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Jointly Estimating multiple Components and Population Properties of Astrophysical Gravitational-Wave Background

Presented by
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Outline

- Introduction
  - What is a stochastic-gravitational wave background (SGWB)?
  - How to search for an SGWB with ground-based detectors?
  - How to account for multiple components being present at the same time?

- This work
  - Joint estimation of the amplitudes and ensemble properties of astrophysical SGWBs from compact binary coalescences (CBCs), r-modes, and magnetars
  - Injection study to validate the method for realistic SGWB spectral shapes
  - Results with the data from the LIGO-Virgo-KAGRA first three observing runs

What is a SGWB? – Definition and related quantities

“Textboook” definition [1]

A random gravitational-wave signal produced by a large number of weak, independent and unresolved sources.

Characterisable only statistically

Depending on details of the observation

Not decomposable into separate and individually detectable sources

[1] Romano, J.D., Cornish, N.J. *Living Rev Relativ* **20**, 2 (2017)

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GW Energy density ratio

$$\Omega_{\rm gw}(f) = \frac{f}{\rho_c}\frac{d\rho_{gw}}{df}(f), \qquad \rho_c = \frac{3H_0^2 c^2}{8\pi G}$$ Critical density

$$\rho_{\rm GW} = \frac{\langle \dot{h}_{ij}(t,\vec{\boldsymbol{x}})\, \dot{h}_{ij}(t,\vec{\boldsymbol{x}})\rangle}{32\pi G/c^2} = \int_{f=0}^{f_{max}} f\,\frac{d\rho_{gw}}{df}\,df$$

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Gaussian, stationary, unpolarized, isotropic background

Spacial homogeneity and isotropy

Stationarity

$$\langle h_A^*(f,\hat{\boldsymbol{n}})h_A(f',\hat{\boldsymbol{n}}')\rangle = \frac{1}{16\pi}S_h(f)\,\delta(f-f')\,\delta_{AA'}\,\delta^2(\hat{\boldsymbol{n}},\hat{\boldsymbol{n}'})$$

Unpolarized

$$S_h(f) = \frac{3H_0^2}{2\pi^2}\frac{\Omega_{\rm gw}(f)}{f^3}$$

One-sided GW strain power spectral density
(summed over polarizations and integrated over the sky)

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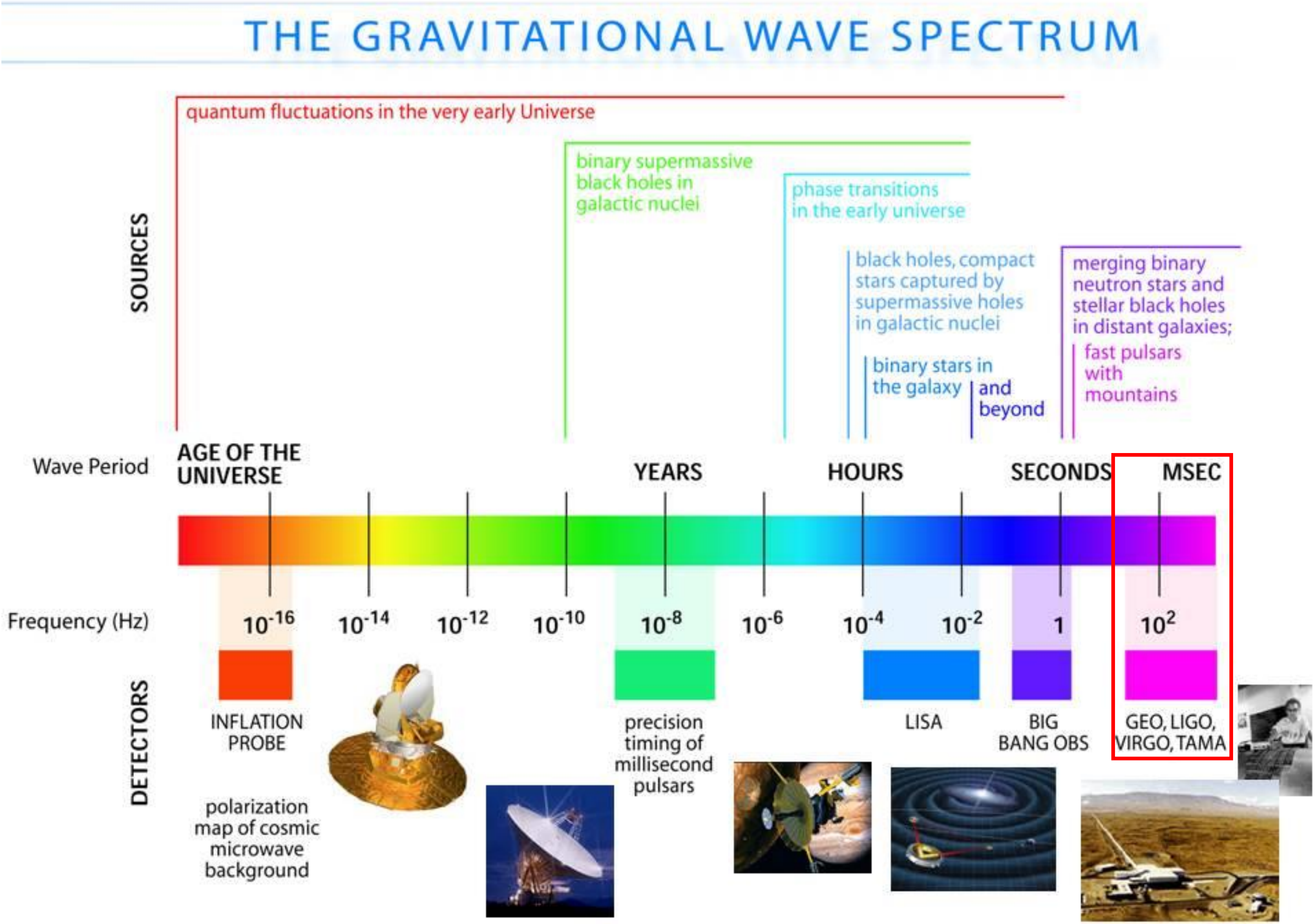
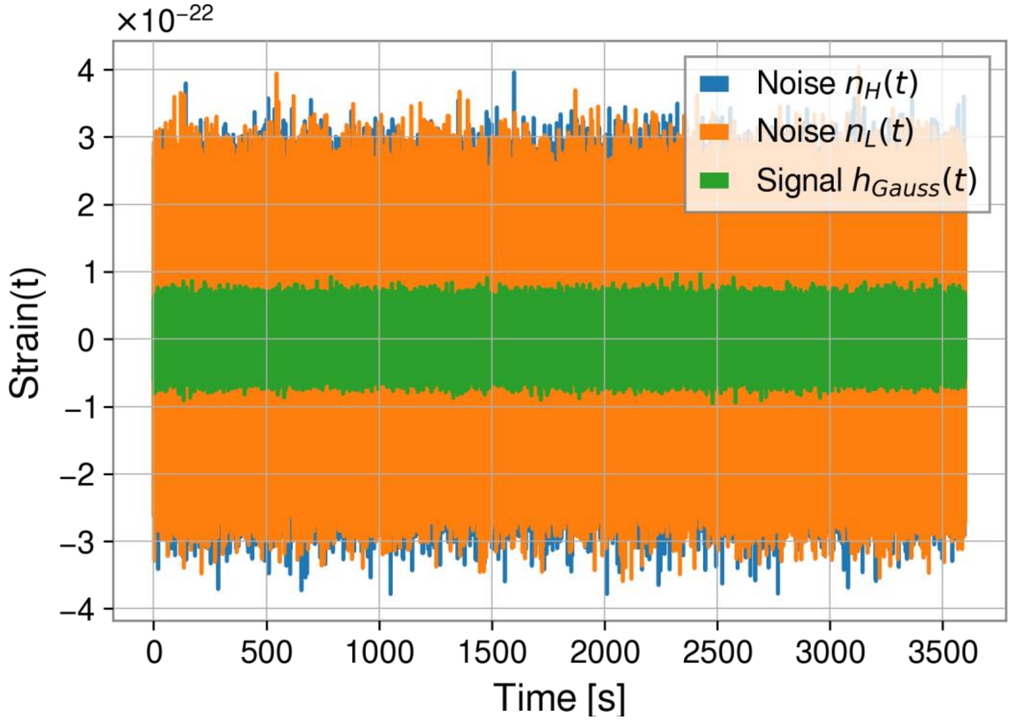


http://www.thespectrumofriemannium.com/2019/10/19/log237-gw-music/

Search for SGWB with ground-based detectors

Cross-correlation statistic: basic ideas

Answer to the question:
''How to deal with the fact that SGWB is indistinguishable from unidentified instrumental noise in a single detector?''



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2 different detectors data $d_1 = h + n_1, \qquad d_2 = h + n_2$

Cross-correlated

$$\langle \hat{C}_{12} \rangle = \langle d_1 d_2 \rangle = \langle h^2 \rangle + \underbrace{\langle h n_2 \rangle}_{0} + \underbrace{\langle n_1 h \rangle}_{0} + \langle n_1 n_2 \rangle = \langle h^2 \rangle \equiv S_h$$

Cross-correlation as **estimator of the GW power spectral density**

Non-zero, in general (e.g. Schumann resonances, see Stavros Venikoudis' poster), yet distinguishable from SGWB

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Estimator for $\boldsymbol{\Omega_{\mathrm{ref}} \equiv \Omega_{\mathrm{gw}}(f_{\mathrm{ref}} = 25\ \mathrm{Hz})}$

Frequency power-law model

$$\Omega_{\mathrm{gw}}(f) = \Omega_{\mathrm{ref},\alpha} \left(\frac{f}{f_{\mathrm{ref}}} \right)^{\alpha}$$

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$$\Omega_{\mathrm{gw}}(f) = \Omega_{\mathrm{ref},\alpha} \left(\frac{f}{f_{\mathrm{ref}}} \right)^{\alpha}$$

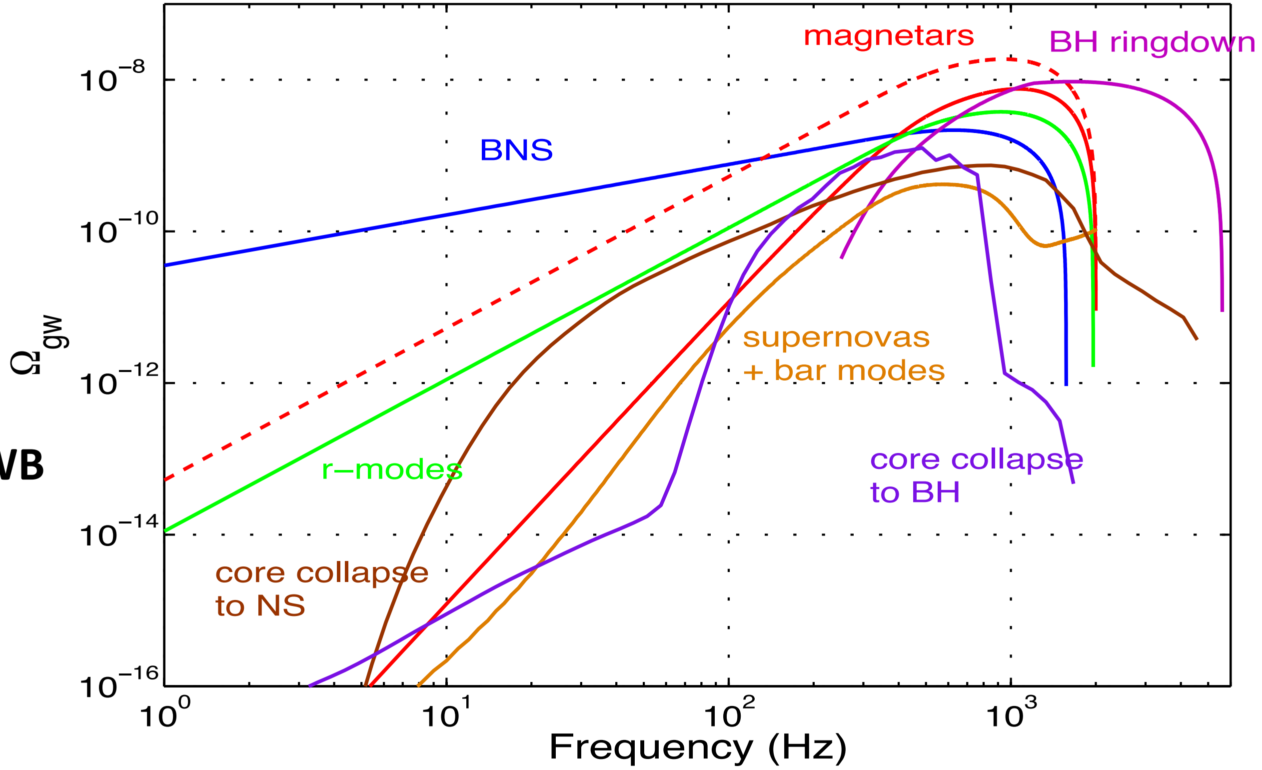
$$\boxed{\widehat{\Omega}_{\mathrm{ref},\alpha}(\mathrm{f}) \equiv \frac{2}{T} \frac{Re\left[\tilde{d}_1(f)\tilde{d}_2^*(f)\right]}{\gamma_{12}(f) S_\alpha(f)}}$$

$$S_\alpha(f) \equiv \frac{3H_0^2}{10\pi^2} \frac{1}{f_{\mathrm{ref}}^3} \left(\frac{f}{f_{\mathrm{ref}}} \right)^{\alpha - 3}$$

Isotropic overlap reduction function

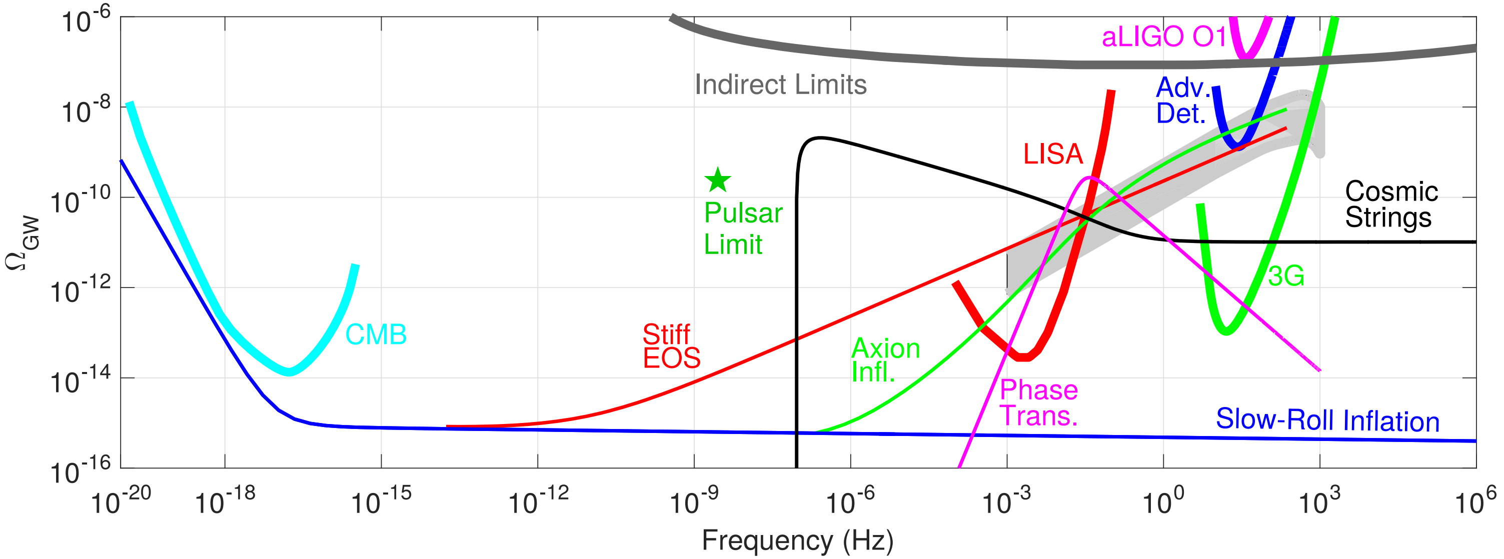
How to deal with multiple SGWBs at the same time?

Astrophysical SGWB



Regimbau T., 2011

Cosmological SGWB
and sensitivities of the experiments



Sathyaprakash B.S. et al., 2019

Limits of the standard formalism

- It estimates (and hence assumes the presence of) a single component at a time
- Still possible to include multiple components at the parameter estimation (PE) stage, assuming in the likelihood

$$\Omega_{gw}(f) = \sum_{\{\alpha\}} \Omega_{\alpha}(f) w_{\alpha}(f), \quad w_{\alpha} \equiv \left(\frac{f}{f_{\mathrm{ref}}}\right)^{\alpha}$$

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$$\Omega_{gw}(f) = \sum_{\{\alpha\}} \Omega_\alpha(f) w_\alpha(f), \quad w_\alpha \equiv \left(\frac{f}{f_{\rm ref}}\right)^\alpha$$

- However
  - this may be time consuming
  - it does not consider possible correlations among different α
  - A priori, the single-component estimator $\widehat{\Omega}_\alpha$ is likely to be biased in presence of multiple components (see next slides)

- How to include correlations among different α before PE and get an unbiased, joint estimator for Ω_α?

