



Universiteit
Leiden



Spacetime Oscillation

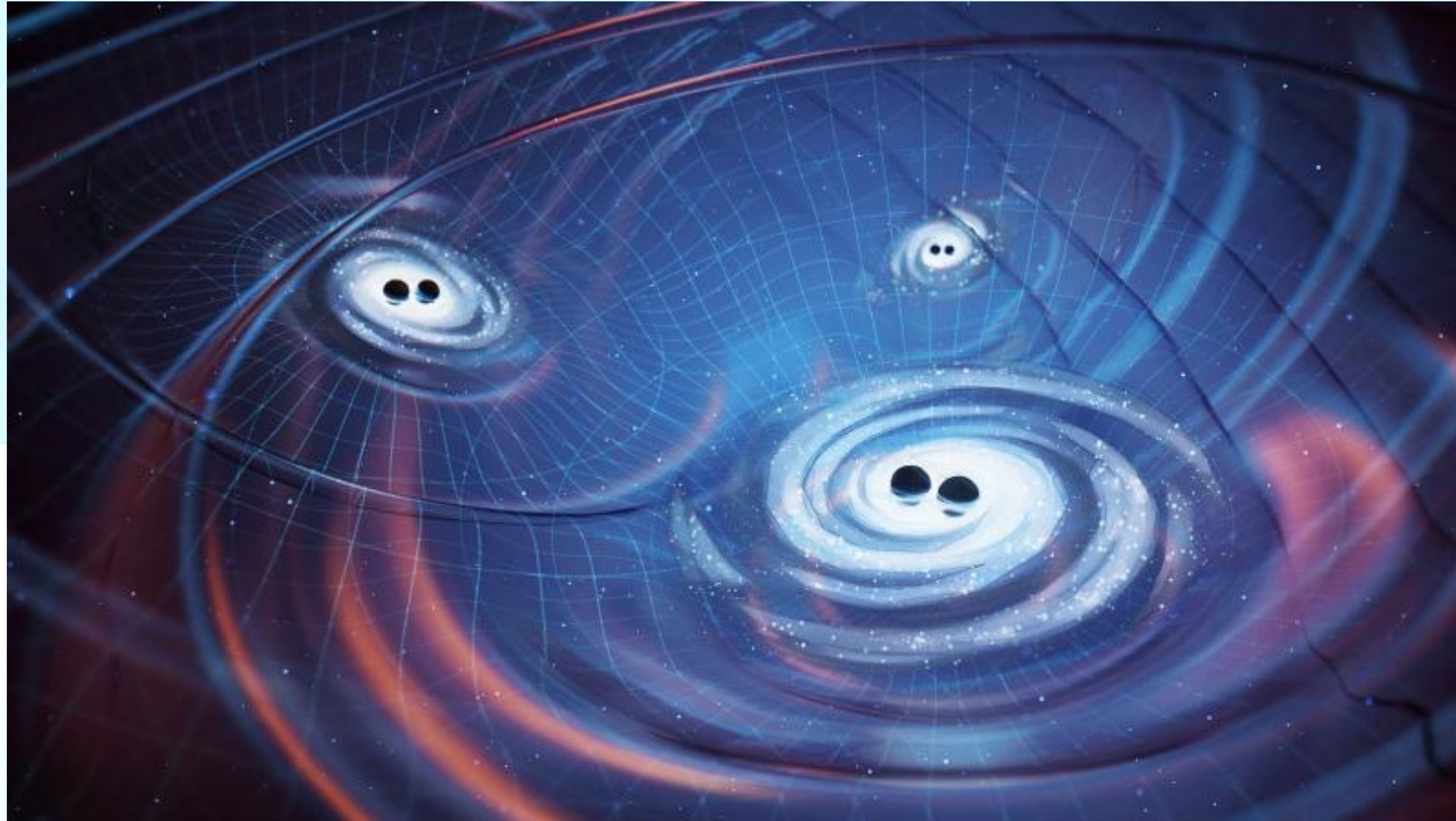
and the

Gravitational Wave Background

arXiv:2307.05455,

Gen Ye and Alessandra Silvestri

Gravitational Wave Background (GWB)



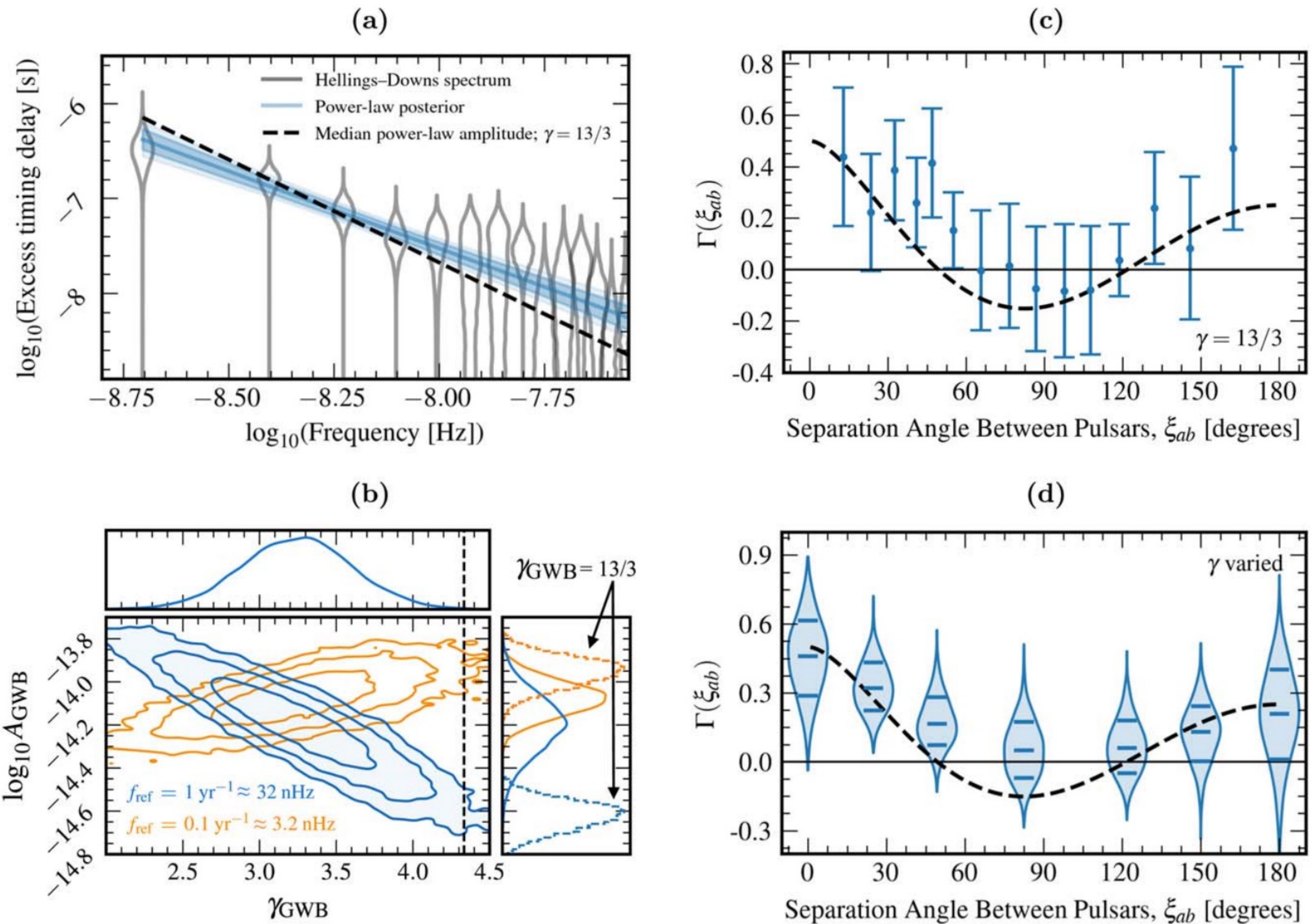
[Low-Frequency Gravitational Waves | NANOGrav](#)

