The metallicity dependence and evolutionary times of merging binary black holes: combined constraints from individual gravitational-wave detections and the stochastic background

Kevin Turbang, Max Lalleman, Thomas Callister, and Nick van Remortel

BE-NL GW meeting, Maastricht - October 23rd, 2023

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Introduction

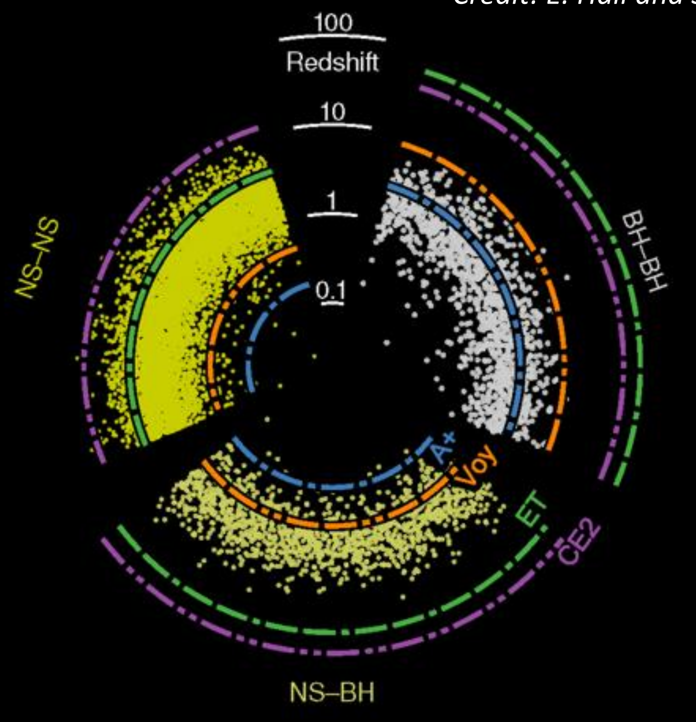
Leverage binary black hole (BBH) demographics as tools to understand :

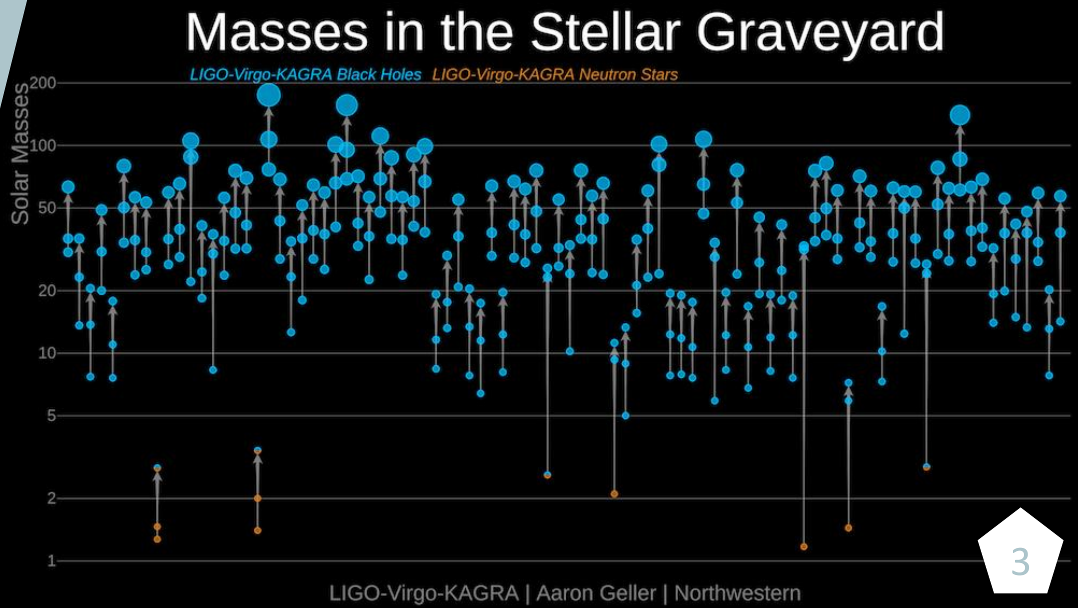
- **Environmental and chemical** conditions required for black hole birth
- **Time delays** experienced by binaries before they merge

