

IMPROVING THEORETICAL PREDICTIONS FOR LHC PROCESSES BY RESUMMATION

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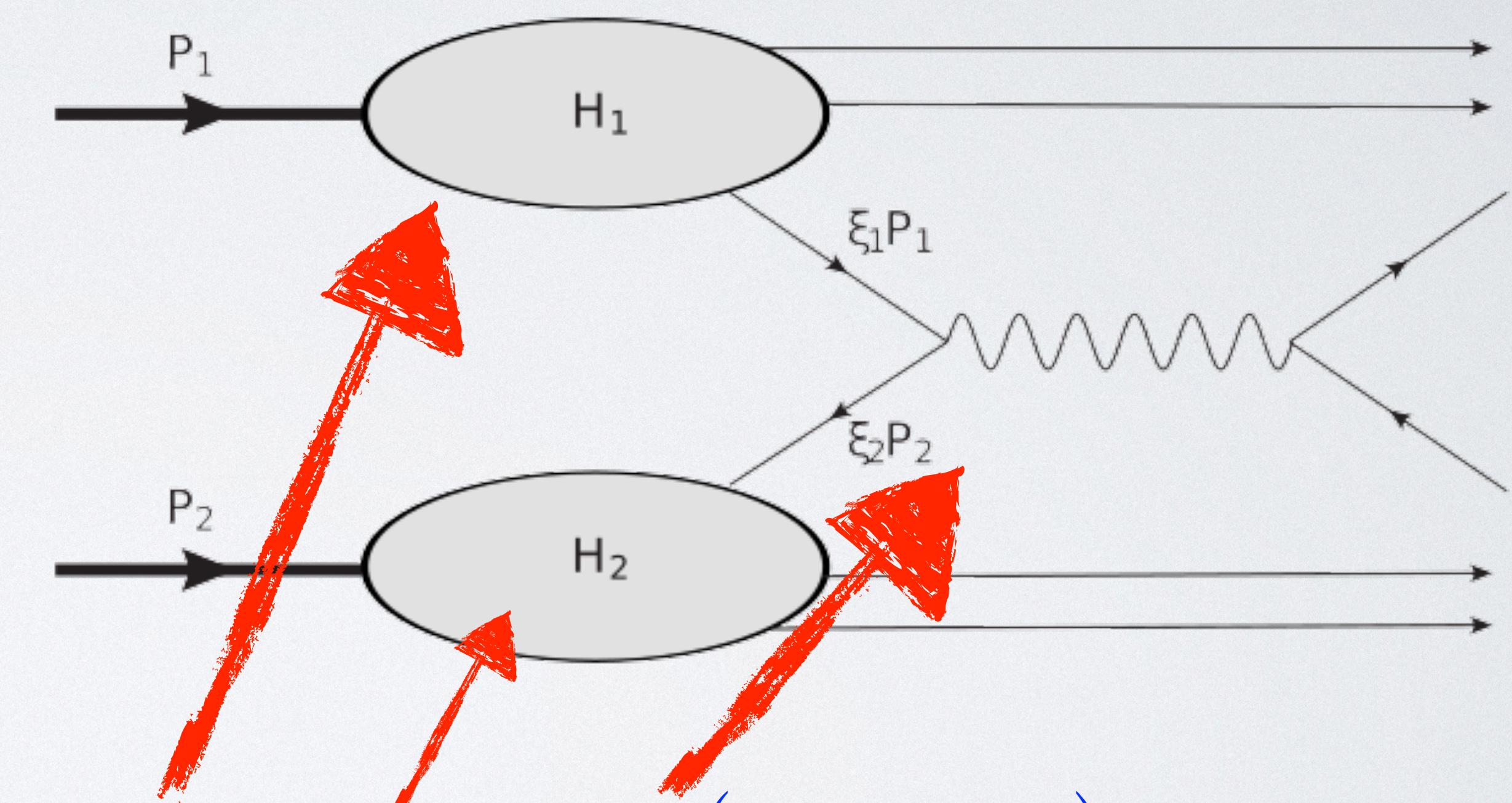
Based on: JHEP 10(2023)126

OUTLINE

- Translate experiment to theory
- Divergences in calculations and their consequences
- All orders at once: resummation
- Rapidity distributions

FROM EXPERIMENT TO THEORY

- Experiment: colliding hadrons
- Theory: colliding partons
- Parton distribution functions $f(\xi)$



$$d\sigma_{H_1 H_2}(\{X_n\}) = \sum_{i,j} \int_{\xi_1,\min}^1 d\xi_1 \int_{\xi_2,\min}^1 d\xi_2 f_{i,H_1}(\xi_1) f_{j,H_2}(\xi_2) d\hat{\sigma}_{ij}(\xi_1, \xi_2, \{X_n\})$$

DIVERGENCES...

