Belgian-dutch GW meeting

First order phase transitions in the early universe and quantizing particles across the wall

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Phase transitions in the early universe

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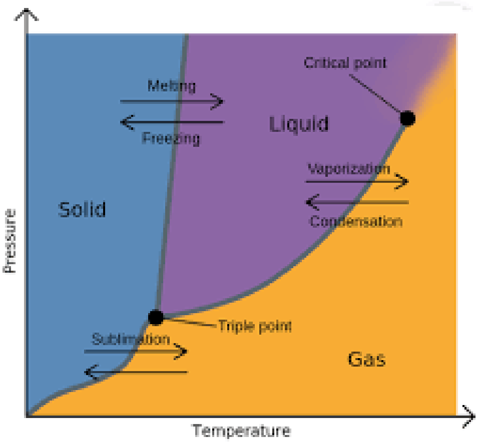
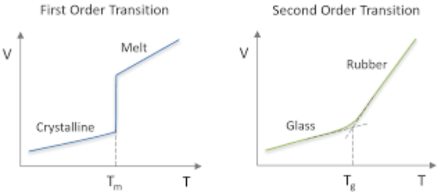
Phase transition in everyday life

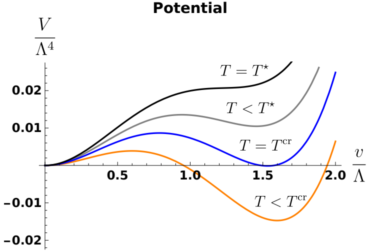


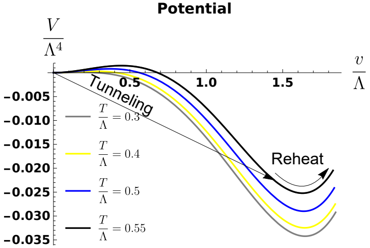


First order phase transition (FOPT) in the early universe

Universe high-T after inflation:cooling of primordial soup

- QFT $=$ landscape of minima $\Rightarrow$ PT





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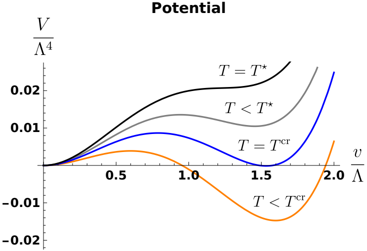

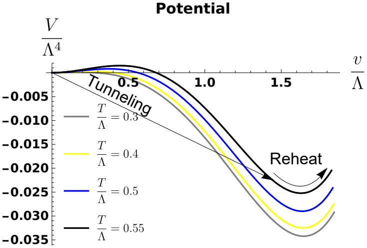

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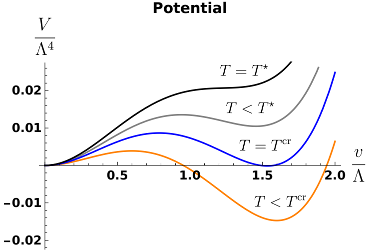

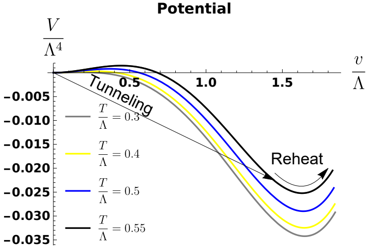

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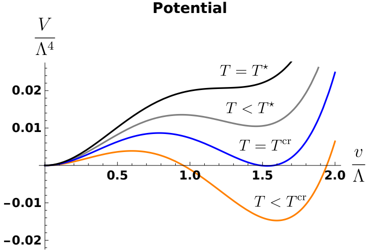

