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# Simulation of the gravitational-wave emission of core-collapse supernovae

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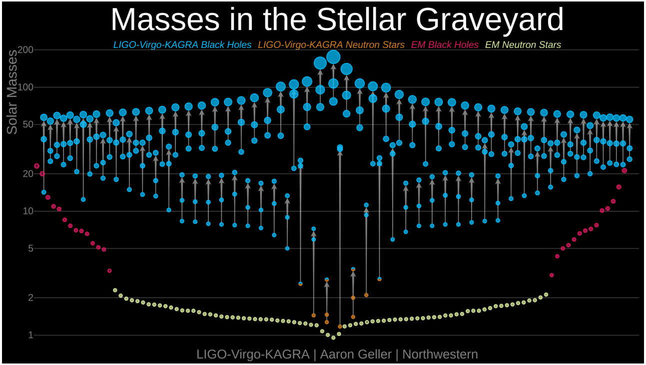
# 1 Simulations and gravitational waves

GR simulations ⟶ waveform templates ⟶ detection + PE

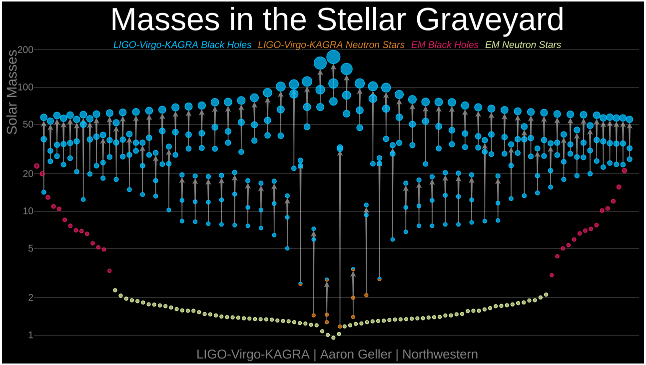



Credit: LIGO-Virgo / Aaron Geller / Northwestern University

# 1 Simulations and gravitational waves

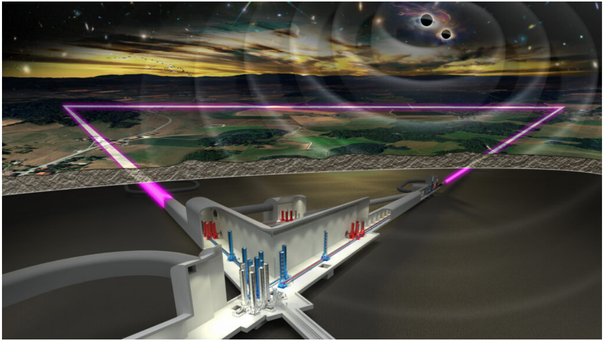
GR simulations ⟶ waveform templates ⟶ detection + PE

⟶ Testing models + understand physics ⟵




Credit: LIGO-Virgo / Aaron Geller / Northwestern University

# 1 The Einstein Telescope



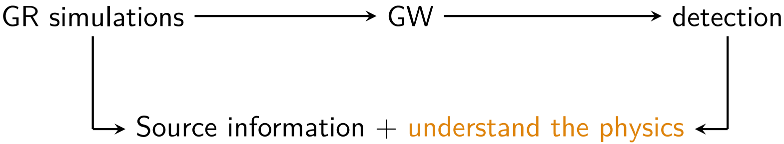
Credit: Nikhef / Marco Kraan

New sources $\Rightarrow$ we need to be ready!

# 1 Goal

Simulation of yet unobserved systems generating gravitational waves

- Isolated neutron stars
- Supernovae
- ...




⇒ Need accurate simulations to understand the system + detect signal and interpret the data

## 1 Need for simulations, but...

Simulations are expensive and challenging!

