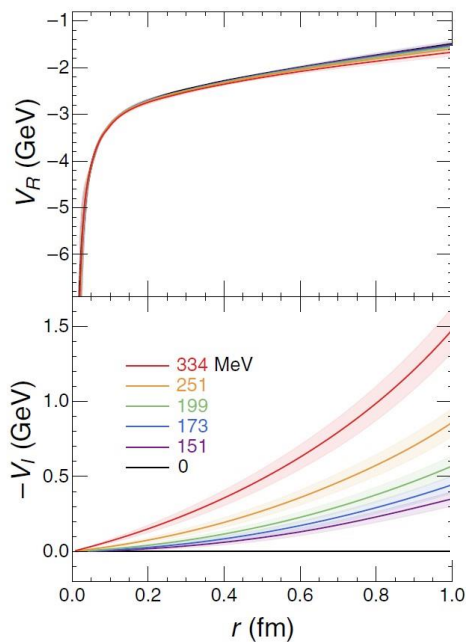
Results with lattice data for mass/width and the reconstructed HQ Potential



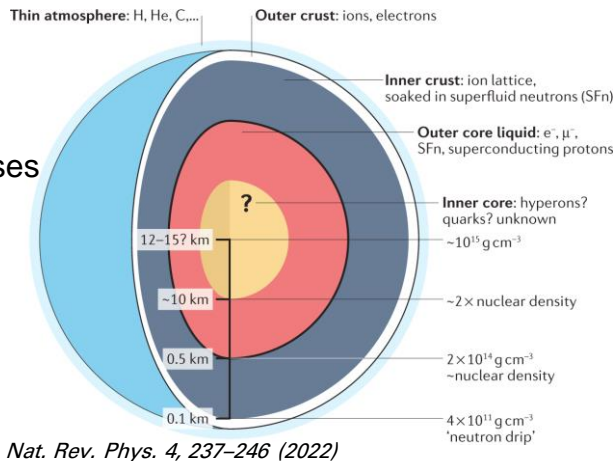
Chi2-per-data=16.5/30



The reconstructed T, r dependent potential



- Mass ~ 2 solar masses
- Radii ~ 10 km
- Densities 5-8 ρ_0



- Gravity $\leftarrow \rightarrow$ Pressure

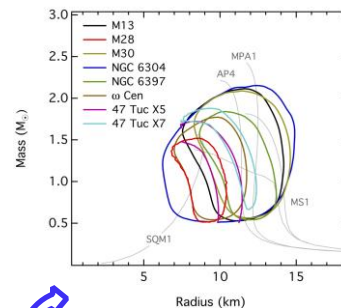
$$\frac{dP}{dr} = -\frac{G}{r^2} \left(\rho + \frac{P}{c^2} \right) \left(m + 4\pi r^3 \frac{P}{c^2} \right) \left(1 - \frac{2Gm}{c^2 r} \right)^{-1}$$

$$M = m(R) = \int_0^R 4\pi r^2 \rho dr$$

- Dense matter Equation of State

$P(\rho)$ \leftarrow \rightarrow

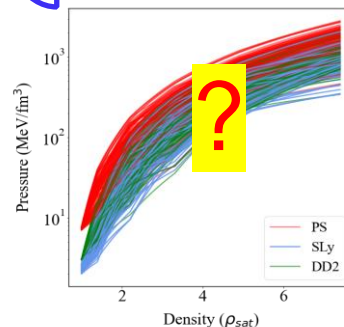
- Noisy/Limited NS Observables to EoS ?