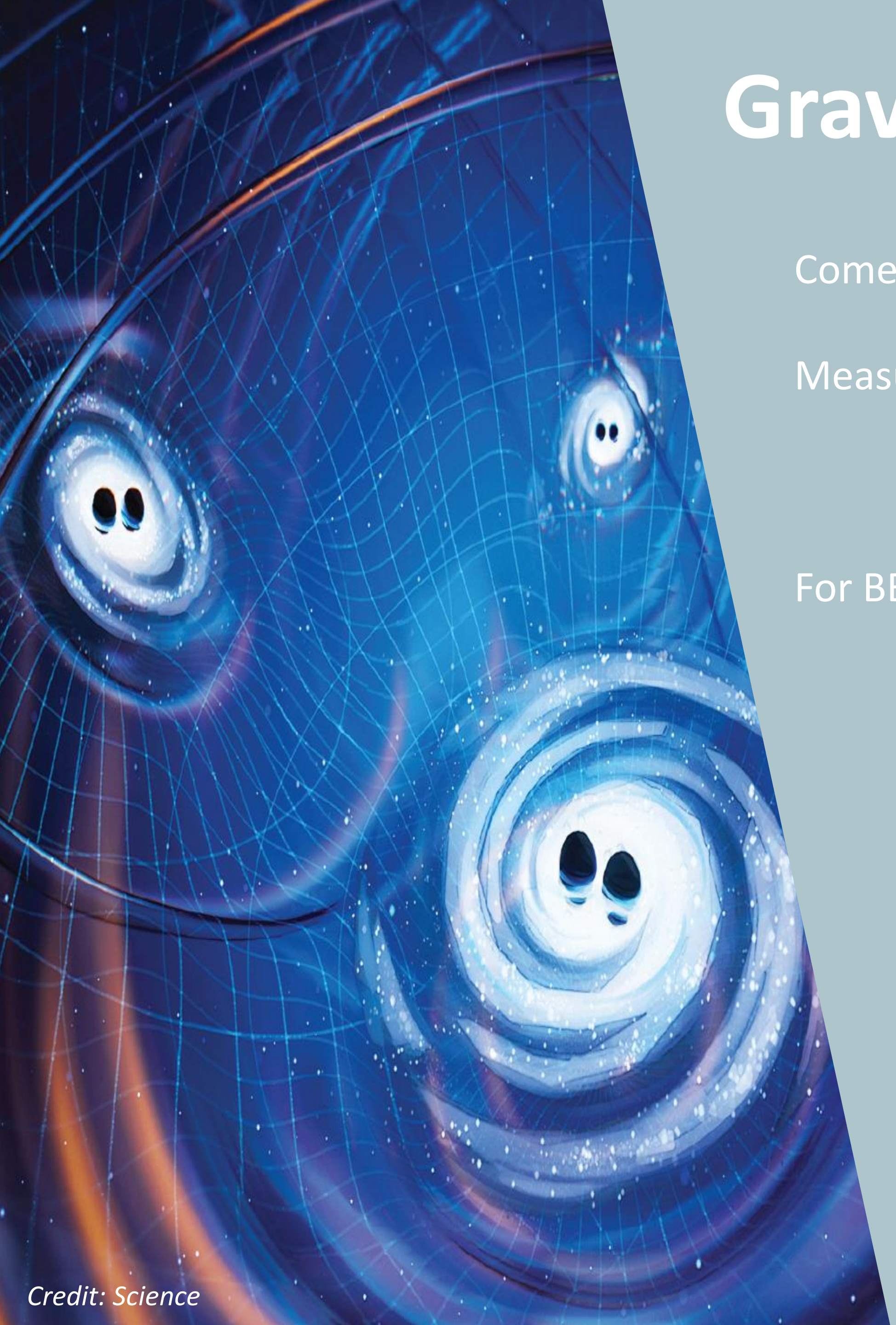
Current gravitational-wave (GW) events occur at low or moderate redshifts ⟶ limited ability to probe high redshift behaviour

Circumvent this by using **joint analysis** that combines individual BBH detections with gravitational-wave background (GWB) data (**Callister+ 2020**)

Credit: E. Hall and S. Vitale







Credit: Science

Gravitational-wave backgrounds

Comes from the **superposition of unresolved** BBH mergers throughout the Universe

Measured in terms of the dimensionless energy density:

$$\Omega(f) = \frac{1}{\rho_c}\frac{d\rho_{\rm GW}}{d\ln f}$$

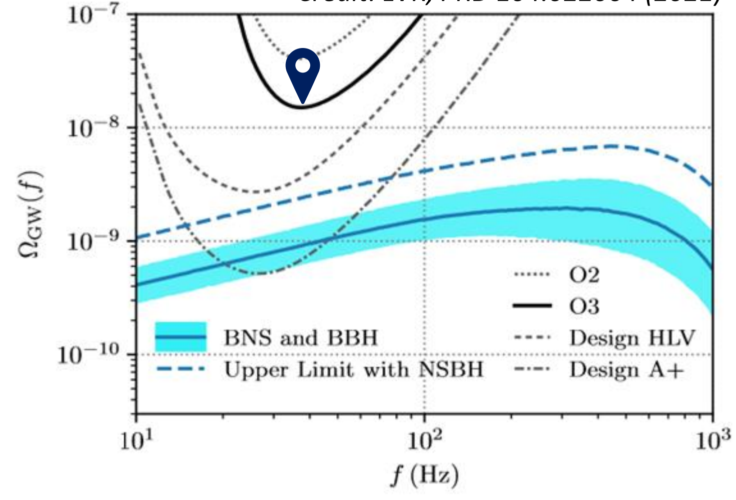
For BBH population:

Merger rate

Population-averaged GW energy spectrum

$$\Omega(f) = \frac{f}{\rho_c}\int_0^{z_{\rm max}} dz\, \frac{\mathcal{R}(z)}{(1+z)H(z)} \left\langle \frac{dE_s}{df_s}\bigg|_{f(1+z)} \right\rangle$$

Credit: LVK, PRD 104.022004 (2021)