Typically multi-scale (length + time) problems. *E.g.* supernovae:

- Turbulence $\sim \mathcal{O}(1)$m, radius of core $\sim \mathcal{O}(10^6)$m
- Small time steps required, but simulation $\sim \mathcal{O}(1)$s

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Challenges:

- Many simulations to span the parameter space
- Need very accurate simulations
- Systems to simulate (multi-scale, …)

$\Rightarrow$ cost and reliability/accuracy of the simulations

## 2 Physics model

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = T_{\mu\nu}, \quad \text{(Einstein equation)}$$

$$\nabla_\mu \left( \rho u^\mu \right) = 0, \quad \text{(cons. rest mass)}$$

$$\nabla_\mu T^{\mu\nu} = 0, \quad \text{(cons. energy/momentum)}$$

$$p = p\left( \rho, \epsilon, \cdots \right), \quad \text{(equation of state)}$$

$$\begin{cases} \nabla_\mu F^{\mu\nu} = \mathcal{J}^\nu, \\ \nabla_\mu {}^* F^{\mu\nu} = 0, \end{cases} \quad \text{(Maxwell equations)}$$

$$\left( p^\mu \frac{\partial}{\partial x^\mu} - \Gamma^\mu_{\alpha\beta} p^\alpha p^\beta \frac{\partial}{\partial p^\mu} \right) f = \left( \frac{\partial f}{\partial \tau} \right)_{\text{coll}} \quad \text{(Boltzmann equation)}$$

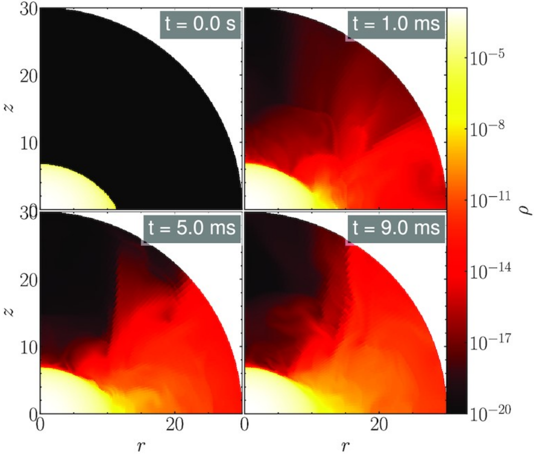
$$\vdots$$

# 2 *Gmunu* solver

1-2-3D general-relativistic (magneto)hydrodynamics (GR(M)HD) (Cheong et al 2020, 2021, 2022, 2023; Ng et al 2023)

Example: rapidly rotating neutron star

Noticeable achievement: NS with $B \sim 10^{17}$G (Leung et al 2022)




Taken from (Cheong et al 2021)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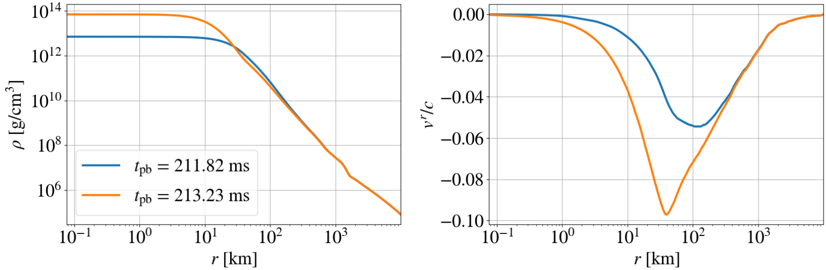
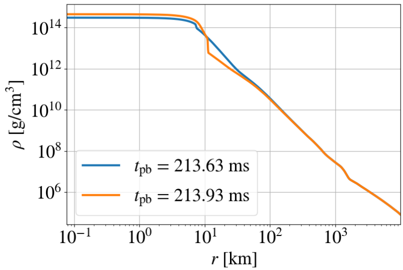
# 3 Core-collapse supernovae