- UV divergence $k^\mu \rightarrow \infty$
- Soft divergence $k^0 \rightarrow 0$
- Collinear divergence $\theta \rightarrow 0$

$$\frac{1}{(p+k)^2} = \frac{1}{2p^0 k^0 (1 - \cos \theta)}$$

... AND THEIR CONSEQUENCES

- KLN theorem
- Invalidates perturbative calculations due to large logarithms
- Consider all orders in coupling constant to regain predictive power

... AND THEIR CONSEQUENCES

$$\frac{d\hat{\sigma}}{dQ^2} = \frac{1}{\hat{s}} \frac{d\hat{\sigma}}{dz} = \sum_{n=0}^{\infty} \alpha_s^n \left(d_n \delta(1-z) + \sum_{m=0}^{2n-1} c_{nm}^{\text{LP}} \left[\frac{\log^m(1-z)}{(1-z)} \right]_+ + c_{nm}^{\text{NLP}} \log^m(1-z) + c_{nm}^{\text{NNLP}} (1-z) \log^m(1-z) + \dots \right)$$

- Expand around $z \equiv Q^2/\hat{s} \rightarrow 1$, equivalent to $k_{\text{gluon}}^\mu \rightarrow 0$
- Leading power: resummation known
- Next-to-leading power: quite involved, working on it...

LET'S BREAK IT DOWN

$$\frac{d\hat{\sigma}}{dQ^2} = \frac{1}{\hat{s}} \frac{d\hat{\sigma}}{dz} = \sum_{n=0}^{\infty} \alpha_s^n \left(d_n \delta(1-z) + \sum_{m=0}^{2n-1} c_{nm}^{\text{LP}} \left[\frac{\log^m(1-z)}{(1-z)} \right]_+ + c_{nm}^{\text{NLP}} \log^m(1-z) + c_{nm}^{\text{NNLP}} (1-z) \log^m(1-z) + \dots \right)$$

	LL	NLL	NNLL
LO	1		
NLO	$\alpha_s L^2$	$\alpha_s L$	α_s
NNLO	$\alpha_s^2 L^4$	$\alpha_s^2 L^3$	$\alpha_s^2 L^2$
N ⁿ LO	$\alpha_s^n L^{2n}$	$\alpha_s^n L^{2n-1}$	$\alpha_s^n L^{2n-2}$

LET'S BREAK IT DOWN

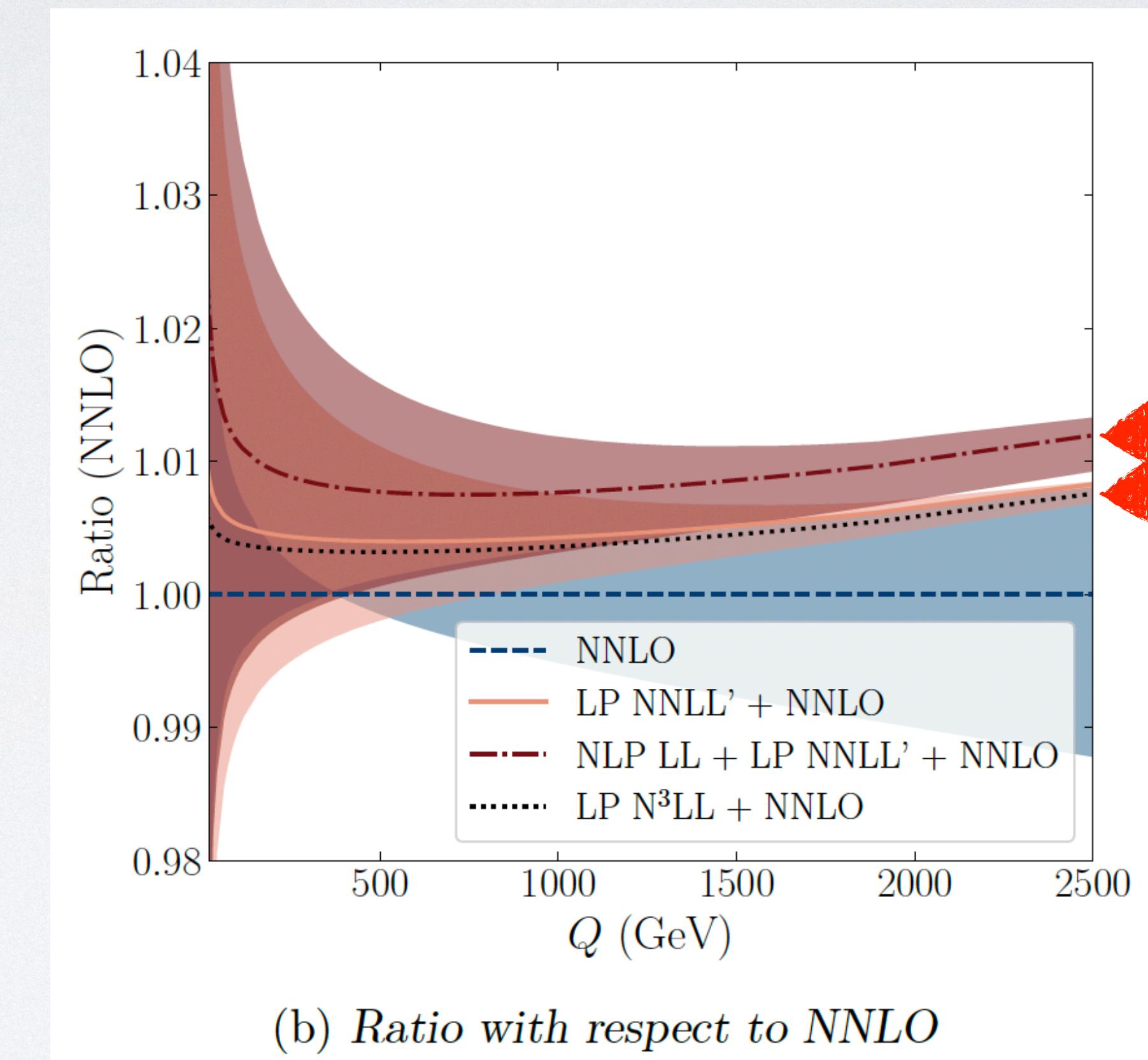
$$\frac{d\hat{\sigma}}{dQ^2} = \frac{1}{\hat{s}} \frac{d\hat{\sigma}}{dz} = \sum_{n=0}^{\infty} \alpha_s^n \left(d_n \delta(1-z) + \sum_{m=0}^{2n-1} c_{nm}^{\text{LP}} \left[\frac{\log^m(1-z)}{(1-z)} \right]_+ + c_{nm}^{\text{NLP}} \log^m(1-z) + c_{nm}^{\text{NNLP}} (1-z) \log^m(1-z) + \dots \right)$$