Current sensitivity

Binary black hole merger rate

Star formation rate * **Metallicity dependence** * **Time-delay distribution**

=

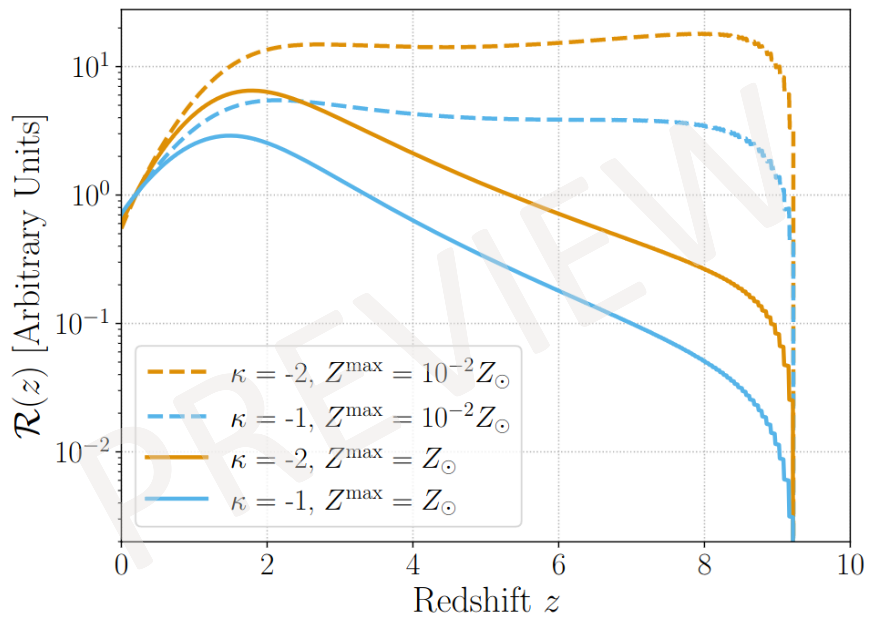
Merger rate $\mathcal{R}(z)$

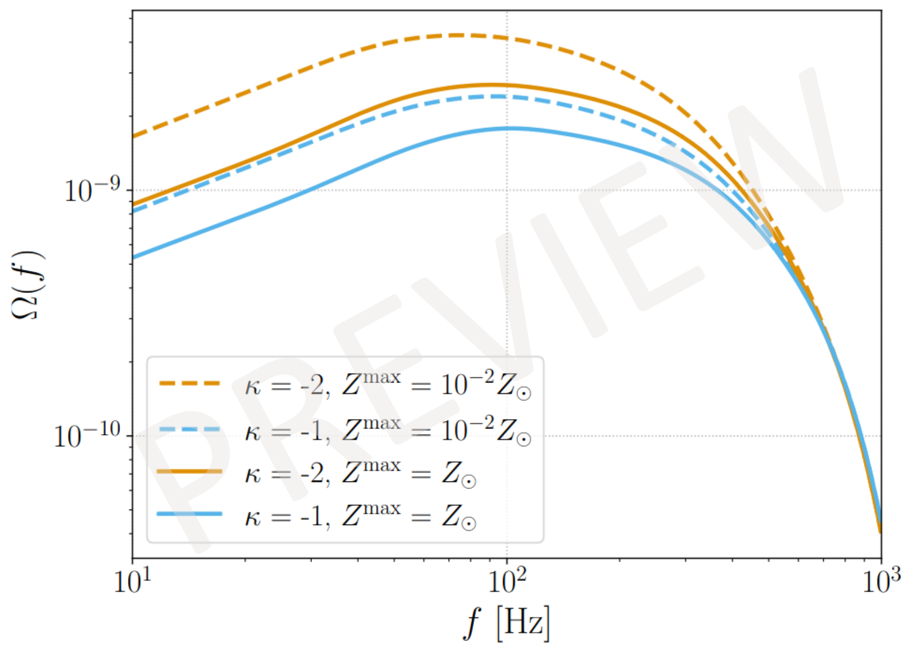
$$p\left(t_d|\kappa, t_d^{\min}\right) = \begin{cases} (t_d)^{\kappa} & \left(t_d^{\min} \leq t_d \leq t_d^{\max}\right) \\ 0 & (else) \end{cases}$$

Fraction of star formation occurring below **metallicity** $Z^{\max}$

Other examples of hyperparameters to be inferred from the data:

- Slope of mass distribution
- Position of Gaussian peak
- Merger rate amplitude





Analysis method

Individual BBH detections

BBH events

↓

Posterior on BBH parameters (e.g., masses)