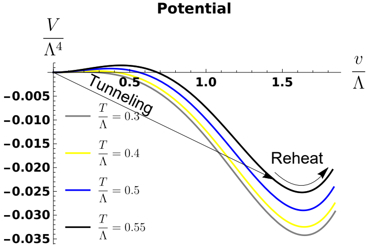

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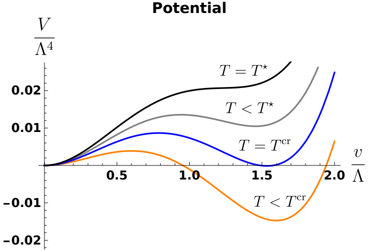

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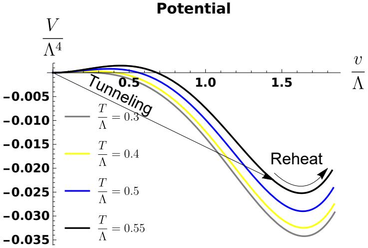
- Nucleation controlled by bounce solution

$$\Gamma \sim T^4 Exp\left[-\frac{S_3}{T}\right]$$

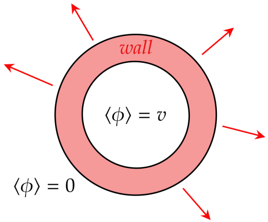


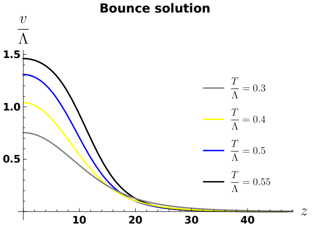
Nucleation and early expansion

- Energy released $\Delta V \Rightarrow$ Driving energy:

$$E_{\text{driving}} = -\frac{4}{3}\pi \Delta V R^3 \text{ VS } \Delta\mathcal{P}_{\text{tension}} = 4\pi\sigma R^2$$

Expansion when $R_{\text{initial}} > R_c \sim \sigma/\Delta V$





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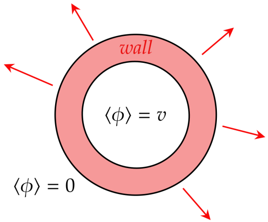

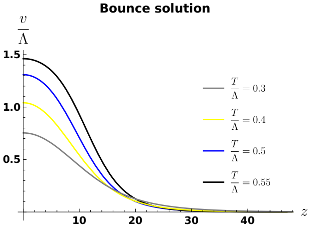

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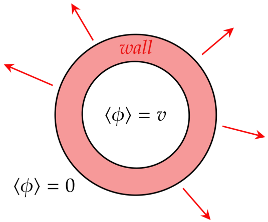

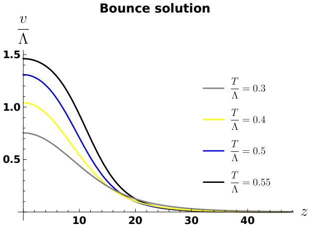

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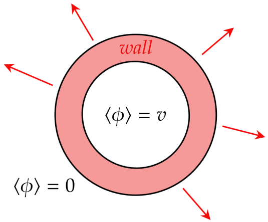
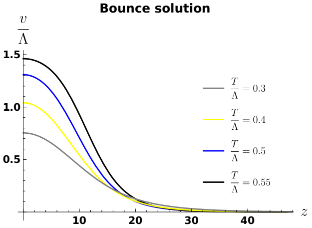
- Duration of the FOPT

