Normalising flows for dense matter equation of state inference from gravitational wave observations of neutron star mergers

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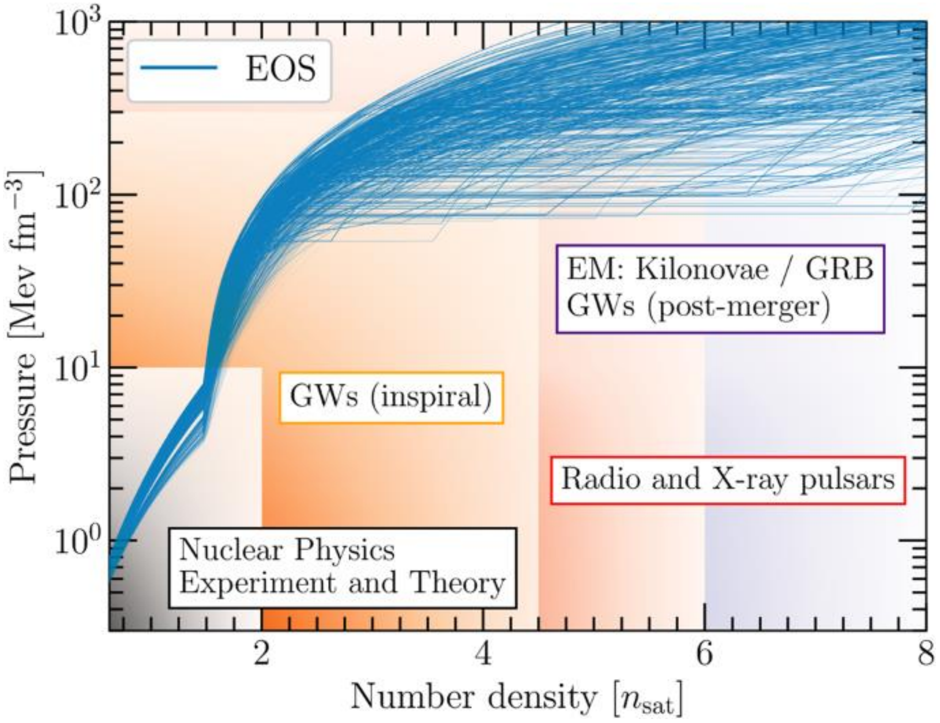
Inferring the NS EOS with GWs

- Matter at densities inside neutron stars (NS) is mostly unexplored

- The equation of state (EOS) describes the pressure-density relationship

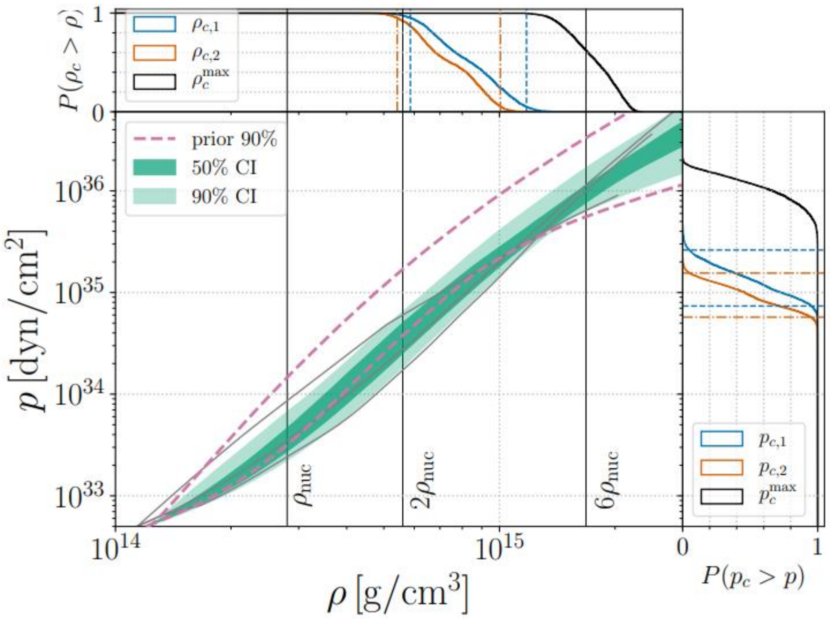
- EOS encodes numerous properties of nuclear matter parameters, e.g., symmetry energy



P.T.H. Pang, T. Dietrich, M.W. Coughlin, et al. Nat Commun 14, 8352 (2023)

Inferring the NS EOS with GWs

- We use GW to probe the NS EOS from a new perspective.
- GWs provide measurements of the gravitational mass, m, and tidal deformability, Λ, of each component of a binary neutron star (BNS) merger
- We can infer nuclear properties of the high density NS EOS and make statements on matter composition.