Stochastic background

GW data from multiple detectors

↓

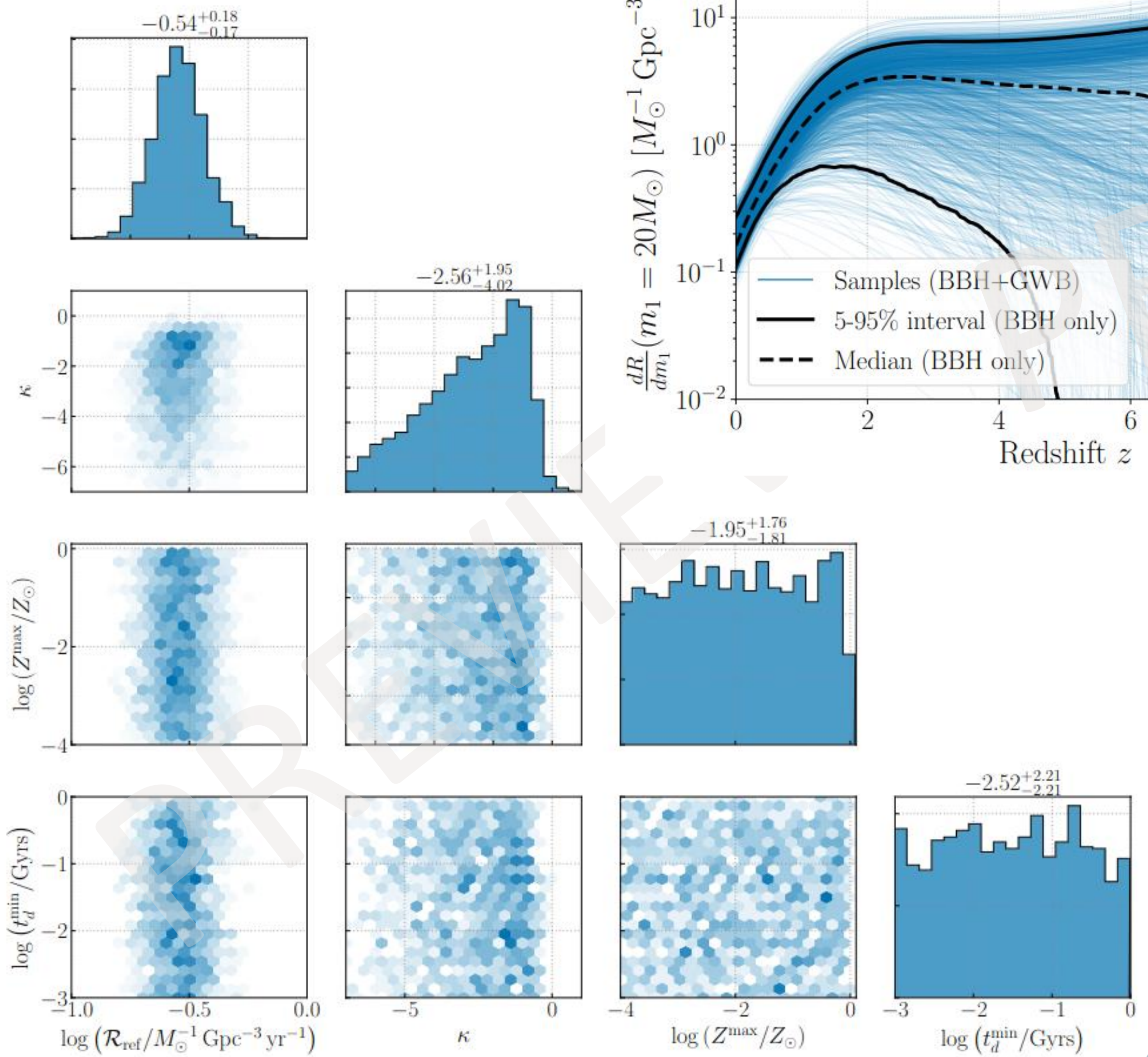
Optimal estimator $\hat{C}$ for GWB spectrum $\Omega(f)$

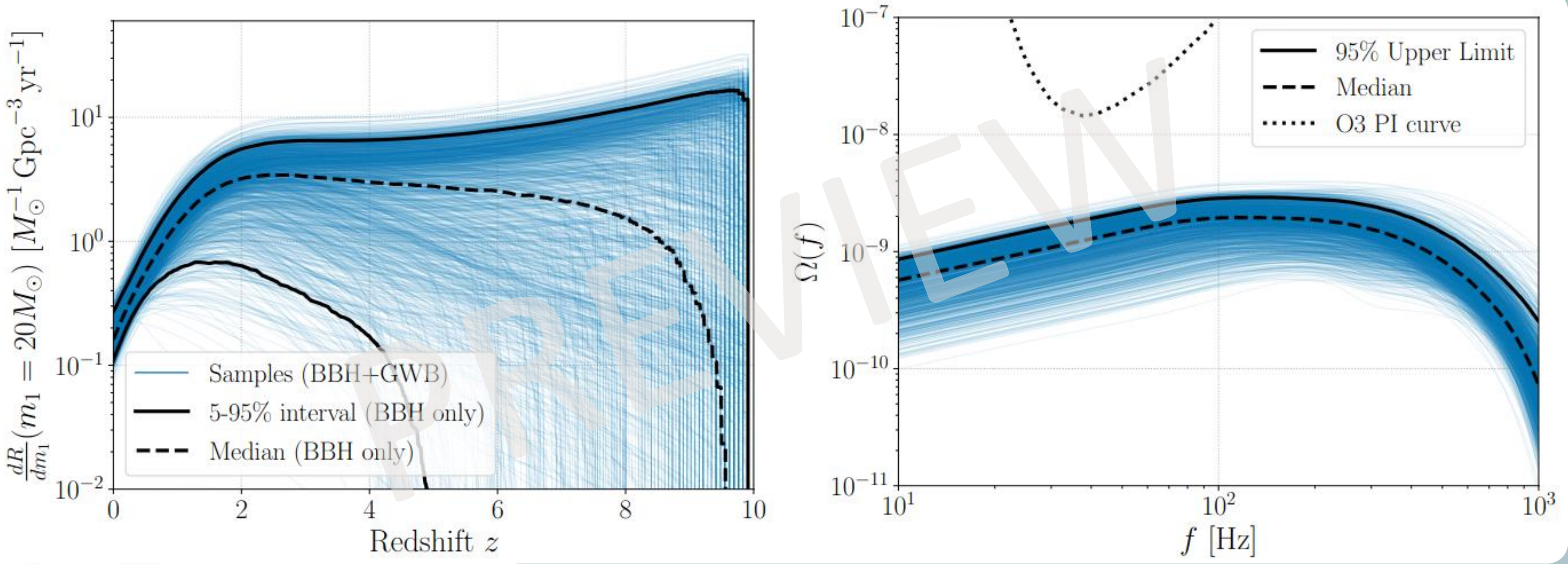
Joint likelihood: $$p(\{d_i\}, \hat{C}|\Lambda) = p_{BBH}(\{d_i\}|\Lambda)\, p_{GWB}(\hat{C}|\Lambda)$$

Want to infer **hyperparameters** Λ **consistent** with the individual **BBH** events and the upper limits/measurements on the **GWB**

Results using O3 LVK data

*Note: at current sensitivity, the addition of the **GWB** to the likelihood is **not yet informative**. Displayed results are entirely dominated by individual BBH detections*

