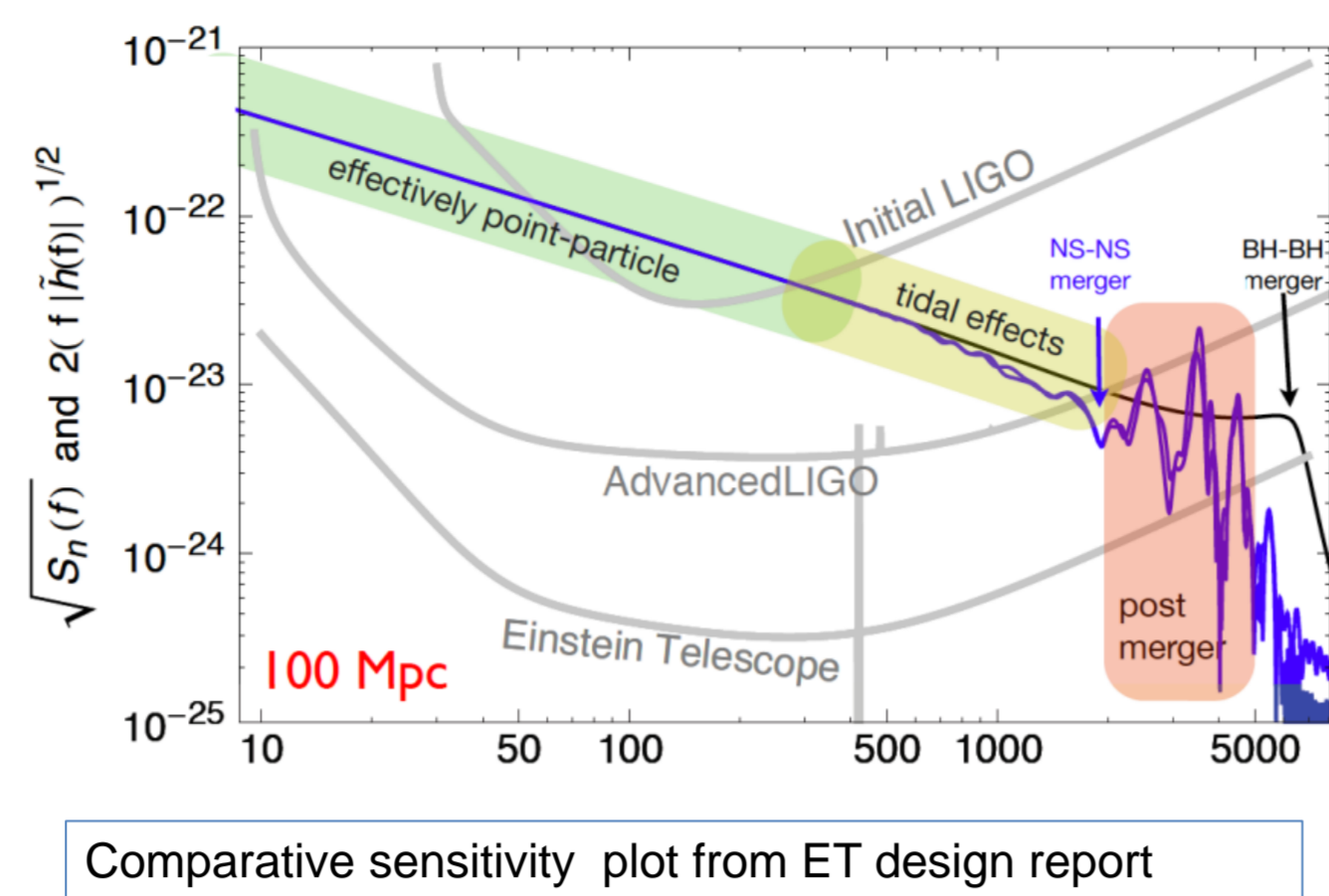


## Motivation: new capabilities of 3G detectors

3G detectors are going to be sensitive to:

- BBH
- Coalescing neutron stars binaries
- Continuous waves from asymmetric NS
- NS/magnetars bursts
- Stochastic GW background (Cosmological and Astrophysical)
- Long separation in-spirals
- Core collapse supernovae
- Exotic objects (i.e. Axions stars)
- Physics near BH horizon
- Multiband observations with LISA
- Multi-wavelength and multi-messenger observations

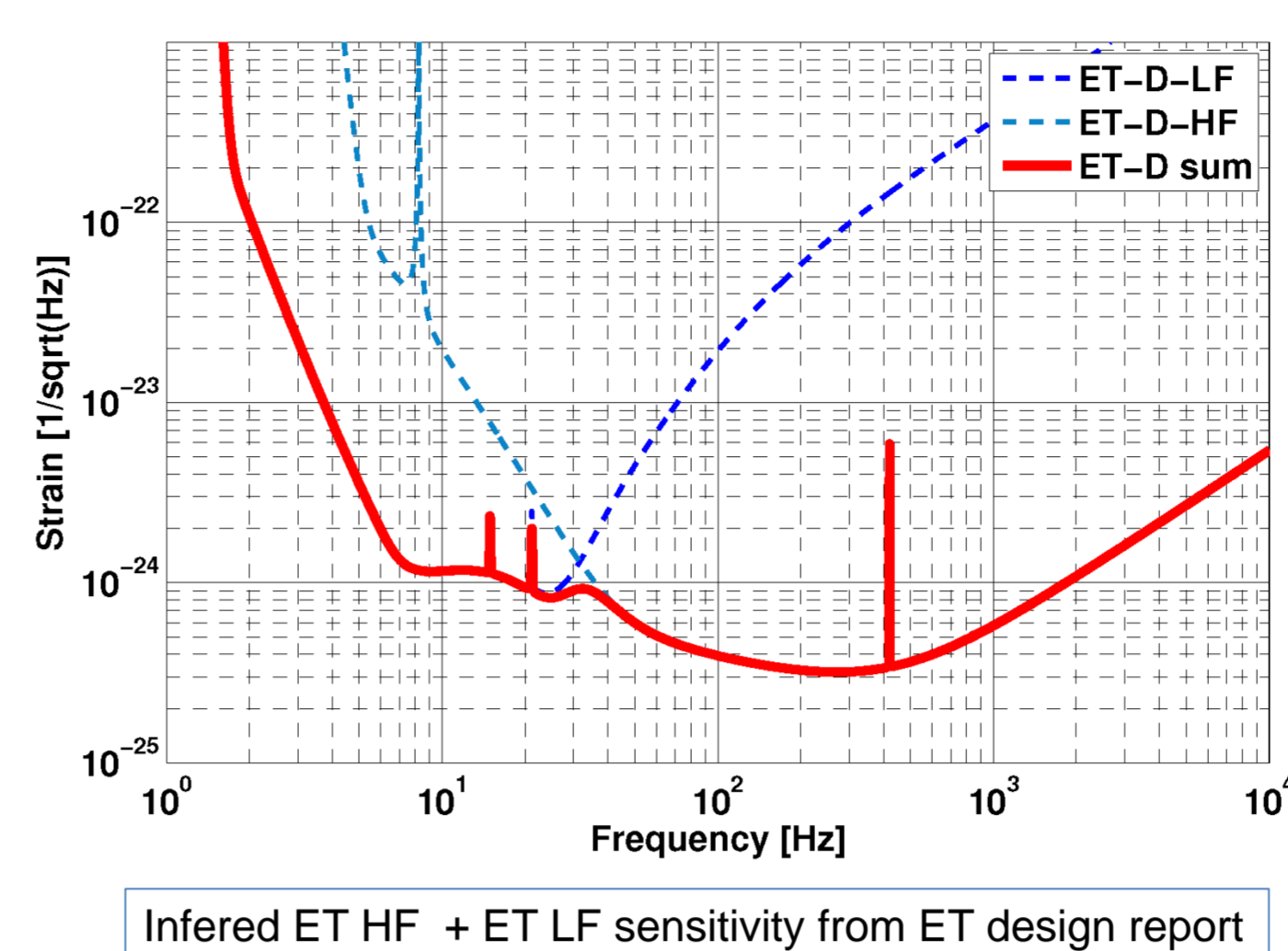


Next generation (3G) of GW Interferometers are going to be extremely sensitive in terms of strain measurement which translates in to being sensitive to entire new range of GW sources i.e. stochastic background, long separation in-spirals, core-collapse supernovae and BH-NS systems.