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$N^n \text{LO}$	$\alpha_s^n L^{2n}$	$\alpha_s^n L^{2n-1}$	$\alpha_s^n L^{2n-2}$

NLP LL EFFECT

- DY cross section

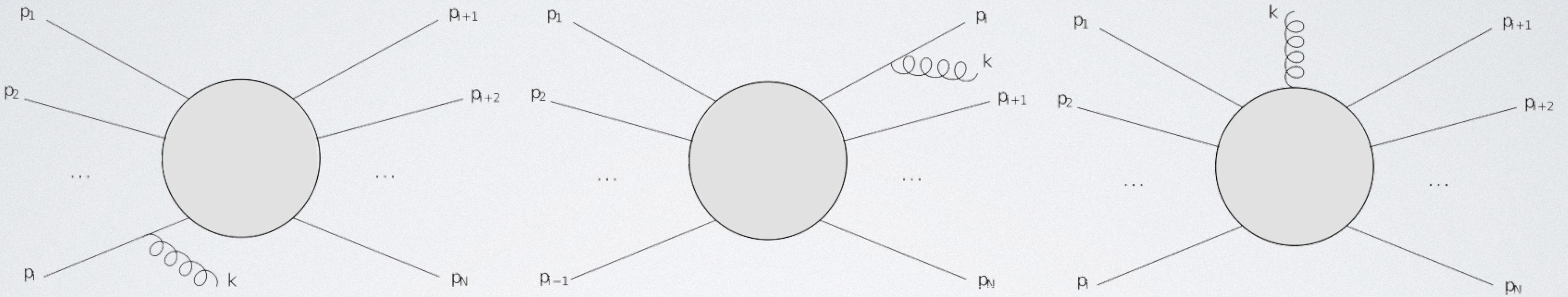
$$\frac{d\sigma_{pp \rightarrow \ell^+ \ell^- + X}}{dQ^2}$$