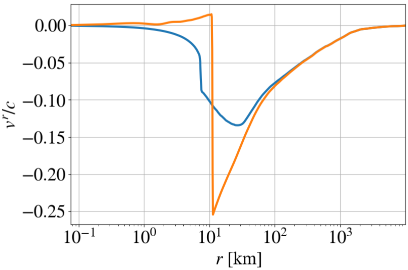
- Collapse of a star when gravity $>$ pressure



# 3 Core-collapse supernovae

- Collapse of a star when gravity > pressure
- Creation of a shock when the core cannot contract anymore = core bounce



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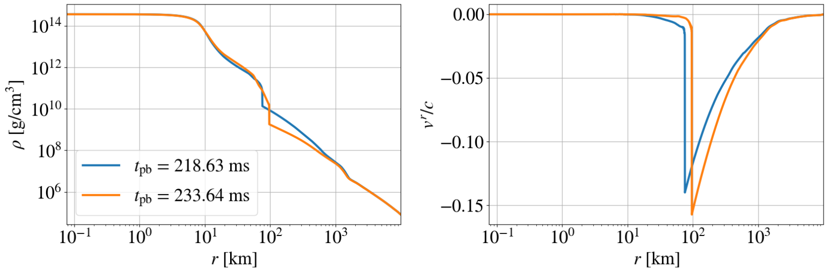

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$\Rightarrow$ Extreme event: very bright, but only $\lesssim 1\%$ of energy in light!
$\sim 99\%$ of energy in neutrinos

## 3 Simulation of core-collapse supernovae

Get gravitational wave signal $\rightarrow$ 2D or 3D simulation to at least few ms after the core bounce

In *Gmunu*

- Simplest 1D simulation (16 cpus): $< 1$h to core bounce ($\sim 213$ms), $\sim 23$h to $250$ms
- Simplest 2D simulation (64 cpus): $\sim 14$h to core bounce

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Advantages of *Gmunu*

- Simulation of the neutron star
- Recenlty improved microphysics (Ng et al 2023)

## 3 Simplifcations

Very expensive simulation $\rightarrow$ simplifications needed

- No magnetic field
- Approximation of Boltzmann equation (6D!) $\rightarrow$ M1 scheme
- Limited simulation domain $\sim 10^4$km ($\sim 100$ NS radius)

Several different codes with different levels of approximation: 1-2-3D, with or without $B$, Newtonian + GR corrections, . . . (see *e.g.* [1–4])

But what do we lose with these approximations?

# 4 Conclusion

- We need simulations to detect new sources + interpret data
- Many simulations to span parameter space, but expensive
- Use the code *Gmunu* to simulate core-collapse supernovae
- Simplifications $\rightarrow$ what do we lose ?

## 6 References I


[1] Shota Shibagaki et al. 2023. arXiv: 2309.05161.

[2] M. Aaron Skinner et al. *The Astrophysical Journal Supplement Series* 241.1 (2019), p. 7.

[3] J Matsumoto et al. *Monthly Notices of the Royal Astronomical Society* 516.2 (2022), pp. 1752–1767.

[4] Evan P. O'Connor and Sean M. Couch. *The Astrophysical Journal* 854.1 (2018), p. 63.

[5] Patrick Chi-Kit Cheong, Lap-Ming Lin, and Tjonnie Guang Feng Li. *Classical and Quantum Gravity* 37.14 (2020), p. 145015.

[6] Patrick Chi-Kit Cheong et al. *Monthly Notices of the Royal Astronomical Society* 508.2 (2021), pp. 2279–2301.

[7] Patrick Chi-Kit Cheong et al. *The Astrophysical Journal Supplement Series* 261.2 (2022), p. 22.

[8] Patrick Chi-Kit Cheong et al. *The Astrophysical Journal Supplement Series* 267.2 (2023), p. 38.

# 6 References II

[10] Christian Y. Cardall, Eirik Endeve, and Anthony Mezzacappa. *Phys. Rev. D* 88 (2 2013), p. 023011.

[11] H.-Th Janka et al. *Phys. Rep.* 442 (2007).

