

TUTORIAL

# Precision Ephemerides for Gravitational-wave Searches

Tom Killestein, Danny Steeghs

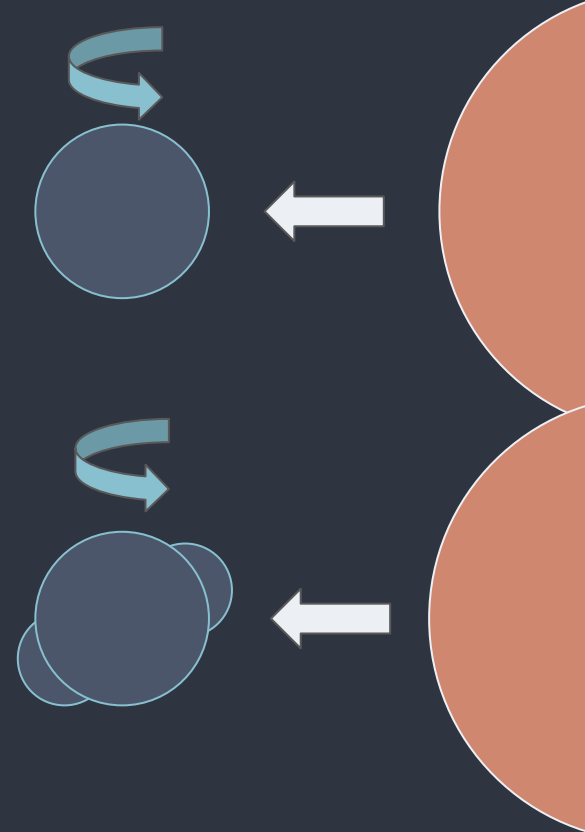
University of Warwick

# LMXBs as CW sources

Low-mass X-ray binaries: donor is low-mass, compact object can be black hole or neutron star.

Mass accretion from donor onto compact object, forming accretion disk. Accretion rate sets geometry (geometric thin/thick) of disk, with strong EM emission from where disk meets compact object.

Time-varying mass quadrupole from NS mountains, *r*-modes, precession -> continuous wave emission on spin period.



# The cornerstone LMXB

First extrasolar X-ray source discovered  
(Giacconi+1962)

Thought to host a neutron star and  
evolved main-sequence donor, accreting  
at  $\sim$ Eddington rate.

Extensively studied at all wavelengths  
(484 papers!)

And yet significant uncertainties remain.

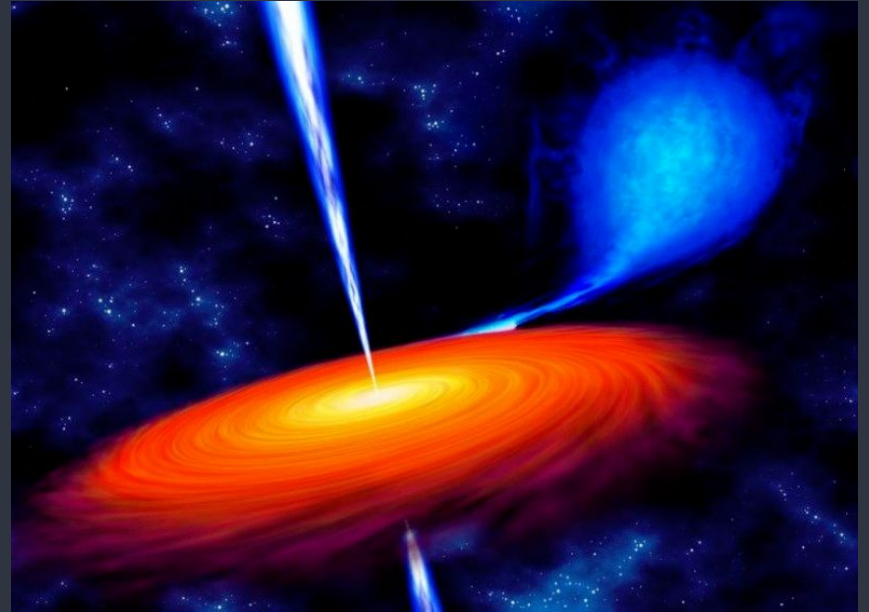


Image credit: Ralf Schoofs

# Why Sco X-1?

- > Among the highest accretion rates in LMXBs, at Eddington limit -> maximal spin-up torque -> maximal cGW emission.
- > Relatively nearby -  $1/r$  dependence on strain.
- > Well-characterised binary orbit, and well-studied means some of the most competitive constraints on orbital/systemic parameters (see latter slides)

# Why not Sco X-1?

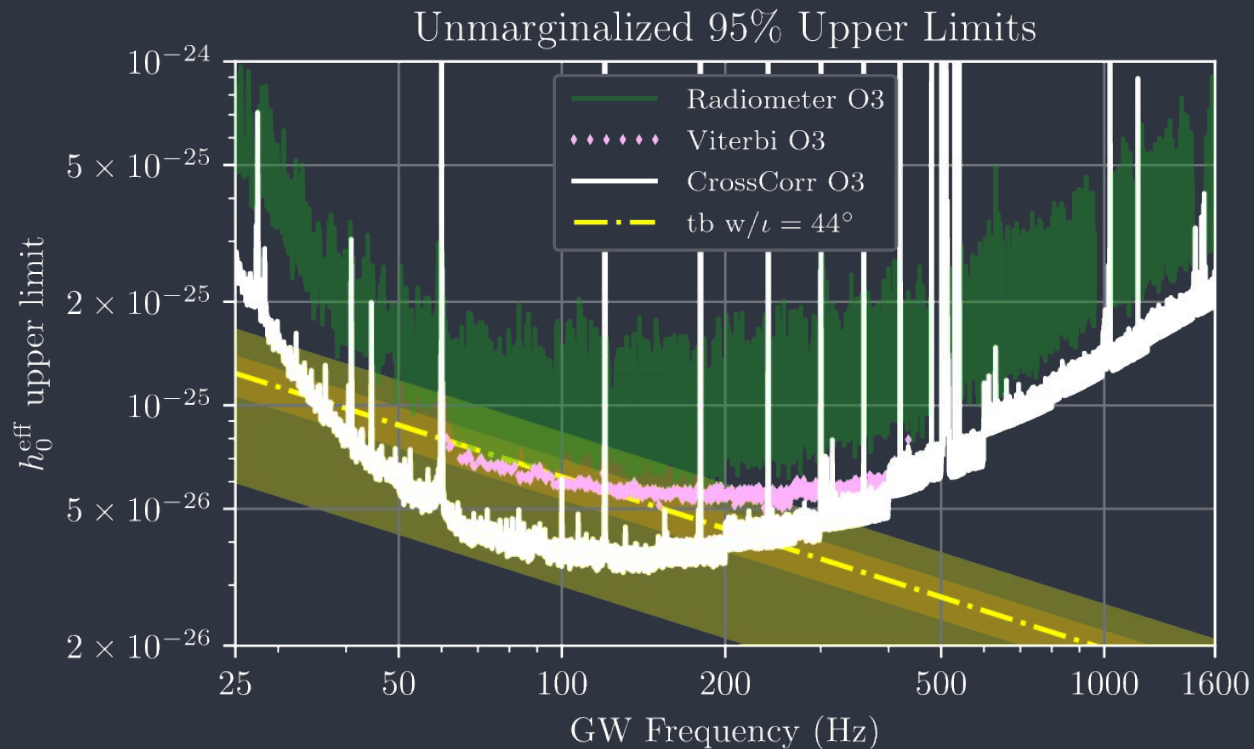
> No detection of the NS spin period down to increasingly deep limits in X-ray stars