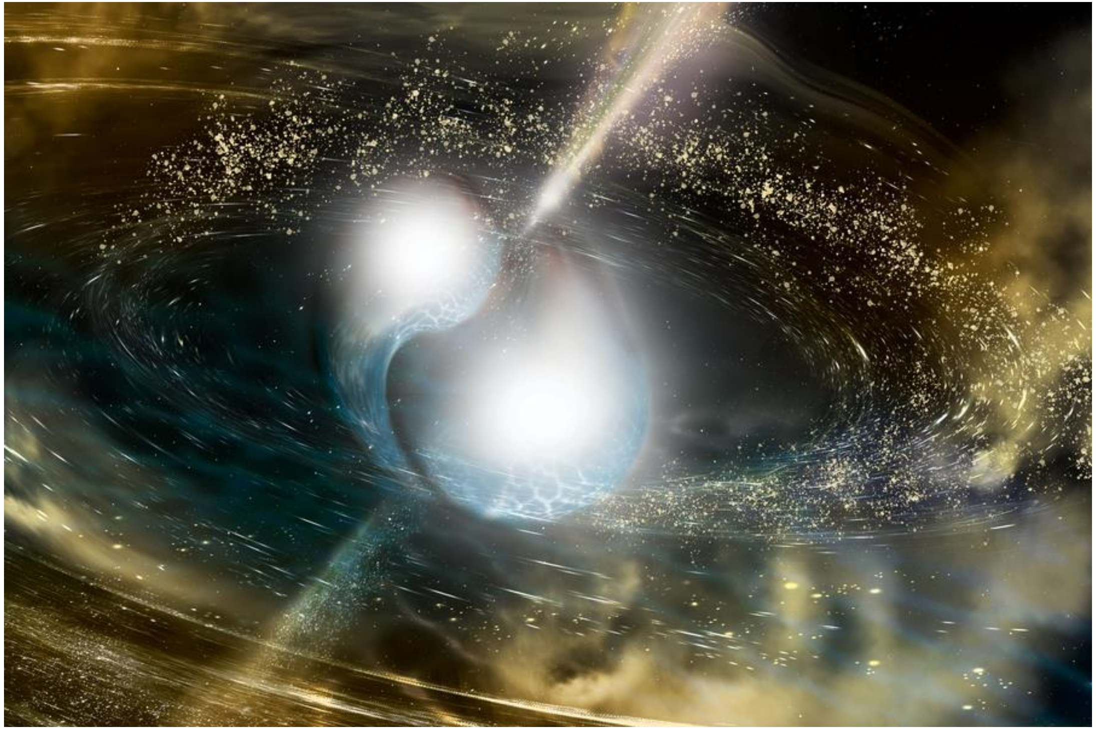
B.P. Abbott et al. (The LIGO Scientific Collaboration and the Virgo Collaboration) Phys. Rev. Lett. 121, 161101

Neutron star tidal deformability

Tidal deformation that each star's gravitational field induces on its companion

$\Lambda \equiv \frac{2}{3} k_2 C^{-5}$, where k_2 is tidal love number and C is compactness