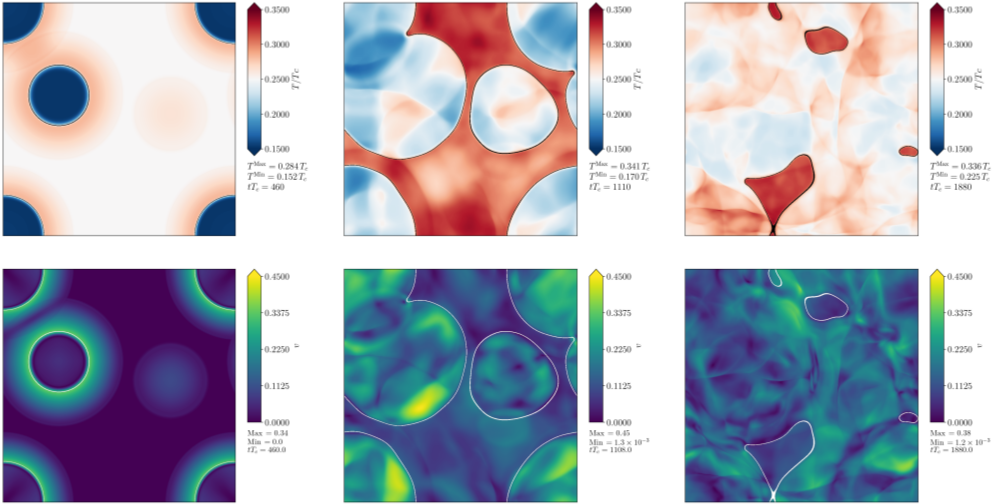
$$\beta \equiv \frac{t_{\text{exp}}}{t_{PT}} = \frac{1}{t_{PT} H} \propto R_{\text{collision}}^{-1}$$



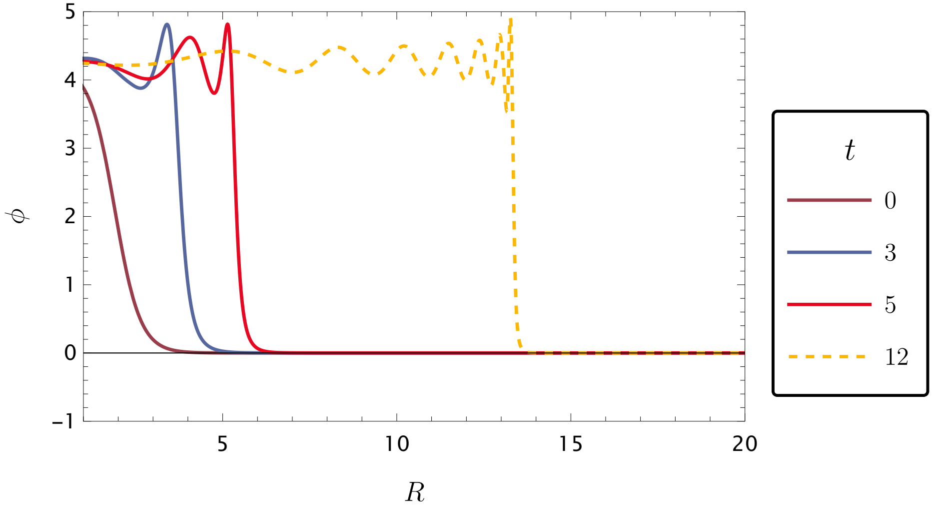


FOPT pictorially

Cutting, Hindmarsh and Weir:[1906.00480]: video in https://vimeo.com/showcase/5968055



The bubble wall in time



FOPT: What is the interest? Baryogenesis

1. Bubbles can create baryon anti-baryon asymmetry

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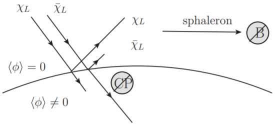
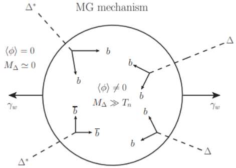
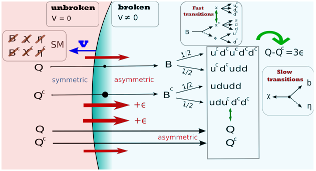


Figure: Credit:T.Konstandin [1302.6713]

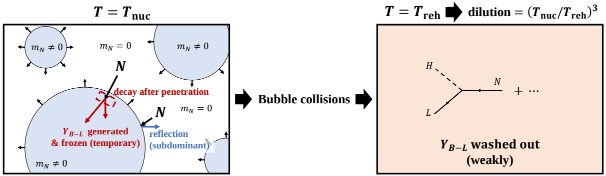


Baldes I., Blasi S., Mariotti A., Sevrin A., Turbang K.