“Chorus” of Gravitational Waves

Individual Singer (Sources):

- Supermassive black hole binaries
- Remainings of cosmological phase transitions:
 - Bubbles
 - Cosmic strings
 - Domain walls
- “Echos” from the primordial Universe:
 - Scalar-induced GW
 - Primordial tensor modes

Evidence of GWB



NANOGrav *Astrophys.J.Lett.* 951 (2023) 1, L8

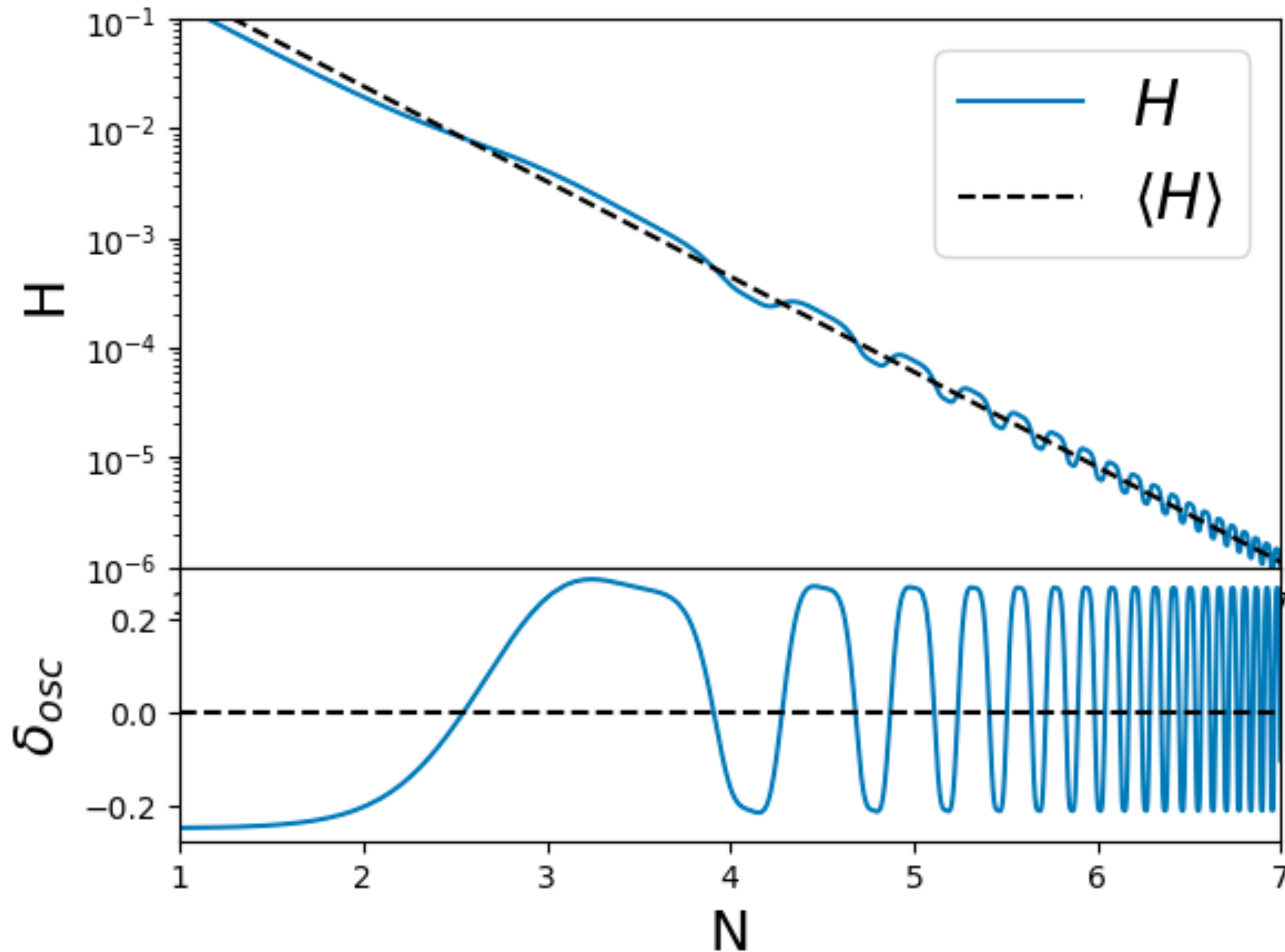
EPTA arXiv:2306.16214

PPTA *Astrophys.J.Lett.* 951 (2023) 1, L6

CPTA *Res.Astron.Astrophys.* 23 (2023) 7, 075024

- Hellings-Downs spatial correlation
- Continuous spectrum $\Omega_{\text{GW}} \propto f^2$
- $3 \sim 4 \sigma$

Spacetime Wiggle and GW Resonance



$$\mathcal{S} = \int dx^4 \sqrt{-g} \left[\frac{M_p^2}{2} G_4(\phi) R - \frac{1}{2} (\partial\phi)^2 - \lambda\phi^4 \right] + \mathcal{S}_m \quad G_4(\phi) = 1 + \alpha \left(\frac{\phi}{M_p} \right)$$

$$\ddot{\gamma} + 2\Gamma\dot{\gamma} + k^2\gamma = 0$$

$$\mathcal{H} = \langle \mathcal{H} \rangle + \mathcal{H}_{osc} \equiv \bar{\mathcal{H}}(1 + \delta_{osc}), \quad \delta_{osc} < 1$$

- Generality:
 - $\Gamma \supset$ modified gravity
 - $c_T = 1$
- Source of δ_{osc} :
 - Oscillating field $O(H^2/k^2)$
 - Nonminimal coupling
 - ...