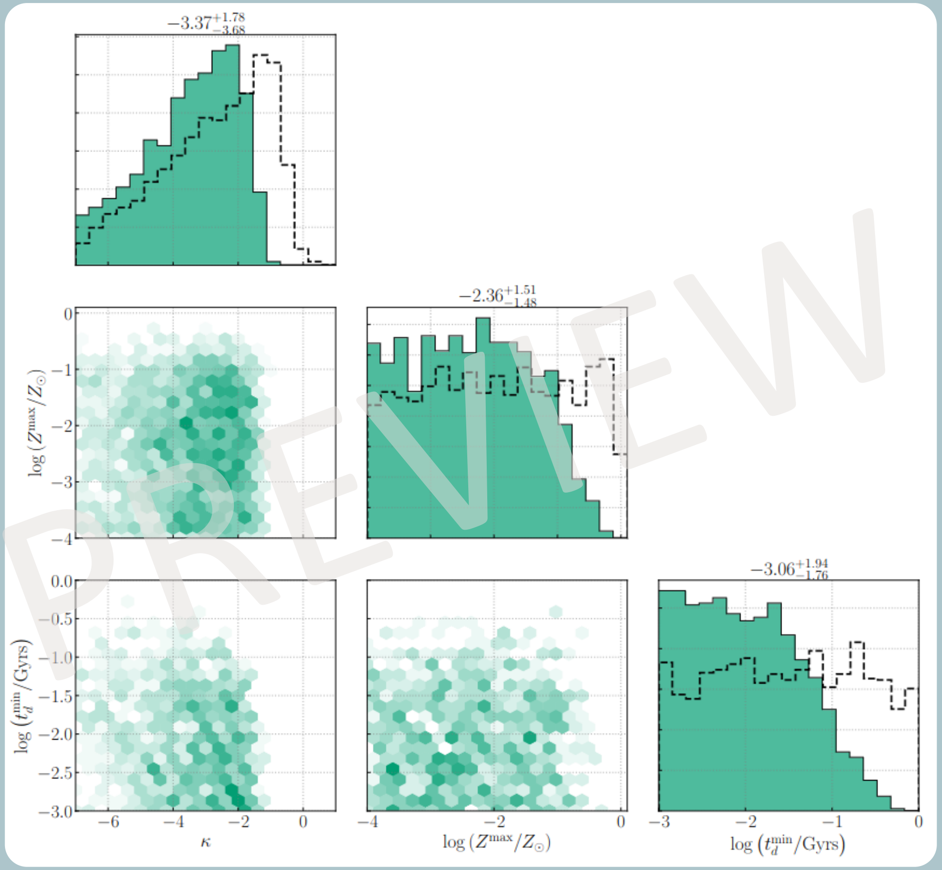




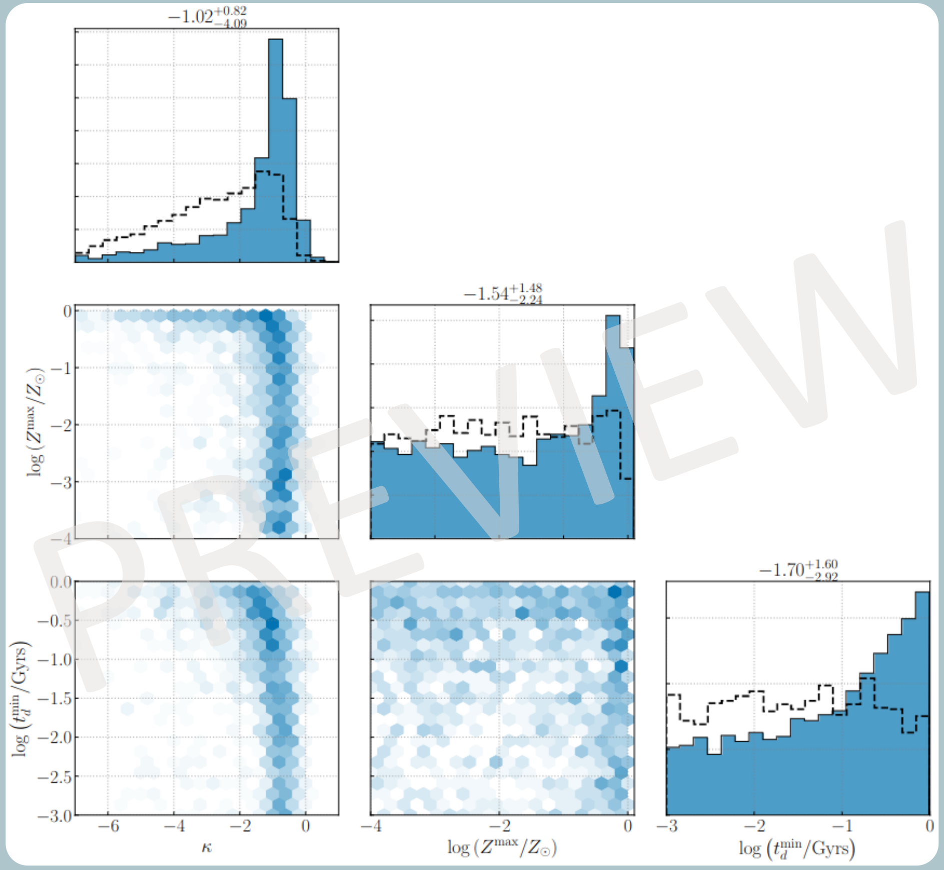
- Local merger rate amplitude entirely determined by individual BBHs
- **Slope** of time-distribution constrained to **negative** values
  - To not contradict observed merger rate at low redshifts
- **Metallicity** and **minimum time-delay** parameter space remains **unconstrained**

