Machine learning algorithms for the conservative-to-primitive conversion in relativistic hydrodynamics

Thibeau Wouters

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Utrecht University

KU LEUVEN

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General relativistic magnetohydrodynamics

New systems in 3G era (isolated neutron star, supernovae)

- Need simulations...
- ... but simulations are costly: cf. Arthur Offerman's talk

What makes simulations so expensive?

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Simulations numerically evolve

$$\partial_t \left(\sqrt{\gamma}\mathcal{C}\right) + \partial_i \left(\sqrt{\gamma}\mathcal{F}^i\right) = \mathcal{S}$$

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$$\partial_t \left(\sqrt{\gamma} \mathcal{C} \right) + \partial_i \left(\sqrt{\gamma} \mathcal{F}^i \right) = \mathcal{S}$$

and depend on 2 sets of fluid variables (energy, momentum, density):

- $\mathcal{C}$: conserved variables (evolved)
- $\mathcal{P}$: primitive variables (computed from $\mathcal{C}$)

The C2P bottleneck

Going from $\mathcal{C}$ to $\mathcal{P}$ (C2P) is a major **bottleneck** [1, 2]:

- No analytic relation
- $\sim$ 40% of total simulation cost
- $>$300MB of external data (equation of state)

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- We use the `Gmunu` solver [3–7].
- "Optimize"? Criteria to evaluate numerical methods [2]:
  1. Speed
  2. Accuracy
  3. Robustness

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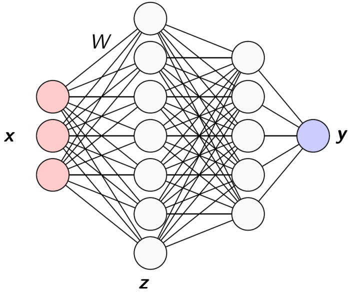
Neural networks

- Neural networks can represent any function: $f : \mathcal{X} \to \mathcal{Y} : \boldsymbol{x} \mapsto \boldsymbol{y}$



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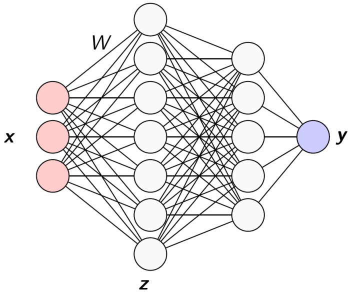

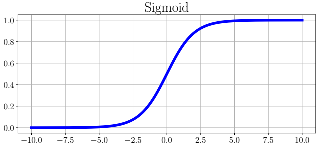

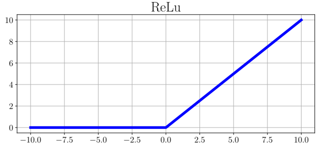

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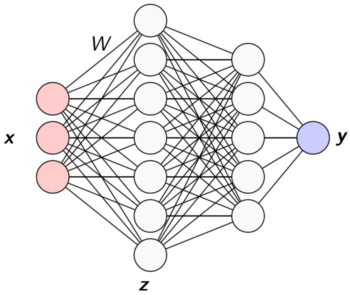
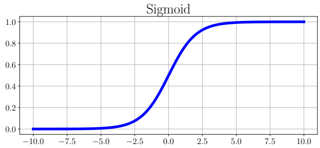
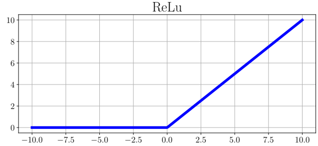
- At each layer: $\boldsymbol{z} = \varphi\left(W^T \boldsymbol{x}\right)$, $\varphi =$ activation function
- Easy to implement in `Gmunu` (Fortran)







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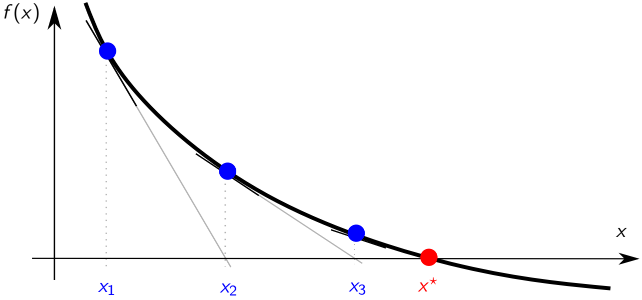
Existing methods: root-finding algorithms

Current C2P methods find root $x^\star$ of master function f by iteratively improving estimates x_i (*e.g.*, Newton-Raphson).