Astrophysical inference of Λ provides constraints on the NS EOS



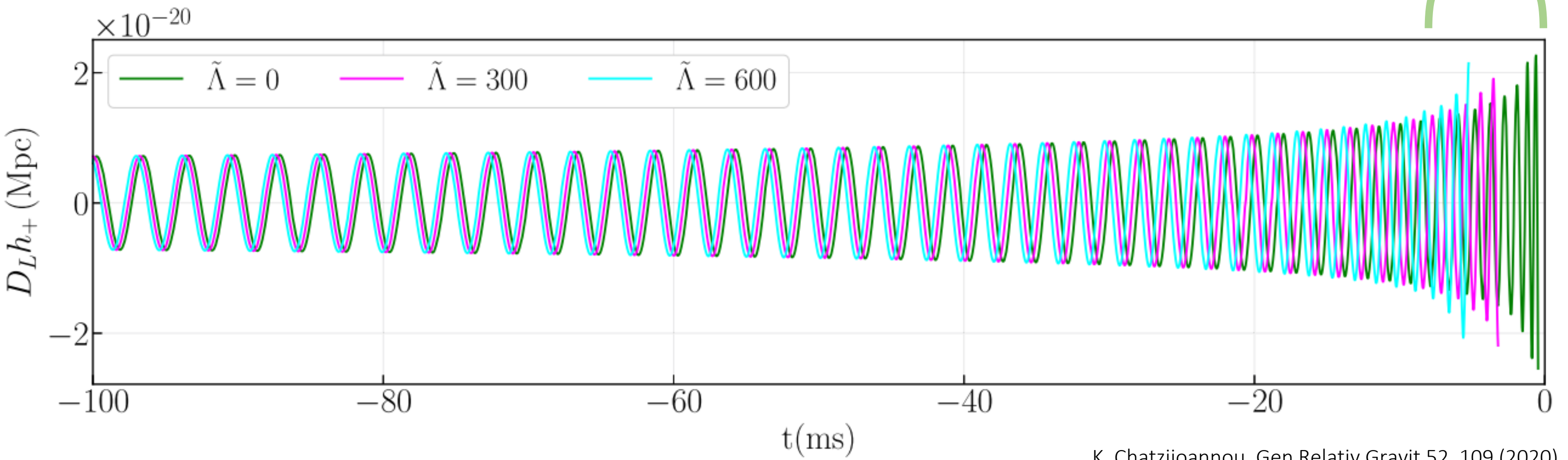
MIT News: Neutron star collisions are a "goldmine" of heavy elements, study finds

Measuring tidal deformability with GWs

Combined dimensionless tidal deformability:

$$\tilde{\Lambda} \equiv \frac{16}{3} \frac{(m_1 + 12m_2)m_1^4\Lambda_1 + (m_2 + 12m_1)m_2^4\Lambda_2}{(m_1 + m_2)^5}$$

Tidal effects significant at late stages of inspiral



K. Chatziioannou, Gen Relativ Gravit 52, 109 (2020)

Neutron star equation of state inference

- We develop a Machine Learning (ML) tool that can perform Bayesian inference of the NS EOS

- Input: posterior samples of component masses (m_1, m_2) and dimensionless tidal deformability parameters (Λ_1, Λ_2) as input

- Output: posterior EOS distribution, i.e., ρ-P dependence

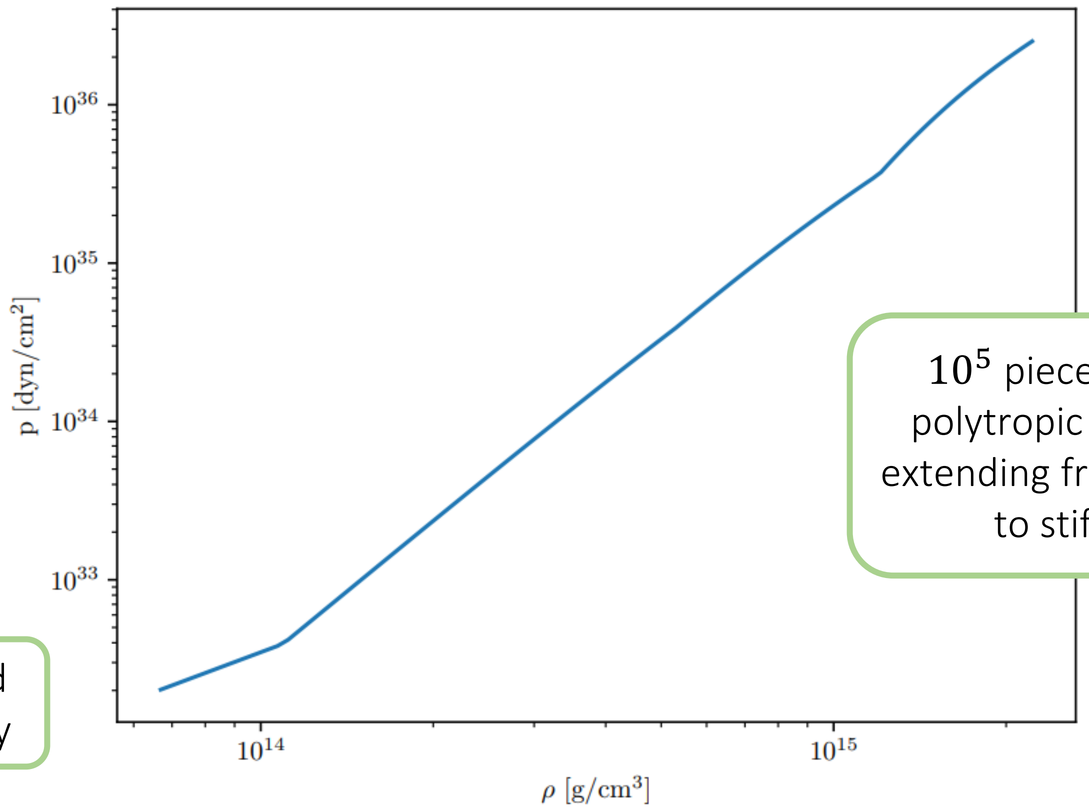
- The method is **intrinsically model independent**