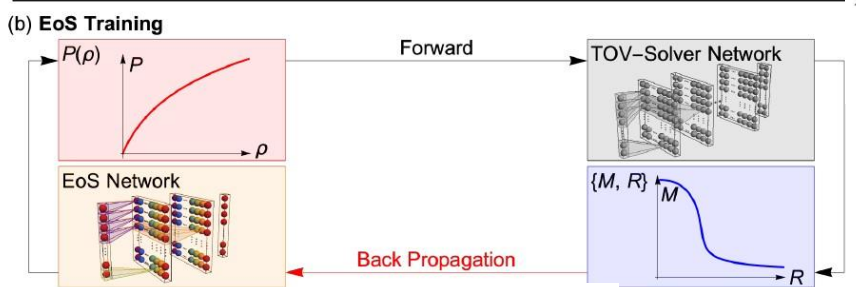
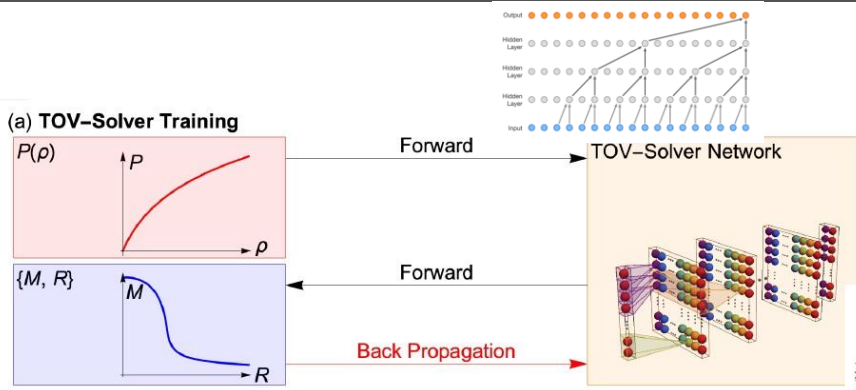
TOV equations

$$\frac{dP}{dr} = \frac{[\epsilon(r) + P(r)][M(r) + 4\pi r^3 P(r)]}{r[r - 2M(r)]}$$

$$\frac{dM(r)}{dr} = 4\pi r^2 \epsilon(r)$$



Mock Test with noise



Loss function

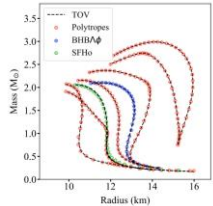
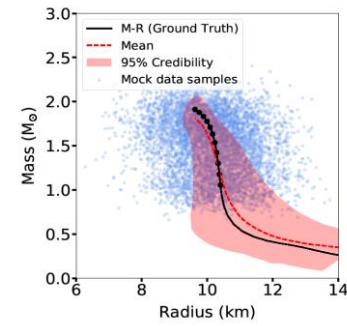
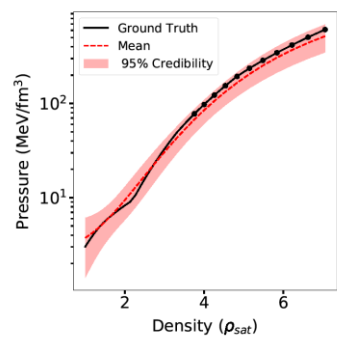
$$\mathcal{L} = \chi^2 = \sum_{i=1}^{N_{\text{obs}}} \frac{(M_i - M_{\text{obs},i})^2}{\Delta M_i^2} + \frac{(R_i - R_{\text{obs},i})^2}{\Delta R_i^2}$$

Gradients

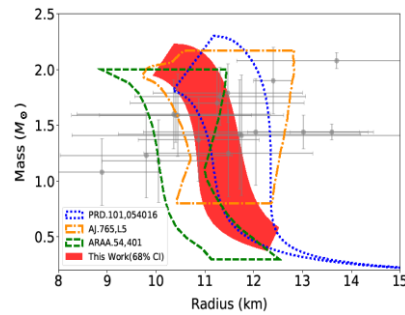
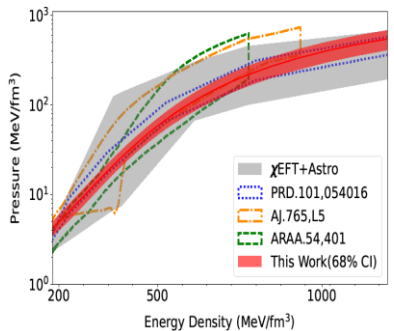
$$g_i = \frac{\delta \chi^2}{\delta \theta_i} = \frac{\delta \chi^2}{\delta z} \frac{\delta z}{\delta x} \frac{\delta x}{\delta \theta_i}$$