**e.g. Messenger+2015, Galaudage+2022**

> Intense X-ray luminosity limits our ability to study it well (i.e. too bright for normal Chandra/XMM!)

> Donor remains un-identified, thought to be an evolved MS star but poorly constrained.

> Highly variable accretion rate potentially drives spin wandering – not a problem for HMM searches!



Significant limiting factor: parameter space to explore is huge and multi-dimensional. EM constraints crucial to make sure search is actually constraining.

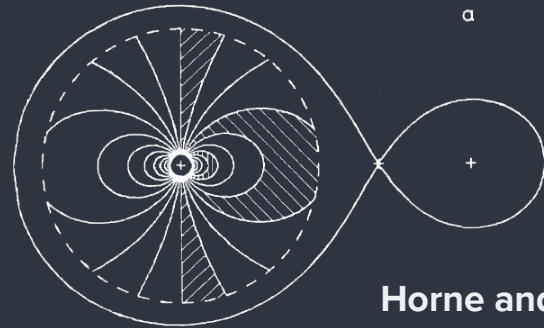
# Tracing the motion of components in LMXBs

**Ideally:** trace motion of neutron star  
centre of mass through the system

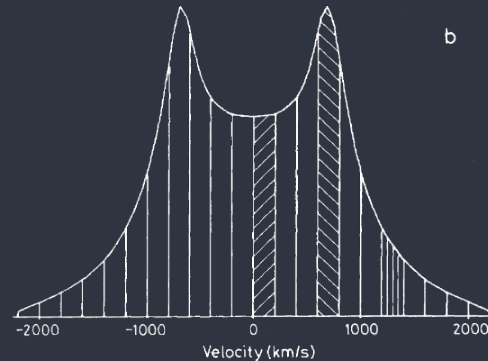
But:

- Luminosity (often) dominated by emission from the accretion disk, that is highly velocity-broadened. We also see both sides of the disk at the same time in spectra.

Hard to robustly infer NS from this - what about the donor star?

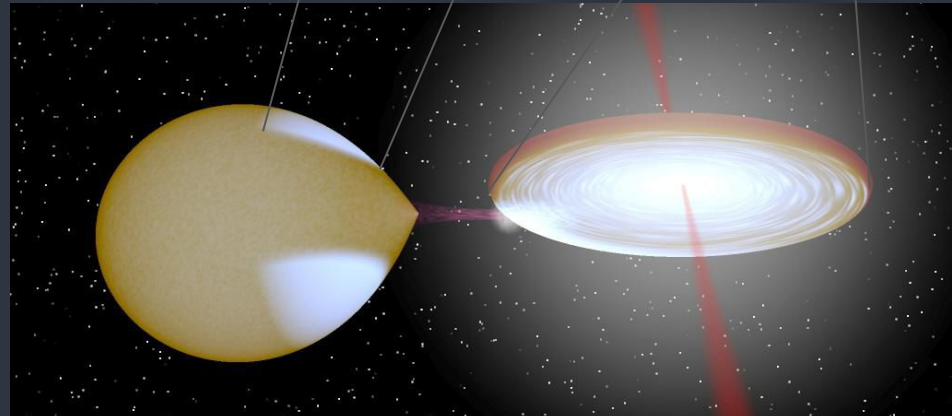
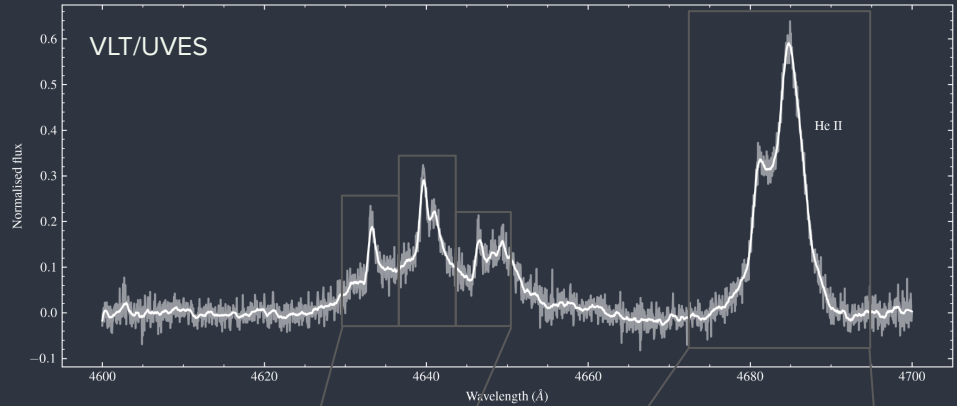


Horne and Marsh  
(1986)