[12] C.D. Ott et al. *Nuclear Physics B - Proceedings Supplements* 235-236 (2013), pp. 381–387.

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# Back-up slides

# 7 Finite volume discretization

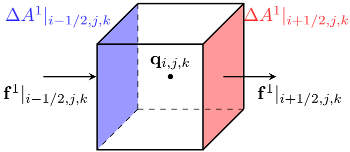
General-relativistic version of conserved equation:

$$\partial_t \boldsymbol{q} + \frac{1}{\sqrt{\hat{\gamma}}}\partial_j \left[\sqrt{\hat{\gamma}}\boldsymbol{f}^j\right] = \boldsymbol{s} + \boldsymbol{s}_{\text{geom}}$$

volume-average $\langle \bullet \rangle \equiv \frac{1}{\Delta V} \int_{\text{cell}} \bullet \sqrt{\hat{\gamma}} dx^1 dx^2 dx^3$ ,

$$\frac{d}{dt}\langle \boldsymbol{q} \rangle_{\mathtt{i,j,k}} = \frac{-1}{\Delta V_{\mathtt{i,j,k}}} \left\{ \left[ \left( \langle \boldsymbol{f} \rangle^1 \Delta A^1 \right) \Big|_{\mathtt{i+1/2,j,k}} - \left( \langle \boldsymbol{f} \rangle^1 \Delta A^1 \right) \Big|_{\mathtt{i-1/2,j,k}} \right] + \cdots \right\} + \langle \boldsymbol{s} \rangle_{\mathtt{i,j,k}} + \langle \boldsymbol{s}_{\text{geom}} \rangle_{\mathtt{i,j,k}} ,$$

Solved with *high-resolution shock-capturing (HRSC)* method



# 7 Primitive and conservative (ideal MHD)

Primitive: $\rho, v_j, e, B^i$
Conservative: $D, S_j, \tau, \mathcal{B}^i$

$$
\begin{aligned}
D &\propto \rho W \\
S_j &\propto \rho h^* W^2 v_j - \alpha b^0 b_j \\
\tau &\propto \rho h^* W^2 - p^* - (\alpha b^0)^2 - D
\end{aligned}
$$

with

$$
\begin{aligned}
W &= \frac{1}{\sqrt{1-v^2}} \\
p^* &= p + \frac{b^2}{2} \\
h^* &= 1 + \epsilon + \frac{p + b^2}{\rho}
\end{aligned}
$$

## 7 Divergence-free magnetic field

One of Maxwell's equations is $\nabla \cdot \mathbf{B} = 0$. To satisfy this constrain equation, *Gmunu* can use elliptic divergence cleaning by soling Poisson's equation

$$\nabla^2 \Phi = \nabla \cdot \mathcal{B}_{\text{old}},$$

and then using the resulting $\Phi$ to enforce a divergence-free magnetic field with

$$\mathcal{B}_{\text{new}} = \mathcal{B}_{\text{old}} - \nabla \Phi$$

Indeed, taking the divergence we get

$$\nabla \cdot \mathcal{B}_{\text{new}} = \nabla \cdot \mathcal{B}_{\text{old}} - \nabla^2 \Phi,$$

which is $0$ by definition of $\Phi$.