Isotropic multi-component formalism

- See Boileau et al. 2021 for a summary of possible methods, here we follow Parida et al. 2016
- Power-law model: $\Omega_{\rm gw}(f) = \sum_{\{\alpha\}} \Omega_\alpha(f) w_\alpha(f)\,, \quad w_\alpha \equiv \left(\frac{f}{f_{\rm ref}}\right)^\alpha$
- Maximum Likelihood estimator for $\mathbf{\Omega} \equiv \Omega_\alpha$, and covariance matrix $\mathbf{\Sigma}$:

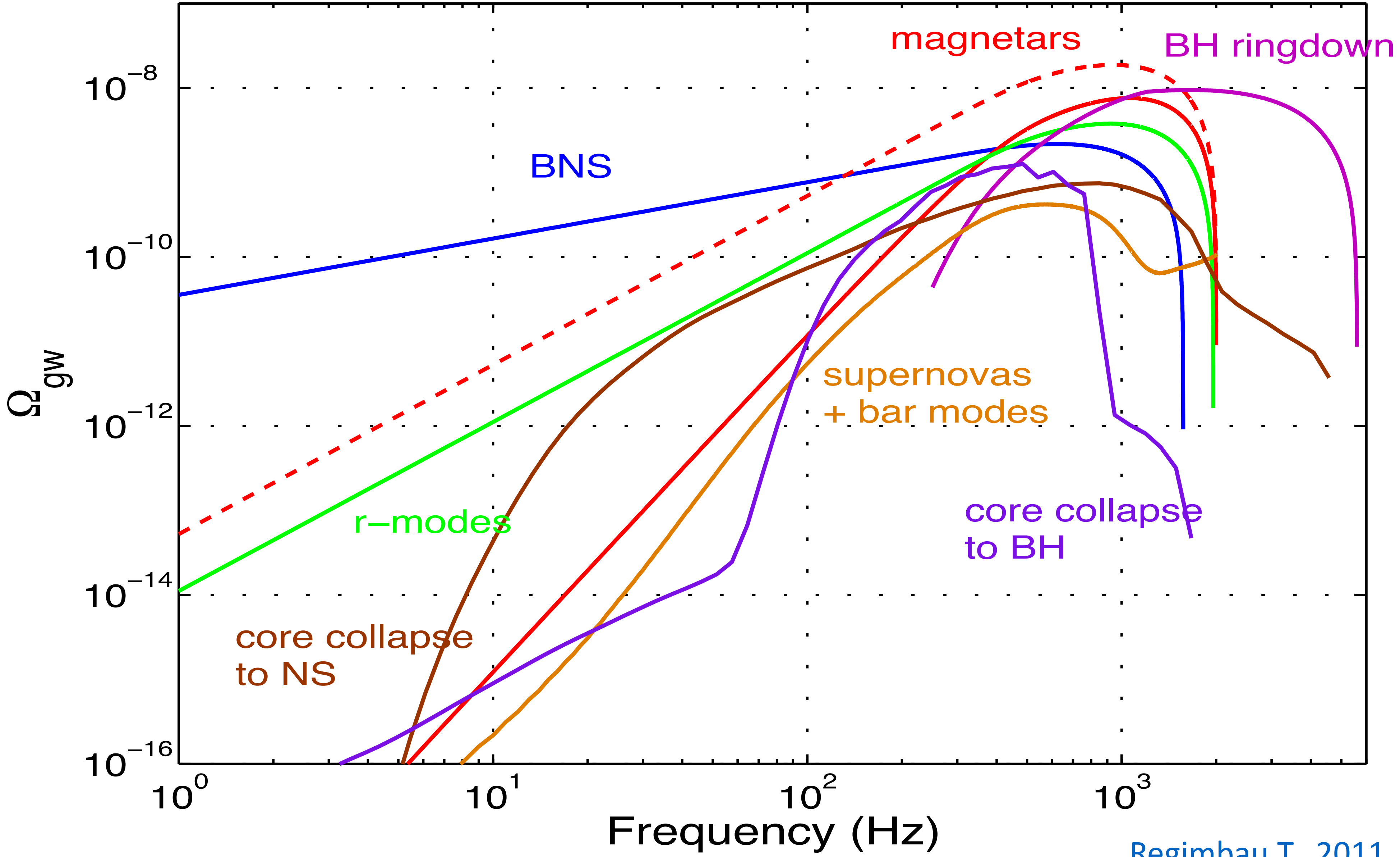
$$\widehat{\mathbf{\Omega}} = \mathbf{\Gamma}^{-1} \cdot \boldsymbol{X}, \qquad \mathbf{\Sigma} = \mathbf{\Gamma}^{-1}, \qquad \boldsymbol{\sigma}^2 = \mathrm{diag}(\mathbf{\Sigma})$$

- (Broad-band) “Dirty map” and Fisher matrix:

$$X_\alpha = \sum_{t,f} 4\Delta f \, \frac{\tilde{s}_I^*(t,f)\tilde{s}_J(t,f)}{P_I(f)P_J(f)} \, \gamma_{IJ}(f)\, S_0(f)\, w_\alpha(f)$$

$$\Gamma_{\alpha\alpha'} = \sum_{t,f} 2T\Delta f \, \frac{\left|\gamma_{IJ}(f)\right|^2 S_0^2(f)}{P_I(f)P_J(f)} \, w_\alpha(f) w_{\alpha'}(f)$$

Astrophysical SGWBs



Regimbau T., 2011

Astrophysical SGWBs

- Astrophysical SGWB master formula(Phinney 2001, Regimbau 2011)

$$\Omega_{\rm gw}(f) = \frac{f}{\rho_c} \int_{\Theta} \underbrace{\mathrm{d}\theta\, p(\theta)}_{\text{Source parameters}} \int_{z_{min}}^{z_{max}} \frac{\overbrace{R(z,\Theta)}^{\text{Cosmic rate}}}{(1+z)\, H_0\, E(z)} \underbrace{\frac{dE_{gw}}{df}}_{\text{GW energy spectrum in source frame}} \Bigg|_{f=f_s(1+z)}$$

Astrophysical SGWBs

- Astrophysical SGWB master formula(Phinney 2001, Regimbau 2011)

$$\Omega_{\rm gw}(f) = \frac{f}{\rho_c}\int_{\Theta} \mathrm{d}\theta\, p(\theta) \int_{z_{min}}^{z_{max}} \frac{R(z,\Theta)}{(1+z)\, H_0\, E(z)} \left.\frac{dE_{gw}}{df}\right|_{f=f_s(1+z)}$$

Source parameters

Cosmic rate

GW energy spectrum in source frame

- Power-law approximation

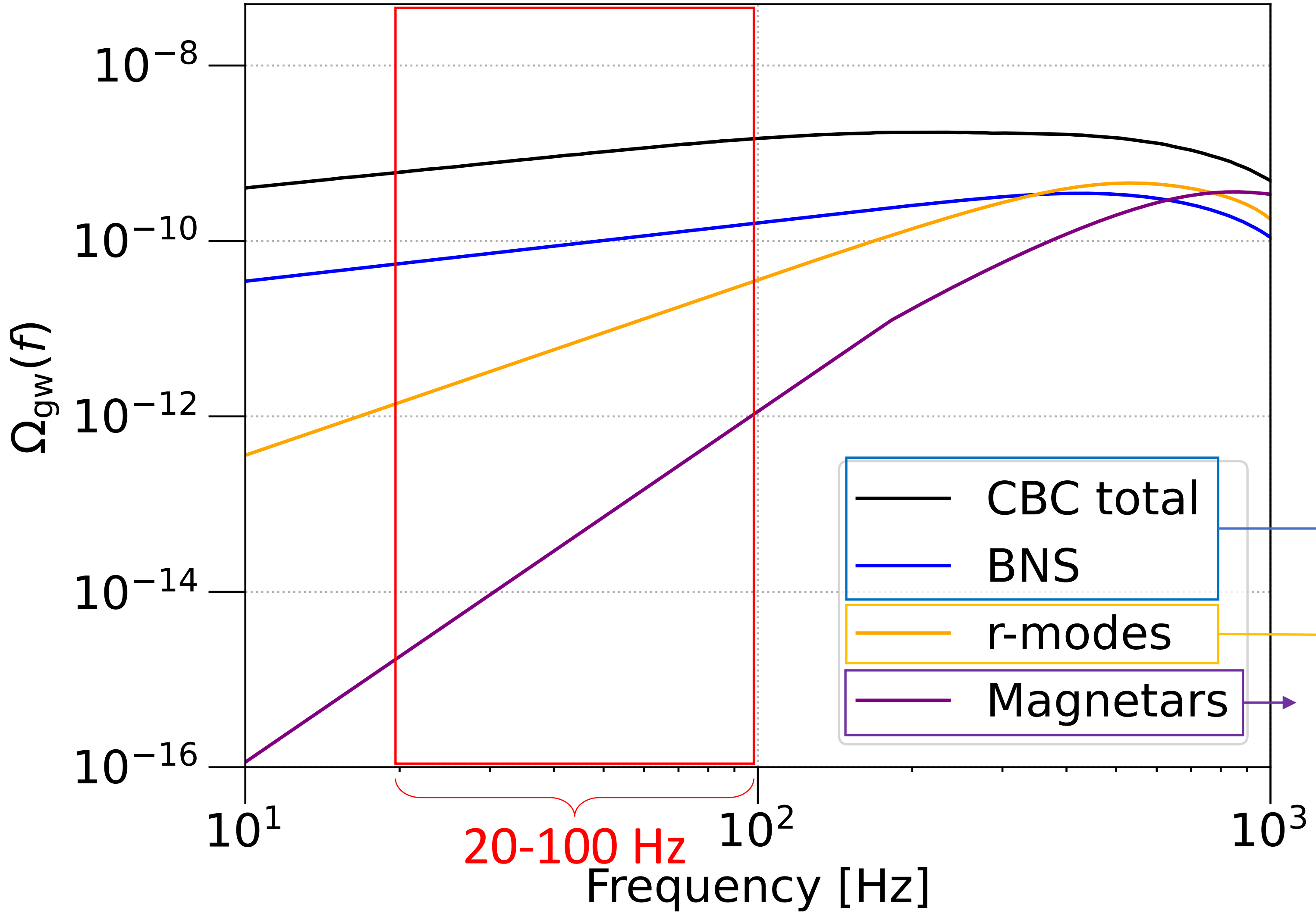
$$\Omega_{\rm gw}(f) \approx \xi \left(\frac{f}{f_{\rm ref}}\right)^{\alpha} \prod_i \langle \theta_i^{c_i} \rangle,$$

Ensemble properties

with

$$\xi \equiv \frac{\Omega_{\rm gw}(f_{\rm ref})}{\prod_{\rm i} \langle \theta_{\rm i}^{c_{\rm i}} \rangle}$$

Astrophysical SGWBs of interest



GWTC-3 Pop

Zhu et al. 2011

Regimbau-Mandic 2008

Astrophysical SGWBs of interest

Population	$\Omega_{\mathrm{gw}}(f)$	**Power law**	**Parameter to constrain**
$j = \mathrm{BBH, BNS, BHNS}$	$\Omega_{\mathrm{gw},j}(f) \approx \xi_j f^{2/3} R_{0,j} \left\langle \mathcal{M}_c^{5/3} \right\rangle_j$	$\alpha = 2/3$	$K_j \equiv R_{0,j} \left\langle \mathcal{M}_c^{5/3} \right\rangle_j$
CBC (see Kevin Turbang's talk)	$\Omega_{\mathrm{gw,CBC}}(f) \approx \xi_{\mathrm{CBC}}\ f^{2/3} \sum_j R_{0,j} \left\langle \mathcal{M}_c^{5/3} \right\rangle_j$	$\alpha = 2/3$	$K_{\mathrm{CBC}} \equiv \sum_j K_j$
r-mode instability in young NSs (Owen et al. 1998, Zhu et al. 2011)			
magnetars (Regimbau-Mandic 2008, Wu et al. 2013)			

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r-mode instability in young NSs (Owen et al. 1998, Zhu et al. 2011)	$\Omega_{\rm gw,r-modes}(f) \approx \xi_{\rm r-modes}\ f^2 \langle (K + 2)^{-1} \rangle$ Related to r-modes intensity α	$\alpha = 2$	$K_{\rm r-modes} \equiv \langle (K+2)^{-1} \rangle$
magnetars (Regimbau-Mandic 2008, Wu et al. 2013)			

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j = BBH, BNS, BHNS	$\Omega_{gw,j}(f) \approx \xi_j f^{2/3} R_{0,j} \left\langle \mathcal{M}_c^{5/3} \right\rangle_j$	$\alpha = 2/3$	$K_j \equiv R_{0,j} \left\langle \mathcal{M}_c^{5/3} \right\rangle_j$
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magnetars ([Regimbau-Mandic 2008](), [Wu et al. 2013]())	$\Omega_{gw,magnetars}(f) \approx \xi_{magnetars}\ f^4 \langle \varepsilon^2 \rangle \langle B^{-2} \rangle$ ellipticity and (poloidal) magnetic field	$\alpha = 4$	$K_{magnetars} \equiv \sqrt{\langle \varepsilon^2 \rangle \langle B^{-2} \rangle}$

Validation: astrophysical multi-component injection study

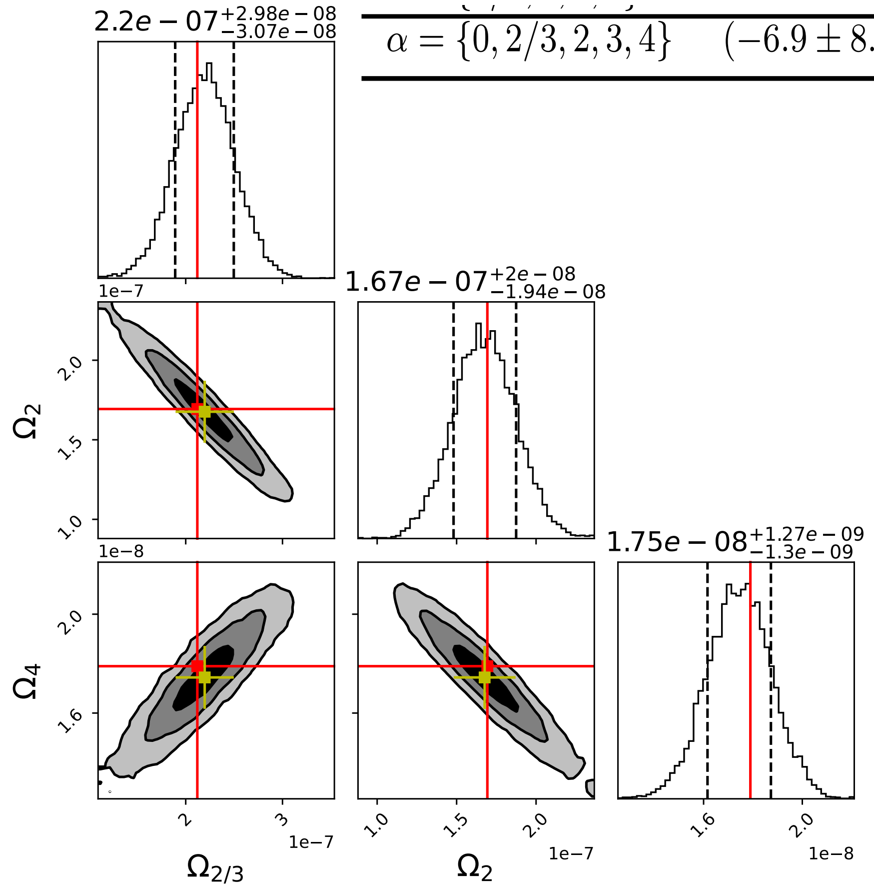
- Injected (detectable) astrophysical SGWB from BNS, r-modes, and magnetars in O3 data
- Goal: recover both the amplitudes (similar and different orders of magnitude) and the population parameters (assuming SGWB being detectable)

Population	BNS	r-modes	Magnetars
Power law	$\alpha = 2/3$	$\alpha = 2$	$\alpha = 4$
Injected parameters	$K_{\mathrm{BNS}} \simeq 7.91 \times 10^5\ M_{\odot}^{5/3}\mathrm{Gpc}^{-3}\mathrm{yr}^{-1}$ $(\mathrm{R}_{0,\mathrm{BNS}} = 3.2 \times 10^5 \mathrm{Gpc}^{-3}\mathrm{yr}^{-1})$	$K_{\mathrm{r-modes}} = 10^3$ $(K = -1.999)$	$K_{\mathrm{magnetars}} = 10^{-11}$ $\left(\frac{\varepsilon}{B} = 10^{-11}\right)$
Resulting Ω_α $(\mathrm{f}_{\mathrm{ref}} = 25\ \mathrm{Hz})$	$\Omega_{\mathrm{BNS}} \simeq 2.12 \times 10^{-7}$	$\Omega_{\mathrm{r-modes}} \simeq 1.69 \times 10^{-7}$	$\Omega_{\mathrm{magnetars}} \simeq 1.79 \times 10^{-8}$