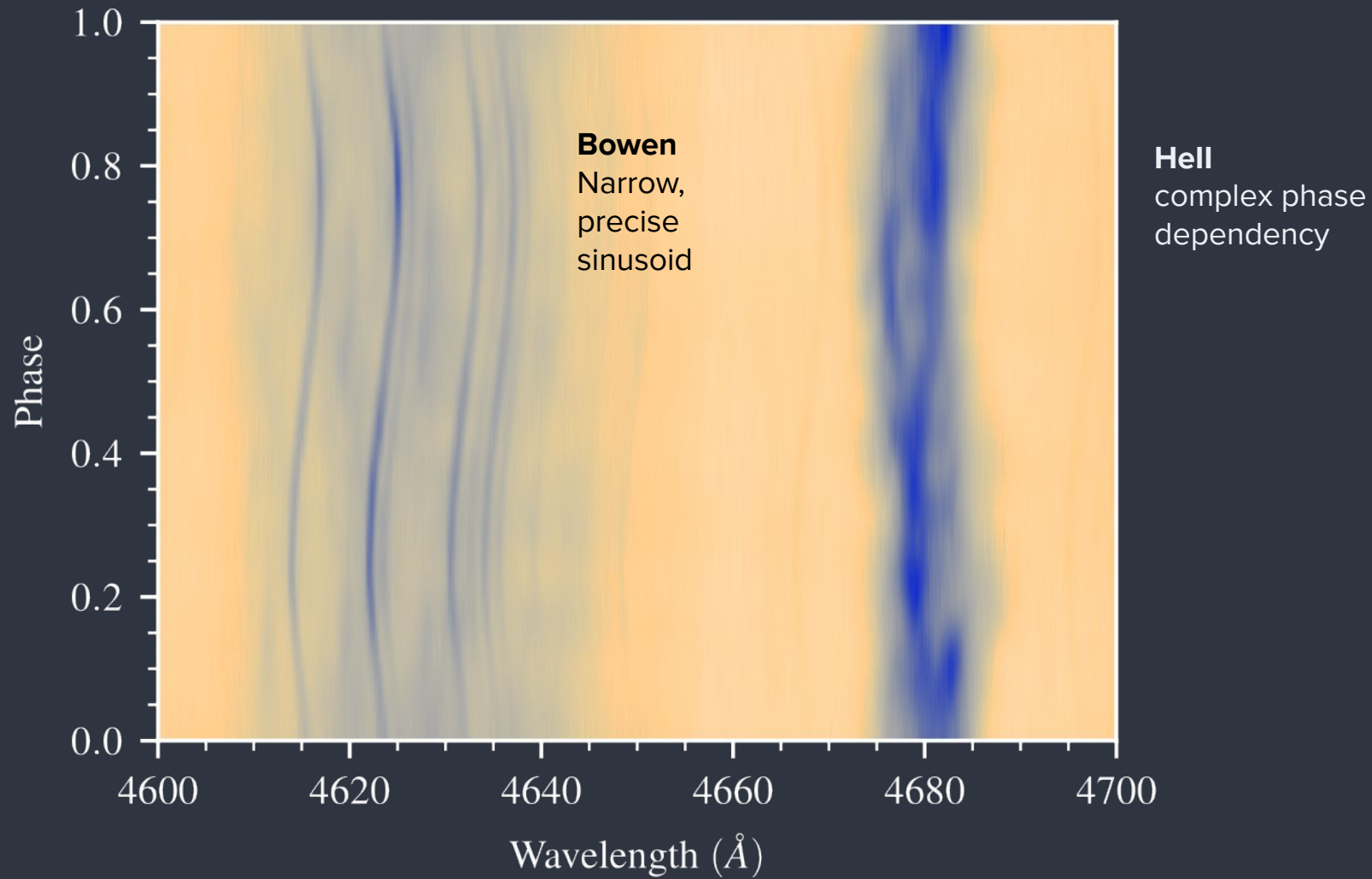


# Bowen fluorescence - measuring the donor star velocity

- > NS ionises visible face of the donor star, exciting He into high ionisation states.
- > He de-excites with correct energies to excite specific C/N/O transitions - the Bowen lines.
- > Localised nature of the emission precisely ( $\sim$ km/s) traces donor star motion, in comparison to broad H/He disk lines with  $\sim$ 300 km/s broadening.







# Bowen line modelling for Sco X-1

## 4 narrow-line components:

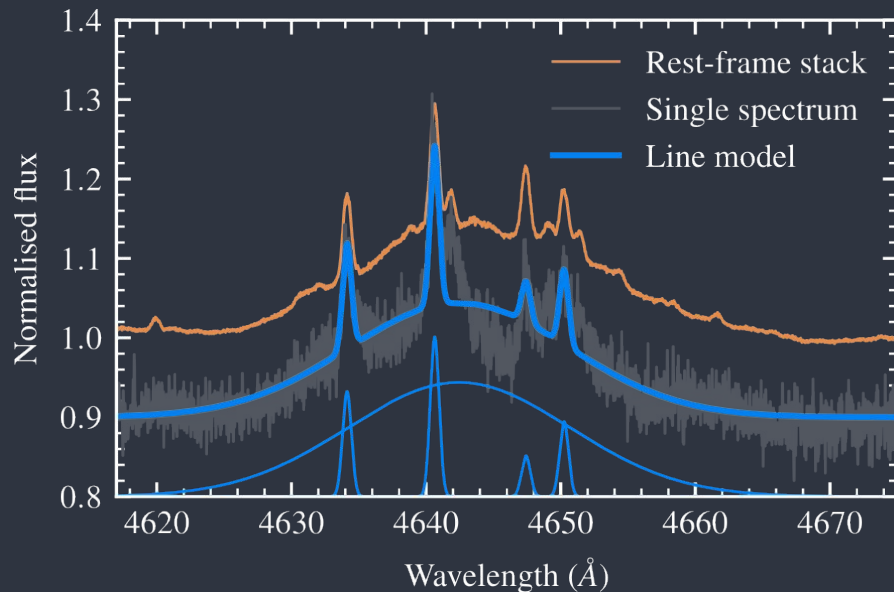
Doppler-shifted by common velocity (common origin on heated face). Fixed width (average), and positive-definite amplitude

## 1 broad-line component

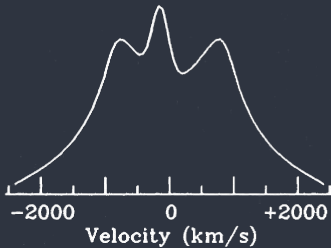
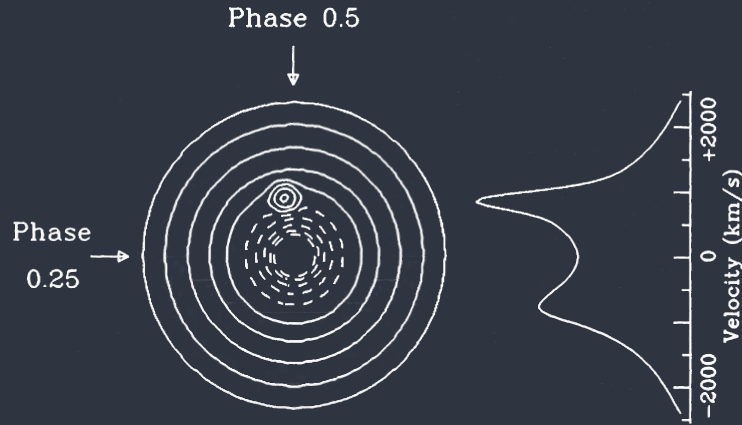
Fixed width, variable centroid and amplitude - fits smeared Bowen disk

Can achieve sub-km/s precision velocity measurements with the joint line fit.