Physical conditions on equation of state

10^5 NS EOSs for training, ensuring **thermodynamic stability** and **causality**

Fixed low density EOS is SLy4

Matched at fixed crossover density



10^5 piecewise polytropic EOSs, extending from soft to stiff

EOS training data preparation

Select the equation of state

Each equation of state consists of energy density computed on a fixed grid of pressure.
Each equation of state consists of 105 points in the pressure-density space, truncated to retain high density information.

EOS training data preparation

Select the equation of state

Define component mass prior range and sample uniformly

Each EOS has a defined maximum mass. We define a uniform prior between $0.5M_{\odot}$ and the maximum mass allowed by this EOS. Component masses are then sampled ensuring that $m_1 \geq m_2$.

EOS training data preparation

EOS and maximum mass determines maximum energy density → component masses → component central energy densities → tidal deformability

Select the equation of state

↓

Define component mass prior range and sample uniformly

↓

Determine central densities and tidal deformability of components

EOS training data preparation

Select the equation of state

Define component mass prior range and sample uniformly

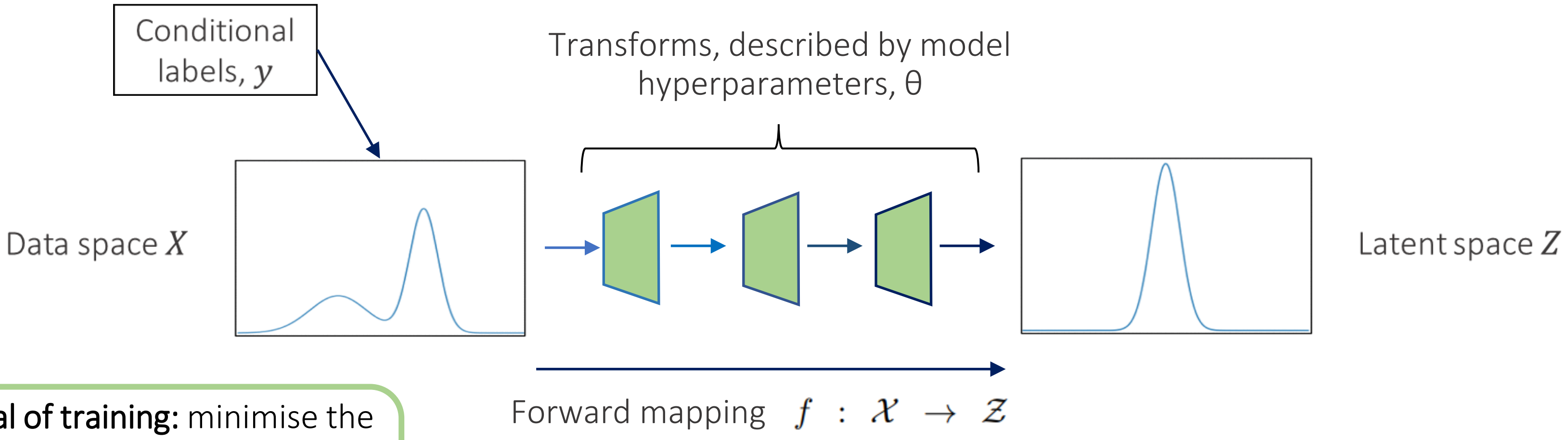
Determine central densities and tidal deformability of components

Training data: an EOS in energy density ρ on fixed grid of pressure P with corresponding auxiliary parameters; central densities ρ_1 and ρ_2 and maximum allowed density ρ_{max}

Conditional data: an associated label $[m_1, m_2, \Lambda_1, \Lambda_2]$

Normalising Flows

Normalising flows map a complex distribution to a standard Gaussian, with zero mean and unit variance, through a series of invertible transforms.