Van Beekveld et al., JHEP 05(2021)|14

SOFT LIMIT

- Soft theorem



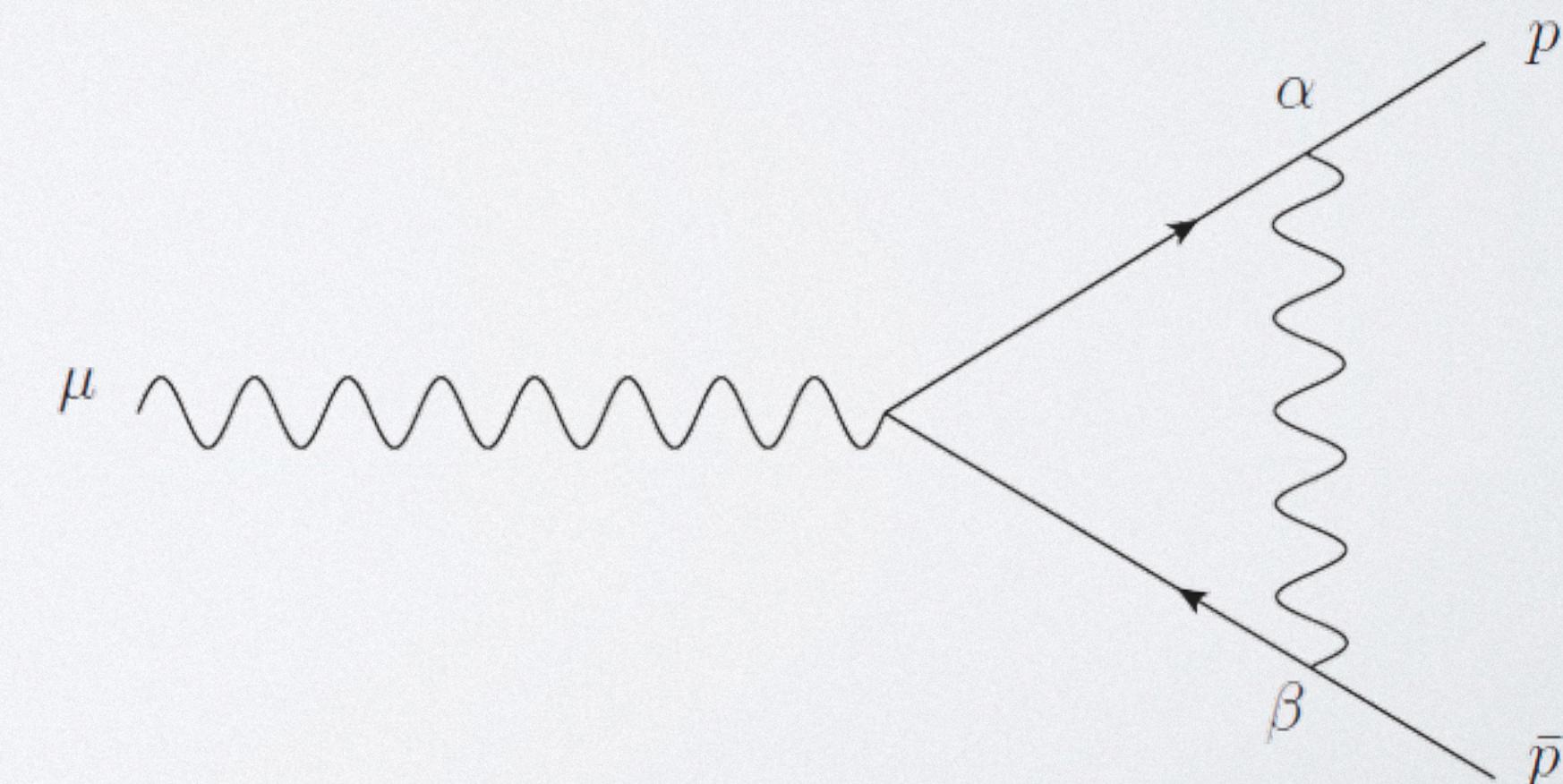
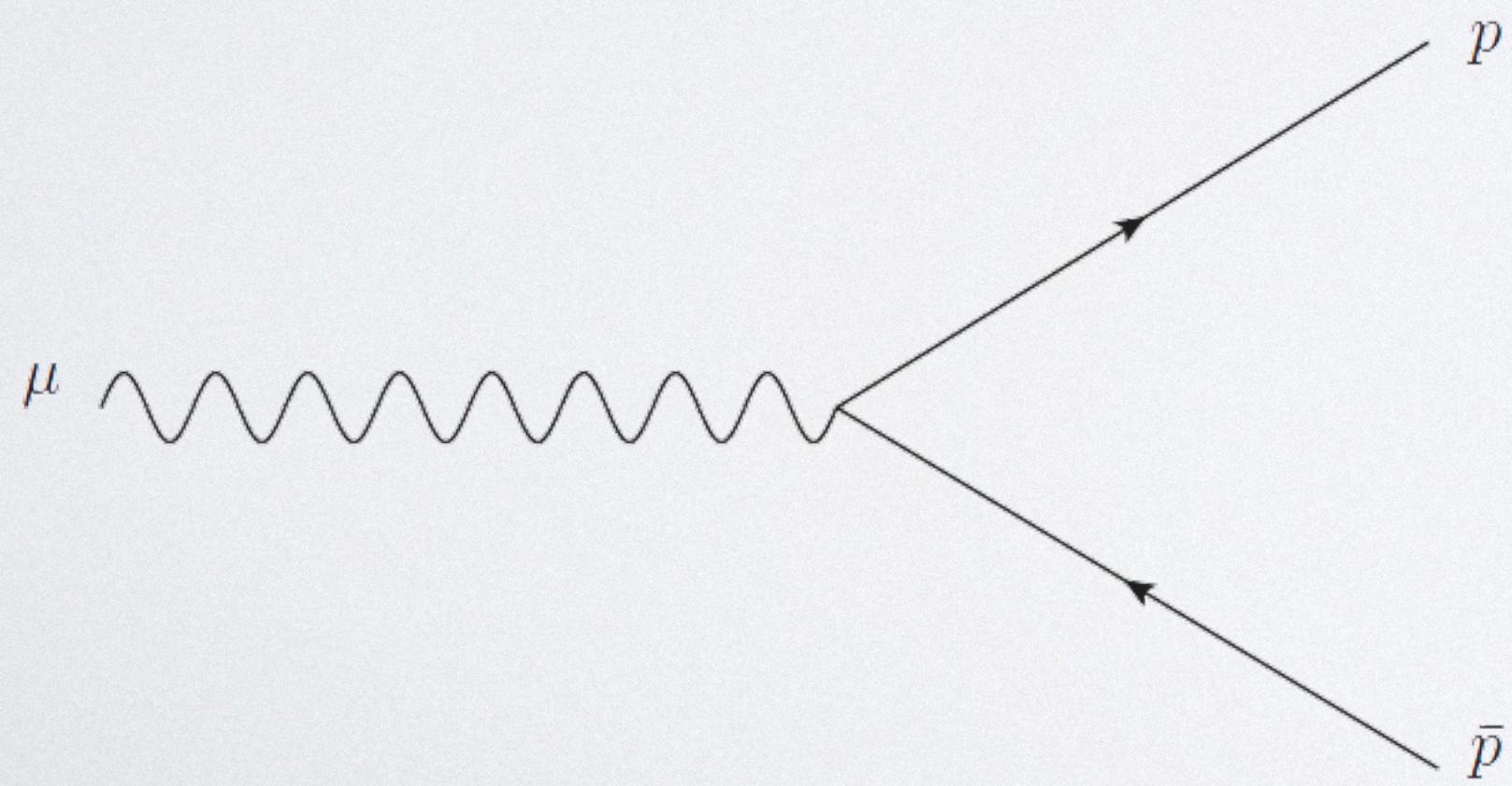
$$\mathcal{M}_{N+1}^{\mu \text{ NLP}} = \sum_{i=1}^N \frac{g_s T_i}{2p_i \cdot k} \left(\underbrace{2p_i^\mu + k^\mu - 2ik_\nu S_{(i)}^{\mu\nu} + 2ik_\nu L_{(i)}^{\mu\nu}}_{\text{scalar}} \right) \otimes \mathcal{M}_N(p_1, \dots, p_N)$$