Astrophysical multi-component injections: SGWB intensity

Estimators	$\hat{\Omega}_0$	$\hat{\Omega}_{2/3} = 2.1 \times 10^{-7}$	$\hat{\Omega}_2 = 1.7 \times 10^{-7}$	$\hat{\Omega}_3$	$\hat{\Omega}_4 = 1.8 \times 10^{-8}$
$\alpha = \{0\}$	$(8.68 \pm 0.08) \times 10^{-7}$	-	-	-	-
$\alpha = \{2/3\}$	-	$(7.12 \pm 0.06) \times 10^{-7}$	-	-	-
$\alpha = \{2\}$	-	-	$(3.61 \pm 0.03) \times 10^{-7}$	-	-
$\alpha = \{3\}$	-	-	-	$(1.44 \pm 0.01) \times 10^{-7}$	-
$\alpha = \{4\}$	-	-	-	-	$(4.28 \pm 0.03) \times 10^{-8}$
$\alpha = \{2/3, 2, 4\}$	-	$(2.2 \pm 0.3) \times 10^{-7}$	$(1.7 \pm 0.2) \times 10^{-7}$	-	$(1.7 \pm 0.1) \times 10^{-8}$
$\alpha = \{0, 2/3, 2, 3, 4\}$	$(-6.9 \pm 8.9) \times 10^{-7}$	$(1.2 \pm 1.4) \times 10^{-6}$	$(-2.6 \pm 7.1) \times 10^{-7}$	$(1.3 \pm 2.6) \times 10^{-7}$	$(4.3 \pm 30.3) \times 10^{-9}$

20-100 Hz



- Ω_α well recovered only when the right α=2/3, 2, 4 combination is considered
- Capability of disentangling SGWB with similar intensities or spanning order of magnitudes

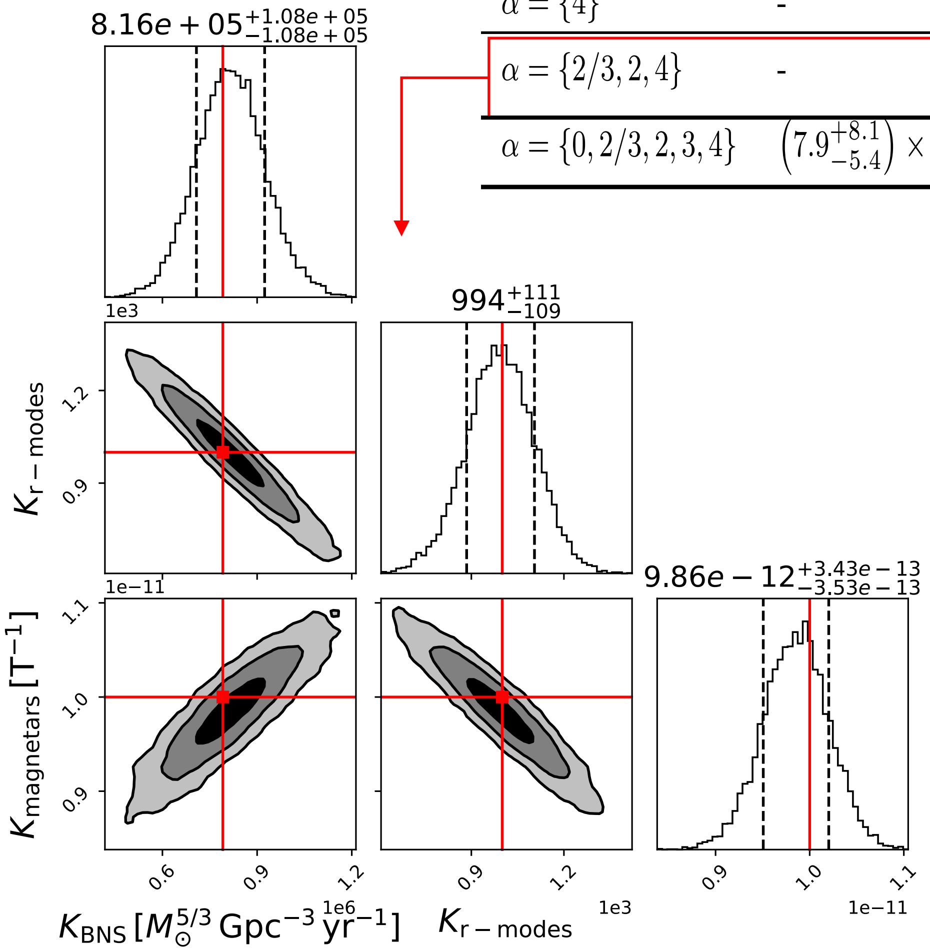
PE results (α=2/3, 2, 4): Red lines/boxes are the injections. Yellow error bars refer to the estimators.

Astrophysical multi-component injections: astrophysical parameters

	Ω_0	Ω_3	$K_{\mathrm{BNS}} = 7.9 \times 10^5\, M_\odot^{5/3}\, \mathrm{Gpc}^{-3}\, \mathrm{yr}^{-1}$	$K_{\mathrm{r-modes}} = 1 \times 10^3$	$K_{\mathrm{magnetars}} = 1 \times 10^{-11}\, \mathrm{T}^{-}$
$\alpha = \{0\}$	$\left(8.68^{+0.08}_{-0.08}\right) \times 10^{-7}$	-	-	-	-
$\alpha = \{2/3\}$	-	-	$\left(2.66^{+0.02}_{-0.02}\right) \times 10^{6}$	-	-
$\alpha = \{2\}$	-	-	-	$\left(2.13^{+0.01}_{-0.01}\right) \times 10^{3}$	-
$\alpha = \{3\}$	-	$\left(1.44^{+0.01}_{-0.01}\right) \times 10^{-7}$	-	-	-
$\alpha = \{4\}$	-	-	-	-	$\left(1.546^{+0.006}_{-0.006}\right) \times 10^{-11}$
$\alpha = \{2/3, 2, 4\}$	-	-	$\left(8.2^{+1.1}_{-1.1}\right) \times 10^{5}$	$\left(9.9^{+1.1}_{-1.1}\right) \times 10^{2}$	$\left(9.9^{+0.3}_{-0.4}\right) \times 10^{-12}$
$\alpha = \{0, 2/3, 2, 3, 4\}$	$\left(7.9^{+8.1}_{-5.4}\right) \times 10^{-8}$	$\left(3.9^{+3.9}_{-2.7}\right) \times 10^{-8}$	$\left(6.3^{+2.8}_{-3.6}\right) \times 10^{5}$	$\left(6.9^{+3.2}_{-4.0}\right) \times 10^{2}$	$\left(7.7^{+1.4}_{-2.9}\right) \times 10^{-12}$

Astro PE results

20-100 Hz



- Parameters well recovered for right α combination
- Importance of the frequency range where power-law approximation is valid: best recovery for frequencies in 20-100 Hz band (in contrast to the 20-1726 Hz used in O3 analysis)

Astro PE results (α=2/3, 2, 4): Red lines/boxes are the injections.

Analysis with real data

- Considered SGWBs, $\alpha = 0, 2/3\,, 2, 3, 4$
- Estimating $\Omega_\alpha$for all combinations
- Astrophysical implications for CBC ($\alpha = 2/3$), r-mode ($\alpha = 2$), and magnetar ($\alpha = 4$) SGWBs
- No further implications for $\alpha = 0 \text{ and } 3$ (cosmological), but useful for comparison with LIGO-Virgo-KAGRA analysis (single-component analysis, $\alpha = 0, 2/3\,, 3$)

Analysis with real data (20-100 Hz): estimators

	$\hat{\Omega}_0$	$\hat{\Omega}_{2/3}$	$\hat{\Omega}_2$	$\hat{\Omega}_3$	$\hat{\Omega}_4$
$\alpha=\{0\}$	$(1.5\pm7.5)\times10^{-9}$	-	-	-	-
$\alpha=\{2/3\}$	-	$(2.3\pm56.2)\times10^{-10}$	-	-	-
$\alpha=\{2\}$	-	-	$(-1.3\pm2.5)\times10^{-9}$	-	-
$\alpha=\{3\}$	-	-	-	$(-9.8\pm10.3)\times10^{-10}$	-
$\alpha=\{4\}$	-	-	-	-	$(-4.0\pm3.4)\times10^{-10}$
$\alpha=\{0,2/3\}$	$(4.4\pm4.6)\times10^{-8}$	$(-3.2\pm3.4)\times10^{-8}$	-	-	-
$\alpha=\{0,2\}$	$(1.6\pm1.4)\times10^{-8}$	-	$(-5.8\pm4.6)\times10^{-9}$	-	-
$\alpha=\{0,3\}$	$(9.5\pm9.5)\times10^{-9}$	-	-	$(-1.8\pm1.3)\times10^{-9}$	-
$\alpha=\{0,4\}$	$(6.1\pm8.2)\times10^{-9}$	-	-	-	$(-5.1\pm3.7)\times10^{-10}$
$\alpha=\{2/3,2\}$	-	$(1.7\pm1.4)\times10^{-8}$	$(-8.3\pm6.1)\times10^{-9}$	-	-
$\alpha=\{2/3,3\}$	-	$(8.4\pm8.1)\times10^{-9}$	-	$(-2.1\pm1.5)\times10^{-9}$	-
$\alpha=\{2/3,4\}$	-	$(5.0\pm6.6)\times10^{-9}$	-	-	$(-5.6\pm4.0)\times10^{-10}$
$\alpha=\{2,3\}$	-	-	$(7.6\pm7.2)\times10^{-9}$	$(-3.9\pm2.9)\times10^{-9}$	-
$\alpha=\{2,4\}$	-	-	$(3.1\pm4.3)\times10^{-9}$	-	$(-7.4\pm5.8)\times10^{-10}$
$\alpha=\{3,4\}$	-	-	-	$(2.4\pm3.7)\times10^{-9}$	$(-1.2\pm1.2)\times10^{-9}$
$\alpha=\{0,2/3,2\}$	$(-7.9\pm11.4)\times10^{-8}$	$(9.5\pm11.3)\times10^{-8}$	$(-1.8\pm1.5)\times10^{-8}$	-	-
$\alpha=\{0,2/3,3\}$	$(-3.2\pm8.3)\times10^{-8}$	$(3.5\pm7.0)\times10^{-8}$	-	$(-2.9\pm2.7)\times10^{-9}$	-
$\alpha=\{0,2/3,4\}$	$(-9.9\pm69.8)\times10^{-9}$	$(1.3\pm5.6)\times10^{-8}$	-	-	$(-6.2\pm6.1)\times10^{-10}$
$\alpha=\{0,2,3\}$	$(-1.0\pm30.0)\times10^{-9}$	-	$(8.3\pm22.6)\times10^{-9}$	$(-4.1\pm6.4)\times10^{-9}$	-
$\alpha=\{0,2,4\}$	$(4.3\pm25.0)\times10^{-9}$	-	$(1.0\pm12.9)\times10^{-9}$	-	$(-5.9\pm10.5)\times10^{-10}$
$\alpha=\{0,3,4\}$	$(6.5\pm17.8)\times10^{-9}$	-	-	$(-1.8\pm81.2)\times10^{-10}$	$(-4.6\pm23.2)\times10^{-10}$
$\alpha=\{2/3,2,3\}$	-	$(1.7\pm38.8)\times10^{-9}$	$(6.1\pm34.4)\times10^{-9}$	$(-3.6\pm8.4)\times10^{-9}$	-
$\alpha=\{2/3,2,4\}$	-	$(7.7\pm30.1)\times10^{-9}$	$(-1.7\pm19.5)\times10^{-9}$	-	$(-4.5\pm12.7)\times10^{-10}$
$\alpha=\{2/3,3,4\}$	-	$(8.1\pm18.3)\times10^{-9}$	-	$(-1.9\pm10.4)\times10^{-9}$	$(-5.6\pm280.6)\times10^{-11}$
$\alpha=\{2,3,4\}$	-	-	$(1.6\pm2.8)\times10^{-8}$	$(-1.1\pm2.5)\times10^{-8}$	$(1.5\pm4.9)\times10^{-9}$
$\alpha=\{0,2/3,2,3\}$	$(-3.2\pm3.5)\times10^{-7}$	$(4.1\pm4.6)\times10^{-7}$	$(-1.2\pm1.5)\times10^{-7}$	$(1.9\pm2.6)\times10^{-8}$	-
$\alpha=\{0,2/3,2,4\}$	$(-2.5\pm2.7)\times10^{-7}$	$(3.0\pm3.3)\times10^{-7}$	$(-6.8\pm7.6)\times10^{-8}$	-	$(2.1\pm3.1)\times10^{-9}$
$\alpha=\{0,2/3,3,4\}$	$(-1.5\pm1.8)\times10^{-7}$	$(1.6\pm1.9)\times10^{-7}$	-	$(-2.3\pm2.8)\times10^{-8}$	$(4.5\pm6.3)\times10^{-9}$
$\alpha=\{0,2,3,4\}$	$(-4.0\pm6.3)\times10^{-8}$	-	$(7.6\pm9.9)\times10^{-8}$	$(-4.7\pm6.2)\times10^{-8}$	$(7.1\pm10.2)\times10^{-9}$
$\alpha=\{2/3,2,3,4\}$	-	$(-5.5\pm9.6)\times10^{-8}$	$(9.9\pm14.7)\times10^{-8}$	$(-5.4\pm7.8)\times10^{-8}$	$(7.8\pm12.0)\times10^{-9}$
$\alpha=\{0,2/3,2,3,4\}$	$(-6.9\pm8.9)\times10^{-7}$	$(9.8\pm13.6)\times10^{-7}$	$(-4.3\pm7.1)\times10^{-7}$	$(1.3\pm2.6)\times10^{-7}$	$(-1.4\pm3.0)\times10^{-8}$

Astrophysical implications

- Increasing uncertainty as the component number increases
- Fixed number of components: uncertainty decreases as the α-space distance increases