Results at Advanced LIGO A+ sensitivity

GWB detection



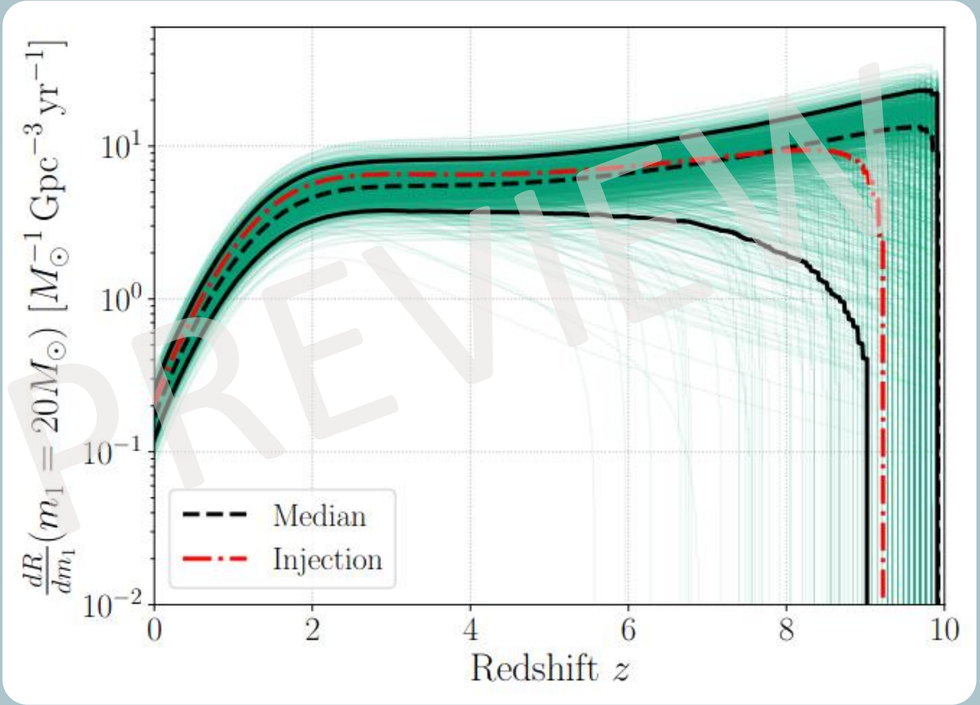
GWB non-detection



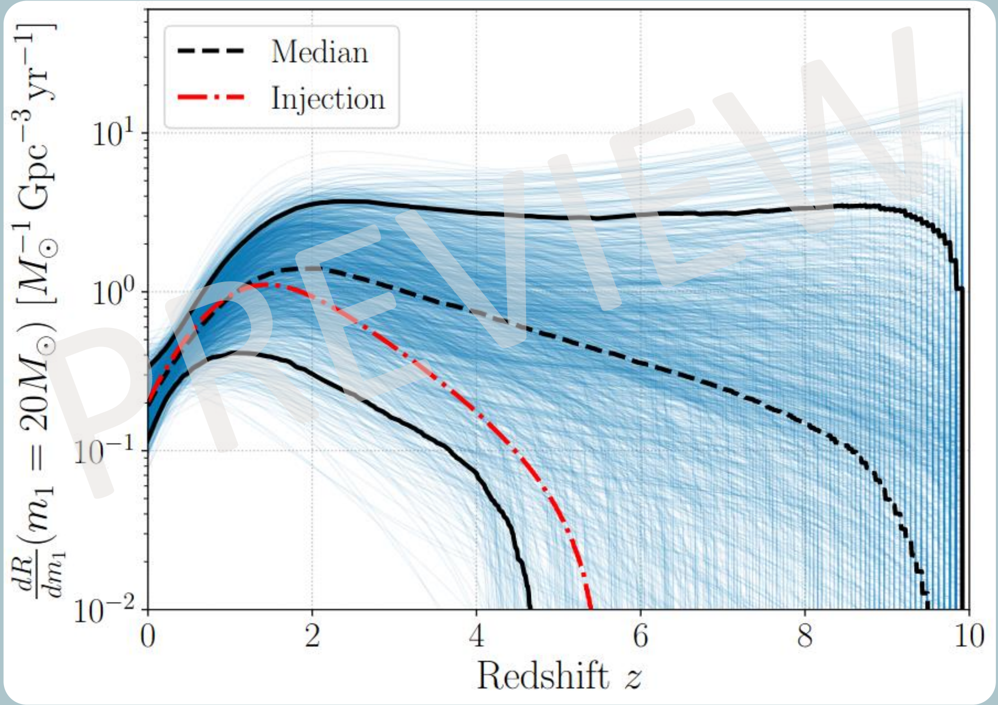
The detection or non-detection of a GWB at O5-like sensitivity will provide **distinct constraints** on the parameter space of interest