SOFT LIMIT AT LEADING POWER

- Use effective Feynman rules

$$i\mathcal{M}^{(0)\mu} = -ie\bar{u}(p)\gamma^\mu v(\bar{p})$$

$$i\mathcal{M}^{(1)\mu} = [-ie\bar{u}(p)\gamma^\mu v(\bar{p})] \left[(-ie)^2 \int \frac{d^d k}{(2\pi)^d} \left(-\frac{p^\alpha}{p \cdot k} \right) \left(\frac{\bar{p}^\beta}{\bar{p} \cdot k} \right) \left(\frac{-i\eta_{\alpha\beta}}{k^2} \right) \right]$$



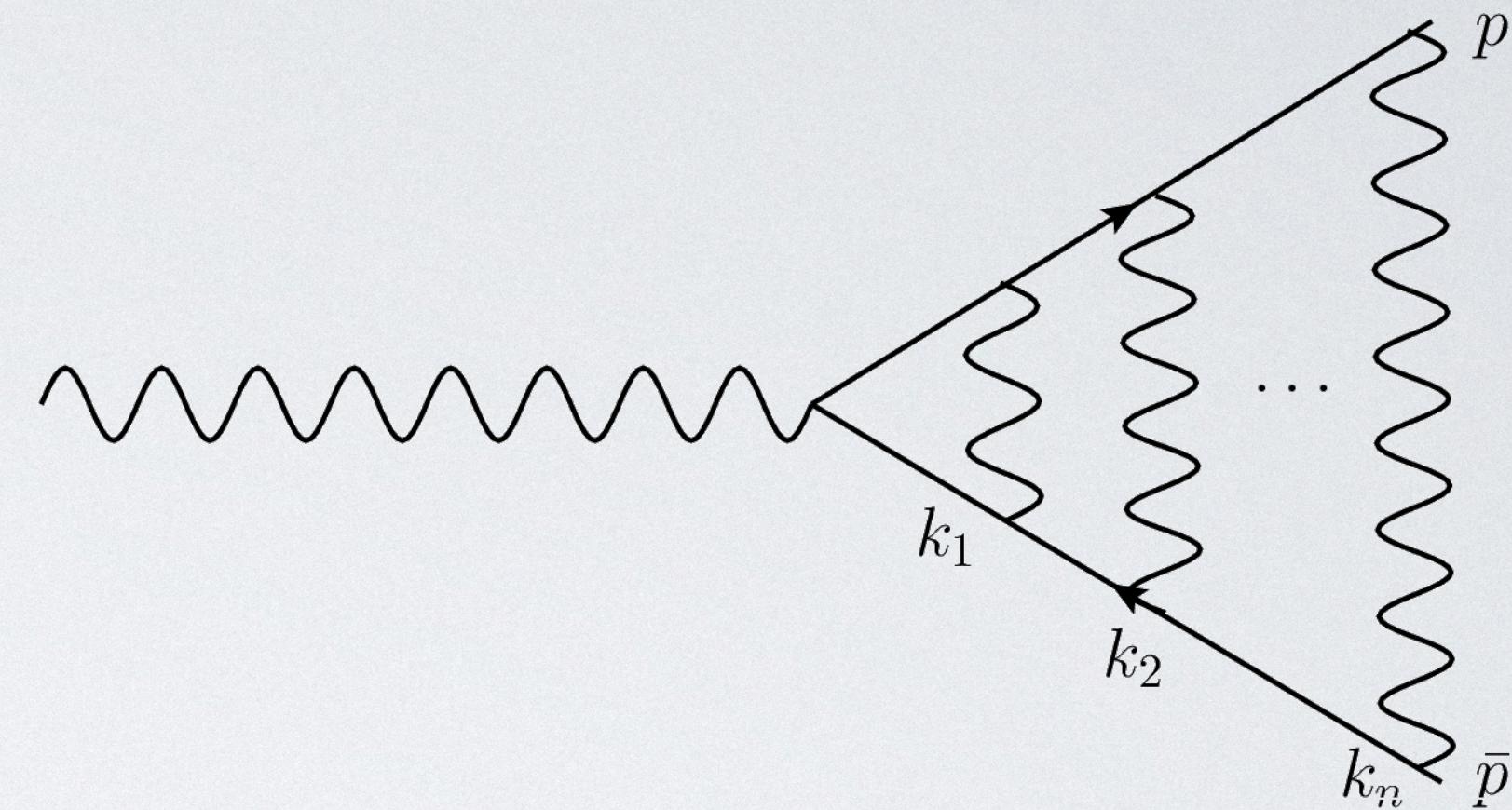
SOFT LIMIT AT LEADING POWER

- Photon emissions are independent
- This result exponentiates

$$i\mathcal{M}^{(n)\mu} = -ie\bar{u}(p)\gamma^\mu v(\bar{p}) \frac{1}{n!} \left[(-ie)^2 \int \frac{d^d k}{(2\pi)^d} \left(\frac{-p^\alpha}{p \cdot k} \right) \left(\frac{\bar{p}^\beta}{\bar{p} \cdot k} \right) \left(\frac{-in_{\alpha\beta}}{k^2} \right) \right]^n$$

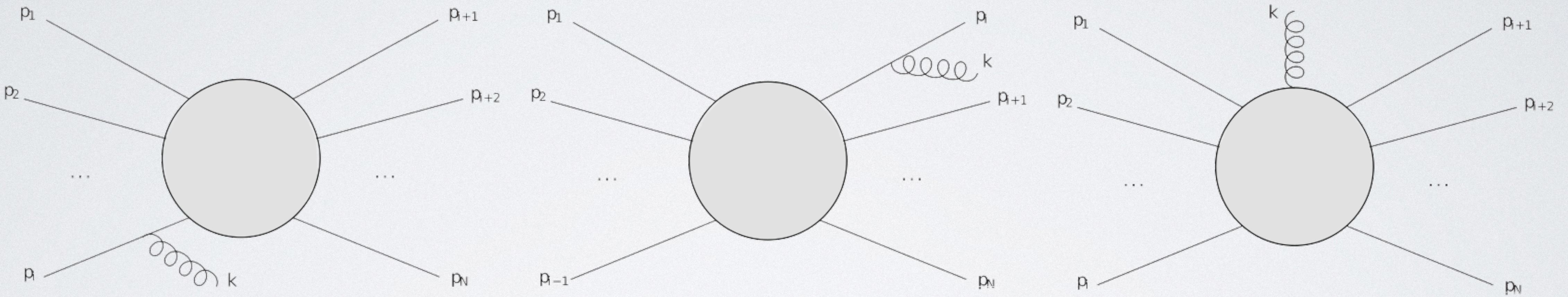


$$i\mathcal{M}^\mu = -ie\bar{u}(p)\gamma^\mu v(\bar{p}) \exp \left[(-ie)^2 \int \frac{d^d k}{(2\pi)^d} \left(\frac{-p^\alpha}{p \cdot k} \right) \left(\frac{\bar{p}^\beta}{\bar{p} \cdot k} \right) \left(\frac{-in_{\alpha\beta}}{k^2} \right) \right]$$



SOFT LIMIT

- Soft theorem



$$\mathcal{M}_{N+1}^{\mu \text{ NLP}} = \sum_{i=1}^N \frac{g_s \mathbf{T}_i}{2p_i \cdot k} \left(\underbrace{2p_i^\mu + k^\mu}_{\text{scalar}} - \underbrace{2ik_\nu S_{(i)}^{\mu\nu}}_{\text{spin}} + \underbrace{2ik_\nu L_{(i)}^{\mu\nu}}_{\text{orbital}} \right) \otimes \mathcal{M}_N(p_1, \dots, p_N)$$