Bowen lines very consistent!



# Doppler tomography: imaging the accretion disk



**Marsh  
and  
Horne  
(1988)**

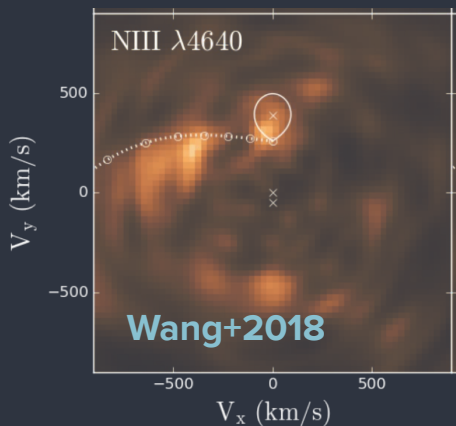
We can infer the disk structure through Doppler tomography - spectra at each phase are projections in velocity space.

Don't need to fit line models, makes use of all spectra simultaneously to infer component velocities - extend to weak lines.

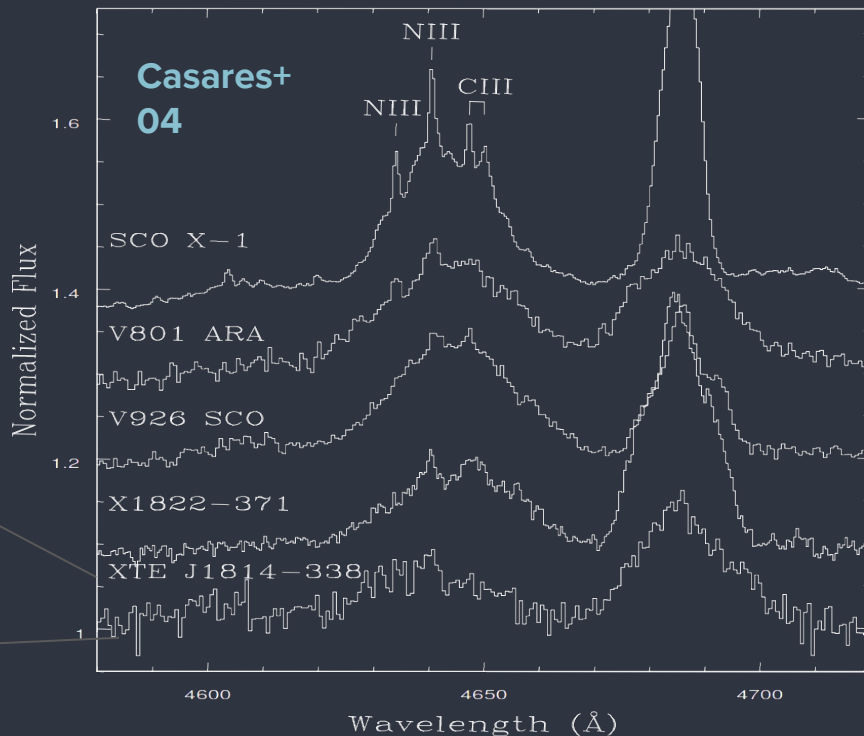
**Very powerful technique**, ubiquitous in binary studies (e.g. **Steeghs+1997** on IP Peg)

# Bowen blend tomography: robustly pushing to lower S/N

Versatile method for dynamical constraints across high  $\dot{M}$  XRBs - including transient sources!

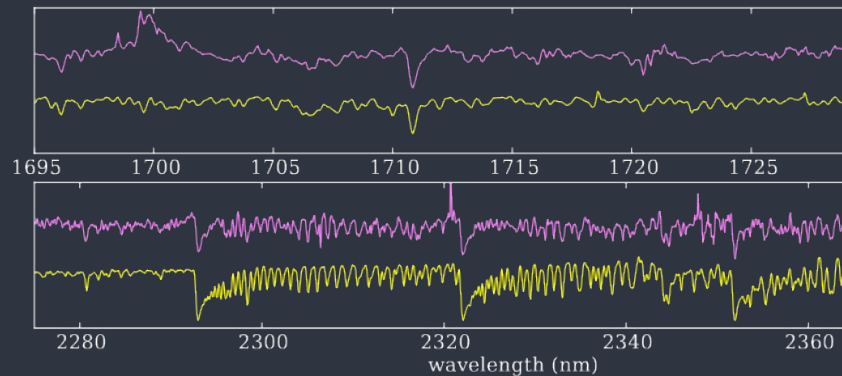
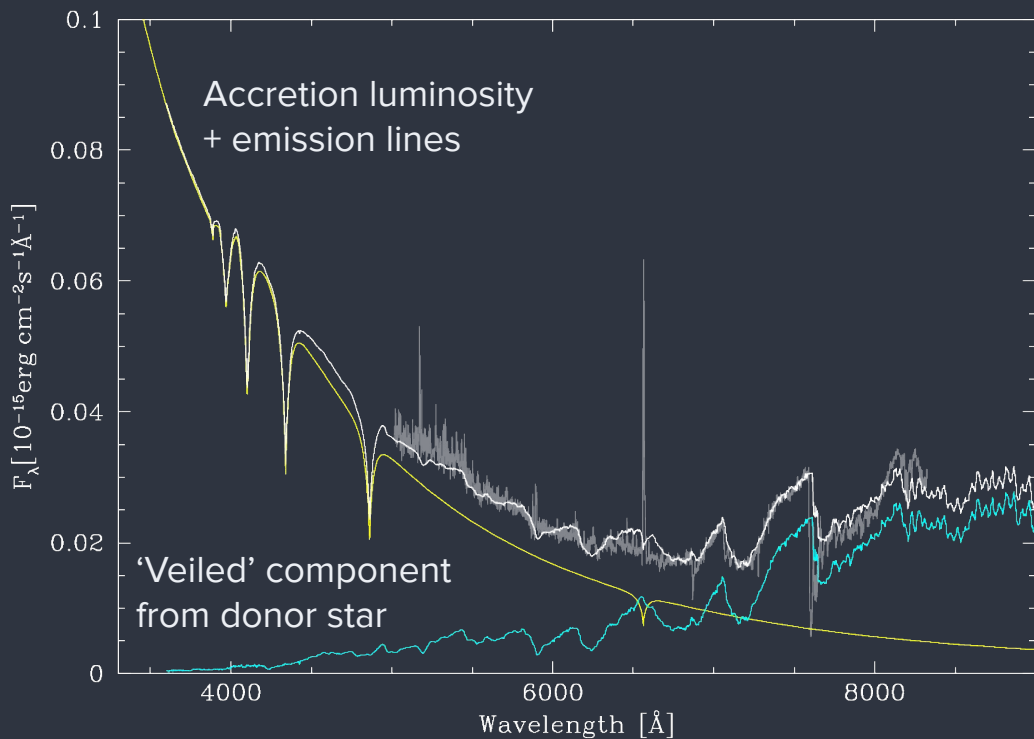