Analysis with real data (20-100 Hz): upper limits

	$\Omega_0^{95\%}$	$\Omega_{2/3}^{95\%}$	$\Omega_2^{95\%}$	$\Omega_3^{95\%}$	$\Omega_4^{95\%}$	$K_{\rm CBC}^{95\%}$	$K_{\rm magnetars}^{95\%}$	$K_{\rm r-modes}^{95\%}$
$\alpha = \{0\}$	1.6×10^{-8}	-	-	-	-	-	-	-
$\alpha = \{2/3\}$	-	1.2×10^{-8}	-	-	-	5.1×10^4	-	-
$\alpha = \{2\}$	-	-	4.1×10^{-9}	-	-	-	-	1.3×10^0
$\alpha = \{3\}$	-	-	-	1.5×10^{-9}	-	-	-	-
$\alpha = \{4\}$	-	-	-	-	4.5×10^{-10}	-	1.4×10^{-12}	-
$\alpha = \{0, 2/3\}$	1.3×10^{-8}	9.5×10^{-9}	-	-	-	4.2×10^4	-	-
$\alpha = \{0, 2\}$	1.4×10^{-8}	-	3.5×10^{-9}	-	-	-	-	1.3×10^0
$\alpha = \{0, 3\}$	1.5×10^{-8}	-	-	1.3×10^{-9}	-	-	-	-
$\alpha = \{0, 4\}$	1.5×10^{-8}	-	-	-	4.1×10^{-10}	-	1.3×10^{-12}	-
$\alpha = \{2/3, 2\}$	-	1.0×10^{-8}	3.5×10^{-9}	-	-	4.9×10^4	-	1.3×10^0
$\alpha = \{2/3, 3\}$	-	1.0×10^{-8}	-	1.3×10^{-9}	-	4.6×10^4	-	-
$\alpha = \{2/3, 4\}$	-	1.0×10^{-8}	-	-	4.1×10^{-10}	4.9×10^4	1.3×10^{-12}	-
$\alpha = \{2, 3\}$	-	-	3.7×10^{-9}	1.3×10^{-9}	-	-	-	1.3×10^0
$\alpha = \{2, 4\}$	-	-	3.9×10^{-9}	-	4.0×10^{-10}	-	1.3×10^{-12}	1.3×10^0
$\alpha = \{3, 4\}$	-	-	-	1.3×10^{-9}	4.0×10^{-10}	-	1.3×10^{-12}	-
$\alpha = \{0, 2/3, 2\}$	1.2×10^{-8}	8.4×10^{-9}	3.1×10^{-9}	-	-	4.3×10^4	-	1.3×10^0
$\alpha = \{0, 2/3, 3\}$	1.2×10^{-8}	8.8×10^{-9}	-	1.2×10^{-9}	-	3.9×10^4	-	-
$\alpha = \{0, 2/3, 4\}$	1.3×10^{-8}	8.9×10^{-9}	-	-	3.9×10^{-10}	4.2×10^4	1.2×10^{-12}	-
$\alpha = \{0, 2, 3\}$	1.3×10^{-8}	-	3.2×10^{-9}	1.1×10^{-9}	-	-	-	1.3×10^0
$\alpha = \{0, 2, 4\}$	1.4×10^{-8}	-	3.3×10^{-9}	-	3.8×10^{-10}	-	1.3×10^{-12}	1.3×10^0
$\alpha = \{0, 3, 4\}$	1.4×10^{-8}	-	-	1.2×10^{-9}	3.8×10^{-10}	-	1.2×10^{-12}	-
$\alpha = \{2/3, 2, 3\}$	-	9.0×10^{-9}	3.2×10^{-9}	1.1×10^{-9}	-	4.6×10^4	-	1.3×10^0
$\alpha = \{2/3, 2, 4\}$	-	9.4×10^{-9}	3.3×10^{-9}	-	3.8×10^{-10}	4.9×10^4	1.3×10^{-12}	1.3×10^0
$\alpha = \{2/3, 3, 4\}$	-	9.5×10^{-9}	-	1.2×10^{-9}	3.8×10^{-10}	4.4×10^4	1.2×10^{-12}	-
$\alpha = \{2, 3, 4\}$	-	-	3.4×10^{-9}	1.2×10^{-9}	3.7×10^{-10}	-	1.3×10^{-12}	1.3×10^0
$\alpha = \{0, 2/3, 2, 3\}$	1.2×10^{-8}	7.8×10^{-9}	2.9×10^{-9}	1.1×10^{-9}	-	3.9×10^4	-	1.3×10^0
$\alpha = \{0, 2/3, 2, 4\}$	1.2×10^{-8}	8.2×10^{-9}	2.8×10^{-9}	-	3.7×10^{-10}	4.1×10^4	1.2×10^{-12}	1.3×10^0
$\alpha = \{0, 2/3, 3, 4\}$	1.2×10^{-8}	8.3×10^{-9}	-	1.1×10^{-9}	3.6×10^{-10}	3.9×10^4	1.2×10^{-12}	-
$\alpha = \{0, 2, 3, 4\}$	1.3×10^{-8}	-	2.9×10^{-9}	1.1×10^{-9}	3.6×10^{-10}	-	1.2×10^{-12}	1.3×10^0
$\alpha = \{2/3, 2, 3, 4\}$	-	8.7×10^{-9}	3.0×10^{-9}	1.1×10^{-9}	3.5×10^{-10}	4.4×10^4	1.2×10^{-12}	1.3×10^0
$\alpha = \{0, 2/3, 2, 3, 4\}$	1.1×10^{-8}	7.6×10^{-9}	2.8×10^{-9}	1.0×10^{-9}	3.4×10^{-10}	3.8×10^4	1.2×10^{-12}	1.3×10^0

- More stringent bounds as the number of components increase: expected, given that (noise) power split among multiple components (Callister et al. 2017)

Analysis with real data (20-100 Hz): upper limits

	$\Omega_0^{95\%}$	$\Omega_{2/3}^{95\%}$	$\Omega_2^{95\%}$	$\Omega_3^{95\%}$	$\Omega_4^{95\%}$	$K_{\rm CBC}^{95\%}$	$K_{\rm magnetars}^{95\%}$	$K_{\rm r-modes}^{95\%}$
$\alpha = \{0\}$	1.6×10^{-8}	-	-	-	-	-	-	-
$\alpha = \{2/3\}$	-	1.2×10^{-8}	-	-	-	5.1×10^{4}	-	-
$\alpha = \{2\}$	-	-	4.1×10^{-9}	-	-	-	-	1.3×10^{0}
$\alpha = \{3\}$	-	-	-	1.5×10^{-9}	-	-	-	-
$\alpha = \{4\}$	-	-	-	-	4.5×10^{-10}	-	1.4×10^{-12}	-
$\alpha = \{0, 2/3\}$	1.3×10^{-8}	9.5×10^{-9}	-	-	-	4.2×10^{4}	-	-
$\alpha = \{0, 2\}$	1.4×10^{-8}	-	3.5×10^{-9}	-	-	-	-	1.3×10^{0}
$\alpha = \{0, 3\}$	1.5×10^{-8}	-	-	1.3×10^{-9}	-	-	-	-
$\alpha = \{0, 4\}$	1.5×10^{-8}	-	-	-	4.1×10^{-10}	-	1.3×10^{-12}	-
$\alpha = \{2/3, 2\}$	-	1.0×10^{-8}	3.5×10^{-9}	-	-	4.9×10^{4}	-	1.3×10^{0}
$\alpha = \{2/3, 3\}$	-	1.0×10^{-8}	-	1.3×10^{-9}	-	4.6×10^{4}	-	-
$\alpha = \{2/3, 4\}$	-	1.0×10^{-8}	-	-	4.1×10^{-10}	4.9×10^{4}	1.3×10^{-12}	-
$\alpha = \{2, 3\}$	-	-	3.7×10^{-9}	1.3×10^{-9}	-	-	-	1.3×10^{0}
$\alpha = \{2, 4\}$	-	-	3.9×10^{-9}	-	4.0×10^{-10}	-	1.3×10^{-12}	1.3×10^{0}
$\alpha = \{3, 4\}$	-	-	-	1.3×10^{-9}	4.0×10^{-10}	-	1.3×10^{-12}	-
$\alpha = \{0, 2/3, 2\}$	1.2×10^{-8}	8.4×10^{-9}	3.1×10^{-9}	-	-	4.3×10^{4}	-	1.3×10^{0}
$\alpha = \{0, 2/3, 3\}$	1.2×10^{-8}	8.8×10^{-9}	-	1.2×10^{-9}	-	3.9×10^{4}	-	-
$\alpha = \{0, 2/3, 4\}$	1.3×10^{-8}	8.9×10^{-9}	-	-	3.9×10^{-10}	4.2×10^{4}	1.2×10^{-12}	-
$\alpha = \{0, 2, 3\}$	1.3×10^{-8}	-	3.2×10^{-9}	1.1×10^{-9}	-	-	-	1.3×10^{0}
$\alpha = \{0, 2, 4\}$	1.4×10^{-8}	-	3.3×10^{-9}	-	3.8×10^{-10}	-	1.3×10^{-12}	1.3×10^{0}
$\alpha = \{0, 3, 4\}$	1.4×10^{-8}	-	-	1.2×10^{-9}	3.8×10^{-10}	-	1.2×10^{-12}	-
$\alpha = \{2/3, 2, 3\}$	-	9.0×10^{-9}	3.2×10^{-9}	1.1×10^{-9}	-	4.6×10^{4}	-	1.3×10^{0}
$\alpha = \{2/3, 2, 4\}$	-	9.4×10^{-9}	3.3×10^{-9}	-	3.8×10^{-10}	4.9×10^{4}	1.3×10^{-12}	1.3×10^{0}
$\alpha = \{2/3, 3, 4\}$	-	9.5×10^{-9}	-	1.2×10^{-9}	3.8×10^{-10}	4.4×10^{4}	1.2×10^{-12}	-
$\alpha = \{2, 3, 4\}$	-	-	3.4×10^{-9}	1.2×10^{-9}	3.7×10^{-10}	-	1.3×10^{-12}	1.3×10^{0}
$\alpha = \{0, 2/3, 2, 3\}$	1.2×10^{-8}	7.8×10^{-9}	2.9×10^{-9}	1.1×10^{-9}	-	3.9×10^{4}	-	1.3×10^{0}
$\alpha = \{0, 2/3, 2, 4\}$	1.2×10^{-8}	8.2×10^{-9}	2.8×10^{-9}	-	3.7×10^{-10}	4.1×10^{4}	1.2×10^{-12}	1.3×10^{0}
$\alpha = \{0, 2/3, 3, 4\}$	1.2×10^{-8}	8.3×10^{-9}	-	1.1×10^{-9}	3.6×10^{-10}	3.9×10^{4}	1.2×10^{-12}	-
$\alpha = \{0, 2, 3, 4\}$	1.3×10^{-8}	-	2.9×10^{-9}	1.1×10^{-9}	3.6×10^{-10}	-	1.2×10^{-12}	1.3×10^{0}
$\alpha = \{2/3, 2, 3, 4\}$	-	8.7×10^{-9}	3.0×10^{-9}	1.1×10^{-9}	3.5×10^{-10}	4.4×10^{4}	1.2×10^{-12}	1.3×10^{0}
$\alpha = \{0, 2/3, 2, 3, 4\}$	1.1×10^{-8}	7.6×10^{-9}	2.8×10^{-9}	1.0×10^{-9}	3.4×10^{-10}	3.8×10^{4}	1.2×10^{-12}	1.3×10^{0}

- Uniform priors:
  - $K_{\rm CBC} \in [0, 10^7]\, M_\odot^{5/3}{\rm Gpc}^{-3}{\rm yr}^{-1}$
  - $K_{\rm r-modes} \in [10^{-13}, 4/3]$
  - $K_{\rm magnetars} \in [0, 10^{-10}]\ {\rm T}^{-1}$
- Quite mild dependence on the alpha combinations: current data are not informative
- $K_{\rm CBC}$ → difficult to compare with inference from GWTC-3: limit of the current approach
- $K_{\rm r-modes}$ → approximately $\langle K \rangle \geq -1.23$, not very informative
- $K_{\rm magnetars}$ → possible implication for distortion parameter β (poloidal-dominated magnetic field) and parameter k (twisted-toroidal field): not competitive with the existing ones

- More stringent bounds as the number of components increase: expected, given that (noise) power split among multiple components (Callister et al. 2017)

Summary

- Search for a Gaussian, stationary, unpolarised, isotropic SGWB, relaxing the hypothesis of a single component being present at a time
- Method extended to astrophysical SGWBs and implications about ensemble properties
- Analysis performed over the data from the first three LIGO-Virgo-KAGRA observing runs for $\alpha = 0, 2/3, 2, 3, 4$, assuming a power law SGWB in the 20-100 Hz band of the analysis
- No signal was found, 95% Bayesian upper limits on the Ω_α were drawn
- Implications about astrophysical ensemble properties for CBC ($\alpha = 2/3$), r-mode ($\alpha = 2$), and magnetar ($\alpha = 4$) SGWBs: limits not yet very informative or competitive with the existing ones
- Important take away from the injection study: this method will avoid bias and overestimating the components when getting closer to a detection (assuming power-law assumption holds)
- Possible improvements: need to go beyond the simple PL approximation to allow more flexibility (e.g., broader frequency range and remove degeneracy for CBC subpopulations)

Thank you for your attention!

BACKUP SLIDES

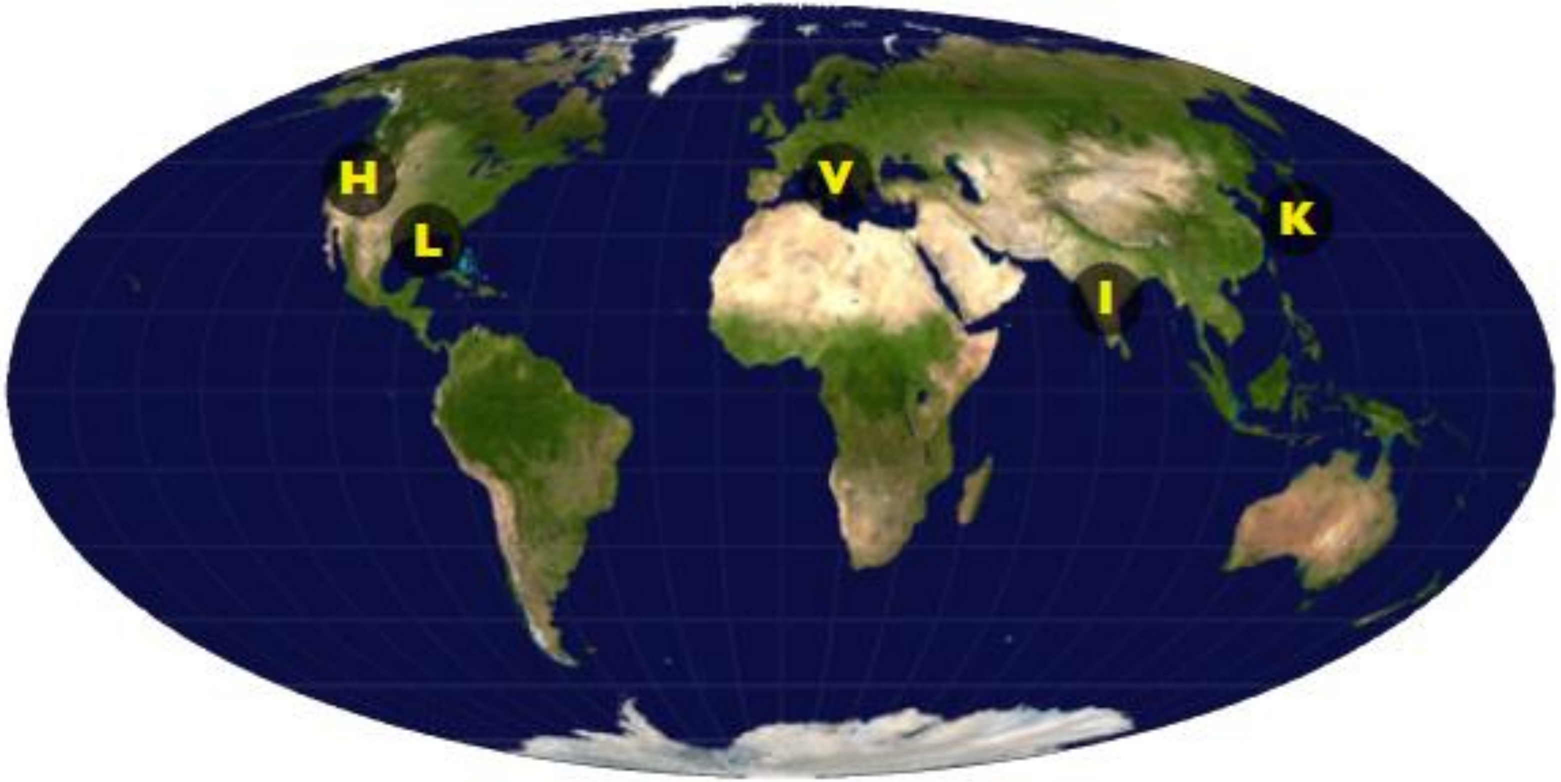
2nd generation ground based detectors network

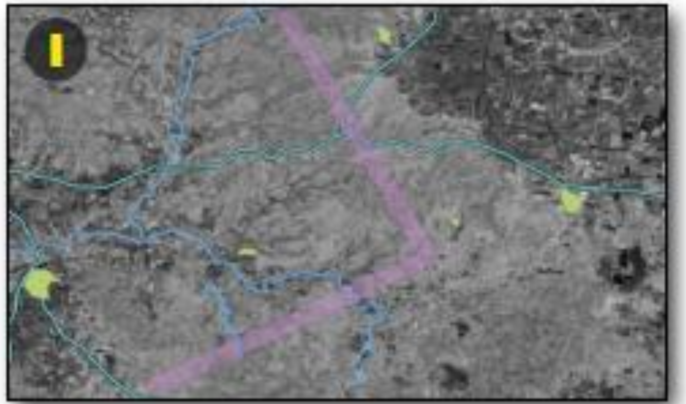
Credits: Readapted from Neil Cornish GWMess2021 presentation

LIGO-Hanford, Washington, USA(2015)

LIGO-Livingston, Louisiana, USA (2015)

VIRGO, Cascina (PI), Italy (2017)





LIGO-India, Hingoli District, Maharashtra,India(202X)

KAGRA, Kamioka, Japan (2020)

Isotropic search: multi-component formalism (1)

- Model: $\Omega_{\mathrm{gw}}(f) = \sum_{\{\alpha\}} \Omega_\alpha(f) w_\alpha(f)$, $\qquad w_\alpha(f) \equiv \left(\frac{f}{f_{ref}}\right)^\alpha$
- Cross-correlation

$$\tilde{s}_I^*(t,f)\tilde{s}_J(t',f') \approx \delta(f-f')\delta(t-t')\frac{T}{2}\, S_0\, \gamma_{IJ}(f)\, \Omega_{\mathrm{gw}}(\mathrm{f}) + \tilde{n}_I^*(t,f)\, \tilde{n}_J(t',f'),$$

which, at $t = t'$and $f = f'$, can be rewritten as

$$\boldsymbol{C} = \boldsymbol{K} \cdot \boldsymbol{\Omega} + \mathbf{N} \qquad \text{or} \qquad C_{tf} = \sum_{\{\alpha\}} K_{tf}^\alpha\, \Omega_\alpha + N_{tf},$$

with

$$\boldsymbol{C} = C_{tf} \equiv \tilde{s}_I^*(t,f)\tilde{s}_J(t,f)$$
$$\boldsymbol{\Omega} = \Omega_\alpha \equiv \Omega_{gw}(f_{ref},\alpha)$$
$$\boldsymbol{K} = K_{tf}^\alpha \equiv \frac{T}{2} S_0(f)\, \gamma_{IJ}(f) w_\alpha(f)$$
$$\mathbf{N} = N_{tf} = \tilde{n}_I^*(t,f)\, \tilde{n}_J(t,f)$$