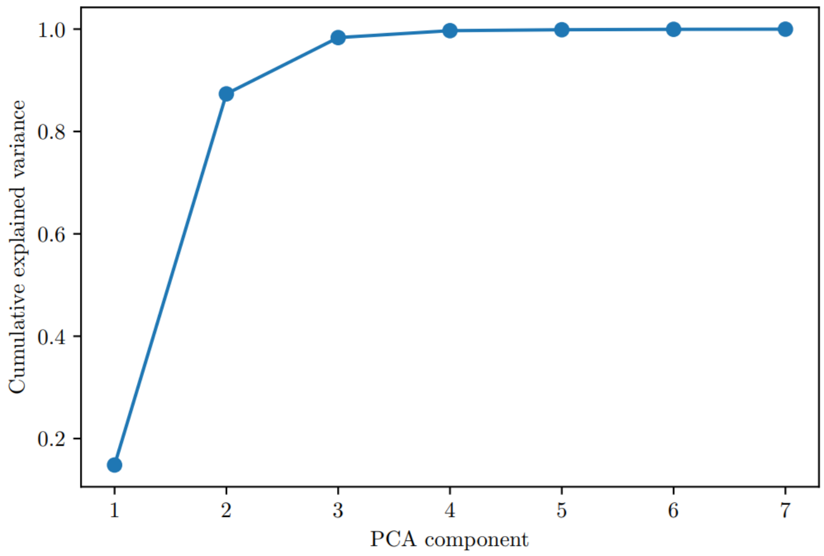


Goal of training: minimise the KL divergence between learned distribution and data space distribution X

Images: I. Kobyzev, S. J. Prince, and M. A. Brubaker, 43, 3964–3979 (2021)

Principal component analysis

- Reduce dimensionality of the training data space with principal component analysis (PCA)
- 7 PCA components represent the 105 data points along each EOS.
- Training data space then consists of 10 dimensions – **7 PCA components plus 3 auxiliary parameters** (central density of each component and maximum allowed density)
- Total explained variance 99.975%