Detection of the donor star + orbital parameters



# Direct detection of the donor in absorption

In cases of lower accretion luminosity, e.g. Cyg X-2



e.g. GRS 1915+105 (Steeghs+2013)

match to template = approximate  
donor mass/binary mass ratio

## From radial velocities -> orbit

$$v(t) = K \sin\left(\frac{2\pi(t - T_0)}{P}\right) + \gamma,$$

Fit a simple Keplerian orbit model to the data - pure sinusoid for zero eccentricity, or slightly modified equation in the case of non-zero eccentricity.

**NB:** for comparatively old systems like Sco X-1 we expect zero eccentricity due to tidal dissipation during the MS lifetime of the NS progenitor (e.g. **Tassoul+1992**)

# Donor star velocities?

Projection effects/geometry affects the 'true' velocities - luckily well-modelled and understood for compact binaries, but function of binary properties.

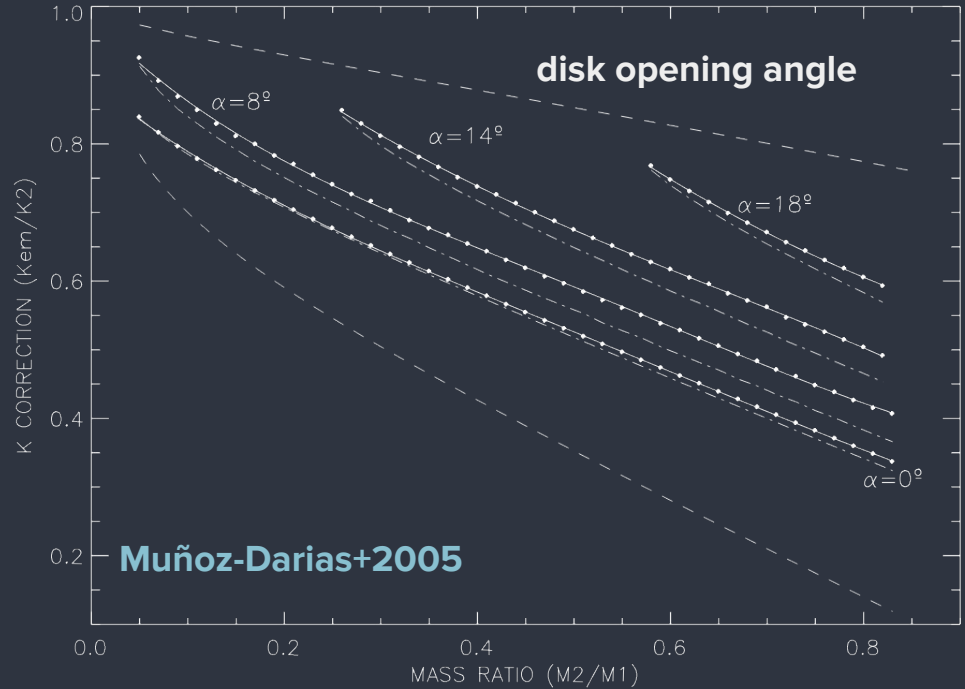
$K_{em}$  (measured)

$v$

$K_2$  (using K correction)

$v$

$K_1$  (using binary mass ratio)



# Subtleties and complexities

## Wavelengths

Need to ensure consistent wavelength calibration throughout study and correct for Earth's motion around SSB. Heliocentric vs Barycentric,

## Timings

Need to move clock to SSB for coherent phase, and have good clock accuracy overall to reach high precision. UTC vs TAI vs TDB vs GPS time.

Gets much harder with multiple different datasets - inter-dataset systematics!



# Precision Ephemerides for Gravitational-wave Searches

Danny Steeghs, Jorge Casares, Duncan Galloway, Louise Wang, Tom Marsh, Matt Mould, Sammanani Premachandra, Rémon Cornelisse, Tom Killestein

## A brief timeline

**Steeghs and Casares  
(2002)**

First Bowen-line ephemeris  
for the system

**PEGS II  
(Premachandra+2016)**

Cyg X-2 ephemeris

**PEGS IV (Killestein+2023)**

Corrected and refined  
ephemeris for Sco X-1

**PEGS I (Galloway+2014)**

Sco X-1 - first ephemeris  
with GW in mind.

**PEGS III (Wang+2018)**

Sco X-1 revisited, updated  
binary constraints

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# Over 20 years of data for Sco X-1



1999 and 2011 - 4.2m  
William Herschel  
Telescope

ISIS spectrograph



2011 - 2019 - 8.2m  
Very Large Telescope

UVES spectrograph

Superb instrumentation + long baseline = high precision ephemeris.

# What does a homogeneous re-analysis entail?

Starting from as close to the raw data as possible (i.e. wavelength vs flux)

- **Double-check** propagated metadata to ensure consistency between facilities
- **Recompute** all timing and velocity corrections using a consistent Solar System ephemeris (DE440)
- **Rescale flux errors** to match the true variance of the data and ensure correct weighting between datasets
- Apply precisely the **same line model** to each spectrum to mitigate systematics from model choice

# Treating systematics with Bayesian inference

How can we combine multiple independent datasets in a principled way?