Results at Advanced LIGO A+ sensitivity

GWB detection



GWB non-detection

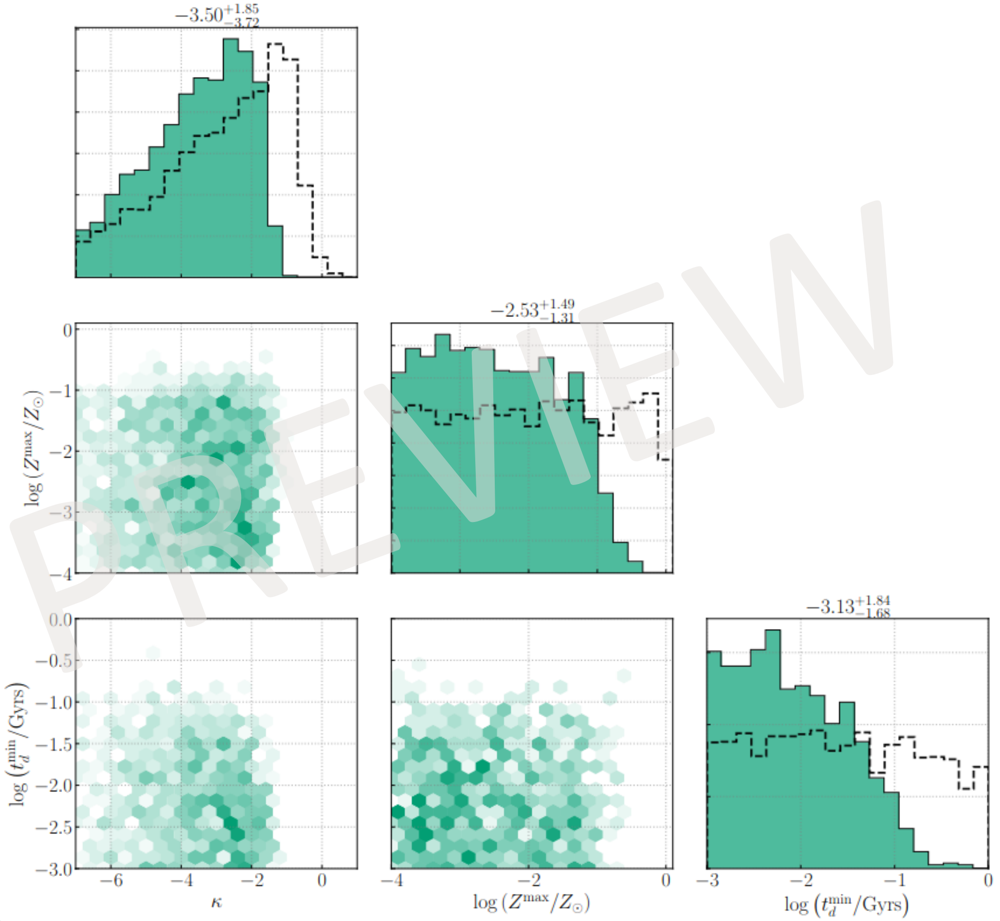


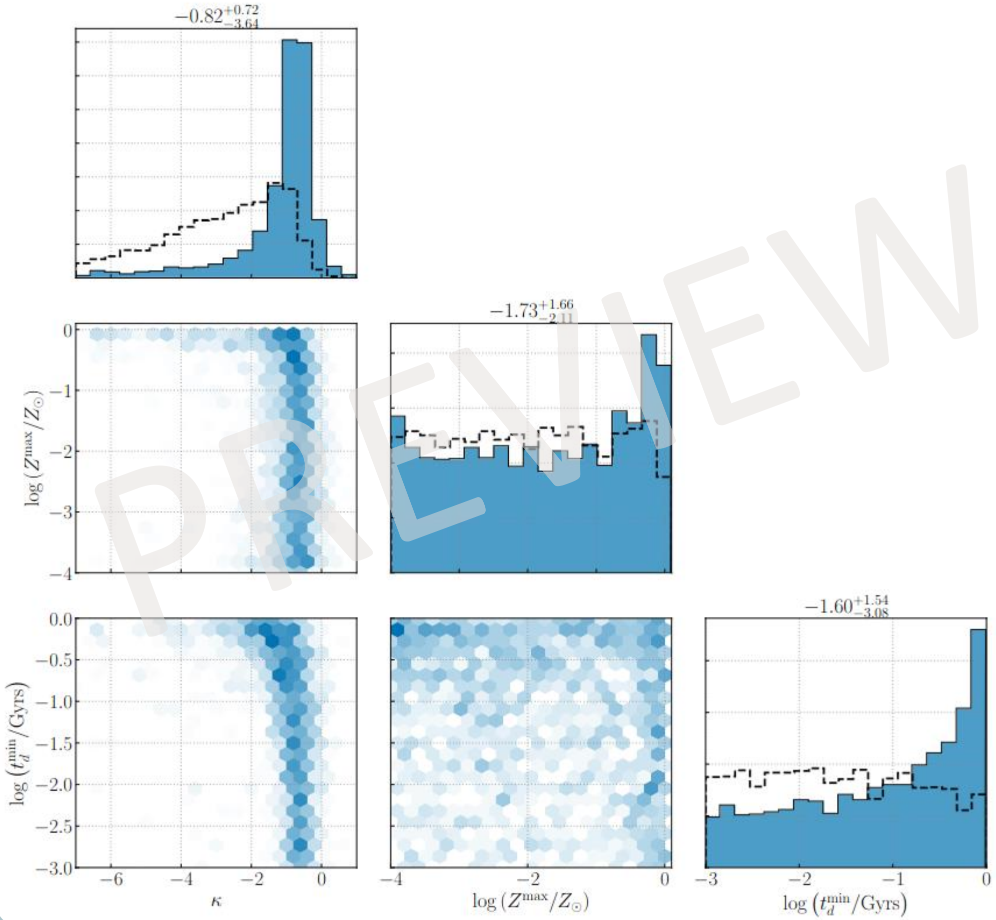
Conclusion

- Constrain slope of the time-delay distribution to **negative** values with O3 data
- Current GWB upper limits are **not informative yet**
- A GWB detection or non-detection at Advanced LIGO A+ sensitivity provides **complementary constraints** and allows to exclude parts of the time-delay and metallicity parameter space
- Keep an eye out for our paper