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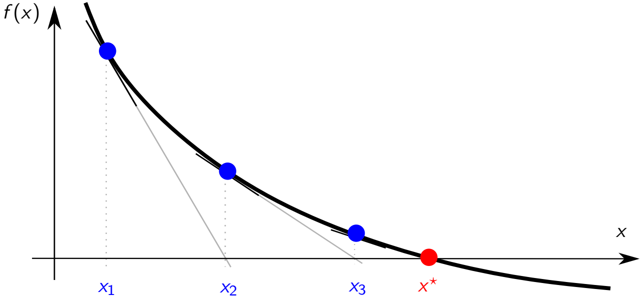
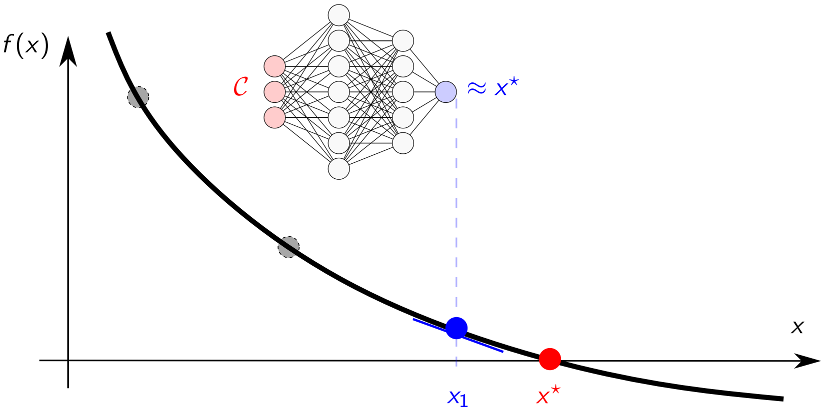


1. **Slow**: evaluating $f(x)$ is costly.
2. **Accurate**: accuracy tolerance as stopping criterion
3. **Robust**: well-designed master function (Kastaun *et al.* [8])

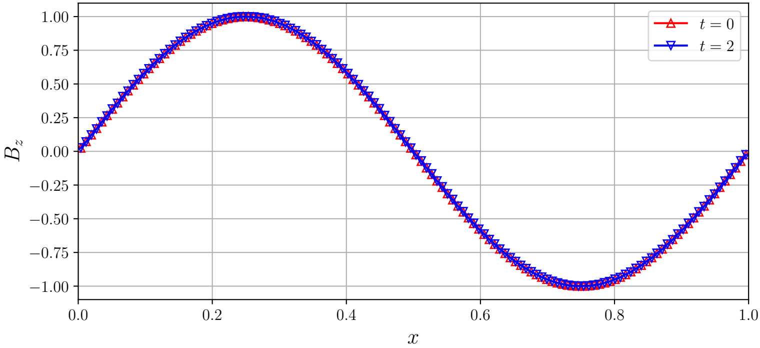
Hybrid approach: idea

Neural network gives an initial guess, to be refined with the root-finding algorithm.



Hybrid approach: proof of concept

Test case: magnetic field B_z of Alfvén wave:



Small neural network: 2 hidden layers, each 20 hidden neurons.

Hybrid approach: proof of concept

Faster! Simulation time:

- Standard: (23.48 ± 0.54) seconds
- Hybrid, ReLU activation function: (18.84 ± 0.19) seconds
- Speed-up of $\sim 25\%$
- Same accuracy and robustness!

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Future work:

- Consider neutron star simulation
- Add non-trivial equation of state
- Train during simulation

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Conclusion

- Future detectors need templates obtained with expensive simulations: the C2P is a major bottleneck to be tackled
- Existing methods using root-finding algorithms are guaranteed to be accurate and robust
- Hybrid approaches can speed up simulations $> 25\%$ without sacrificing accuracy or robustness
- Future work: simulate neutron star with non-trivial equation of state (ongoing)

References

[1] Tobias Dieselhorst et al. "Machine learning for conservative-to-primitive in relativistic hydrodynamics". In: *Symmetry* 13.11 (2021), p. 2157.

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BACK-UP SLIDES

Training data?

The primitive-to-conservative (P2C) transformation is analytic, e.g. in GRHD:

$$\begin{aligned} D &= \rho W(v) \\ S_i &= \rho h W(v)^2 v_i \\ \tau &= \rho h W(v)^2 - p - D \,, \end{aligned}$$

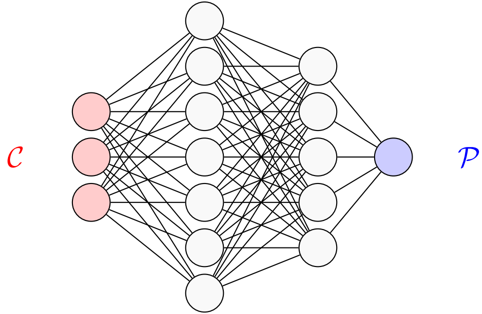
with $W(v)$ the Lorentz factor and $h(p, \rho, \varepsilon)$ the enthalpy.

This is easy for low-dimensional, flat space-times with simple equations of state.

Can also sample directly from simulations: easier for high-dimensional, curved space-times with complicated equations of state in GRMHD.

Naïve approach

1st (naïve) idea: Approximate $f : \mathcal{C} \to \mathcal{P}$ with a neural network.



- Data generated with the *analytic* $f^{-1} : \mathcal{P} \to \mathcal{C}$
- MLP with 504, 127 hidden neurons; sigmoid activation functions
- Trained with Adam & adaptable learning rate

Results of naïve approach

1. Speed: $\sim 5\times$ slower than existing methods
2. Accuracy: Squared difference: $\sim 10^{-3}$, vs. $\sim 10^{-8}$ for existing methods
3. Robustness: Not guaranteed by machine learning (*e.g.*, performance outside training domain).