They are going to provides us with observations of high energy, high z events with unprecedented SNR and localisation capabilities. All of this comes with the use od most advanced and cutting edge technologies as in the case of input/output resonant filter cavities

NOTE: Order of magnitude better suppression of noise leads to doubling of GW BBH coalescence signal!

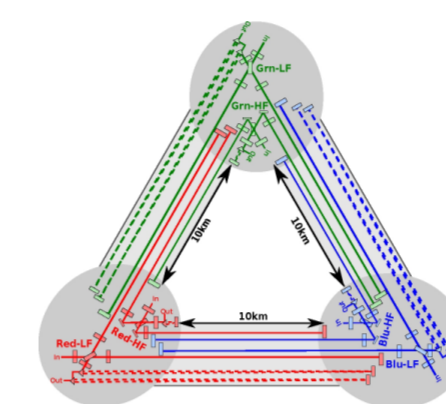
## Problem: new technologies led to unforeseen consequences



We need state of the art I/O cavities for next generation of GW interferometers.

3G detectors are going to use not only fundamental TEM00 mode but also Higher order modes (HOM's). Importance of exploring of many different parameter combinations (Lengths, RoC's, Finesse) to arrive at the (globally) optimal solution.

3G detectors are going to use squeezed sources for subquantum measurements so they are going to need squeezed compatible Output mode cleaner = specific topology and low loss.



ET is going to have multiple input/output mode cleaner cavities. It is going to have bowtie OPO's in squeezers and linear filter cavities all on different laser wavelengths so we need a tool for fast prototyping.

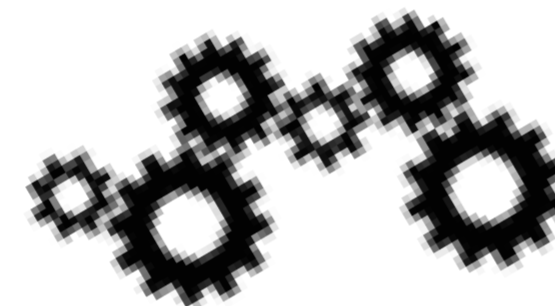
## Idea and realization:

### Assumptions for this code are

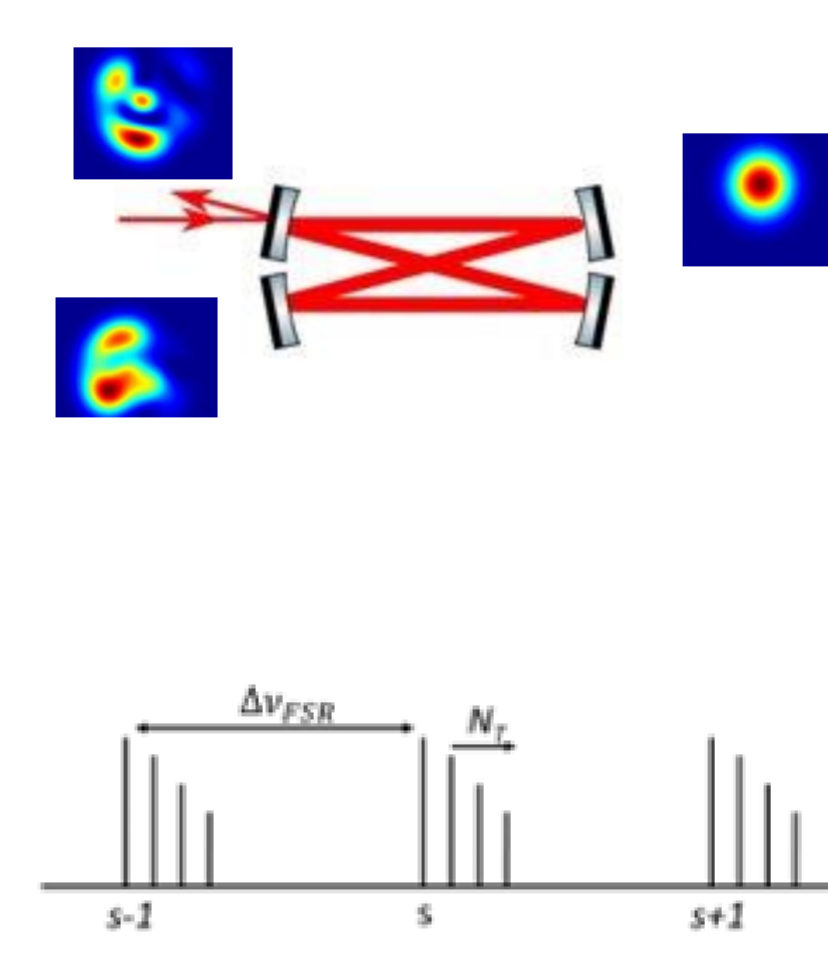
- Modeling of HOM power distribution as power law:  $(\text{sum of mode indices}) \exp(-\text{const})$ .
- Linearization of Hermite – Gauss polynomial for calculation of modal crosstalk coefficients (expansion up to second order).
- Include cavity geometry induced mismatch and misalignment – we initially treat the beam as perfect Gaussian and al of the aberrant behavior is attributed to the cavity.
- Bundle up all of the mismatch and misalignment in nondimensional coefficients that depend of the HOM order.
- Model resonances as FR airy functions.
- Exchange power overlap integrals with sums up to the selected mode order (i.e. up to mode 10).
- Power distribution vector and depth of sidebands modulation is to be intuited from previous GW interferometers.

### Desired code outputs

- Graphical representation of evasion regions. Those are regions of high transmission of sidebands and HOM's.
- It should give us possibility to calculate and display cavity content and to separate influence of dc, sidebands and HOM's.
- It should allow independent variation of critical parameters in order to (first manually but later automatically) search through parameter space to find the optimal geometrical and optical parameters.
- Regions of low noise that are desired for resonant filter cavity parameters selection.



## Anatomy of an Output mode cleaner: ABCD matrixes and HOM's



$$\begin{pmatrix} A_1 & B_1 \\ C_1 & D_1 \end{pmatrix} = \begin{pmatrix} 1 & \frac{L}{2} \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -\frac{2}{R} & 1 \end{pmatrix} \begin{pmatrix} 1 & L_1 + 2L_2 \\ 0 & 1 \end{pmatrix}$$

$$\psi_{out}(x) = 2\text{arccos}\left(\frac{\text{sign}(B)\sqrt{A+D+2}}{2}\right)$$

$$\psi_{out}(x) = 2\text{arccos}\left(\frac{\text{sign}(B)\sqrt{A+D+2}}{2}\right)$$

Property Symmetric ring

$$g_1 = 1 - \frac{d_1 + 2d_2}{R}$$

$$g_2 = 1 - \frac{d_2}{R}$$

Path lengths  $L = d_1 + 2d_2 + d_3$

$$W_{out}(x) = \left(\frac{\partial W}{\partial x}\right) \sqrt{\frac{2R(1-m)}{m}}$$

$$W_{out}^2 = \left(\frac{\partial W}{\partial x}\right) \sqrt{\frac{2R(1-m)}{m}}$$

$$r_{out} = \left(\frac{q + (n+m+1)\frac{m-1}{2}\sqrt{\frac{2R}{m}}}{2}\right) \xi$$

$$\text{FSR} = \frac{c}{L}$$

the Gaussian beam parameter. It is defined as:

$$\frac{1}{q(z)} = \frac{1}{R(z)} - i \frac{\lambda}{2\pi w^2(z)}$$

and can also be written as:

$$q(z) = \frac{z - z_0}{\cos(\theta)} + i \frac{z_0}{\sin(\theta)} = z - z_0 + i z_0 \theta$$

where  $z_0 = 1/z_0$

$$w(z) = \frac{1}{q(z)} \exp\left(-i \frac{\pi}{\lambda} \frac{z^2}{2}\right)$$

$$E(x, y, z) = E_0 \exp(-ikz) \exp\left(-i \frac{\pi}{\lambda} \frac{x^2 + y^2}{2}\right) \exp(-i \frac{\pi}{\lambda} \frac{z^2}{2})$$

$$w(z) = \frac{w_0}{z} \exp\left(-i \frac{\pi}{\lambda} \frac{z^2}{2}\right) \exp\left(-i \frac{\pi}{\lambda} \frac{z^2}{2}\right)$$

The Guoy phase can be written as:

$$\psi(z) = \text{arctan}\left(\frac{z - z_0}{z_0}\right) = \psi(z) = \text{arctan}\left(\frac{z - z_0}{z_0}\right)$$

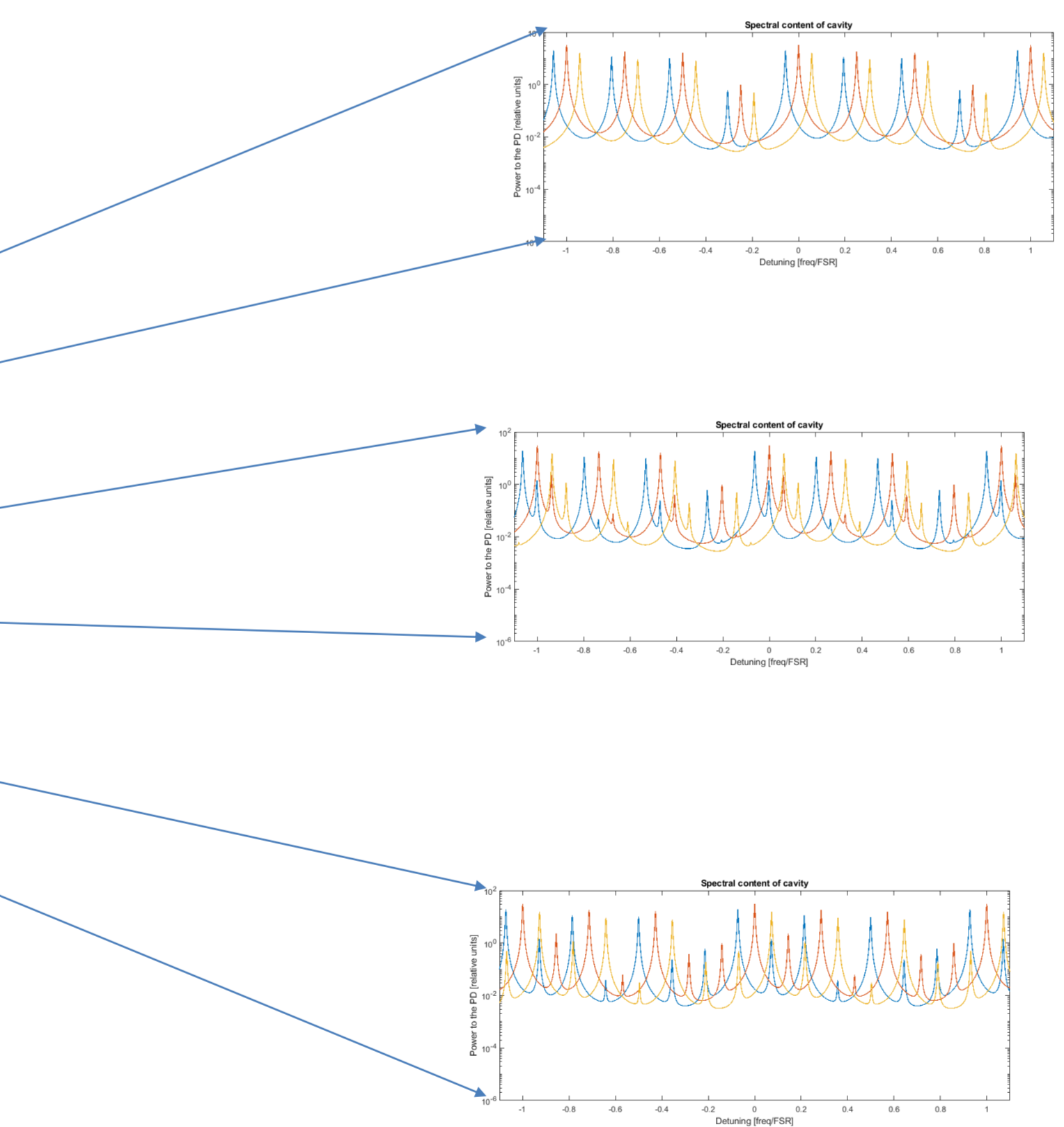
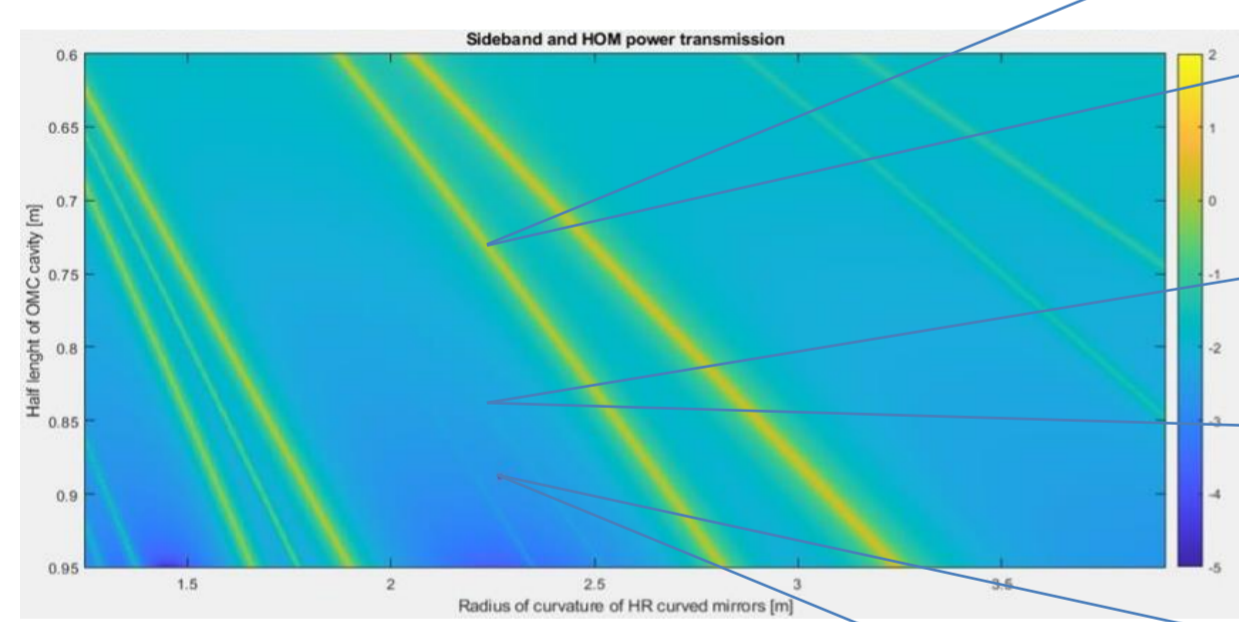
Compared to a plane wave, the phase lag  $\psi$  of a Hermite-Gauss mode  $\psi = (n + m + 1)\psi(z)$ .