THRESHOLD RESUMMATION

- $q\bar{q}$ or gg initial state
- No QCD in final state
- Use soft theorem

$$\alpha_s = \frac{g_s^2}{4\pi}$$

$$\left| \mathcal{M}_{\text{NLO,NLP}} \right|^2 \sim \alpha_s \frac{p_1 \cdot p_2}{(p_1 \cdot k)(p_2 \cdot k)} \left| \mathcal{M}_{\text{LO}}(p_1 + \delta p_1, p_2 + \delta p_2) \right|^2$$

$$\frac{d\hat{\sigma}_{\text{NLO, NLP}}^{(q\bar{q}/gg)}}{dz} = C_{F/A} K_{\text{NLP}}(z, \alpha_s, \epsilon) \hat{\sigma}_{\text{LO}}^{(q\bar{q}/gg)}(Q^2)$$

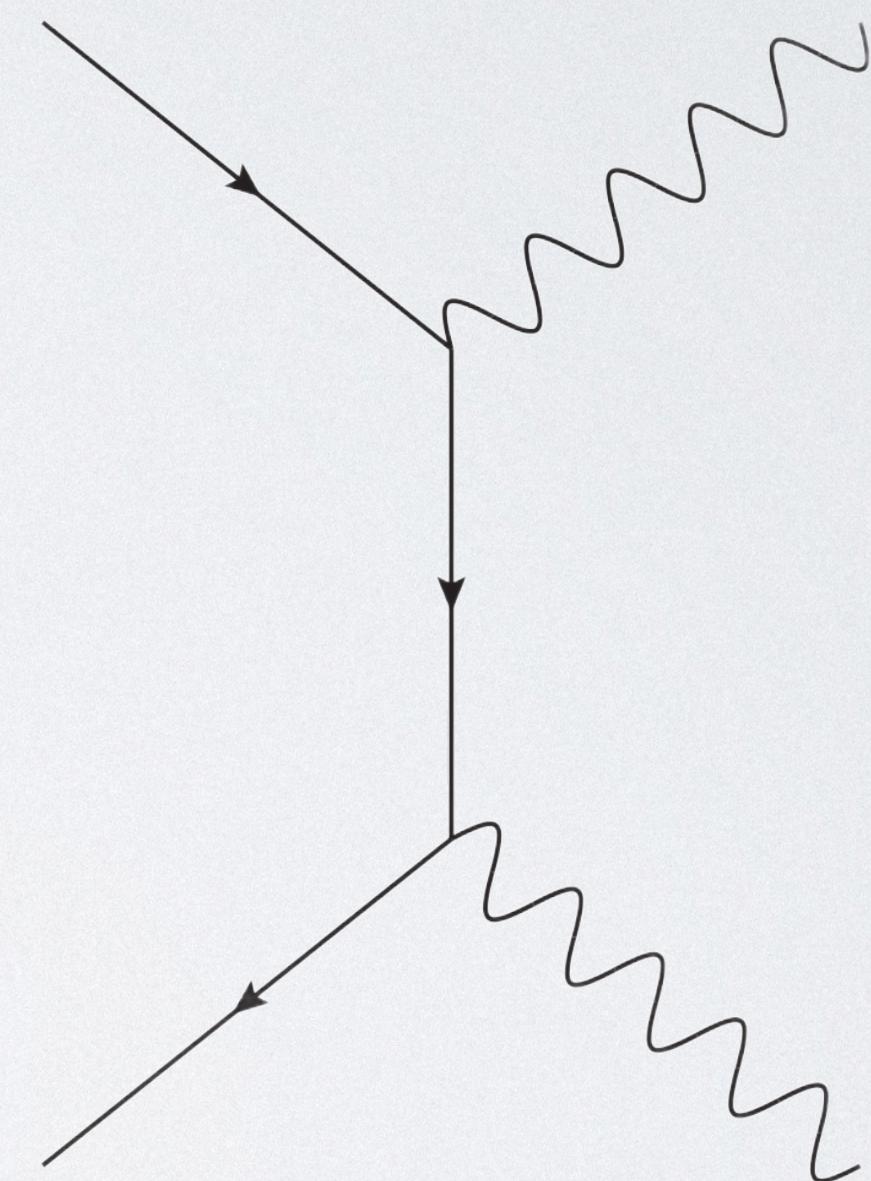
THRESHOLD RESUMMATION

- Generalise K-factor
- Mellin transform $f(N) = \int_0^1 dz z^{N-1} f(z)$
- Resummation for leading logarithms at LP and NLP

$$\frac{d\hat{\sigma}_{\text{NLP}}^{(q\bar{q}/gg)}}{dz} = \hat{\sigma}_{\text{LO}}^{(q\bar{q}/gg)}(Q^2) \exp \left[\frac{2\alpha_s C_{F/A}}{\pi} \log^2 N \right] \left(1 + \frac{2\alpha_s C_{F/A}}{\pi} \frac{\log N}{N} \right)$$