- Noise covariance matrix:

$$\boldsymbol{\mathcal{N}} = \mathcal{N}_{tft'f'} \equiv \delta(f-f')\delta(t-t')\left(\frac{T}{2}\right)^2 P_I(f)\, P_J(f)$$

Isotropic search: multi-component formalism (2)

- Maximum Likelihood estimator for $\boldsymbol{\Omega}$, and covariance matrix:

$$\widehat{\boldsymbol{\Omega}} = \boldsymbol{\Gamma}^{-1} \cdot \boldsymbol{X}, \qquad \boldsymbol{\Sigma} = \boldsymbol{\Gamma}^{-1}, \qquad \boldsymbol{\sigma^2} = \mathrm{diag}(\boldsymbol{\Sigma})$$

where

$$\boldsymbol{X} = \boldsymbol{K}^\dagger \cdot \boldsymbol{\mathcal{N}}^{-1} \cdot \boldsymbol{C}\,, \qquad \boldsymbol{\Gamma} = \boldsymbol{K}^\dagger \cdot \boldsymbol{\mathcal{N}}^{-1} \cdot \boldsymbol{K}.$$

- (Broad-band) Dirty map and Fisher matrix:

$$X_\alpha = \sum\nolimits_{t,f} \frac{2}{T} \frac{\tilde{s}_I^*(t,f)\tilde{s}_J(t,f)}{P_I(f)P_J(f)}\, \gamma_{IJ}(f)\, S_0(f)\, w_\alpha(f)$$

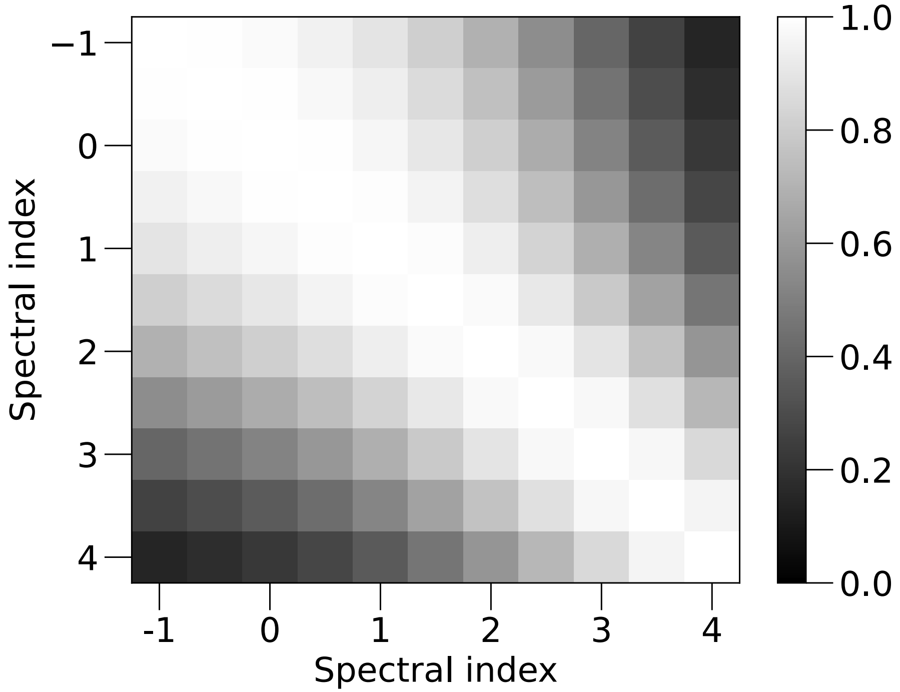
$$\Gamma_{\alpha\alpha'} = \sum\nolimits_{t,f} \frac{2}{T} \frac{\left|\gamma_{IJ}(f)\right|^2 S_0^2(f)}{P_I(f)P_J(f)}\, w_\alpha(f) w_{\alpha'}(f)$$

Isotropic search: multi-component formalism (3)

- Preconditioning may be necessary to avoid numerical errors when inverting $\mathbf{\Gamma}$:

$$\Gamma_{\alpha\alpha'} \to \Gamma'_{\alpha\alpha'} \equiv \frac{\Gamma_{\alpha\alpha'}}{\sqrt{\Gamma_\alpha}\sqrt{\Gamma_{\alpha'}}}, \qquad X_\alpha \to X'_\alpha \equiv \frac{X_\alpha}{\sqrt{\Gamma_\alpha}}, \qquad \Gamma_\alpha \equiv [\mathrm{diag}(\mathbf{\Gamma})]_\alpha$$

$$\Omega_\alpha = \frac{(\Gamma')^{-1}_{\alpha\alpha'} X'_{\alpha'}}{\sqrt{\Gamma_\alpha}}, \qquad \Sigma_{\alpha\alpha'} = \frac{(\Gamma')^{-1}_{\alpha\alpha'}}{\sqrt{\Gamma_\alpha}\sqrt{\Gamma_{\alpha'}}}, \qquad \sigma_\alpha^2 = [\mathrm{diag}(\mathbf{\Sigma})]_\alpha$$



Example of coupling matrix for O1+O2+O3 dataset.

Astrophysical SGWBs of interest

- SGWB from CBCs (see Kevin Turbang's talk for BBH):

$$\Omega_{\mathrm{gw},j}(f) \approx \xi_j f^{2/3} R_{0,j} \left\langle \mathcal{M}_c^{5/3} \right\rangle_j \equiv \xi_j f^{2/3} K_j \,, \qquad j = \mathrm{BBH}, \mathrm{BNS}, \mathrm{BHNS}$$

$$\Omega_{\mathrm{gw,CBC}}(f) \approx \xi_{\mathrm{CBC}}\, f^{2/3}\, K_{\mathrm{CBC}}, \qquad K_{\mathrm{CBC}} \equiv \sum_j K_j$$

- r-mode instability in young NSs (Owen et al. 1998, Zhu et al. 2011)

$$\Omega_{\mathrm{gw,r-modes}}(f) \approx \xi_{\mathrm{r-modes}}\, f^2 \langle (K + 2)^{-1} \rangle \equiv \xi_{\mathrm{r-modes}}\, f^2 K_{\mathrm{r-modes}}$$

Related to r-modes intensity α

- Magnetars (Regimbau-Mandic 2008, Wu et al. 2013)

$$\Omega_{\mathrm{gw,magnetars}}(f) \approx \xi_{\mathrm{magnetars}}\, f^4 \langle \varepsilon^2 \rangle \langle B^{-2} \rangle \equiv \xi_{\mathrm{magnetars}}\, f^4 K^2_{\mathrm{magnetars}}$$

ellipticity and (poloidal)
magnetic field

Validation: astrophysical multi-component injection study

- Injected (detectable) astrophysical SGWB from BNS, r-modes, and magnetars in O3 data

- Astrophysical (unphysical) population parameters

$$K_{\mathrm{BNS}} \simeq 7.91 \times 10^5\, M_{\odot}^{5/3}\mathrm{Gpc}^{-3}\mathrm{yr}^{-1} (\mathrm{R}_{0,\mathrm{BNS}} = 3.2 \times 10^5 \mathrm{Gpc}^{-3}\mathrm{yr}^{-1})$$

$$K_{\mathrm{r-modes}} = 10^3, (K = -1.999)$$

$$K_{\mathrm{magnetars}} = 10^{-11}, \left(\frac{\varepsilon}{B} = 10^{-11}\right)$$

- Corresponding Ω_{ref} ($\mathrm{f}_{\mathrm{ref}} = 25$ Hz)

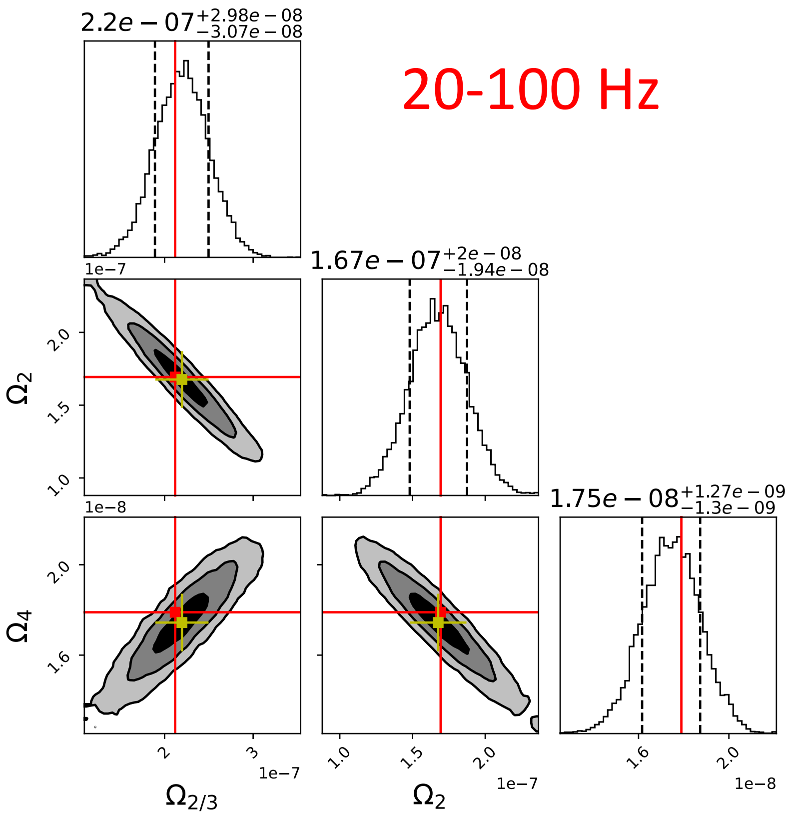
$$\Omega_{\mathrm{BNS}} \simeq 2.12 \times 10^{-7}, \qquad \Omega_{\mathrm{r-modes}} \simeq 1.69 \times 10^{-7}, \qquad \Omega_{\mathrm{magnetars}} \simeq 1.79 \times 10^{-8}$$

- Goal: recover both the amplitudes (similar and different orders of magnitude) and the population parameters (assuming SGWB being detectable)

Astrophysical multi-component injections: SGWB intensity

Estimators	$\hat{\Omega}_0$	$\hat{\Omega}_{2/3} = 2.1 \times 10^{-7}$	$\hat{\Omega}_2 = 1.7 \times 10^{-7}$	$\hat{\Omega}_3$	$\hat{\Omega}_4 = 1.8 \times 10^{-8}$
$\alpha = \{0\}$	$(8.68 \pm 0.08) \times 10^{-7}$	-	-	-	-
$\alpha = \{2/3\}$	-	$(7.12 \pm 0.06) \times 10^{-7}$	-	-	-
$\alpha = \{2\}$	-	-	$(3.61 \pm 0.03) \times 10^{-7}$	-	-
$\alpha = \{3\}$	-	-	-	$(1.44 \pm 0.01) \times 10^{-7}$	-
$\alpha = \{4\}$	-	-	-	-	$(4.28 \pm 0.03) \times 10^{-8}$
$\alpha = \{0, 2/3\}$	$(-2.67 \pm 0.05) \times 10^{-6}$	$(2.68 \pm 0.03) \times 10^{-6}$	-	-	-
$\alpha = \{0, 2\}$	$(-1.3 \pm 0.1) \times 10^{-7}$	-	$(3.96 \pm 0.05) \times 10^{-7}$	-	-
$\alpha = \{0, 3\}$	$(3.55 \pm 0.1) \times 10^{-7}$	-	-	$(1.14 \pm 0.01) \times 10^{-7}$	-
$\alpha = \{0, 4\}$	$(5.79 \pm 0.08) \times 10^{-7}$	-	-	-	$(3.21 \pm 0.04) \times 10^{-8}$
$\alpha = \{2/3, 2\}$	-	$(-1.5 \pm 0.1) \times 10^{-7}$	$(4.21 \pm 0.06) \times 10^{-7}$	-	-
$\alpha = \{2/3, 3\}$	-	$(3.03 \pm 0.08) \times 10^{-7}$	-	$(1.04 \pm 0.01) \times 10^{-7}$	-
$\alpha = \{2/3, 4\}$	-	$(4.73 \pm 0.07) \times 10^{-7}$	-	-	$(2.79 \pm 0.04) \times 10^{-8}$
$\alpha = \{2, 3\}$	-	-	$(2.61 \pm 0.07) \times 10^{-7}$	$(4.4 \pm 0.3) \times 10^{-8}$	-
$\alpha = \{2, 4\}$	-	-	$(3.06 \pm 0.04) \times 10^{-7}$	-	$(9.2 \pm 0.6) \times 10^{-9}$
$\alpha = \{3, 4\}$	-	-	-	$(2.62 \pm 0.04) \times 10^{-7}$	$(-4.1 \pm 0.1) \times 10^{-8}$
$\alpha = \{0, 2/3, 2\}$	$(1.4 \pm 0.1) \times 10^{-6}$	$(-1.5 \pm 0.1) \times 10^{-6}$	$(5.9 \pm 0.2) \times 10^{-7}$	-	-
$\alpha = \{0, 2/3, 3\}$	$(4.4 \pm 8.3) \times 10^{-8}$	$(2.7 \pm 0.7) \times 10^{-7}$	-	$(1.05 \pm 0.03) \times 10^{-7}$	-
$\alpha = \{0, 2/3, 4\}$	$(-6.0 \pm 0.7) \times 10^{-7}$	$(9.5 \pm 0.6) \times 10^{-7}$	-	-	$(2.4 \pm 0.06) \times 10^{-8}$
$\alpha = \{0, 2, 3\}$	$(2.6 \pm 0.3) \times 10^{-7}$	-	$(7.6 \pm 2.3) \times 10^{-8}$	$(9.3 \pm 0.6) \times 10^{-8}$	-
$\alpha = \{0, 2, 4\}$	$(1.8 \pm 0.2) \times 10^{-7}$	-	$(2.2 \pm 0.1) \times 10^{-7}$	-	$(1.5 \pm 0.1) \times 10^{-8}$
$\alpha = \{0, 3, 4\}$	$(3.2 \pm 0.2) \times 10^{-7}$	-	-	$(1.35 \pm 0.08) \times 10^{-7}$	$(-6.1 \pm 2.3) \times 10^{-9}$
$\alpha = \{2/3, 2, 3\}$	-	$(3.4 \pm 0.4) \times 10^{-7}$	$(-3.8 \pm 3.4) \times 10^{-8}$	$(1.13 \pm 0.08) \times 10^{-7}$	-
$\alpha = \{2/3, 2, 4\}$	-	$(2.2 \pm 0.3) \times 10^{-7}$	$(1.7 \pm 0.2) \times 10^{-7}$	-	$(1.7 \pm 0.1) \times 10^{-8}$
$\alpha = \{2/3, 3, 4\}$	-	$(3.3 \pm 0.2) \times 10^{-7}$	-	$(8.8 \pm 1.0) \times 10^{-8}$	$(4.4 \pm 2.8) \times 10^{-9}$
$\alpha = \{2, 3, 4\}$	-	-	$(5.0 \pm 0.3) \times 10^{-7}$	$(-1.8 \pm 0.2) \times 10^{-7}$	$(4.4 \pm 0.5) \times 10^{-8}$
$\alpha = \{0, 2/3, 2, 3\}$	$(-8.0 \pm 3.5) \times 10^{-7}$	$(1.4 \pm 0.5) \times 10^{-6}$	$(-3.6 \pm 1.5) \times 10^{-7}$	$(1.7 \pm 0.3) \times 10^{-7}$	-
$\alpha = \{0, 2/3, 2, 4\}$	$(-2.4 \pm 2.7) \times 10^{-7}$	$(5.1 \pm 3.3) \times 10^{-7}$	$(1.0 \pm 0.8) \times 10^{-7}$	-	$(2.0 \pm 0.3) \times 10^{-8}$
$\alpha = \{0, 2/3, 3, 4\}$	$(-3.6 \pm 1.8) \times 10^{-7}$	$(7.0 \pm 1.9) \times 10^{-7}$	-	$(3.8 \pm 2.8) \times 10^{-8}$	$(1.5 \pm 0.6) \times 10^{-8}$
$\alpha = \{0, 2, 3, 4\}$	$(9.9 \pm 6.3) \times 10^{-8}$	-	$(3.5 \pm 1.0) \times 10^{-7}$	$(-8.6 \pm 6.2) \times 10^{-8}$	$(2.9 \pm 1.0) \times 10^{-8}$
$\alpha = \{2/3, 2, 3, 4\}$	-	$(1.6 \pm 1.0) \times 10^{-7}$	$(2.7 \pm 1.5) \times 10^{-7}$	$(-5.4 \pm 7.8) \times 10^{-8}$	$(2.6 \pm 1.2) \times 10^{-8}$
$\alpha = \{0, 2/3, 2, 3, 4\}$	$(-6.9 \pm 8.9) \times 10^{-7}$	$(1.2 \pm 1.4) \times 10^{-6}$	$(-2.6 \pm 7.1) \times 10^{-7}$	$(1.3 \pm 2.6) \times 10^{-7}$	$(4.3 \pm 30.3) \times 10^{-9}$