Traditional EWBG



Yin W. Azatov A. and MV (2021)



Eung C., Dutka T., Jung T., Nagels X. and MV (2023)

FOPT: What is the interest? DM

1. FOPT can modify and set the DM abundance via

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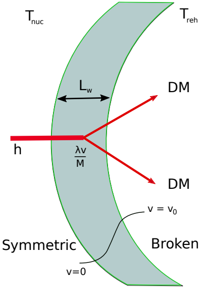
Supercool Dark Matter: Hambye, Strumia, Tesi (2018)



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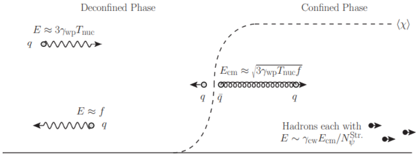
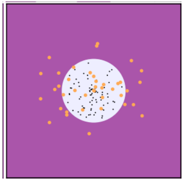


Baldes I.,Gouttenoire Y., Sala F., Servant G. (2022)



Asadi, Kramer, Kuflik, Ridgway Slatyer, Smirnov (2022)

FOPT: What is the interest? GW

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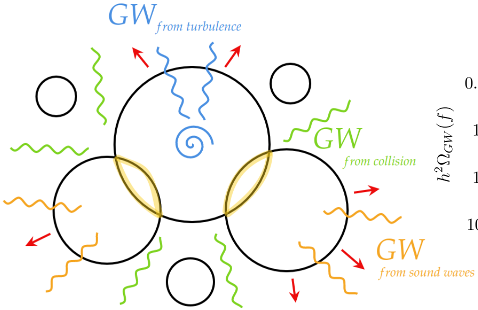
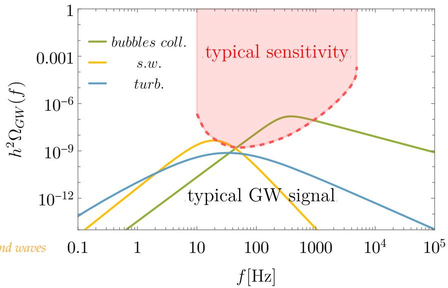
- bubble collision
- sound waves
- turbulence

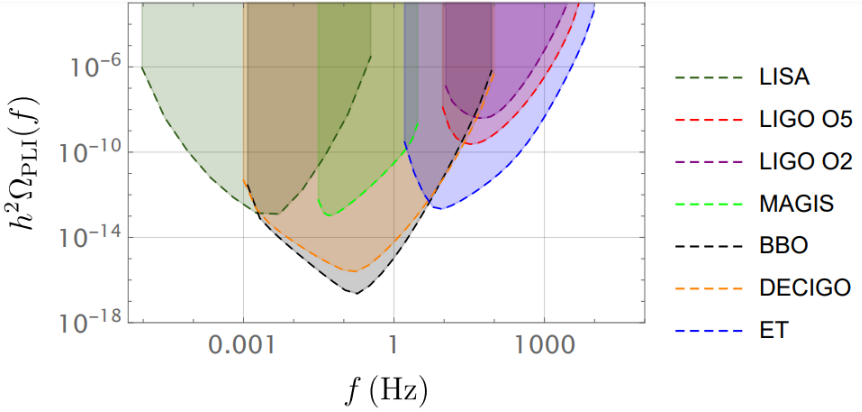




Primordial GWs could be observed soon: Frequency $\Rightarrow$ information about $T_{\rm reh}$: $f_{\rm peak} \propto T_{\rm reh}$