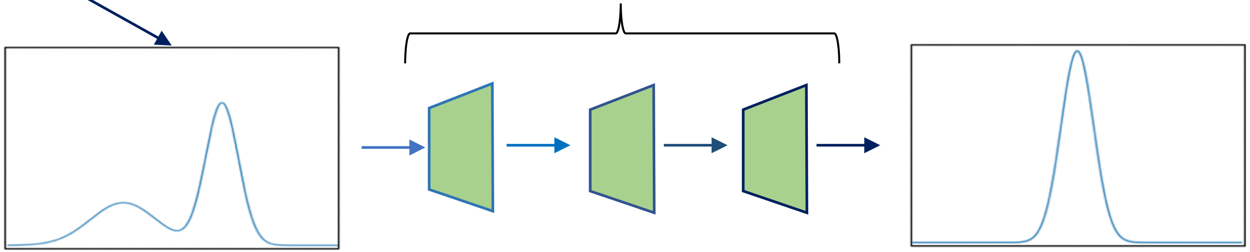


Training ASTREOS

Conditional labels, y: $[m_1, m_2, \Lambda_1, \Lambda_2]$

3 transforms, each with 2 residual blocks consisting of 151 neurons

Data space X: EOS data (7 PCA) and 3 auxiliary parameters

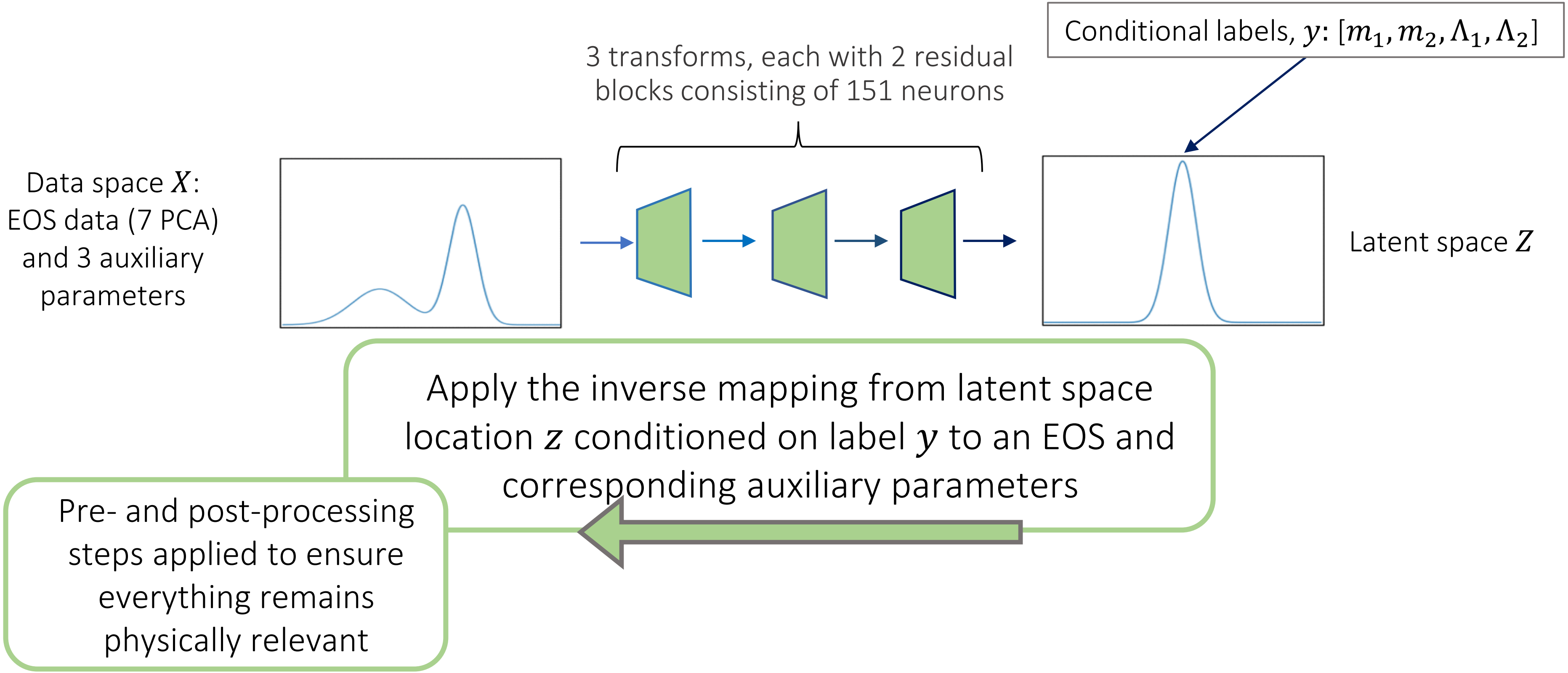


Latent space Z

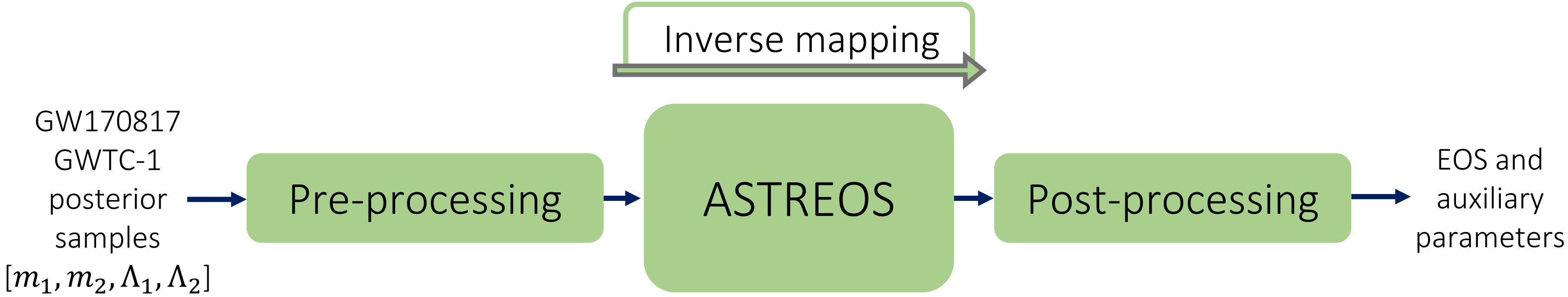
Flow learns the **forward mapping** from **EOS and auxiliary data** conditioned on NS parameters to **multi-dimensional standard Gaussian**