20-100 Hz

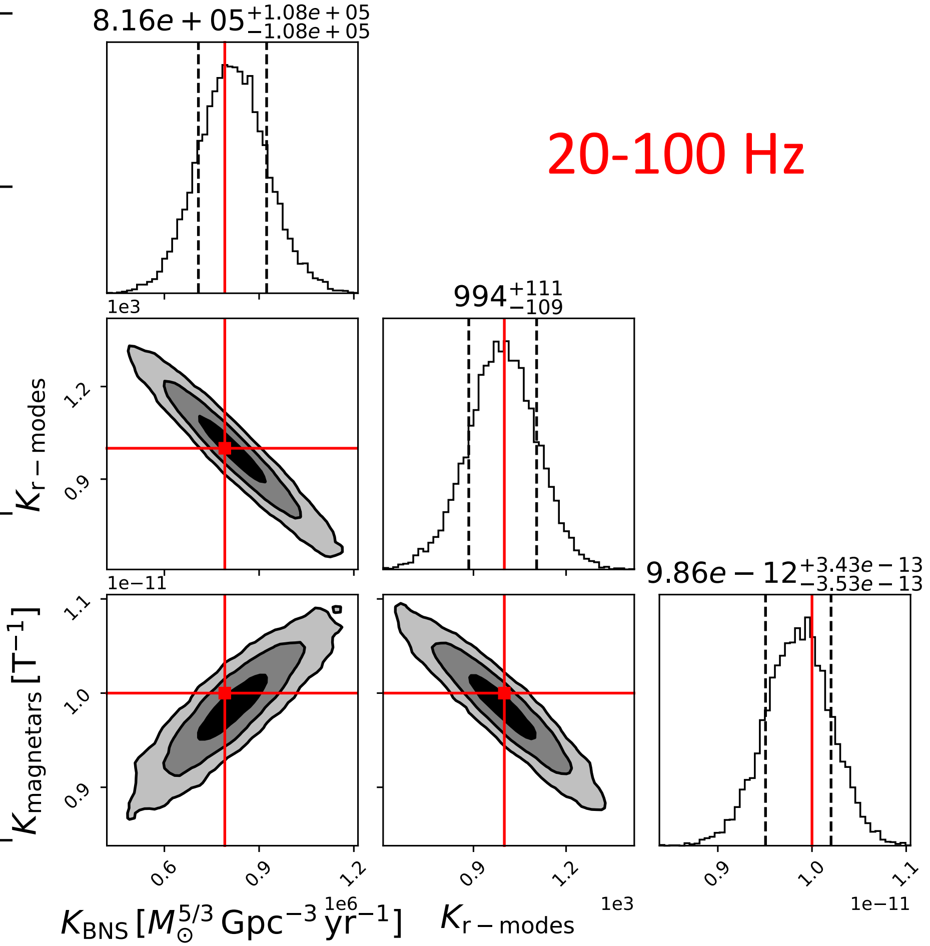


PE results (α=2/3, 2, 4): Red lines/boxes are the injections. Yellow error bars refer to the estimators.

- Ω_α well recovered only when the right α=2/3, 2, 4 combination is considered
- Capability of disentangling SGWB with similar intensities or spanning order of magnitudes

Astrophysical multi-component injections: astrophysical parameters

	Ω_0	Ω_3	$K_{\mathrm{BNS}} = 7.9 \times 10^5\, M_\odot^{5/3}\,\mathrm{Gpc}^{-3}\,\mathrm{yr}^{-1}$	$K_{\mathrm{r-modes}} = 1 \times 10^3$	$K_{\mathrm{magnetars}} = 1 \times 10^{-11}\,\mathrm{T}^{-}$
$\alpha = \{0\}$	$\left(8.68^{+0.08}_{-0.08}\right) \times 10^{-7}$	-	-	-	-
$\alpha = \{2/3\}$	-	-	$\left(2.66^{+0.02}_{-0.02}\right) \times 10^{6}$	-	-
$\alpha = \{2\}$	-	-	-	$\left(2.13^{+0.01}_{-0.01}\right) \times 10^{3}$	-
$\alpha = \{3\}$	-	$\left(1.44^{+0.01}_{-0.01}\right) \times 10^{-7}$	-	-	-
$\alpha = \{4\}$	-	-	-	-	$\left(1.546^{+0.006}_{-0.006}\right) \times 10^{-11}$
$\alpha = \{0, 2/3\}$	$\left(5.4^{+9.2}_{-4.2}\right) \times 10^{-10}$	-	$\left(2.65^{+0.02}_{-0.02}\right) \times 10^{6}$	-	-
$\alpha = \{0, 2\}$	$\left(9.9^{+16.6}_{-7.4}\right) \times 10^{-10}$	-	-	$\left(2.13^{+0.01}_{-0.01}\right) \times 10^{3}$	-
$\alpha = \{0, 3\}$	$\left(3.55^{+0.1}_{-0.1}\right) \times 10^{-7}$	$\left(1.14^{+0.01}_{-0.01}\right) \times 10^{-7}$	-	-	-
$\alpha = \{0, 4\}$	$\left(5.79^{+0.08}_{-0.08}\right) \times 10^{-7}$	-	-	-	$\left(1.339^{+0.008}_{-0.008}\right) \times 10^{-11}$
$\alpha = \{2/3, 2\}$	-	-	$\left(3.2^{+5.2}_{-2.4}\right) \times 10^{3}$	$\left(2.13^{+0.01}_{-0.01}\right) \times 10^{3}$	-
$\alpha = \{2/3, 3\}$	-	$\left(1.04^{+0.01}_{-0.01}\right) \times 10^{-7}$	$\left(1.13^{+0.03}_{-0.03}\right) \times 10^{6}$	-	-
$\alpha = \{2/3, 4\}$	-	-	$\left(1.76^{+0.02}_{-0.02}\right) \times 10^{6}$	-	$\left(1.248^{+0.009}_{-0.009}\right) \times 10^{-11}$
$\alpha = \{2, 3\}$	-	$\left(4.4^{+0.3}_{-0.3}\right) \times 10^{-8}$	-	$\left(1.54^{+0.04}_{-0.04}\right) \times 10^{3}$	-
$\alpha = \{2, 4\}$	-	-	-	$\left(1.81^{+0.03}_{-0.03}\right) \times 10^{3}$	$\left(7.1^{+0.2}_{-0.2}\right) \times 10^{-12}$
$\alpha = \{3, 4\}$	-	$\left(1.438^{+0.01}_{-0.01}\right) \times 10^{-7}$	-	-	$\left(2.9^{+2.2}_{-1.3}\right) \times 10^{-13}$
$\alpha = \{0, 2/3, 2\}$	$\left(1.0^{+1.6}_{-0.8}\right) \times 10^{-9}$	-	$\left(3.2^{+5.3}_{-2.4}\right) \times 10^{3}$	$\left(2.12^{+0.01}_{-0.02}\right) \times 10^{3}$	-
$\alpha = \{0, 2/3, 3\}$	$\left(7.7^{+6.7}_{-5.1}\right) \times 10^{-8}$	$\left(1.06^{+0.02}_{-0.02}\right) \times 10^{-7}$	$\left(8.8^{+1.6}_{-2.1}\right) \times 10^{5}$	-	-
$\alpha = \{0, 2/3, 4\}$	$\left(5.5^{+9.1}_{-4.1}\right) \times 10^{-9}$	-	$\left(1.74^{+0.03}_{-0.03}\right) \times 10^{6}$	-	$\left(1.249^{+0.009}_{-0.009}\right) \times 10^{-11}$
$\alpha = \{0, 2, 3\}$	$\left(2.6^{+0.3}_{-0.3}\right) \times 10^{-7}$	$\left(9.3^{+0.6}_{-0.6}\right) \times 10^{-8}$	-	$\left(4.5^{+1.3}_{-1.3}\right) \times 10^{2}$	-
$\alpha = \{0, 2, 4\}$	$\left(1.8^{+0.3}_{-0.2}\right) \times 10^{-7}$	-	-	$\left(1.3^{+0.08}_{-0.08}\right) \times 10^{3}$	$\left(9.3^{+0.3}_{-0.3}\right) \times 10^{-12}$
$\alpha = \{0, 3, 4\}$	$\left(3.58^{+0.1}_{-0.1}\right) \times 10^{-7}$	$\left(1.13^{+0.02}_{-0.02}\right) \times 10^{-7}$	-	-	$\left(1.1^{+1.1}_{-0.7}\right) \times 10^{-12}$
$\alpha = \{2/3, 2, 3\}$	-	$\left(1.01^{+0.03}_{-0.04}\right) \times 10^{-7}$	$\left(1.07^{+0.05}_{-0.07}\right) \times 10^{6}$	$\left(7.8^{+9.5}_{-5.6}\right) \times 10^{1}$	-
$\alpha = \{2/3, 2, 4\}$	-	-	$\left(8.2^{+1.1}_{-1.1}\right) \times 10^{5}$	$\left(9.9^{+1.1}_{-1.1}\right) \times 10^{2}$	$\left(9.9^{+0.3}_{-0.4}\right) \times 10^{-12}$
$\alpha = \{2/3, 3, 4\}$	-	$\left(9.2^{+0.9}_{-1.1}\right) \times 10^{-8}$	$\left(1.21^{+0.07}_{-0.06}\right) \times 10^{6}$	-	$\left(4.4^{+1.6}_{-2.2}\right) \times 10^{-12}$
$\alpha = \{2, 3, 4\}$	-	$\left(2.3^{+3.9}_{-1.7}\right) \times 10^{-9}$	-	$\left(1.79^{+0.03}_{-0.03}\right) \times 10^{3}$	$\left(6.9^{+0.3}_{-0.4}\right) \times 10^{-12}$
$\alpha = \{0, 2/3, 2, 3\}$	$\left(1.3^{+0.8}_{-0.8}\right) \times 10^{-7}$	$\left(1.0^{+0.05}_{-0.07}\right) \times 10^{-7}$	$\left(6.1^{+2.9}_{-3.3}\right) \times 10^{5}$	$\left(1.7^{+1.9}_{-1.2}\right) \times 10^{2}$	-
$\alpha = \{0, 2/3, 2, 4\}$	$\left(7.2^{+6.8}_{-5.0}\right) \times 10^{-8}$	-	$\left(4.9^{+2.4}_{-3.0}\right) \times 10^{5}$	$\left(1.1^{+0.1}_{-0.1}\right) \times 10^{3}$	$\left(9.6^{+0.4}_{-0.4}\right) \times 10^{-12}$
$\alpha = \{0, 2/3, 3, 4\}$	$\left(4.9^{+6.1}_{-3.6}\right) \times 10^{-8}$	$\left(9.8^{+0.6}_{-1.1}\right) \times 10^{-8}$	$\left(1.0^{+0.1}_{-0.2}\right) \times 10^{6}$	-	$\left(3.3^{+1.9}_{-2.0}\right) \times 10^{-12}$
$\alpha = \{0, 2, 3, 4\}$	$\left(2.1^{+0.4}_{-0.3}\right) \times 10^{-7}$	$\left(2.8^{+3.9}_{-2.0}\right) \times 10^{-8}$	-	$\left(1.0^{+0.2}_{-0.4}\right) \times 10^{3}$	$\left(7.8^{+1.2}_{-2.8}\right) \times 10^{-12}$
$\alpha = \{2/3, 2, 3, 4\}$	-	$\left(3.6^{+3.5}_{-2.6}\right) \times 10^{-8}$	$\left(9.8^{+1.6}_{-1.5}\right) \times 10^{5}$	$\left(6.0^{+2.9}_{-3.9}\right) \times 10^{2}$	$\left(8.1^{+1.3}_{-2.2}\right) \times 10^{-12}$
$\alpha = \{0, 2/3, 2, 3, 4\}$	$\left(7.9^{+8.1}_{-5.4}\right) \times 10^{-8}$	$\left(3.9^{+3.9}_{-2.7}\right) \times 10^{-8}$	$\left(6.3^{+2.8}_{-3.6}\right) \times 10^{5}$	$\left(6.9^{+3.2}_{-4.0}\right) \times 10^{2}$	$\left(7.7^{+1.4}_{-2.9}\right) \times 10^{-12}$



20-100 Hz

- Parameters well recovered for right α combination
- Importance of the frequency range where power-law approximation is valid: best recovery for frequencies in 20-100 Hz band (in contrast to the 20-1726 Hz used in O3 analysis)

Analysis with real data (20-100 Hz): estimators

	$\hat{\Omega}_0$	$\hat{\Omega}_{2/3}$	$\hat{\Omega}_2$	$\hat{\Omega}_3$	$\hat{\Omega}_4$
$\alpha = \{0\}$	$(1.5 \pm 7.5) \times 10^{-9}$	-	-	-	-
$\alpha = \{2/3\}$	-	$(2.3 \pm 56.2) \times 10^{-10}$	-	-	-
$\alpha = \{2\}$	-	-	$(-1.3 \pm 2.5) \times 10^{-9}$	-	-
$\alpha = \{3\}$ Astrophysical	-	-	-	$(-9.8 \pm 10.3) \times 10^{-10}$	-
$\alpha = \{4\}$ implications	-	-	-	-	$(-4.0 \pm 3.4) \times 10^{-10}$
$\alpha = \{2/3, 2, 4\}$	-	$(7.7 \pm 30.1) \times 10^{-9}$	$(-1.7 \pm 19.5) \times 10^{-9}$	-	$(-4.5 \pm 12.7) \times 10^{-10}$
$\alpha = \{0, 2/3, 2, 3, 4\}$	$(-6.9 \pm 8.9) \times 10^{-7}$	$(9.8 \pm 13.6) \times 10^{-7}$	$(-4.3 \pm 7.1) \times 10^{-7}$	$(1.3 \pm 2.6) \times 10^{-7}$	$(-1.4 \pm 3.0) \times 10^{-8}$

Standard analysis

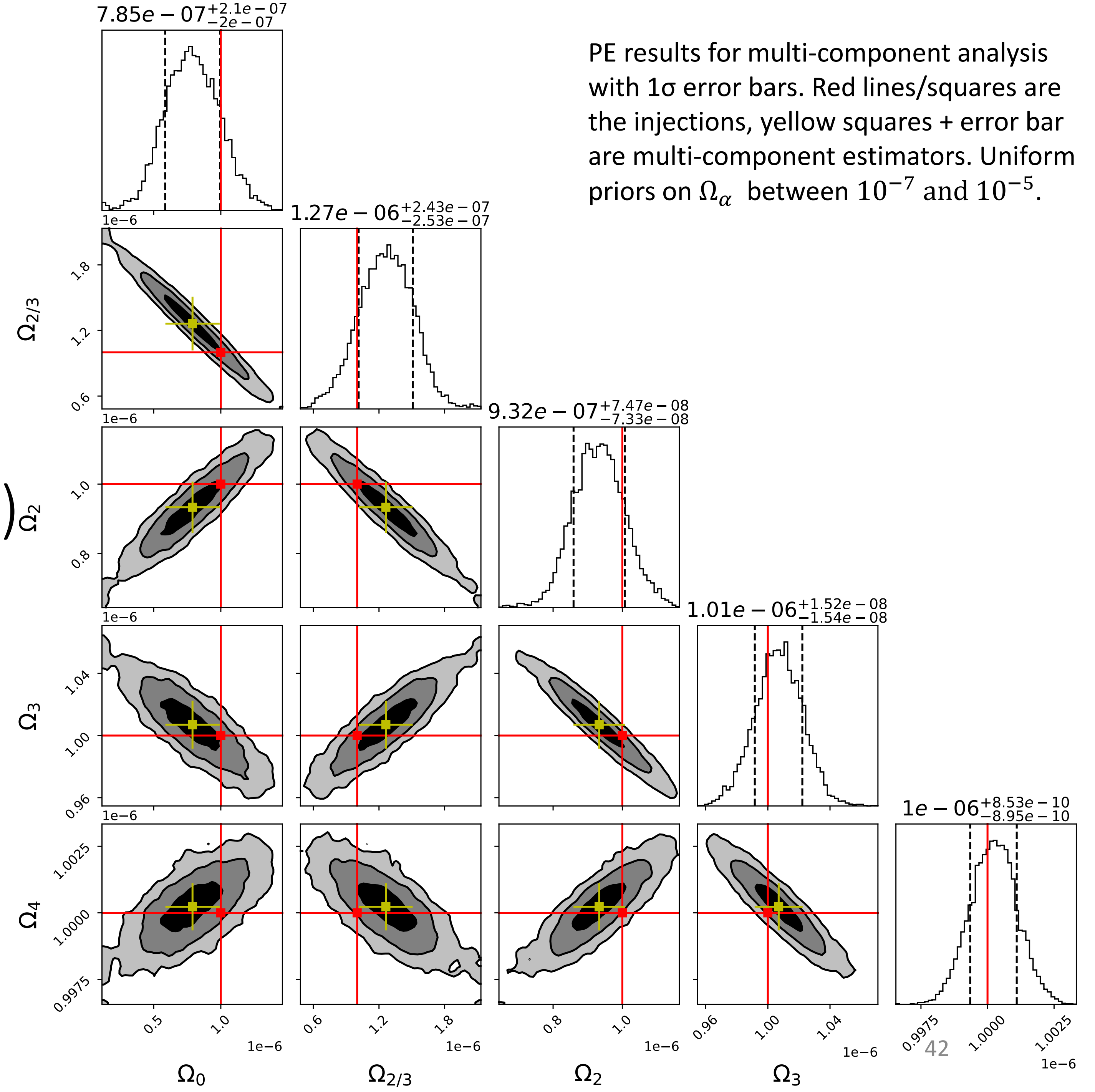
- Increasing uncertainty as the component number increases
- Fixed number of components: uncertainty decreases as the α-space distance increases