Spacetime Wiggle and GW Resonance

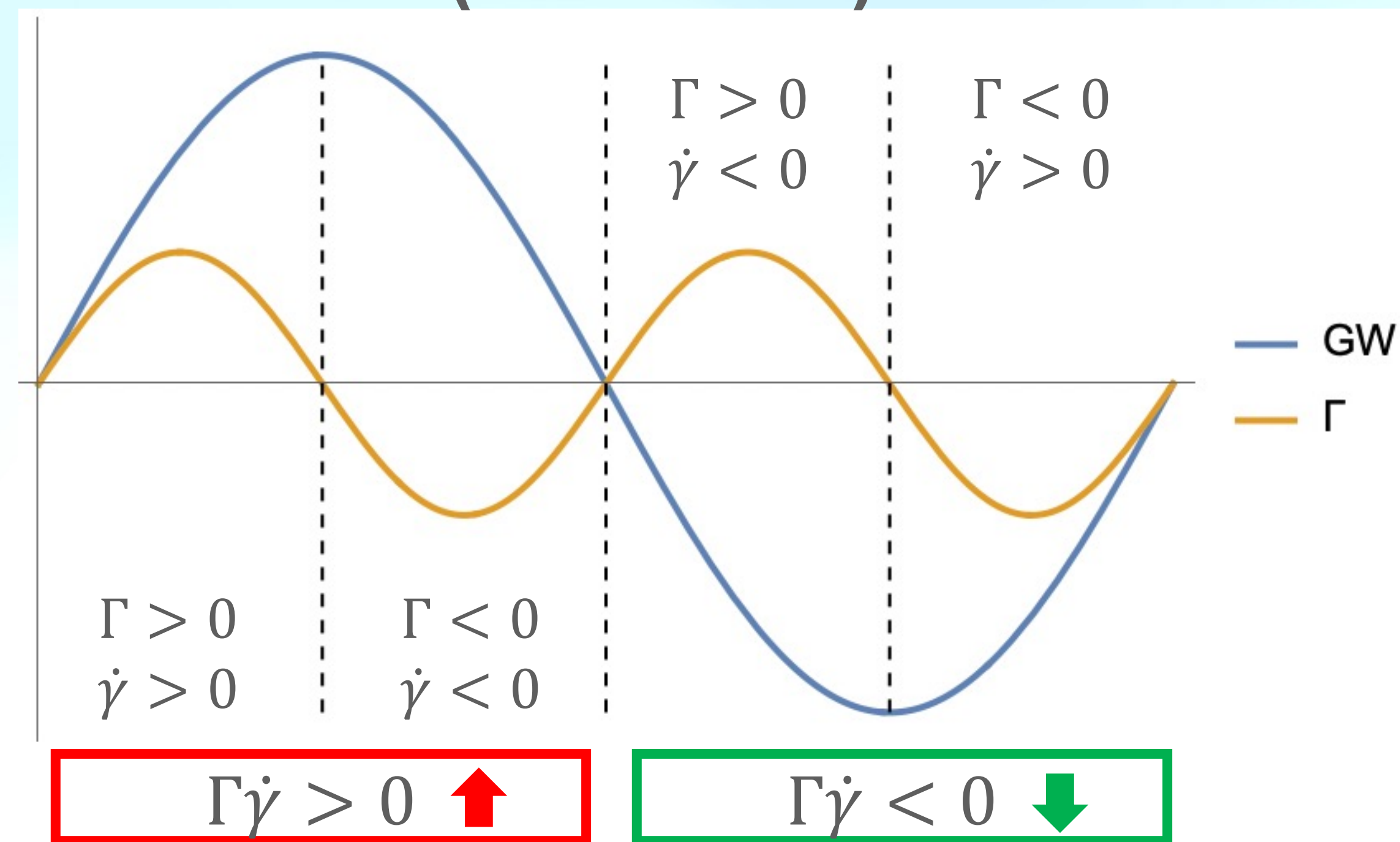
$$\ddot{\gamma} + 2\Gamma\dot{\gamma} + k^2\gamma = 0$$

Broad Spectrum Shape

$$\Gamma = \langle \mathcal{H} \rangle \simeq \frac{1}{\tau}$$

$$\gamma \sim \frac{\sin k\tau}{k\tau}$$

Local Spectrum Feature
(Resonance)



Resonance:

$$f_c = f_{osc}/2$$

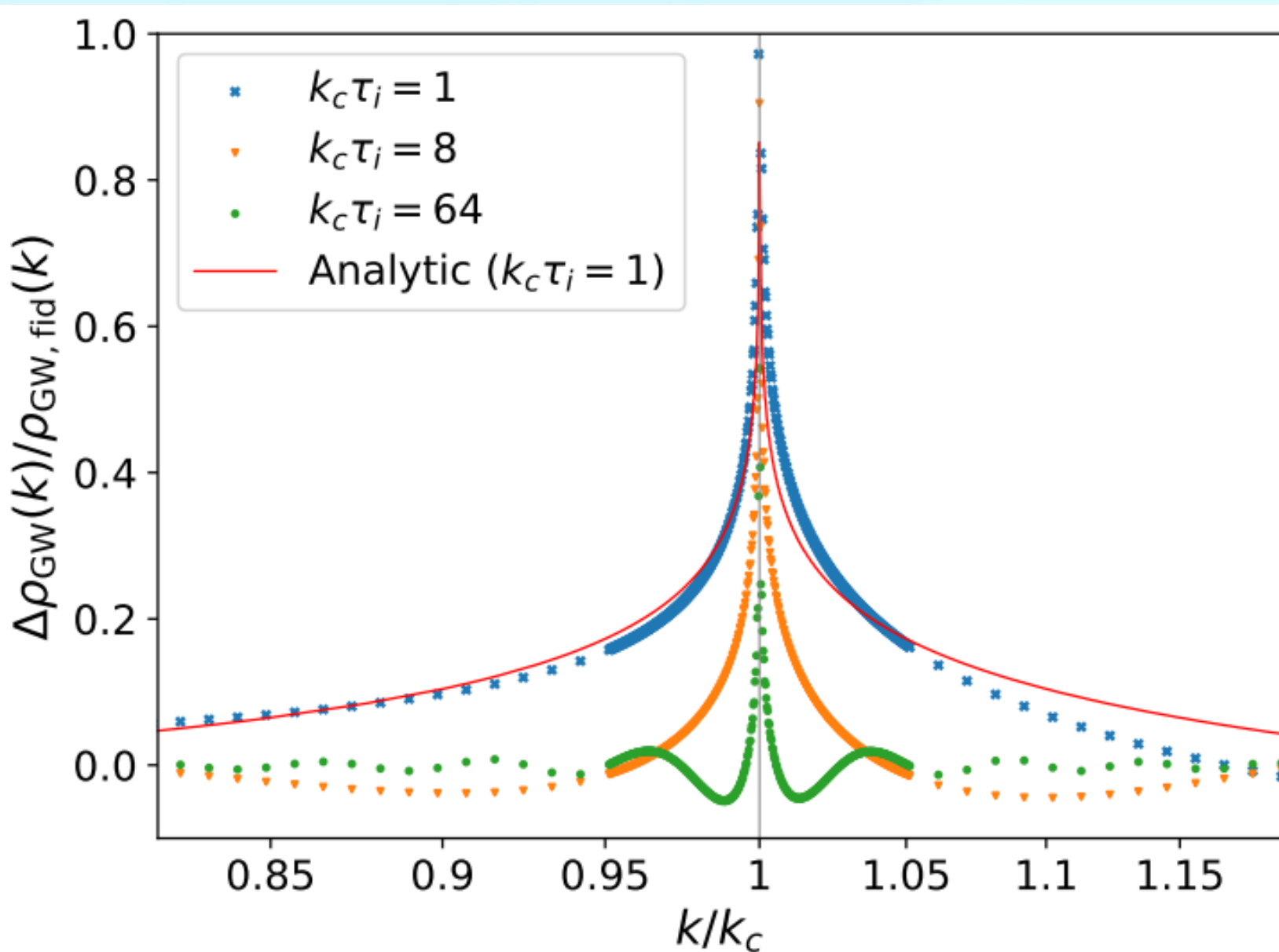
Spacetime Wiggle and GW Resonance

$$\rho_{GW}(k) \simeq \left[1 + 2f(\Delta k)\psi_0 \sin(2\beta - \alpha) + f^2(\Delta k)\psi_0^2 \right] \frac{M_p^2}{8\pi^2} \frac{k^3}{a^2\tau^2}$$