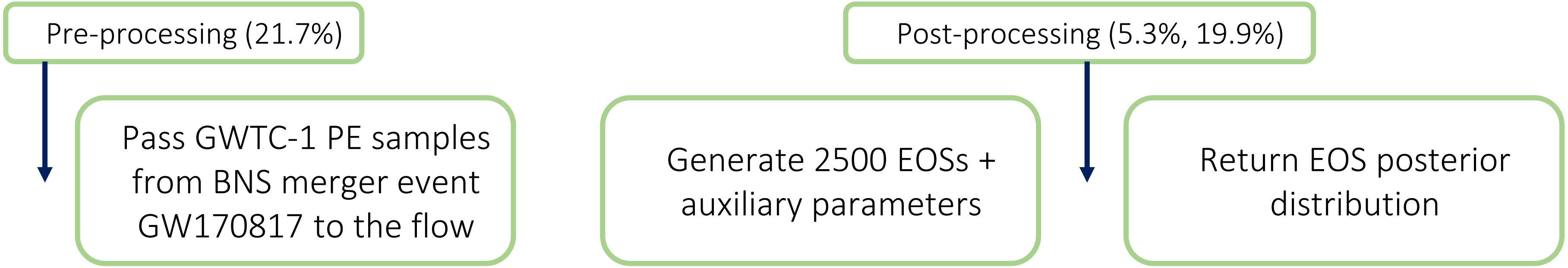
Training requires ~2 hours using a GPU (NVIDIA Tesla V100, ~2 GB memory footprint