Analysis with real data (20-100 Hz): upper limits

	$\Omega_0^{95\%}$	$\Omega_{2/3}^{95\%}$	$\Omega_2^{95\%}$	$\Omega_3^{95\%}$	$\Omega_4^{95\%}$	$K_{\rm CBC}^{95\%}$	$K_{\rm magnetars}^{95\%}$	$K_{\rm r-modes}^{95\%}$
$\alpha=\{0\}$	1.6×10^{-8}	-	-	-	-	-	-	-
$\alpha=\{2/3\}$	-	1.2×10^{-8}	-	-	-	5.1×10^{4}	-	-
$\alpha=\{2\}$	-	-	4.1×10^{-9}	-	-	-	-	1.3×10^{0}
$\alpha=\{3\}$	-	-	-	1.5×10^{-9}	-	-	-	-
$\alpha=\{4\}$	-	-	-	-	4.5×10^{-10}	-	1.4×10^{-12}	-
$\alpha=\{2/3,2,4\}$	-	9.4×10^{-9}	3.3×10^{-9}	-	3.8×10^{-10}	4.9×10^{4}	1.3×10^{-12}	1.3×10^{0}
$\alpha=\{0,2/3,2,3,4\}$	1.1×10^{-8}	7.6×10^{-9}	2.8×10^{-9}	1.0×10^{-9}	3.4×10^{-10}	3.8×10^{4}	1.2×10^{-12}	1.3×10^{0}

- More stringent bounds as the number of components increases: expected, given that (noise) power split among multiple components (Callister et al. 2017)
- Uniform priors:
  - $K_{\rm CBC}\in[0,10^7]\,M_\odot^{5/3}\,{\rm Gpc}^{-3}{\rm yr}^{-1}$
  - $K_{\rm r-modes}\in[10^{-13},4/3]$
  - $K_{\rm magnetars}\in[0,10^{-10}]\,{\rm T}^{-1}$
- Quite mild dependence on the alpha combinations: current data are not informative
- $K_{\rm CBC}$ → difficult to compare with inference from GWTC-3: limit of the current approach
- $K_{\rm r-modes}$ → approximately $\langle K\rangle\geq-1.23$, not very informative
- $K_{\rm magnetars}$ → possible implication for distortion parameter β (poloidal-dominated magnetic field) and parameter k (twisted-toroidal field): not competitive with the existing ones

Strong PL injections (20-1726 Hz) {α=0, α=2/3, α=2, α=3, α=4}



PE results for multi-component analysis with 1σ error bars. Red lines/squares are the injections, yellow squares + error bar are multi-component estimators. Uniform priors on Ω_α between 10^{-7} and 10^{-5}.

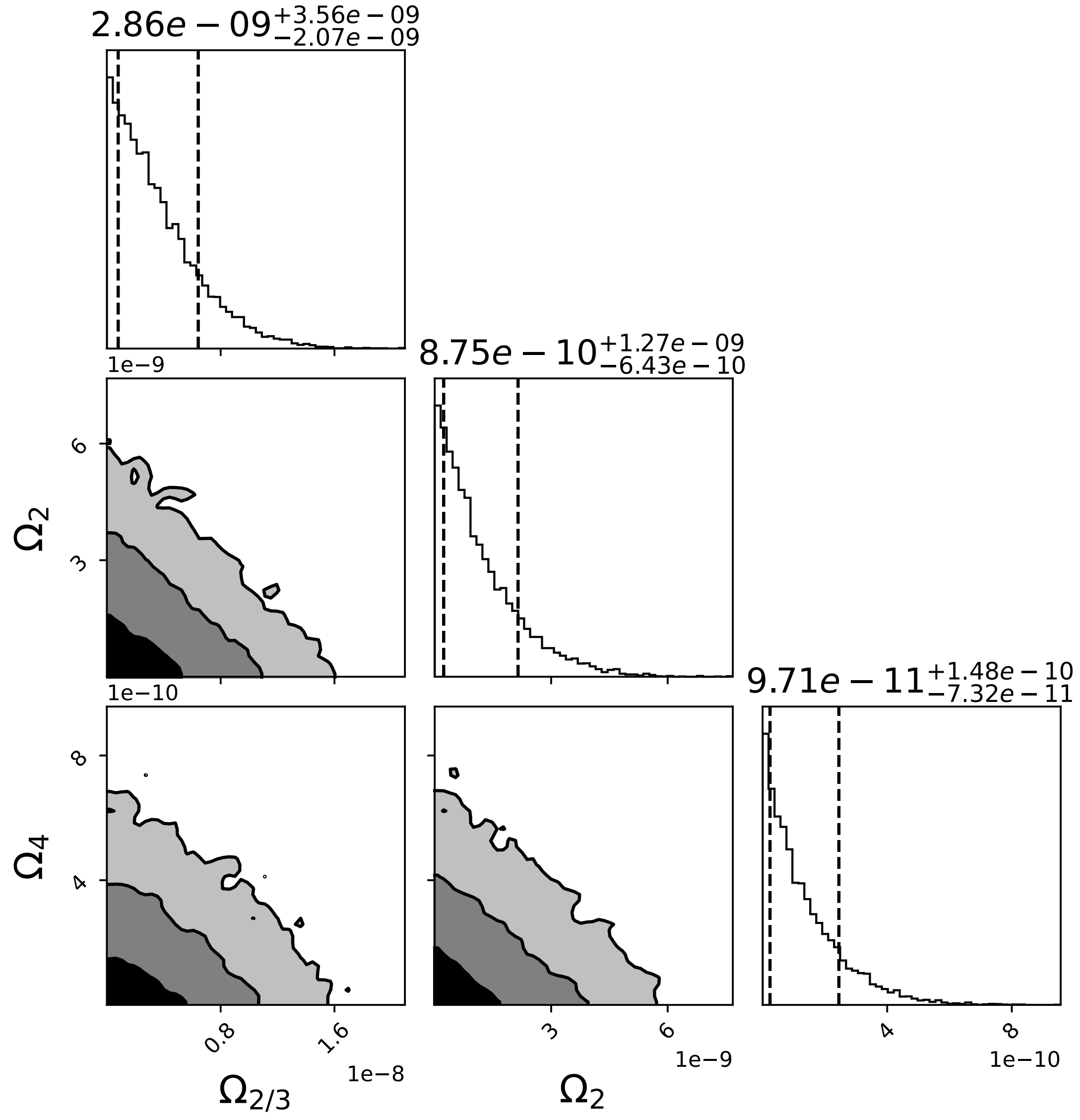
Strong PL injections (20-1726 Hz) {α=0, α=2/3, α=2, α=3, α=4}: $\widehat{\Omega}_\alpha$

	$\hat{\Omega}_0 = 1\times10^{-6}$	$\hat{\Omega}_{2/3} = 1\times10^{-6}$	$\hat{\Omega}_2 = 1\times10^{-6}$	$\hat{\Omega}_3 = 1\times10^{-6}$	$\hat{\Omega}_4 = 1\times10^{-6}$
$\alpha = \{0\}$	$1.94\times10^{-5} \pm 7.51\times10^{-9}$	-	-	-	-
$\alpha = \{2/3\}$	-	$1.81\times10^{-5} \pm 5.6\times10^{-9}$	-	-	-
$\alpha = \{2\}$	-	-	$1.25\times10^{-5} \pm 2.39\times10^{-9}$	-	-
$\alpha = \{3\}$	-	-	-	$5.66\times10^{-6} \pm 8.29\times10^{-10}$	-
$\alpha = \{4\}$	-	-	-	-	$1.23\times10^{-6} \pm 1.67\times10^{-10}$
$\alpha = \{0, 2/3\}$	$-0.000156 \pm 4.44\times10^{-8}$	$0.000133 \pm 3.3\times10^{-8}$	-	-	-
$\alpha = \{0, 2\}$	$-3.59\times10^{-5} \pm 1.28\times10^{-8}$	-	$2.18\times10^{-5} \pm 4.06\times10^{-9}$	-	-
$\alpha = \{0, 3\}$	$-9.51\times10^{-6} \pm 8.76\times10^{-9}$	-	-	$6.2\times10^{-6} \pm 9.66\times10^{-10}$	-
$\alpha = \{0, 4\}$	$7.67\times10^{-6} \pm 7.7\times10^{-9}$	-	-	-	$1.19\times10^{-6} \pm 1.72\times10^{-10}$
$\alpha = \{2/3, 2\}$	-	$-3.87\times10^{-5} \pm 1.23\times10^{-8}$	$2.72\times10^{-5} \pm 5.24\times10^{-9}$	-	-
$\alpha = \{2/3, 3\}$	-	$-9.22\times10^{-6} \pm 7.14\times10^{-9}$	-	$6.51\times10^{-6} \pm 1.06\times10^{-9}$	-
$\alpha = \{2/3, 4\}$	-	$6.3\times10^{-6} \pm 5.87\times10^{-9}$	-	-	$1.17\times10^{-6} \pm 1.75\times10^{-10}$
$\alpha = \{2, 3\}$	-	-	$-9.98\times10^{-6} \pm 5.28\times10^{-9}$	$8.75\times10^{-6} \pm 1.83\times10^{-9}$	-
$\alpha = \{2, 4\}$	-	-	$3.52\times10^{-6} \pm 2.94\times10^{-9}$	-	$1.08\times10^{-6} \pm 2.06\times10^{-10}$
$\alpha = \{3, 4\}$	-	-	-	$1.82\times10^{-6} \pm 1.57\times10^{-9}$	$9.15\times10^{-7} \pm 3.17\times10^{-10}$
$\alpha = \{0, 2/3, 2\}$	$0.000225 \pm 9.6\times10^{-8}$	$-0.000253 \pm 9.23\times10^{-8}$	$5.09\times10^{-5} \pm 1.13\times10^{-8}$	-	-
$\alpha = \{0, 2/3, 3\}$	$9.42\times10^{-5} \pm 6.51\times10^{-8}$	$-8.53\times10^{-5} \pm 5.3\times10^{-8}$	-	$8.15\times10^{-6} \pm 1.55\times10^{-9}$	-
$\alpha = \{0, 2/3, 4\}$	$-2.18\times10^{-5} \pm 5.03\times10^{-8}$	$2.27\times10^{-5} \pm 3.83\times10^{-8}$	-	-	$1.13\times10^{-6} \pm 1.99\times10^{-10}$
$\alpha = \{0, 2, 3\}$	$2.92\times10^{-5} \pm 2.03\times10^{-8}$	-	$-2.59\times10^{-5} \pm 1.22\times10^{-8}$	$1.2\times10^{-5} \pm 2.91\times10^{-9}$	-
$\alpha = \{0, 2, 4\}$	$-9.46\times10^{-7} \pm 1.5\times10^{-8}$	-	$3.83\times10^{-6} \pm 5.72\times10^{-9}$	-	$1.07\times10^{-6} \pm 2.42\times10^{-10}$
$\alpha = \{0, 3, 4\}$	$3.36\times10^{-6} \pm 9.98\times10^{-9}$	-	-	$1.38\times10^{-6} \pm 2.04\times10^{-9}$	$9.73\times10^{-7} \pm 3.61\times10^{-10}$
$\alpha = \{2/3, 2, 3\}$	-	$3.61\times10^{-5} \pm 2.27\times10^{-8}$	$-3.53\times10^{-5} \pm 1.68\times10^{-8}$	$1.33\times10^{-5} \pm 3.38\times10^{-9}$	-
$\alpha = \{2/3, 2, 4\}$	-	$-1.17\times10^{-6} \pm 1.52\times10^{-8}$	$4.06\times10^{-6} \pm 7.6\times10^{-9}$	-	$1.07\times10^{-6} \pm 2.54\times10^{-10}$
$\alpha = \{2/3, 3, 4\}$	-	$2.92\times10^{-6} \pm 8.53\times10^{-9}$	-	$1.25\times10^{-6} \pm 2.28\times10^{-9}$	$9.86\times10^{-7} \pm 3.79\times10^{-10}$
$\alpha = \{2, 3, 4\}$	-	-	$2.61\times10^{-6} \pm 7.78\times10^{-9}$	$5.24\times10^{-7} \pm 4.16\times10^{-9}$	$1.03\times10^{-6} \pm 4.68\times10^{-10}$
$\alpha = \{0, 2/3, 2, 3\}$	$-0.000153 \pm 1.49\times10^{-7}$	$0.000205 \pm 1.66\times10^{-7}$	$-7.07\times10^{-5} \pm 3.84\times10^{-8}$	$1.74\times10^{-5} \pm 5.25\times10^{-9}$	-
$\alpha = \{0, 2/3, 2, 4\}$	$1.17\times10^{-5} \pm 1.14\times10^{-7}$	$-1.29\times10^{-5} \pm 1.15\times10^{-7}$	$5.64\times10^{-6} \pm 1.72\times10^{-8}$	-	$1.05\times10^{-6} \pm 3.01\times10^{-10}$
$\alpha = \{0, 2/3, 3, 4\}$	$-1.55\times10^{-6} \pm 7.87\times10^{-8}$	$4.23\times10^{-6} \pm 6.73\times10^{-8}$	-	$1.2\times10^{-6} \pm 3.58\times10^{-9}$	$9.91\times10^{-7} \pm 4.59\times10^{-10}$
$\alpha = \{0, 2, 3, 4\}$	$1.82\times10^{-6} \pm 2.61\times10^{-8}$	-	$1.3\times10^{-6} \pm 2.04\times10^{-8}$	$9.37\times10^{-7} \pm 7.24\times10^{-9}$	$1\times10^{-6} \pm 6.01\times10^{-10}$
$\alpha = \{2/3, 2, 3, 4\}$	-	$2.21\times10^{-6} \pm 3.17\times10^{-8}$	$6.66\times10^{-7} \pm 2.9\times10^{-8}$	$1.06\times10^{-6} \pm 8.7\times10^{-9}$	$9.98\times10^{-7} \pm 6.55\times10^{-10}$
$\alpha = \{0, 2/3, 2, 3, 4\}$	$7.89\times10^{-7} \pm 2.02\times10^{-7}$	$1.26\times10^{-6} \pm 2.46\times10^{-7}$	$9.33\times10^{-7} \pm 7.43\times10^{-8}$	$1.01\times10^{-6} \pm 1.54\times10^{-8}$	$1\times10^{-6} \pm 8.88\times10^{-10}$