Assuming scale-invariant $P_{T,ini}$

Shape Function

$$\Delta k \equiv k - k_{GW}$$



Signal Strength

$$\psi_0 \simeq |\delta_{osc}|$$

E.g: CMB
cos waves

Phase Factor α – phase of δ_{osc} . β – phase of GW

- GWB of *primordial* origin:

- Superhorizon $h = h_0, \dot{h} = 0 \Rightarrow \beta$ is the same for all GW

- Signal strength $\sim |\delta_{osc}|$

- **Peak and Trough** (Example)

- GWB of *subhorizon* origin:

- β is usually stochastic. E.g: uniform random β

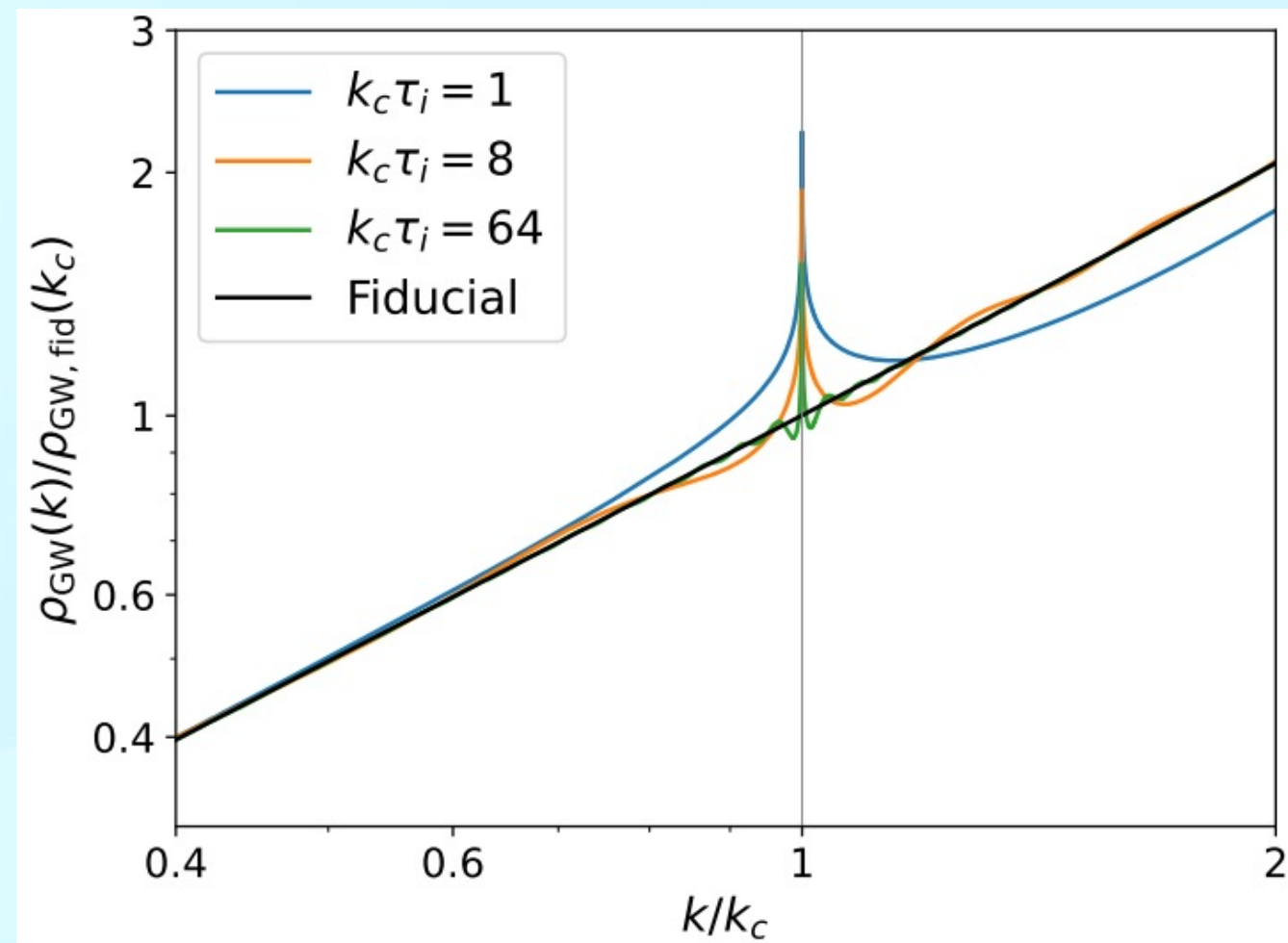
- Signal strength $\sim |\delta_{osc}|^2$

- **Only Peak**

Analytic approximation:

$$f(\Delta k) \sim \frac{1}{1+3w} [\text{Ci}(2|\Delta k|\tau) - \text{Ci}(2|\Delta k|\tau_i)]$$