Using ASTREOS for EOS inference



Results – GW170817 workflow





Results – GW170817 workflow

Inverse mapping

GW170817 GWTC-1 posterior samples $[m_1, m_2, \Lambda_1, \dots$

ASTREOS

Pre-processing

Post-processing

EOS and auxiliary parameters

~0.1 seconds

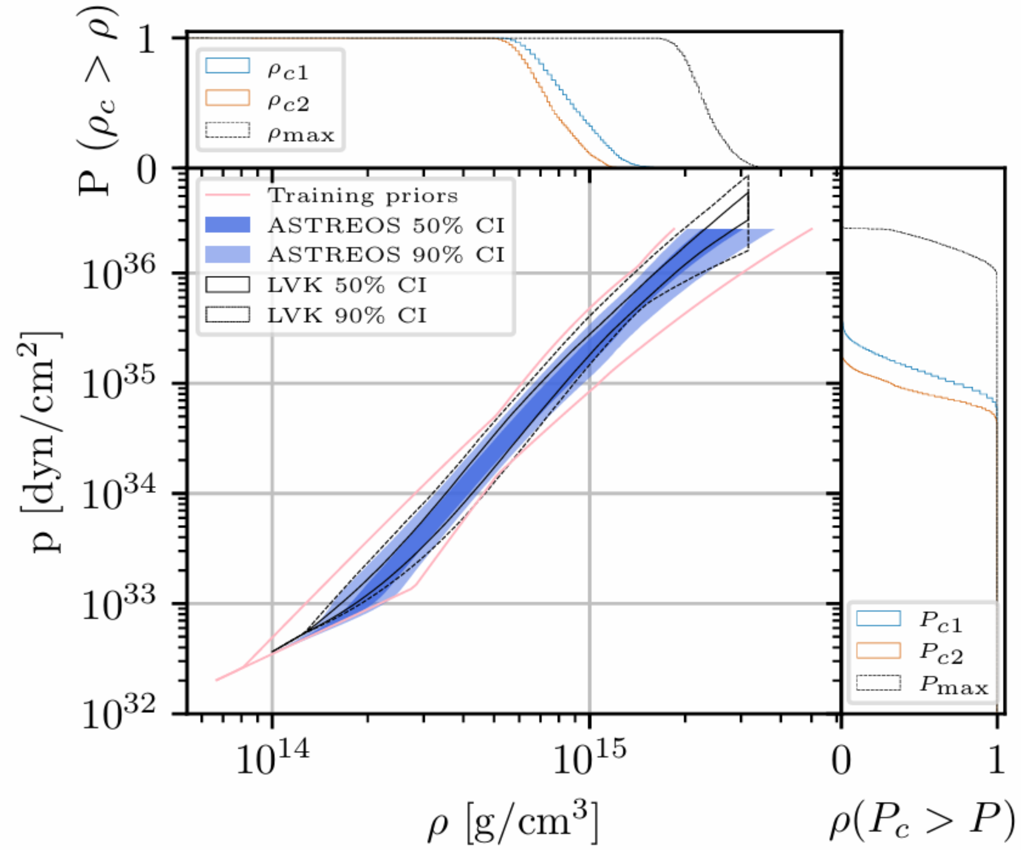
Pre-processing

Pass GW170817 samples from BNS merger event GW170817 to the flow

Generate 2500 EOSs + auxiliary parameters

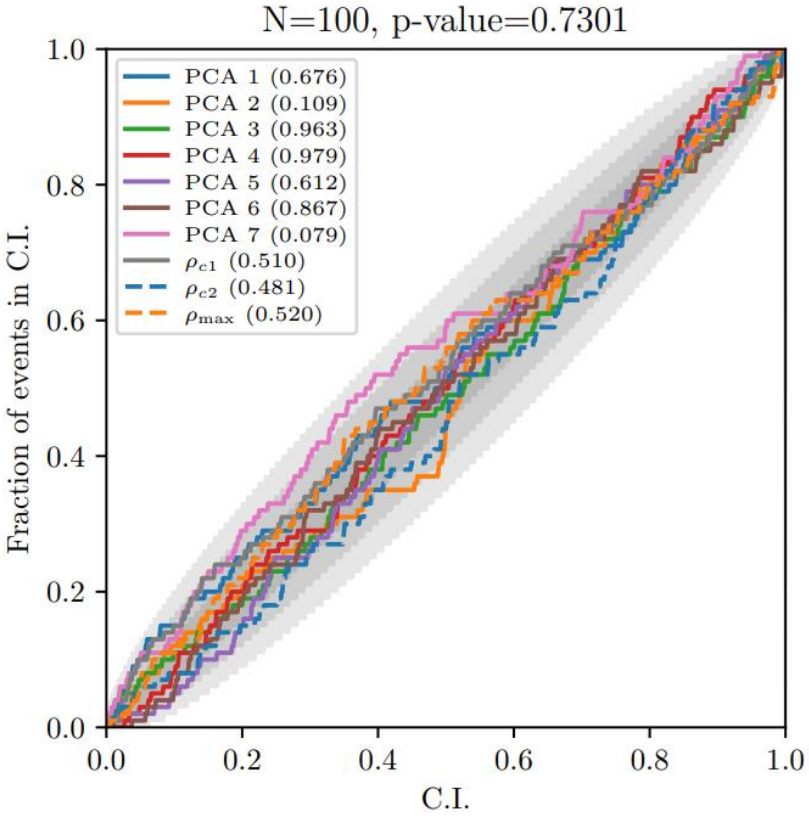
Return EOS posterior distribution

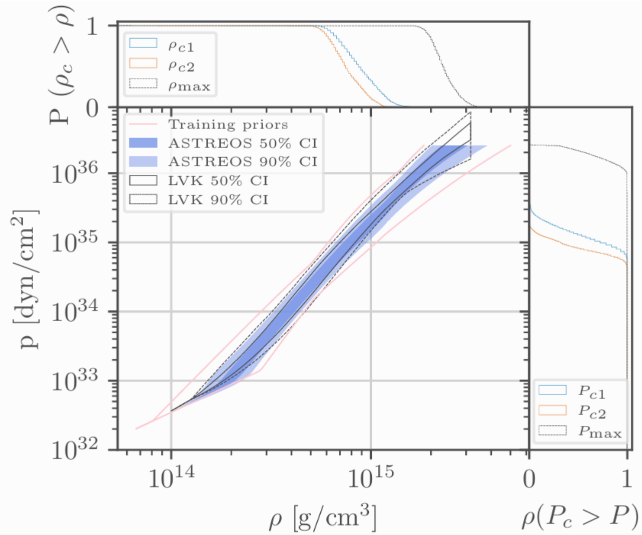
Results – GW170817 EOS posterior



- Confidence intervals of 2500 EOS curves generated by passing GWTC-1 posterior samples to the Flow
- ~0.1 seconds to generate 2500 EOS and auxiliary parameter samples
- ASTREOS demonstrates broad general agreement with LVK

Results and discussion





Pass 2000 samples for conditional label y to the flow to obtain 2000 EOS + corresponding auxiliary parameters

Isolated test demonstrated ASTREOS is statistically robust

Conclusions and future work

- Flows can accurately and rapidly infer the neutron star equation of state
- Needs to be trained only once for repeatedly performing rapid inference for <u>all</u> possible future events
- Explicit model independent approach
- Easily modifiable for alternative conditional statements
- Complements existing literature and developments of ML in low latency GW science
- Easier to combine over multiple GW events and potential implications for BNS population inference

Rapid neutron star equation of state inference with Normalising Flows

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