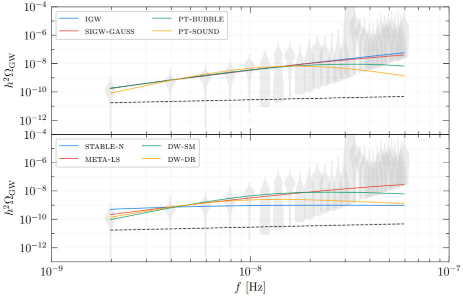
Observation prospects of GW

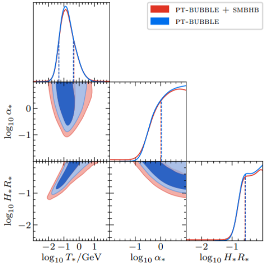
$$\left(T_{\text{reh}}, \quad \beta, \quad \alpha, \quad v_w\right) \quad \Rightarrow \quad (\Omega^{GW}_{\text{peak}}, \quad f^{GW}_{\text{peak}})$$

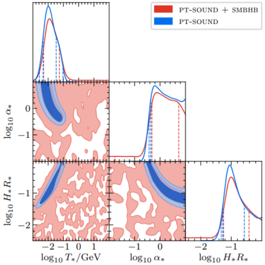


PTA with FOPT?

Nanograv BSM study: arXiv:2306.16219



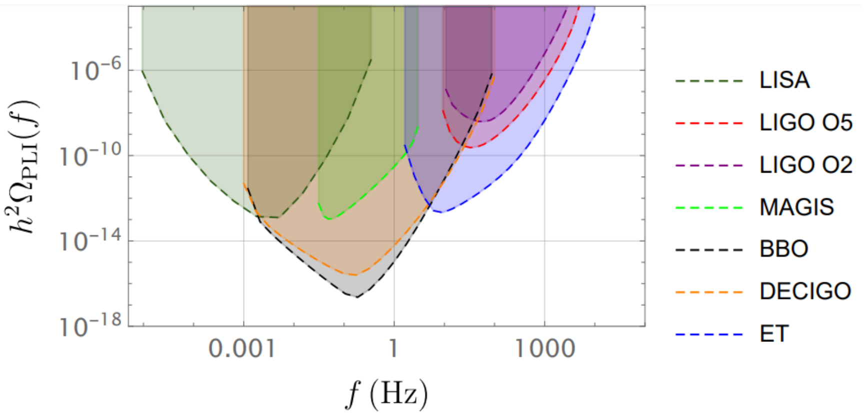




FOPT could explain the signal of PTA if $\boxed{\alpha \gtrsim 0.1, \quad \beta \lesssim 10}$. Need for *strong* and *long* FOPT!

What about v_w??

$$\left(T_{\rm reh}, \quad \beta, \quad \alpha, \quad {\color{red} v_w}\right) \quad \Rightarrow \quad (\Omega^{GW}_{\rm peak}, \quad f^{GW}_{\rm peak})$$



Pressure on the bubble wall in the relativistic regime

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FOPT and bubbles

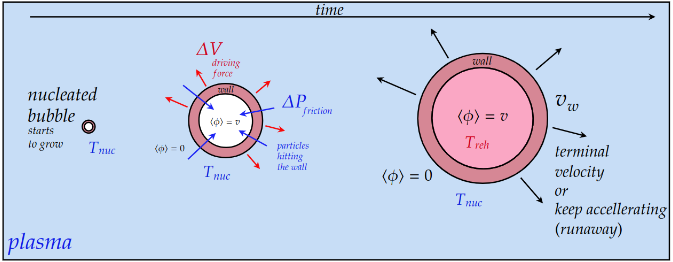


Figure: Credit: Giulio Barni, thanks to him

$$\Delta V = \underbrace{\Delta\mathcal{P}(\gamma = \gamma^{MAX})}_{??} \quad \text{(velocity)}$$

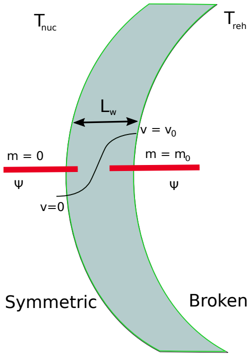
Build the function $\Delta\mathcal{P}(\gamma)$!!