RAPIDITY DISTRIBUTIONS

- So far: only invariant mass distributions
- Rapidity measures the angle: $\eta = -\log \left(\tan \frac{\theta}{2} \right)$
- Extend known methods for e.g. diphoton production $q\bar{q} \rightarrow \gamma\gamma$
- Important for Higgs



RAPIDITY DISTRIBUTIONS

- Want to apply previous results to this new case
- Shift in kinematics also affects rapidity
- Can we factorise the rapidity dependence?
- Gluon emissions are not independent a priori

RAPIDITY DISTRIBUTIONS

- Rapidity dependence completely factors out

$$\frac{d\hat{\sigma}_{q\bar{q} \rightarrow \gamma\gamma}}{dz d\eta} = \hat{\sigma}_{\text{LO}}(Q^2, \eta) \exp \left[\frac{2\alpha_s C_F}{\pi} \log^2 N \right] \left(1 + \frac{2\alpha_s C_F \log N}{\pi} \right)$$

- Cannot go beyond LL at NLP

CONCLUSION

- Large logarithms spoil convergence perturbative series
- Resum these large logs to cure this problem
- Extend known methods to resum cross sections at NLP with additional rapidity dependence
- Next step: extend framework for NLP to include more than LL

THANK YOU!

BACK UP SLIDES

APPROXIMATE NLO CROSS SECTION

$$\delta p_1^\alpha = -\frac{1}{2} \left(\frac{p_2 \cdot k}{p_1 \cdot p_2} p_1^\alpha - \frac{p_1 \cdot k}{p_1 \cdot p_2} p_2^\alpha + k^\alpha \right), \quad \delta p_2^\alpha = -\frac{1}{2} \left(\frac{p_1 \cdot k}{p_1 \cdot p_2} p_2^\alpha - \frac{p_2 \cdot k}{p_1 \cdot p_2} p_1^\alpha + k^\alpha \right)$$

$$\left| \mathcal{M}_{\text{NLP}}^{(q\bar{q})} \right|^2 = \frac{2g_s^2 C_F p_1 \cdot p_2}{(p_1 \cdot k)(p_2 \cdot k)} \left| \mathcal{M}_{\text{LO}}(p_1 + \delta p_1, p_2 + \delta p_2) \right|^2$$

$$K_{\text{NLP}}(z, \alpha_s, \mu^2, \hat{s}, \epsilon) = \frac{\alpha_s}{\pi} \left(\frac{4\pi\mu^2}{\hat{s}} \right)^\epsilon z (1-z)^{-1-2\epsilon} \frac{\Gamma^2(-\epsilon)}{\Gamma(-2\epsilon)\Gamma(1-\epsilon)}$$

Del Duca et al., JHEP 11(2017)057

DIPHOTON

$$\overline{|\mathcal{M}|_{\text{LO}}^2} = \frac{2(ee_q)^4}{N_c} \left[\frac{1 + \cos^2 \theta}{1 - \cos^2 \theta} (1 - \epsilon)^2 - \epsilon(1 - \epsilon) \right]$$

$$\frac{d\hat{\sigma}_{q\bar{q}}^{(0)}}{dz d\eta}(\hat{s}, z, \eta) = \frac{\pi \alpha_{\text{EM}}^2 e_q^4}{N_c \hat{s}} (1 + \tanh^2 \eta) \delta(1 - z)$$

$$\frac{d\sigma}{dQ^2 dY} = \frac{1}{s} \int_{\tau}^1 \frac{dz}{z} \int_{\log\left(\sqrt{\frac{\tau}{z}} e^Y\right)}^{\log\left(\sqrt{\frac{z}{\tau}} e^Y\right)} d\eta \mathcal{L}(z, \eta) \frac{d\hat{\sigma}}{dz d\eta}(\hat{s}, z, \eta)$$

NLP PHASE SPACE

$$\hat{\sigma} = \frac{1}{2\hat{s}} \left[\int d\Phi_{\text{LP}} \left| \mathcal{M} \right|_{\text{LP}}^2 + \int d\Phi_{\text{LP}} \left| \mathcal{M} \right|_{\text{NLP}}^2 + \int d\Phi_{\text{NLP}} \left| \mathcal{M} \right|_{\text{LP}}^2 + \mathcal{O}(\text{NNLP}) \right]$$

$$\left| \mathcal{M} \right|_{\text{LP},n}^2 = f(\alpha_s, \epsilon, \mu^2, \eta) \prod_{i=1}^n \frac{p_1 \cdot p_2}{p_1 \cdot k_i p_2 \cdot k_i}$$

$$\delta(Q^2 - (p_3 + p_4)^2) = \frac{1}{\hat{s}} \delta \left(1 - z - \frac{2 \sum_i k_i \cdot (p_1 + p_2)}{\hat{s}} + \frac{2 \sum_{i < j} k_i \cdot k_j}{\hat{s}} \right)$$