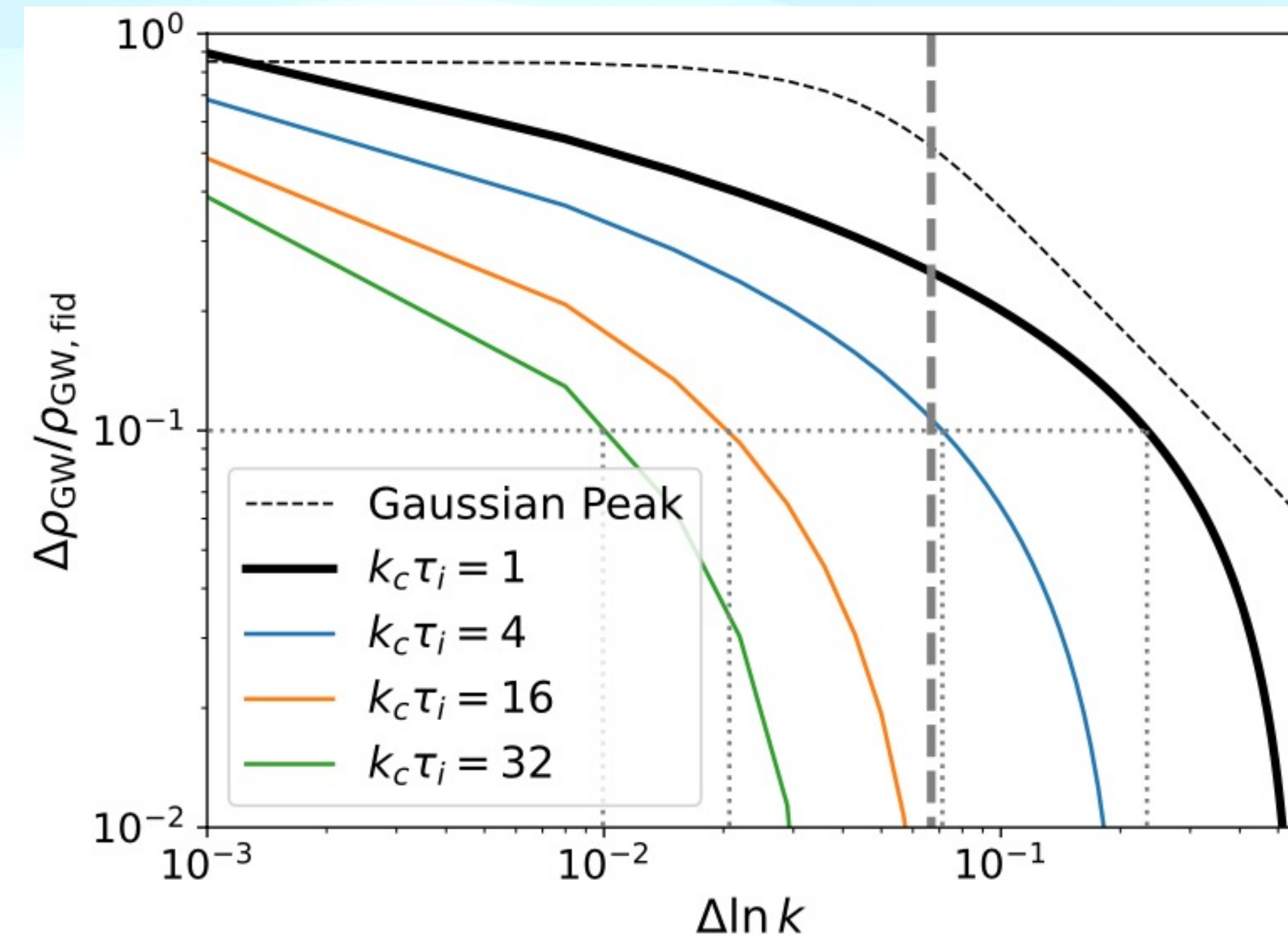
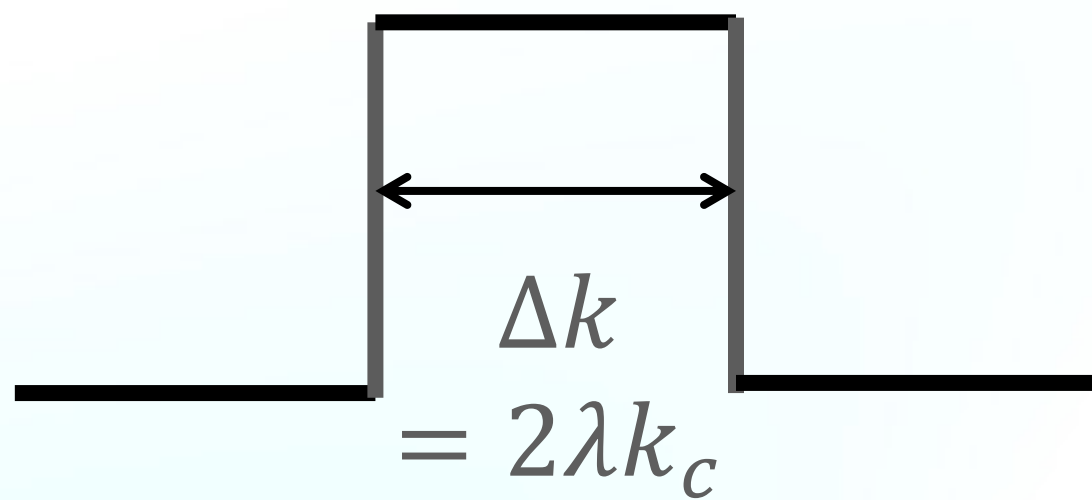
Prospects of observation