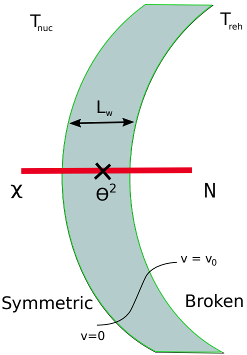
ultra-relativistic limit:

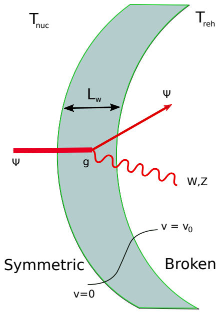
$$v_w \to c, \qquad \gamma \equiv \frac{1}{\sqrt{1 - v_w^2}}$$

Pressure contributions

Bodeker-Moore [0903.4099], [1703.08215], Azatov, MV[2010.02590], Sala, Jinno, Gouttenoire[arXiv:2112.07686]







$$\Delta\mathcal{P}_{\rm mass} \propto \Delta m^2 T^2$$

$$\Delta\mathcal{P}_{\rm mixing} \propto T^2 v^2 \Theta(\gamma T - M^2 L_w)$$

$$\Delta\mathcal{P}^{\tau}_{\rm splittings} \propto \gamma g^3 T^3 v \log v/T$$

$$\Delta\mathcal{P}^{\lambda}_{\rm splittings} = ??$$

QFT in the background of the wall

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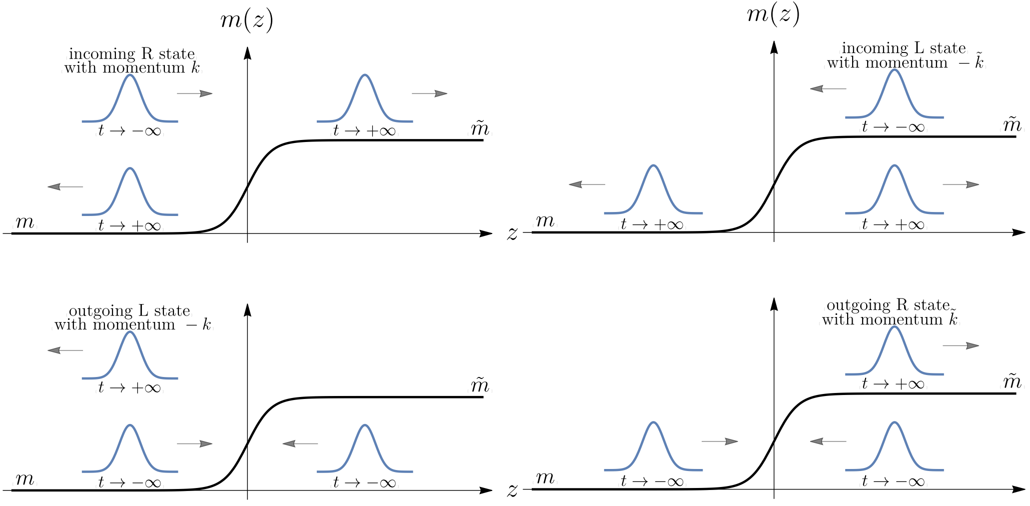
[2310.06972] with *Aleksandr Azatov*(Sissa Trieste), *Giulio Barni*(Sissa Trieste) and *Rudin Petrossian*(ICTP Trieste)

First technicality: What is the complete basis of scattering states?