Keplerian orbit

```
def circular_orbit(t, pars):  
    t0, P, gamma, K = pars  
    return K*jnp.sin(2*jnp.pi*(t - t0)/P) + gamma
```

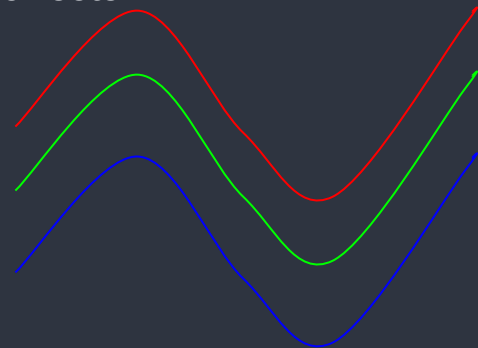


Per-dataset  
error scaling

$$\sigma'_i = \sigma_i \times \text{efac}_i$$



Per-dataset RV zero point  
offsets



**A very basic Bayesian hierarchical model - ‘global’ orbital parameters, and per-dataset calibration params**

Marginalise over calibration uncertainties - ensures we’re fully accounting for sources of error -> for CW searches, don’t want underestimated uncertainties!

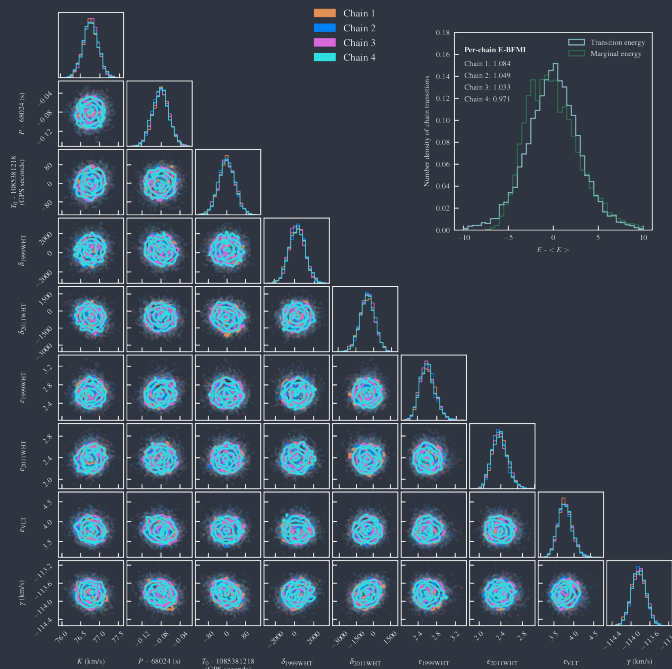
# ML frameworks for scalable Bayesian inference

High-performance linear algebra packages with:

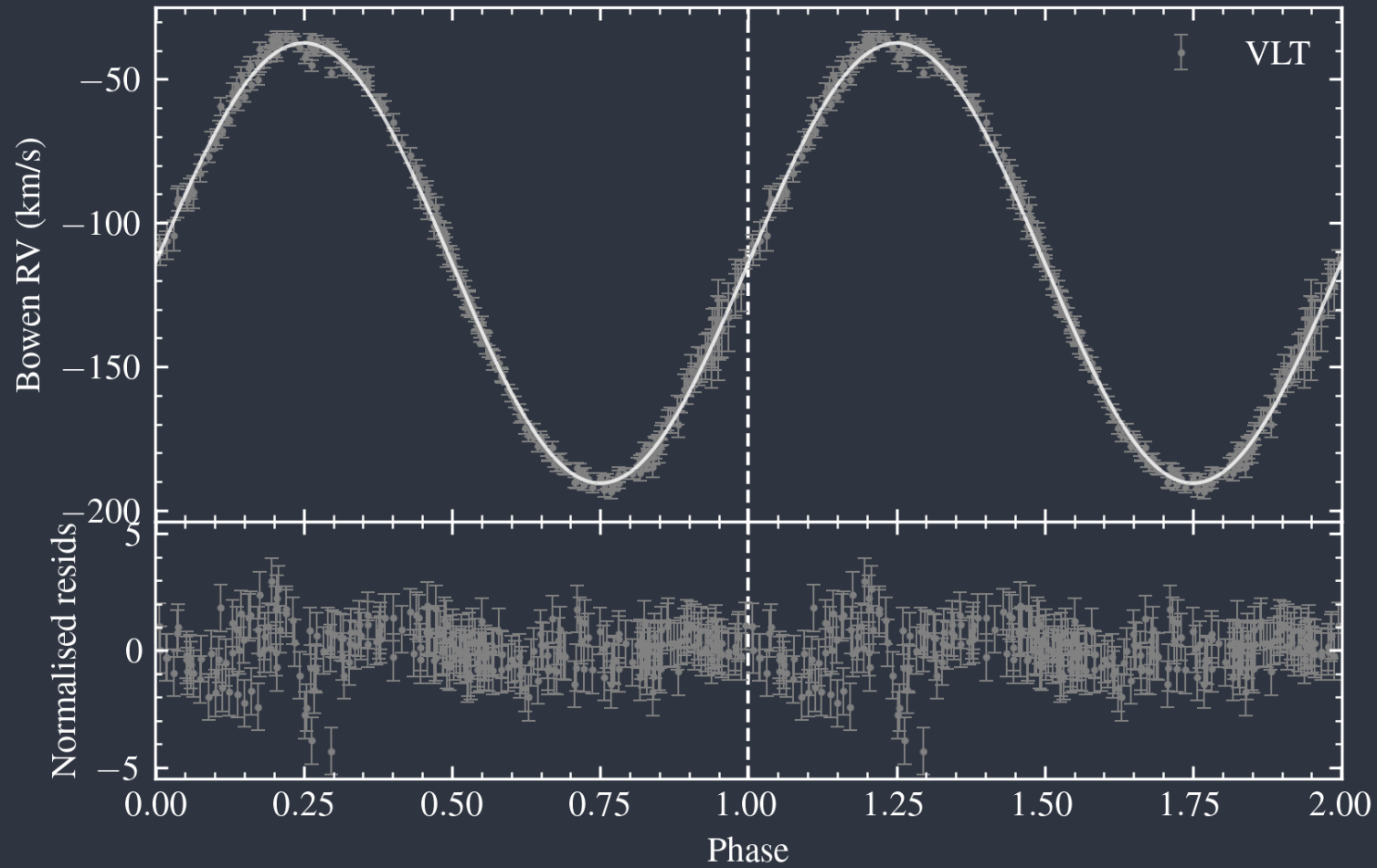
- Automatic differentiation
- JIT compilation
- Seamless(?) execution on GPU