$$\Omega_{GW}(k) \rightarrow [1 + \psi_0 F_{\Delta k}(k, k_c)] \Omega_{GW}(k)$$

convolve

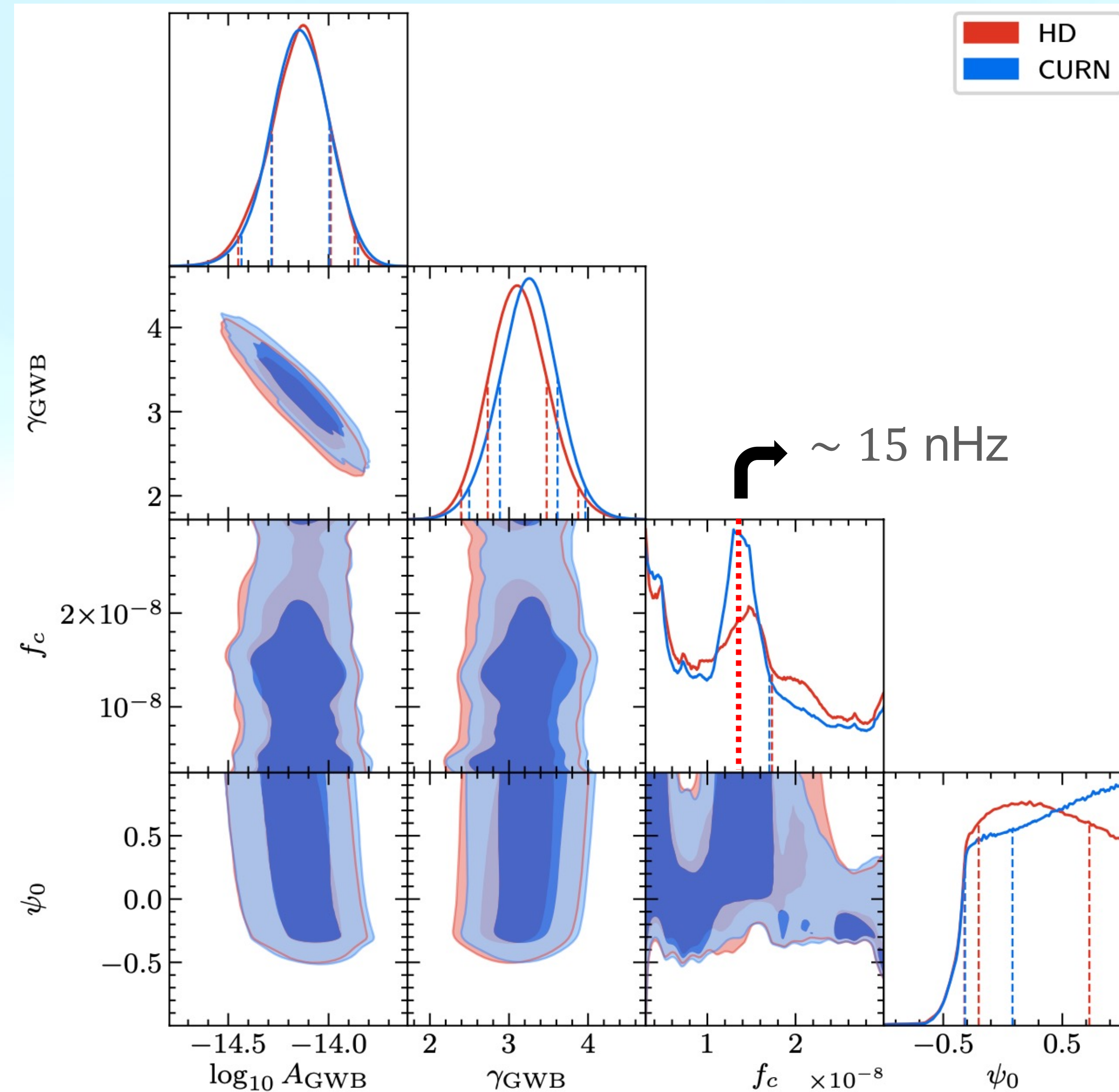
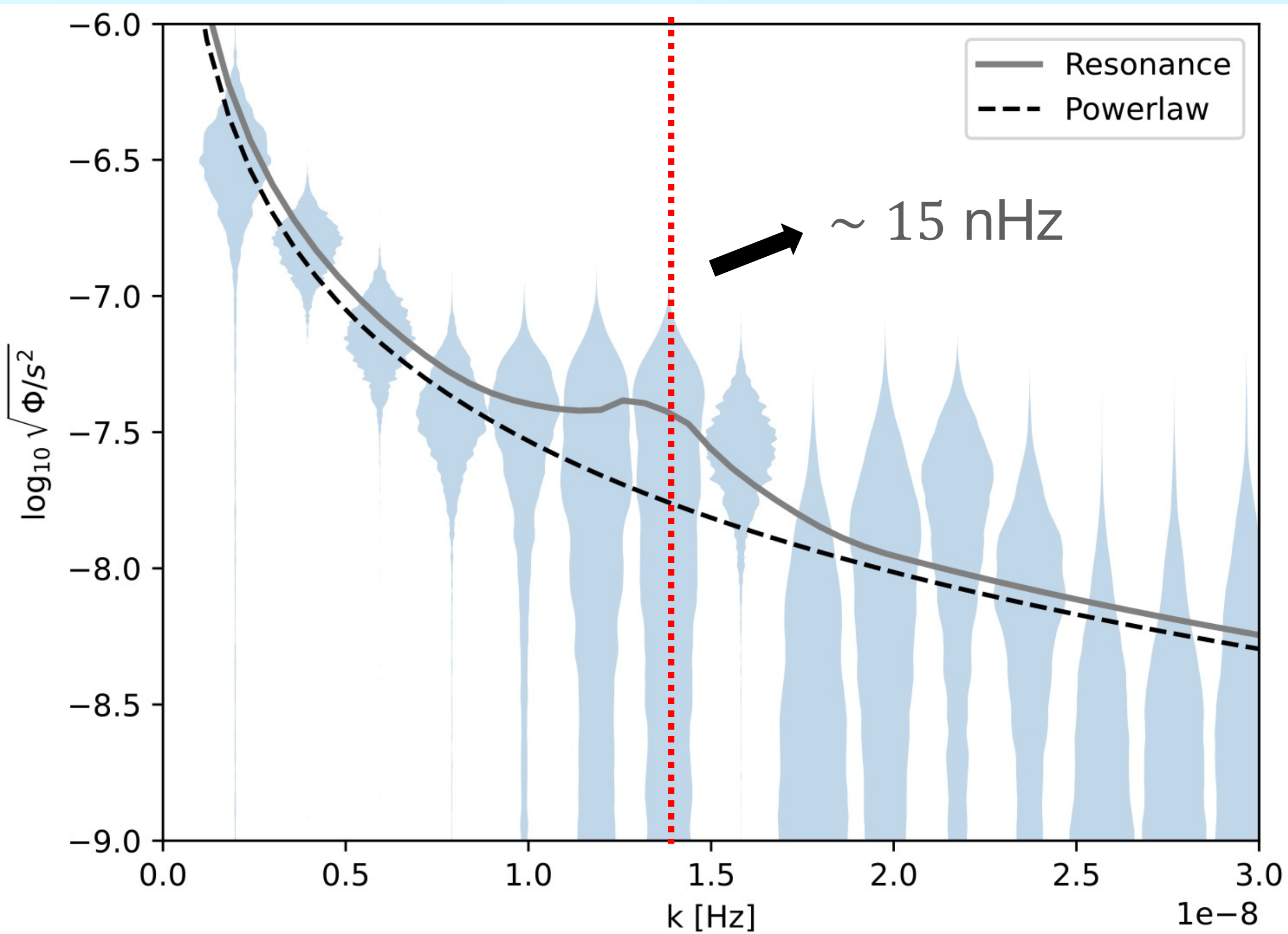
- ψ_0 - Signal Amplitude
- k_c - Peak/Trough Position
- Δk - Frequency Resolution?