Second technicality: Global dof across the wall: $0 \neq v_1 \ll v_2$

- We set the unitary gauge: $\Box h = -V''(v)h \qquad \partial_\nu F^{\mu\nu} = g^2 v^2(z) A^\mu \,.$

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$$A^\mu = \sum_{i=1,2,3} \epsilon_i^\mu a_i(x). \qquad k_\mu = (k_0, k_\perp, 0, k_z)$$

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- Transverse $(A_z = 0)$

$$\mathsf{T} : \epsilon_1 = (0,0,1,0), \qquad \epsilon_2 = (k_\perp, k_0, 0, 0)/\sqrt{k_0^2 - k_\perp^2} \implies \left[E^2 - \partial_z^2 + g^2 v^2(z)\right] A_{\tau_{1,2}}(z) = 0 \ .$$

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- Longitudinal ($A_z \neq 0$)

$$\mathsf{L}: A_\mu^{(z-pol)} = \partial_n \alpha(z) + \lambda(z) \qquad \partial_\mu (v^2 A^\mu) = 0 \quad + \quad \text{transversality}$$

$$\left(-E^2 - \partial_z^2 + U_\lambda(z)\right)\lambda = 0 \qquad \text{with} \qquad U_\lambda(z) = \; g^2 v^2(z) + 2\left(\frac{\partial_z v}{v}\right)^2 - \frac{\partial_z^2 v}{v}$$

λ as the magical dof? $v_1 \to 0$.

- When $v_1 \to 0$, how do we see that

$$\lambda_{z\to-\infty} \to \phi_2?$$

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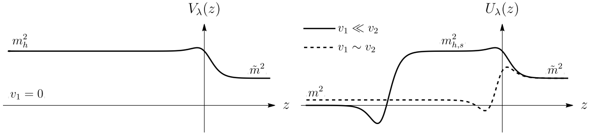

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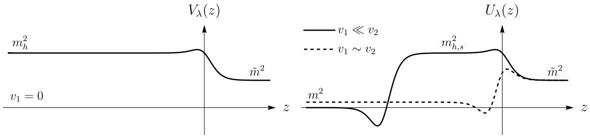

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-

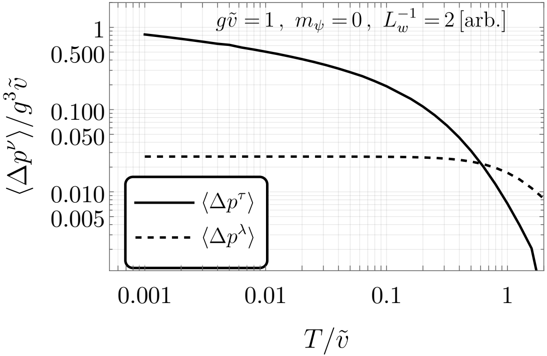
$$\boxed{A_\lambda^\mu\big|_{z\to-\infty} \propto \partial^\mu\left(\frac{\phi_2}{gv}\right)}$$

Going to pressure: Who dominates? λ or τ?

$$\Delta\mathcal{P}^{\tau}_{\text{splittings}} \propto \gamma g^3 T^3 v \log v/T \qquad \Delta\mathcal{P}^{\lambda}_{\text{splittings}} \propto \gamma g^3 T^3 v$$

Relative importance of τ and λ contributions

Symm. $\to$ Broken : thermal case



Take-home message

- FOPT are related to baryogenesis, Dark matter, primordial black holes and observable GW

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- In the relativistic regime, three contributions: i) $1 \to 1$: γ^0, ii) $1 \to$ heavy: γ^0, ii) τ :$1 \to 2$: $\gamma^1 \log v/T$
- Procedure for computing $1 \to 2$: i) define complete basis (LM and RM), ii) define global dof $\lambda \to \phi_2$, iii) split properly the phase space integral
- Conclusion: λ :$1 \to 2$: γ^1. Can dominate for $v/T \sim 1$.