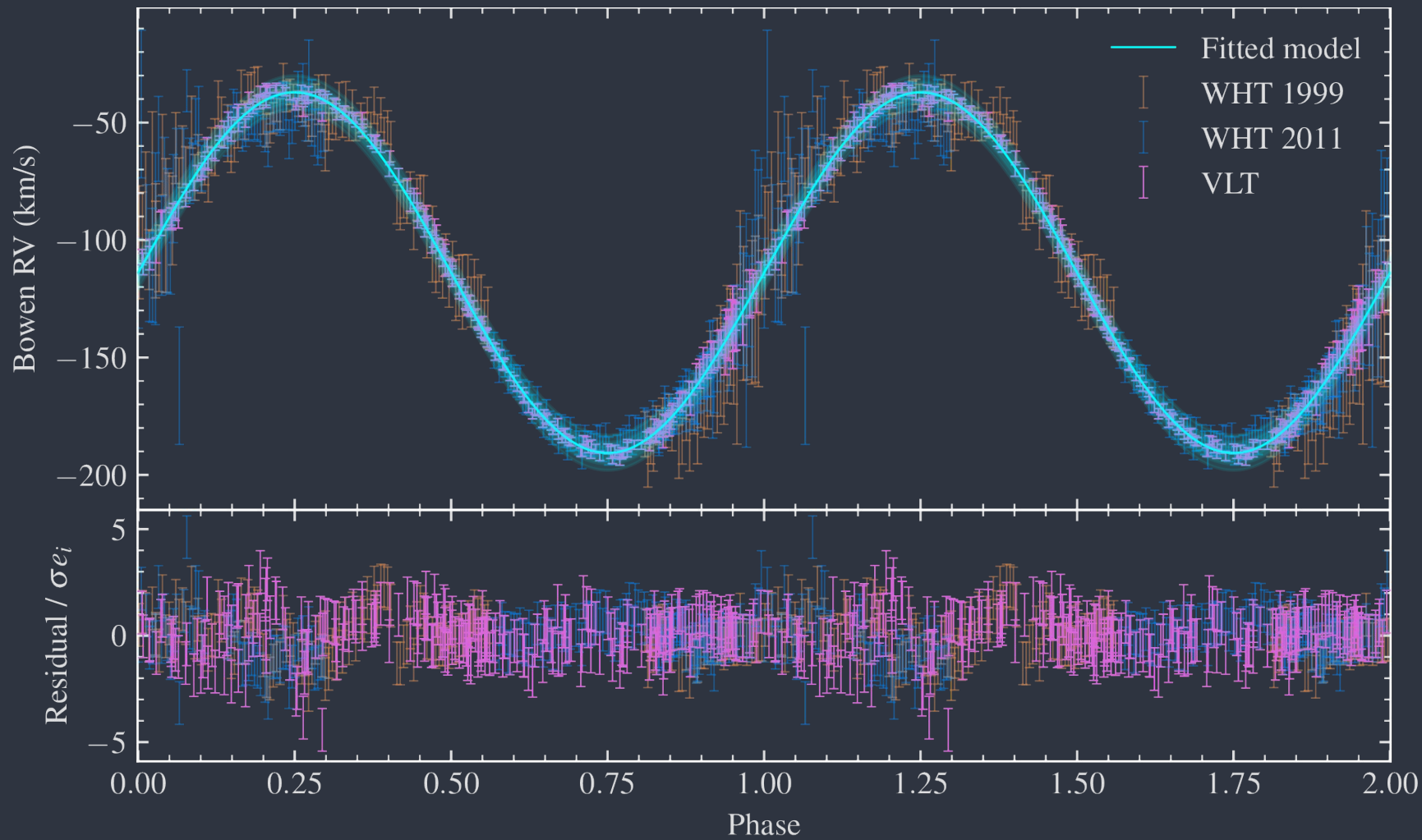
Here use JAX + NumPyro

Gives access to new techniques like Hamiltonian Monte Carlo!