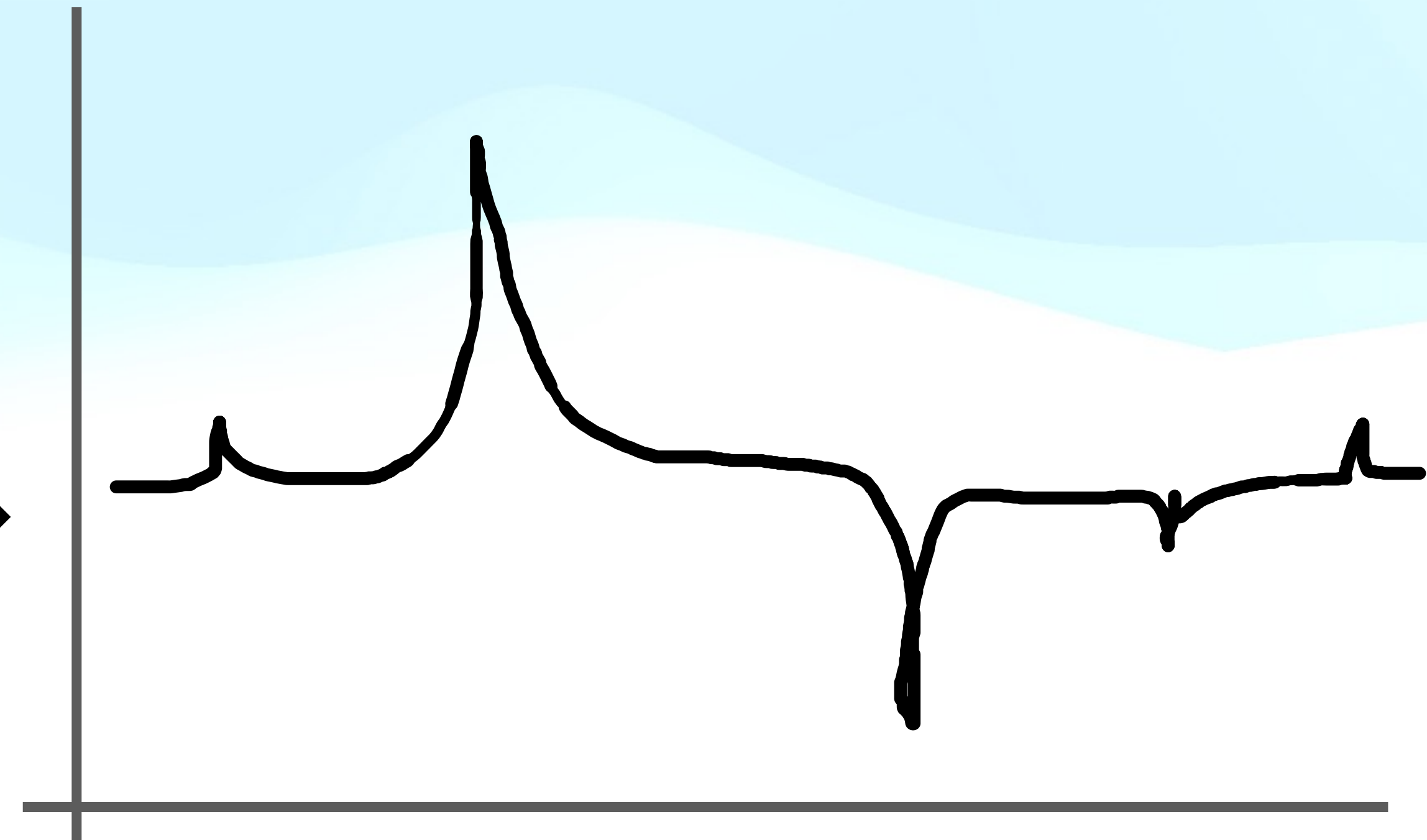
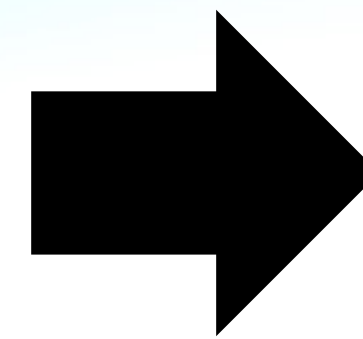
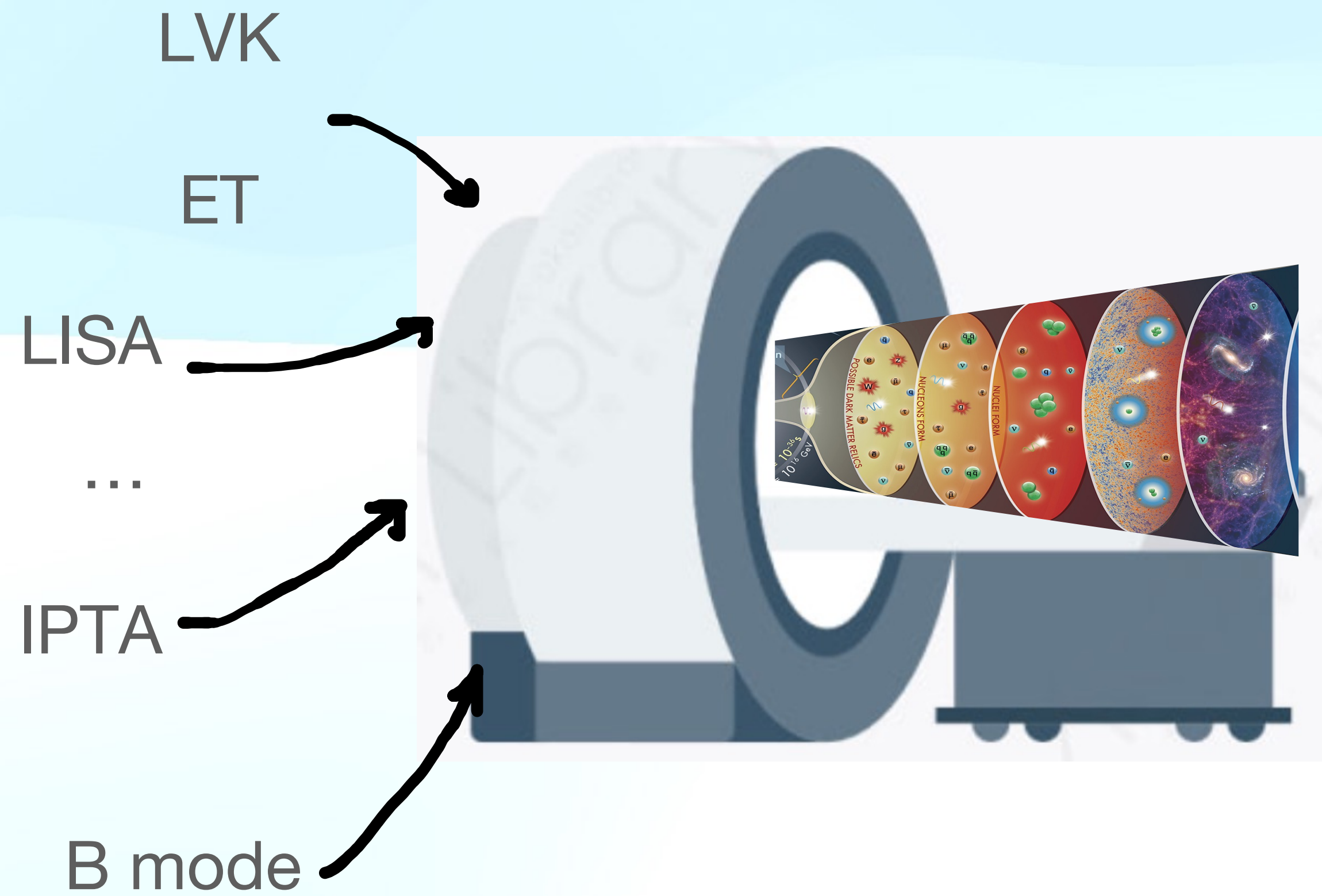
Prospects of observation