Matching our polarisation with *usual* ones

- Usual dof of polarisation

$$\epsilon_{T_1} = (0,0,1,0), \qquad \epsilon_{T_2} = (0,k_z,0,-k_\perp)/\sqrt{k_z^2+k_\perp^2} \qquad \epsilon_L = \left(\frac{k_0^2-m^2}{k_0}, k_\perp, 0, k_z\right)\frac{k_0}{m\sqrt{k_0^2-m^2}}$$

- *Our* dof of polarisation

$$\epsilon_{\tau_1} = (0,0,1,0), \qquad \epsilon_{\tau_2} = (k_\perp,k_0,0,0)/\sqrt{k_0^2-k_\perp^2} \qquad \epsilon_\lambda^\mu = \frac{k^z}{Egv}\,k^\mu + \frac{gv}{E}(0\ ,0\ ,0\ ,1)$$

- Matching matrix between the two

$$\begin{pmatrix} \epsilon_{T_1} \\ \epsilon_{T_2} \\ \epsilon_L \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{k_0 k_z}{E\sqrt{k_z^2+k_\perp^2}} & -\frac{k_\perp m}{E\sqrt{k_z^2+k_\perp^2}} \\ 0 & \frac{k_\perp m}{E\sqrt{k_z^2+k_\perp^2}} & \frac{k_0 k_z}{E\sqrt{k_z^2+k_\perp^2}} \end{pmatrix} \begin{pmatrix} \epsilon_{\tau_1} \\ \epsilon_{\tau_2} \\ \epsilon_\lambda \end{pmatrix}$$

Quantisation in the background of the wall

- System of Equations of motion across the wall with "wall terms" and masses in R_ξ gauges

$$\Box h = -V''(v)h \qquad \text{EoM of the wall}$$

$$\Box \phi_2 = -2gA^\mu(\partial_\mu v) - \xi g^2 v^2 \phi_2 - V' \frac{\phi_2}{v} \qquad \text{EoM of the Goldstone}$$

$$\partial_\nu F^{\mu\nu} = \frac{1}{\xi}\partial^\mu(\partial_\nu A^\nu) + g^2 v^2 A^\mu - 2g\phi_2(\partial^\mu v) \qquad \text{EoM of the Gauge field}$$

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- Field in the symmetric phase: $(h, \phi_2, A_1^\mu, A_2^\mu) \qquad m_h^2 \equiv \partial_h^2 V(0) = \partial_{\phi_2}^2 V(0)$
$\epsilon^\mu_{T_1} = (0,0,1,0)\ , \qquad \epsilon^\mu_{T_2} = \frac{1}{\sqrt{k_\perp^2 + k_z^2}}(0, k_z, 0, -k_\perp)\ , \qquad \mathsf{k}^\mu = (k_0, k_\perp, 0, k^z)\ .$

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- Field in the broken phase $\xi \to \infty$ (Unitary gauge): $\qquad m_h^2 = \partial_h^2 V(v_2) \qquad \tilde{m} \equiv g v_2$

$$\partial_\mu F^{\mu\nu} + \tilde{m}^2 A^\nu = 0\ , \implies \partial^2 A^\mu + \tilde{m}^2 A^\mu = 0\ , \qquad \partial_\mu A^\mu = 0\ .$$

$$\epsilon_L^\mu = \left(\frac{p_0^2 - m^2}{p_0}, p_\perp, 0, p_z\right)\frac{p_0}{m\sqrt{p_0^2 - m^2}}$$

Third technicality: Solving procedure

We cannot apply the same approximations all over in the phase space!

- $k_z < L_w^{-1}$: *Step wall procedure*: non-trivial matching conditions for λ

$$\frac{\partial_z \lambda}{v(z)}\Big|_{<0} = \frac{\partial_z \lambda}{v(z)}\Big|_{>0} \quad , \quad v(z)\lambda|_{<0} = v(z)\lambda|_{>0}$$

discontinuity in λ across the wall!!

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- In the end, three computations needed to evaluate each process

$$\langle \Delta p^{\text{step}}_{\zeta_L} \rangle, \ \ \langle \Delta p^{\text{step}}_{\zeta_R} \rangle, \ \ \langle \Delta p^{WKB}_{\zeta_R} \rangle.$$