# 7 Boltzmann equation (Cardall et al 2013)

$$
\begin{aligned}
S_N \quad &+ \quad M_N = C[f] \\
S_N \quad &= \quad \frac{-p_{\hat{0}}}{\alpha\sqrt{\gamma}} \left[ \frac{\partial}{\partial t} \left( \frac{\gamma}{-p_{\hat{0}}} \mathcal{L}_{\hat{\mu}} p^{\hat{\mu}} f \right) + \frac{\partial}{\partial x^i} \left( \frac{\sqrt{\gamma}}{\left(-p_{\hat{0}}\right)} \left( \alpha\, \ell^i{}_{\hat{\mu}} - \beta^i \mathcal{L}_{\hat{\mu}} \right) p^{\hat{\mu}} f \right) \right] \\
M_N \quad &= \quad \frac{1}{\alpha\sqrt{\gamma}} \frac{\left(-p_{\hat{0}}\right)}{\sqrt{\lambda}} \frac{\partial}{\partial p^{\tilde{i}}} \left\{ \sqrt{\lambda}\, \frac{Q^{\tilde{i}} \left(-p_{\hat{0}}\right)}{p} \left[ \left(R_N\right)^{\hat{0}} + \left(O_N\right)^{\hat{0}} \right] \right. \\
&\qquad \left. + \sqrt{\lambda}\, U^{\tilde{i}}{}_{\hat{i}} \left[ \left(R_N\right)^{\hat{i}} + \left(O_N\right)^{\hat{i}} \right] \right\} \\
\left(R_N\right)^{\hat{\rho}} \quad &= \quad \frac{\alpha\sqrt{\gamma}}{\left(-p_{\hat{0}}\right)} p^{\hat{\nu}} p^{\hat{\mu}} f \left[ \mathcal{L}^{\hat{\rho}}\, \ell^j{}_{\hat{\nu}} \left( \frac{\mathcal{L}_{\hat{\mu}}}{\alpha} \frac{\partial \alpha}{\partial x^j} - \ell^k{}_{\hat{\mu}}\, K_{jk} \right) \right. \\
&\qquad \left. - \ell^{\hat{\rho} j} \left( \frac{\mathcal{L}_{\hat{\nu}} \mathcal{L}_{\hat{\mu}}}{\alpha} \frac{\partial \alpha}{\partial x^j} - \frac{\ell_{k\hat{\nu}}\, \mathcal{L}_{\hat{\mu}}}{\alpha} \frac{\partial \beta^k}{\partial x^j} - \frac{\ell^k{}_{\hat{\nu}}\, \ell^i{}_{\hat{\mu}}}{2} \frac{\partial \gamma_{ki}}{\partial x^j} \right) \right] \\
\left(O_N\right)^{\hat{\rho}} \quad &= \quad \frac{\sqrt{\gamma}}{\left(-p_{\hat{0}}\right)} p^{\hat{\nu}} p^{\hat{\mu}} f \left\{ \mathcal{L}^{\hat{\rho}} \left[ \mathcal{L}_{\hat{\mu}} \frac{\partial \mathcal{L}_{\hat{\nu}}}{\partial t} + \left( \alpha\, \ell^j{}_{\hat{\mu}} - \beta^j \mathcal{L}_{\hat{\mu}} \right) \frac{\partial \mathcal{L}_{\hat{\nu}}}{\partial x^j} \right] \right. \\
&\qquad \left. - \ell^{\hat{\rho} k} \left[ \mathcal{L}_{\hat{\mu}} \frac{\partial \ell_{k\hat{\nu}}}{\partial t} + \left( \alpha\, \ell^j{}_{\hat{\mu}} - \beta^j \mathcal{L}_{\hat{\mu}} \right) \frac{\partial \ell_{k\hat{\nu}}}{\partial x^j} \right] \right\}
\end{aligned}
$$

# 7 Neutrino interactions

6 different species of neutrinos, only $n = 3$ considered.
Each neutrino has an energy $\rightarrow$ energy space divided in $m$ bins.
Different ways to treat interactions

- **Simplest**: no interaction between neutrinos: "ss sg"
  $\rightarrow$ Each species and energy evolved independently: $n \times m$ systems of $(D + 1)$ equations
- **Intermediate**: interactions between neutrinos of same species and different energy: "ss mg"
  $\rightarrow$ Each species evolved independently, energies coupled: $n$ systems of $m \times (D + 1)$ equations
- **Complete**: interactions between all species: "ms mg"
  $\rightarrow$ Species and energies coupled: 1 system of $n \times m \times (D + 1)$ equations