$$\psi = \left(n + \frac{1}{2}\right)\psi(z) + \left(m + \frac{1}{2}\right)\psi(z)$$

General treatment of HG HOM's and expression of Guoy phase in terms of q parameter

## R-L plots: the good and bad parameter choice

Let's see how does cavity content looks like when we vary one of the parameters. Here it is L and it represents a walk across one fringe from L= 0.69m to L=0.877m. RoC is held constant at R=2.3 m. Sideband frequency is 12.3 MHz – for ET Pf



Top panel: for selected R and L the OMC is co-resonant for sidebands, dc and HOM so all of them are transmitted. This is bad behaviour for OMC but may be desirable (in some amount) for IMC in order to transmit through certain sideband modulation signals.

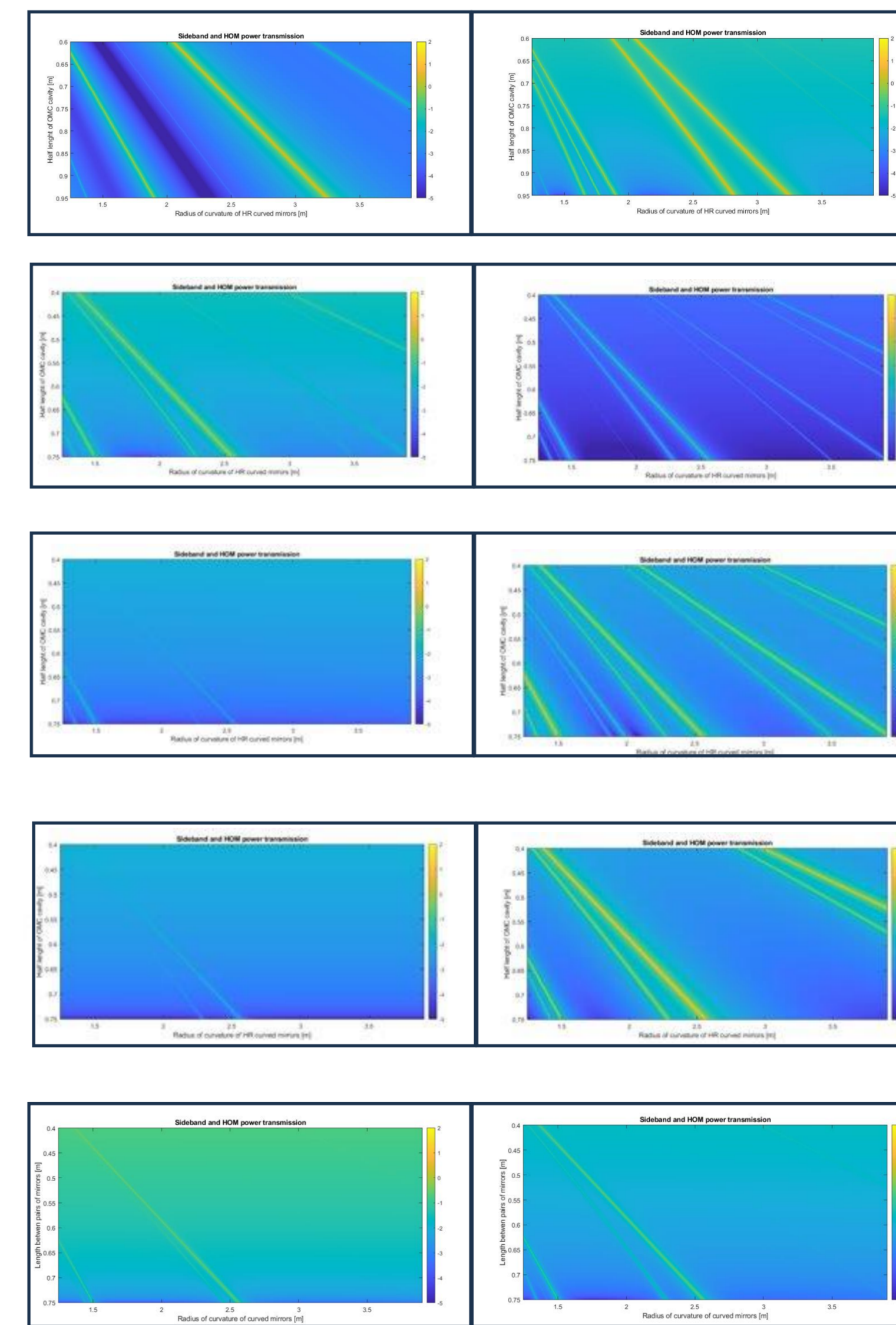
Mid panel: The OMC is not co resonant for the main sidebands but HOM of lower sideband is being transmitted along with the DC – NOT A GOOD CHOICE!