$z = (M_i, R_i), x = P_i(\rho_i)$

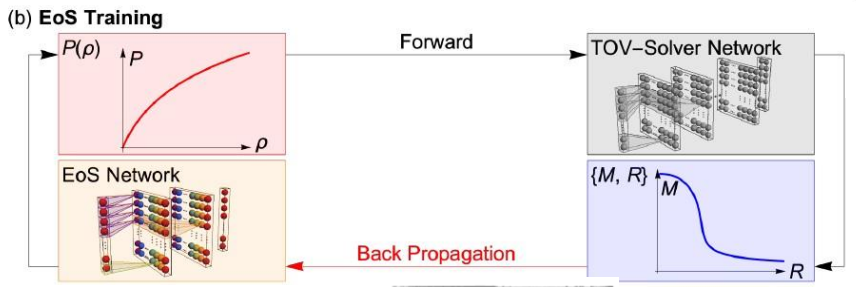
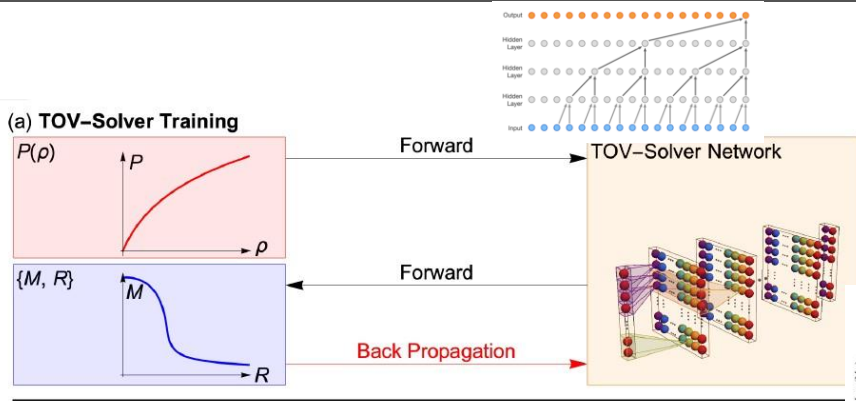
- Well validated through **Mock Tests**



- With **real observable** we reconstruct the NS EoS also



Mock Test with noise



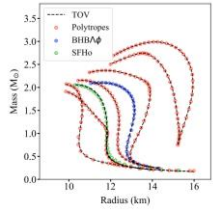
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Loss function

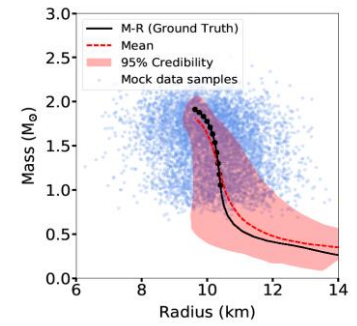
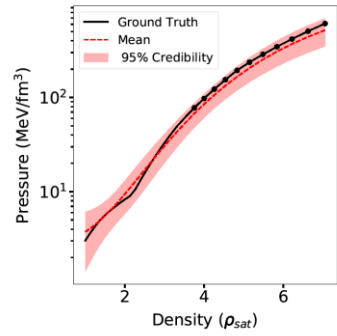
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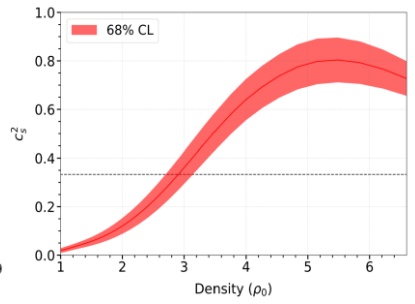
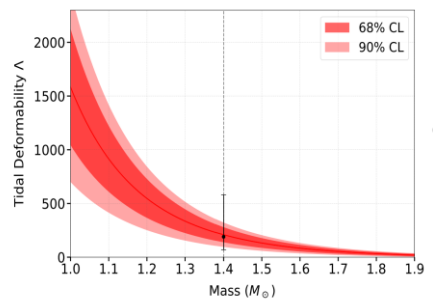
Gradients



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K. Zhou, L. Wang, L. Pang, S. Shi
PPNP 135, 104084



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Review

Exploring QCD matter in extreme conditions with Machine Learning

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ABSTRACT

In recent years, machine learning has emerged as a powerful computational tool and novel problem-solving perspective for physics, offering new avenues for studying strongly interacting QCD matter properties under extreme conditions. This review article aims to provide an overview of the current state of this intersection of fields, focusing on the application of machine learning to theoretical studies in high energy nuclear physics. It covers diverse aspects, including heavy ion collisions, lattice field theory, and neutron stars, and discuss how machine learning can be used to explore and facilitate the physics goals of understanding QCD matter. The review also provides a commonality overview from a methodology perspective, from data-driven perspective to physics-driven perspective. We conclude by discussing the challenges and future prospects of machine learning applications in high energy nuclear physics, also underscoring the importance of incorporating physics priors into the purely data-driven learning toolbox. This review highlights the critical role of machine learning as a valuable computational paradigm for advancing physics exploration in high energy nuclear physics.

Thanks!