Extra slides

Vangioni + SFR





Distributions

$$\pi(m_1) = \frac{f_p}{\sqrt{2\pi\sigma_m^2}} \exp\left(-\frac{(m_1-\mu_m)^2}{2\sigma_m^2}\right) + (1-f_p)\left(\frac{1+\alpha}{(100M_\odot)^{1+\alpha}-(2M_\odot)^{1+\alpha}}\right) m_1^\alpha$$

$$p(q|m_1) = \left(\frac{1+\beta_q}{m_1^{1+\beta_q}-(2M_\odot)^{1+\beta_q}}\right) m_2^{\beta_q}$$

$$\phi(m_1) = \begin{cases} \pi(m_1)\exp\left(\frac{-(m_1-m_{\rm low})^2}{2\delta m_{\rm low}^2}\right) & (m_1 < m_{\rm low}) \\ \pi(m_1) & (m_{\rm low} \le m_1 \le m_{\rm high}) \\ \pi(m_1)\exp\left(\frac{-(m_1-m_{\rm high})^2}{2\delta m_{\rm high}^2}\right) & (m_{\rm high} < m_1), \end{cases}$$

$$\pi(\chi_i) = \sqrt{\frac{2}{\pi\sigma_\chi^2}} \frac{e^{-(\chi_i-\mu_\chi)^2/2\sigma_\chi^2}}{\mathrm{Erf}\left(\frac{1-\mu_\chi}{\sqrt{2\sigma_\chi^2}}\right)+\mathrm{Erf}\left(\frac{\mu_\chi}{\sqrt{2\sigma_\chi^2}}\right)}$$

$$\pi(\cos\theta_i) = \sqrt{\frac{2}{\pi\sigma_u^2}} \frac{e^{-(\cos\theta_i-1)^2/2\sigma_u^2}}{\mathrm{Erf}\left(\frac{-2}{\sqrt{2\sigma_u^2}}\right)}$$

CBC likelihood

$$p_{\rm BBH}\left(\{d_i\}|\Lambda\right) \propto e^{-N_{\rm exp}(\Lambda)} \prod_{i=1}^{N_{\rm obs}} \int p\left(d_i|\lambda\right) \frac{dN}{d\lambda}(\Lambda) d\lambda$$

$$p_{\rm BBH}\left(\{d_i\}|\Lambda\right) \propto \left[N\left(\Lambda\right)\xi(\Lambda)\right]^{N_{\rm obs}} e^{-N(\Lambda)\xi(\Lambda)} \times \prod_{i=1}^{N_{\rm obs}} \frac{1}{\xi(\Lambda)} \left\langle \frac{p\left(\lambda_i|\Lambda\right)}{p_{\rm pe}\left(\lambda_i\right)} \right\rangle_{\rm samples}$$

$$\frac{dN}{dz dm_1 dm_2} = \frac{dV_c}{dz} \frac{1}{1+z} \frac{dR}{dm_1 dm_2}$$

$$\frac{dR}{dm_1 dm_2} = \mathcal{R}_{\rm ref} \frac{\mathcal{R}(z)}{\mathcal{R}(0.2)} \frac{\phi(m_1)}{\phi(20M_\odot)} p(m_2)$$

$$\mathcal{R}_{\rm ref} = \left.\frac{dN}{dz dm_1}\right|_{z=0.2, m_1=20M_\odot}$$

GWB likelihood

$$p_{\rm GWB}\left(\hat{C}|\Lambda\right) \propto \exp\left[-\frac{1}{2}\sum_k \left(\frac{\hat{C}(f_k) - \Omega\left(f_k, \Lambda\right)}{\sigma(f_k)}\right)^2\right]$$

$$\hat{C}(f) = \frac{2}{T} \frac{10\pi^2}{3H_0^2} \frac{f^3}{\gamma_{12}(f)} \tilde{s}_1(f) \tilde{s}_2^*(f)$$

$$\sigma^2(f) \approx \frac{1}{2T\Delta f} \left(\frac{10\pi^2}{3H_0^2}\right)^2 \frac{f^6}{\gamma_{12}^2(f)} P_1(f) P_2(f)$$

Priors

Mass distribution			
Parameter	Prior	Minimum	Maximum
$m_{\rm low}/M_\odot$	Uniform	5	15
$m_{\rm high}/M_\odot$	Uniform	50	100
$\mu_m/M_\odot$	Uniform	20	50
$\sigma_m/M_\odot$	Uniform	1.5	15
$f_{\rm peak}$	Log-Uniform	10^{-3}	1
$\delta m_{\rm low}/M_\odot$	Log-Uniform	10^{-1}	$10^{0.5}$
$\delta m_{\rm high}/M_\odot$	Log-Uniform	$10^{0.5}$	$10^{1.5}$
Parameter	Prior	Mean	Standard deviation
α	Gaussian	-2	3
β_q	Gaussian	0	3
Time-delay distribution			
Parameter	Prior	Minimum	Maximum
$\mathcal{R}_{\rm ref}/M_\odot^{-1}{\rm Gpc}^{-3}{\rm yr}^{-1}$	Log-Uniform	10^{-2}	10
$Z^{\rm max}/Z_\odot$	Log-Uniform	10^{-4}	1
$t_d^{\rm min}/{\rm Gyrs}$	Log-Uniform	10^{-5}	1
Parameter	Prior	Mean	Standard deviation
κ	Gaussian	-1	3
Spin distribution			
Parameter	Prior	Minimum	Maximum
μ_χ	Uniform	0	1
σ_χ	Log-Uniform	10^{-1}	1
σ_u	Uniform	0.3	2