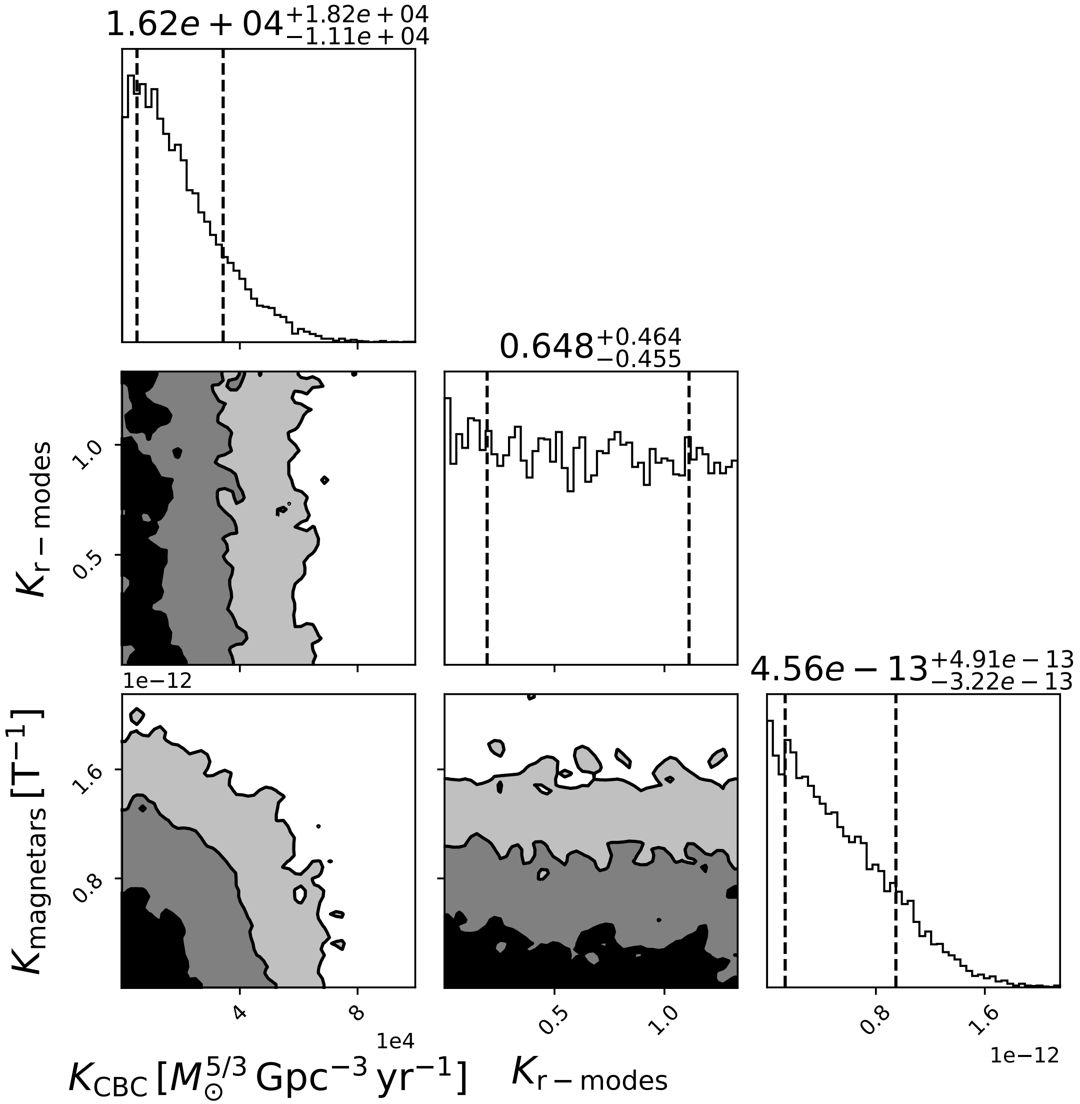
Strong PL injections (20-1726 Hz) {α=0, α=2/3, α=2, α=3, α=4}: PE

	$\Omega_0 = 1\times10^{-6}$	$\Omega_{2/3} = 1\times10^{-6}$	$\Omega_2 = 1\times10^{-6}$	$\Omega_3 = 1\times10^{-6}$	$\Omega_4 = 1\times10^{-6}$
$\alpha=\{0\}$	$1e-05^{3.12e-12}_{6.69e-12}$	-	-	-	-
$\alpha=\{2/3\}$	-	$1e-05^{1.94e-12}_{4.46e-12}$	-	-	-
$\alpha=\{2\}$	-	-	$1e-05^{1.16e-12}_{2.54e-12}$	-	-
$\alpha=\{3\}$	-	-	-	$5.66e-06^{8.31e-10}_{8.24e-10}$	-
$\alpha=\{4\}$	-	-	-	-	$1.23e-06^{1.65e-10}_{1.67e-10}$
$\alpha=\{0,2/3\}$	$6.19e-06^{7.39e-09}_{7.28e-09}$	$1e-05^{4.63e-12}_{1.06e-11}$	-	-	-
$\alpha=\{0,2\}$	$1e-07^{1.08e-11}_{4.78e-12}$	-	$1e-05^{1.17e-12}_{2.57e-12}$	-	-
$\alpha=\{0,3\}$	$1e-07^{9.01e-12}_{4.26e-12}$	-	-	$5.66e-06^{8.16e-10}_{8.48e-10}$	-
$\alpha=\{0,4\}$	$7.67e-06^{7.65e-09}_{7.57e-09}$	-	-	-	$1.19e-06^{1.71e-10}_{1.68e-10}$
$\alpha=\{2/3,2\}$	-	$1e-07^{1.25e-11}_{5.48e-12}$	$1e-05^{1.2e-12}_{2.61e-12}$	-	-
$\alpha=\{2/3,3\}$	-	$1e-07^{6.03e-12}_{2.81e-12}$	-	$5.66e-06^{8.23e-10}_{8.27e-10}$	-
$\alpha=\{2/3,4\}$	-	$6.31e-06^{5.75e-09}_{5.85e-09}$	-	-	$1.17e-06^{1.72e-10}_{1.78e-10}$
$\alpha=\{2,3\}$	-	-	$1e-07^{3.15e-12}_{1.44e-12}$	$5.63e-06^{8.34e-10}_{8e-10}$	-
$\alpha=\{2,4\}$	-	-	$3.52e-06^{2.85e-09}_{2.92e-09}$	-	$1.08e-06^{2.09e-10}_{1.99e-10}$
$\alpha=\{3,4\}$	-	-	-	$1.82e-06^{1.56e-09}_{1.54e-09}$	$9.15e-07^{3.17e-10}_{3.02e-10}$
$\alpha=\{0,2/3,2\}$	$1e-07^{1.02e-11}_{4.72e-12}$	$1e-07^{1.27e-11}_{5.52e-12}$	$1e-05^{1.24e-12}_{2.66e-12}$	-	-
$\alpha=\{0,2/3,3\}$	$1e-07^{8.79e-12}_{4.19e-12}$	$1e-07^{6.22e-12}_{2.87e-12}$	-	$5.65e-06^{8.11e-10}_{8.29e-10}$	-
$\alpha=\{0,2/3,4\}$	$1e-07^{1.3e-10}_{5.92e-11}$	$6.23e-06^{5.77e-09}_{5.85e-09}$	-	-	$1.17e-06^{1.69e-10}_{1.8e-10}$
$\alpha=\{0,2,3\}$	$1e-07^{9.09e-12}_{4.21e-12}$	-	$1e-07^{3.16e-12}_{1.44e-12}$	$5.63e-06^{8.34e-10}_{8.31e-10}$	-
$\alpha=\{0,2,4\}$	$1e-07^{2.47e-10}_{1.12e-10}$	-	$3.49e-06^{2.87e-09}_{2.96e-09}$	-	$1.08e-06^{2.08e-10}_{2.04e-10}$
$\alpha=\{0,3,4\}$	$3.36e-06^{9.76e-09}_{1.01e-08}$	-	-	$1.38e-06^{2.02e-09}_{2.06e-09}$	$9.73e-07^{3.7e-10}_{3.62e-10}$
$\alpha=\{2/3,2,3\}$	-	$1e-07^{6.19e-12}_{2.84e-12}$	$1e-07^{3.14e-12}_{1.42e-12}$	$5.62e-06^{8.28e-10}_{8.14e-10}$	-
$\boxed{\alpha=\{2/3,2,4\}}$	-	$1e-07^{2.1e-10}_{9.3e-11}$	$3.47e-06^{3e-09}_{2.8e-09}$	-	$1.08e-06^{2.03e-10}_{2.03e-10}$
$\alpha=\{2/3,3,4\}$	-	$2.92e-06^{8.23e-09}_{8.7e-09}$	-	$1.25e-06^{2.31e-09}_{2.27e-09}$	$9.86e-07^{3.71e-10}_{3.93e-10}$
$\alpha=\{2,3,4\}$	-	-	$2.61e-06^{7.82e-09}_{7.65e-09}$	$5.24e-07^{4.02e-09}_{4.2e-09}$	$1.03e-06^{4.62e-10}_{4.63e-10}$
$\alpha=\{0,2/3,2,3\}$	$1e-07^{9.01e-12}_{4.19e-12}$	$1e-07^{6.27e-12}_{2.8e-12}$	$1e-07^{3.15e-12}_{1.44e-12}$	$5.62e-06^{8.26e-10}_{8.43e-10}$	-
$\alpha=\{0,2/3,2,4\}$	$1e-07^{2.16e-10}_{9.92e-11}$	$1e-07^{1.89e-10}_{8.52e-11}$	$3.44e-06^{2.83e-09}_{2.86e-09}$	-	$1.08e-06^{1.97e-10}_{2e-10}$
$\alpha=\{0,2/3,3,4\}$	$1.03e-07^{4.46e-09}_{1.96e-09}$	$2.83e-06^{8.83e-09}_{8.99e-09}$	-	$1.25e-06^{2.29e-09}_{2.24e-09}$	$9.85e-07^{3.77e-10}_{3.74e-10}$
$\alpha=\{0,2,3,4\}$	$1.82e-06^{2.55e-08}_{2.5e-08}$	-	$1.3e-06^{1.96e-08}_{1.97e-08}$	$9.37e-07^{7.03e-09}_{7e-09}$	$1e-06^{5.88e-10}_{5.98e-10}$
$\alpha=\{2/3,2,3,4\}$	-	$2.21e-06^{3.21e-08}_{3.18e-08}$	$6.65e-07^{2.91e-08}_{2.91e-08}$	$1.06e-06^{8.86e-09}_{8.64e-09}$	$9.98e-07^{6.41e-10}_{6.78e-10}$
$\alpha=\{0,2/3,2,3,4\}$	$7.9e-07^{2.04e-07}_{2.05e-07}$	$1.26e-06^{2.52e-07}_{2.47e-07}$	$9.34e-07^{7.36e-08}_{7.63e-08}$	$1.01e-06^{1.6e-08}_{1.52e-08}$	$1e-06^{8.67e-10}_{9.02e-10}$

Analysis with real data (20-100 Hz): corner plots



PE plot for Ω_α, with α=2/3, 2, 4



PE plot for CBC, r-mode, and magnetar astrophysical implications

Beyond simple power law: SGWB from magnetars

- Astrophysical SGWB

Star fraction into GW source of interest

Stellar formation rate

$$\Omega_{gw}(f) = \frac{f}{\rho_c}\int_0^{z_{sup}} \frac{\lambda R_*(z)}{(1+z)\, H_0\, E(z)} \left\langle \frac{dE_{gw}}{df}\bigg|_{f=f_s(1+z)} \right\rangle$$

$$z_{sup} = \begin{cases} z_{max}, & f < \dfrac{f_{max}}{1+f_{max}} \\ \dfrac{f_{max}}{f} - 1, & \text{otherwise} \end{cases}$$

- Energy spectrum (magnetar with rotational period T_{rot}, see Regimbau, Mandic 2008)

$$\frac{dE_{gw}}{df} = Kf^3\left(1 + \frac{K}{\pi^2 I_{zz}} f^2\right)^{-1} = \begin{cases} Kf^3, \text{GW emission negligible} \\ \pi^2 I_{zz} f, \text{purely GW spindown} \end{cases}$$

Magnetar moment of inertia

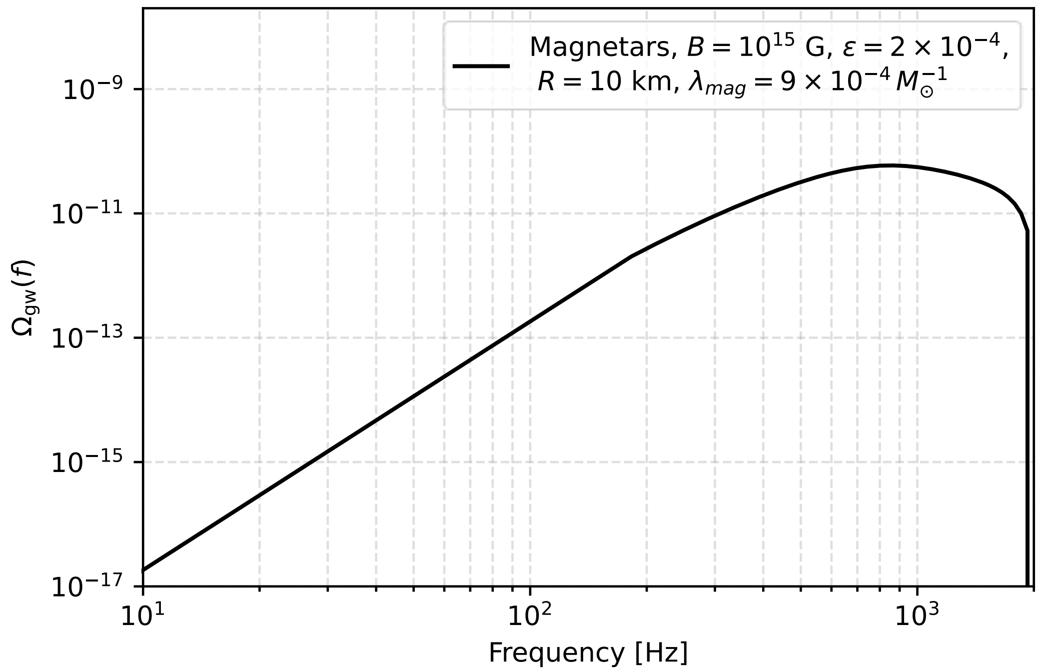
Ellipticity

$$K = \frac{192\pi^4 G\, I_{zz}^3}{5c^2 R^6}\, \frac{\varepsilon^2}{B^2 \sin\alpha} \qquad f \in \left[0, \frac{2}{T_{rot}}\right]$$

Magnetic field

- Focus on the case where GW emission is negligible with respect to magnetic torque and on the (ensemble average of) B and ε parameters:

$$\Omega_{gw}^{mag}(f) \propto \langle \varepsilon^2 \rangle \left\langle \frac{1}{B^2} \right\rangle f^4$$



r-mode instability SGWB

- GW emission drives r-mode instability in young, rotating NSs, carrying away most the of the star angular momentum in $\simeq$ 1yr timescale

- Simple description with parameters Ω (star angular velocity) and α (r-mode-instability amplitude) → Owen et al. 1998

- More recent studies (Sa-Tome 2006, Zhu et al. 2011) use the initial amount of differential rotation $K \in [-5/4, 10^{13}]$, related to the saturation amplitude $\alpha_{sat} \propto (K+2)^{-1/2}$

$$\Omega_{\mathrm{gw,\,r-modes}}(\mathrm{f}) \propto f^2 \langle (K+2)^{-1} \rangle$$

LVK O3 search for isotropic SGWB: results

Power law	$f^{HL}_{99\%}$ [Hz]	$\hat{C}^{HL}/10^{-9}$	$f^{HV}_{99\%}$ [Hz]	$\hat{C}^{HV}/10^{-9}$	$f^{LV}_{99\%}$ [Hz]	$\hat{C}^{LV}/10^{-9}$	$f^{\mathrm{O1+O2+O3}}_{99\%}$ [Hz]	$\hat{C}^{\mathrm{O1+O2+O3}}/10^{-9}$
0	76.1	-2.1 ± 8.2	97.7	229 ± 98	88.0	-134 ± 63	76.6	1.1 ± 7.5
2/3	90.2	-3.4 ± 6.1	117.8	145 ± 60	107.3	-82 ± 40	90.6	-0.2 ± 5.6
3	282.8	-1.3 ± 0.9	375.8	9.1 ± 4.1	388.0	-4.9 ± 3.1	291.6	-0.6 ± 0.8

TABLE I. Search results for an isotropic GWB, using the optimal filter method for power law GWBs with $\alpha = \{0, 2/3, 3\}$. For each of the three baselines IJ, we show the point estimate and 1σ uncertainty for the cross-correlation estimate C_{IJ}, along with the frequency band from 20 Hz to $f^{IJ}_{99\%}$ containing 99% of the sensitivity. We see that the HL baseline is the most sensitive, and the HV and LV baselines are more sensitive at higher frequencies, and for larger spectral indices, due to the longer baseline. In the last two columns, we also present the search result combining all three baselines from O3, as well as the O1 and O2 data. As noted in the main text, the point estimates for the HV and LV are approximately 2σ away from zero, however this is not consistent with a GWB given the result of the much more sensitive HL baseline.

	Uniform prior			Log-uniform prior		
α	O3	O2 [43]	Improvement	O3	O2 [43]	Improvement
0	1.7×10^{-8}	6.0×10^{-8}	3.6	5.8×10^{-9}	3.5×10^{-8}	6.0
2/3	1.2×10^{-8}	4.8×10^{-8}	4.0	3.4×10^{-9}	3.0×10^{-8}	8.8
3	1.3×10^{-9}	7.9×10^{-9}	5.9	3.9×10^{-10}	5.1×10^{-9}	13.1
Marg.	2.7×10^{-8}	1.1×10^{-7}	4.1	6.6×10^{-9}	3.4×10^{-8}	5.1

TABLE II. Upper limits at the 95% credible level on Ω_{ref} under the power law model for the GWB. We show upper limits conditioned on different fixed power law indices α, as well as a marginalized limit obtained by integration over α, using a Gaussian prior with zero mean and a standard deviation of 3.5. We show the results using a prior that is uniform in Ω_{ref}, as well as uniform in $\log \Omega_{\mathrm{ref}}$. As described in the main text, the uniform upper limits are more conservative, while the log uniform priors are more sensitive to weak signals. We also compare with the upper limits from [43], and give the improvement factor we achieve using O3 data.

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