Bottom panel: for selected R and L the OMC is not co-resonant with any unwanted signal and it is possible to build a filter cavity around the central carrier.

Color code: red – carrier; blue – lower sb; yellow – upper sb

## Products of the code: parameter plots as heuristics for designing ET Pf OMC

As output the code gives graphical representation of Sidebands and HOM transmission in R L plane. Independently vary the parameters (power of sidebands, Finesse, cavity induced mode mismatch, cavity induced mode misalignment, vary the sideband frequency) while keep other the same in order to intuit how each of the parameter influences the plots. The bluer the region is the better.



Left panel: only Dc no sb.  
Right panel: DC and +/- sb of equal power

Finesse, Misalignment, mismatch, sidebands frequencies are constant.

Left panel: Finesse 200  
Right panel: Finesse 2000

DC and sb power are constant. Misalignment, mismatch and sb frequencies are constant.

Left panel: Misalignment is 0.01  
Right panel: Misalignment is 0.2

Finesse, mismatch, DC and sideband power, sidebands frequencies are constant.

Left panel: Mismatch is 0.01  
Right panel: Mismatch is 0.02

Finesse, misalignment, DC and sideband power, sidebands frequencies are constant.

Left panel: Sideband at 3 MHz  
Right panel: Sideband at 12 MHz

Finesse, misalignment, mismatch, DC and sideband power, sidebands frequencies are constant.

## Literature

- [1] Einstein Telescope editorial team, *ET Desing report*. 2020
- [2] F. Bayer-Helms, *Coupling coefficients of an incident wave and the modes of a spherical optical resonator in the case of mismatching and misalignment*. Appl. Opt., 23(9):1369-1390, 1984.
- [3] Tao et al. *Power coupling losses for misaligned and mode-matched higher order Hermite-Gauss modes*. Opt. Lett. 1;46(11):2694-2697, 2021.
- [4] Ast et al. *Higher order Hermite-Gauss modes for gravitational wave detectors*. Phys. Rev. D, 103:042008, 2021
- [5] A. W. Jones and A. Freise. *Increased sensitivity of higher order laser beams to mode mismatches*. Optics Letters, 45(20):5876, 2020
- [6] Bond et al. *Interferometer techniques for gravitational-wave detection*. Living Reviews of Relativity, 2017.
- [7] Philip Koh. *Design, Suspension and Optical Characterization of the Beam Splitter and Output Mode Cleaner for the AEI 10 m Prototype*. PhD dissesation, 2022

## Conclusion and future work

We have demonstrated capability of linearized code to generate useful transmission plots in R-L space. Based on this we have chosen a preliminary sets of parameters of ET Pf OMC.

Need more rigorous linearization procedure and have to separate treatment for odd and even modes. We are going to include a robust nearest neighbouring mode plot. It is necessary to include a tool for HOM identification and parameter optimization (i.e. least squares) to find global minima of transmitted noise.

## Support and acknowledgements

This work is supported by:

- iBOF 21/084 *Unlocking the Dark Universe with Gravitational Wave Observations: from Quantum Optics to Quantum Gravity* in order to conduct a research on novel optical configurations for I/O resonant cavities using free-form optics.
- *Interreg ET-Pathfinder* for procurement of capital infrastructure for Silicon mirror fabrication and metrology