$$\Delta f = \frac{1}{15\text{yr}}$$

NANOGrav 15yr data

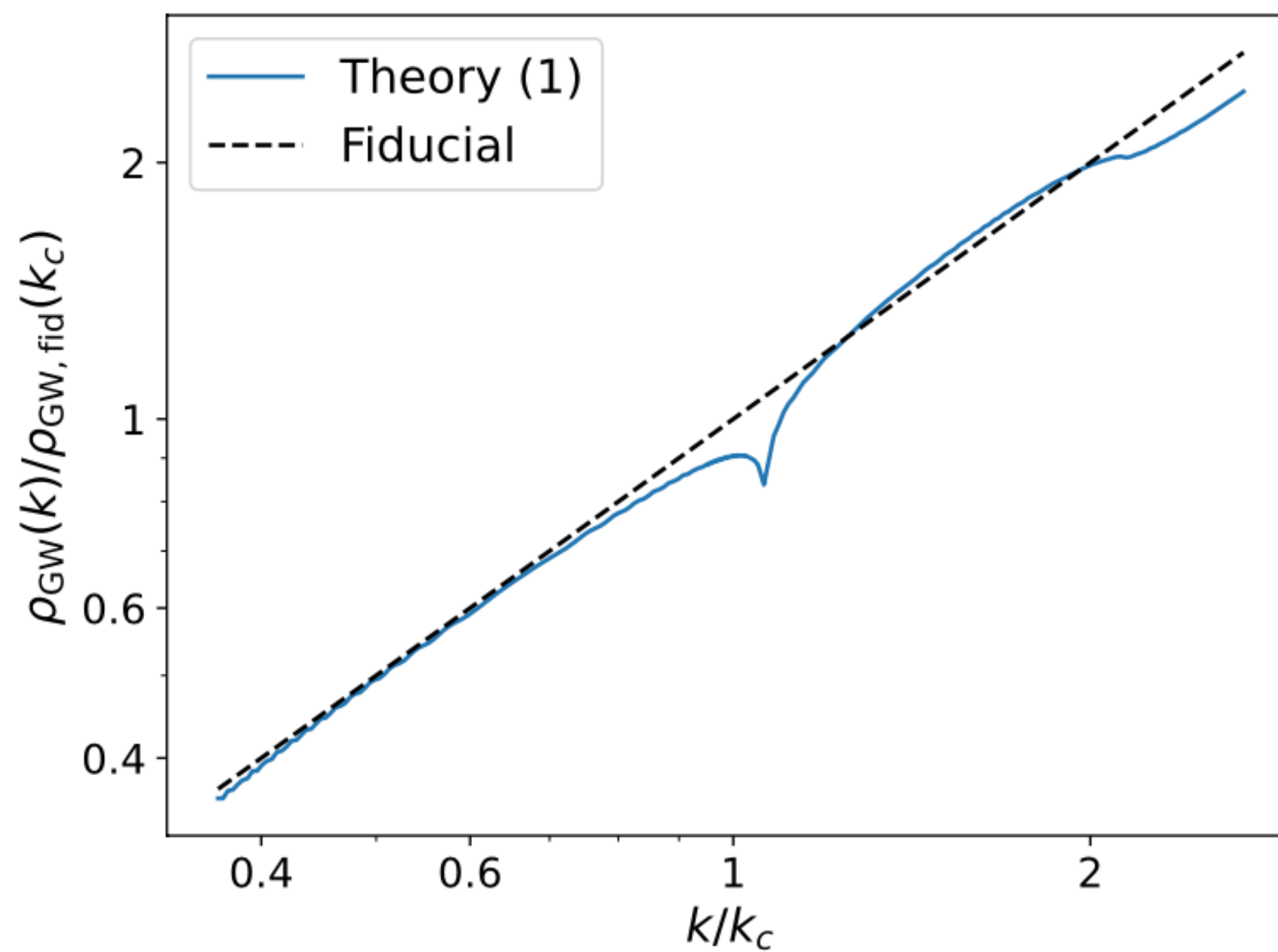
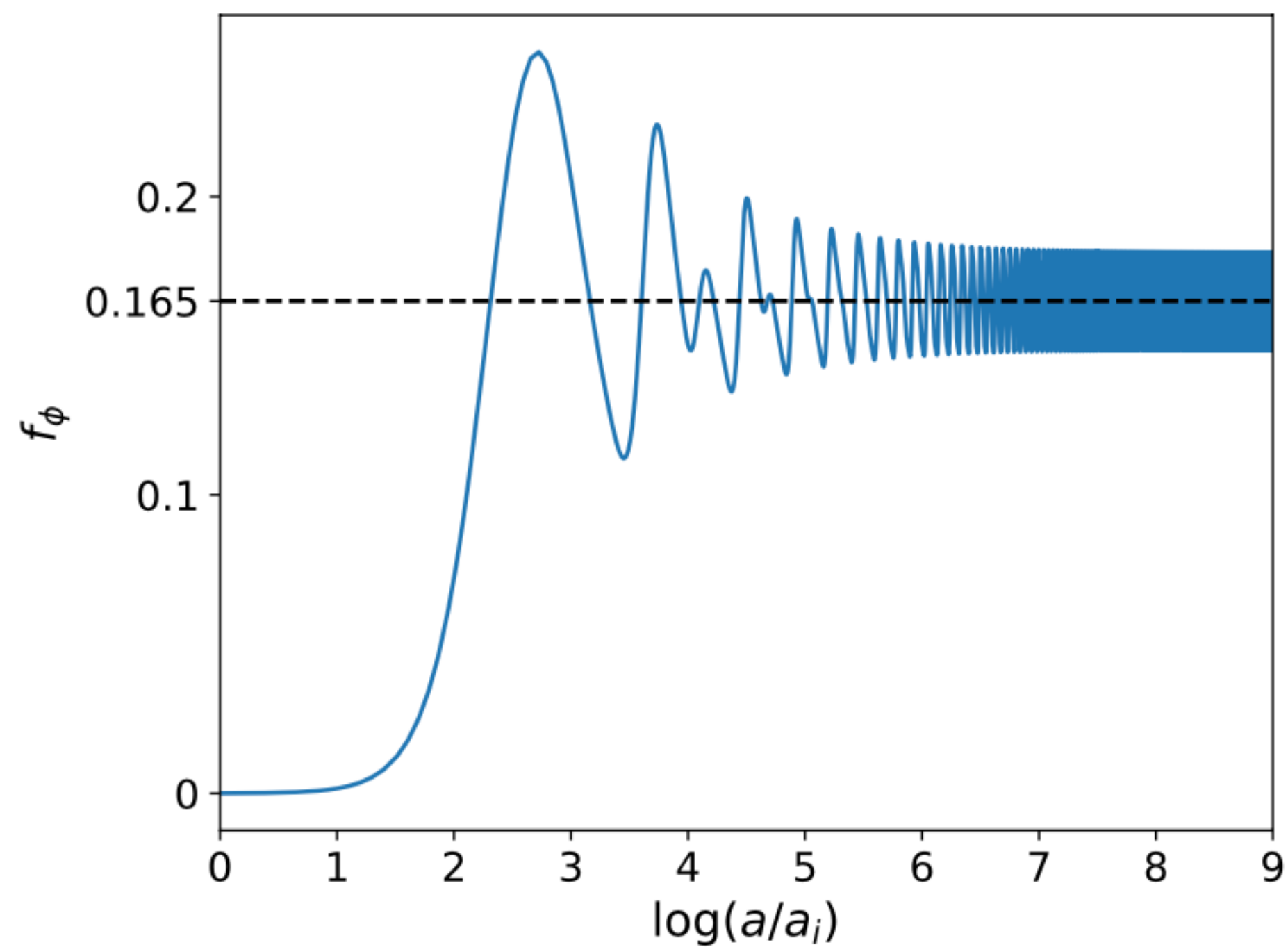


Gravitational Spectroscopy of the Early Universe



$$\mathcal{S} = \int dx^4 \sqrt{-g} \left[\frac{M_p^2}{2} G_4(\phi) R - \frac{1}{2} (\partial\phi)^2 - \lambda\phi^4 \right] + \mathcal{S}_m$$

$$G_4(\phi) = 1 + \alpha \left(\frac{\phi}{M_p} \right)$$



$$\frac{k_\phi}{\mathcal{H}_i} \simeq 2.2(\lambda f_\phi)^{1/4} \quad k_c = k_\phi/2$$