Entire MCMC runs in ~2 mins on laptop



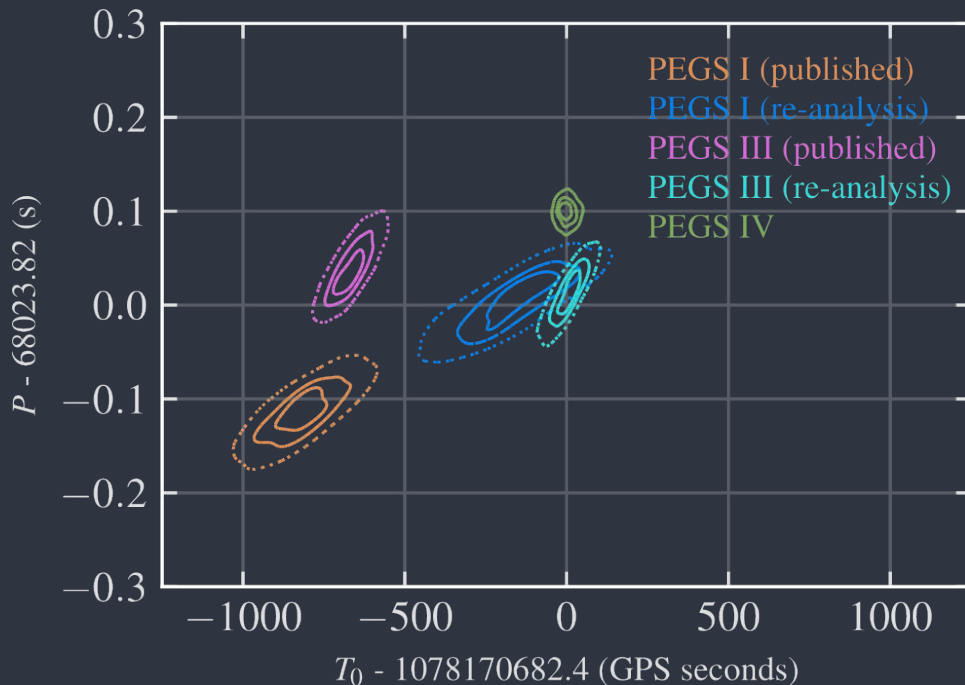




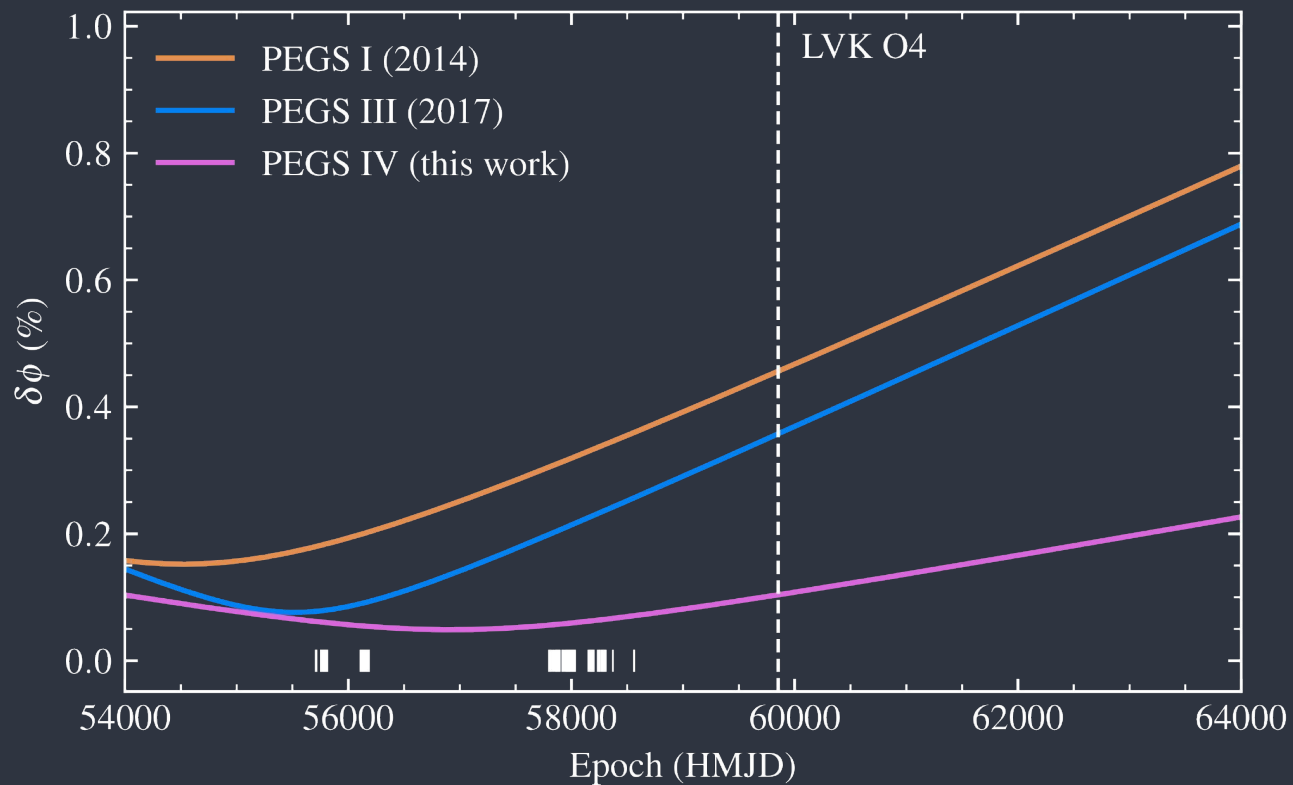
# Ephemeris quality

Significant improvements in size of uncertainty in the time of inferior conjunction and period - with minimal covariance.

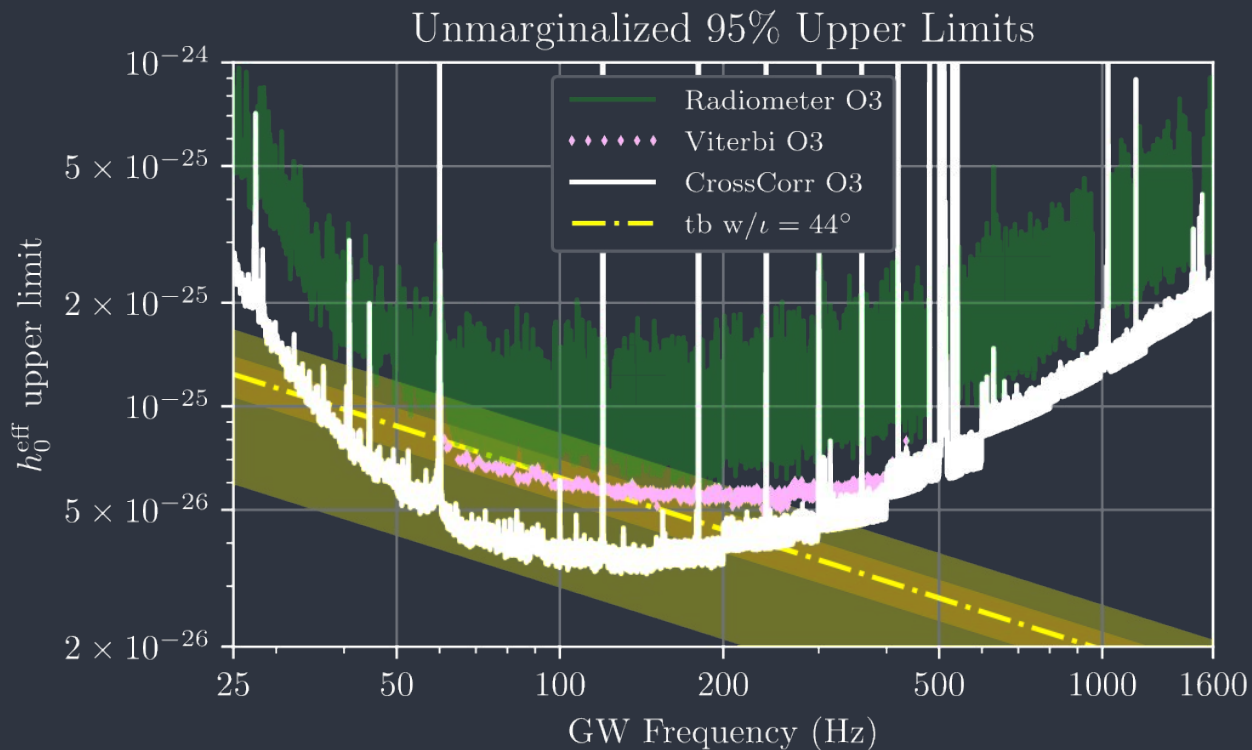
Some minor shift in period owing to the increasing dominance of the VLT dataset - **expected!**



# Pushing to ever better precision



# Still nothing - things starting to get interesting!



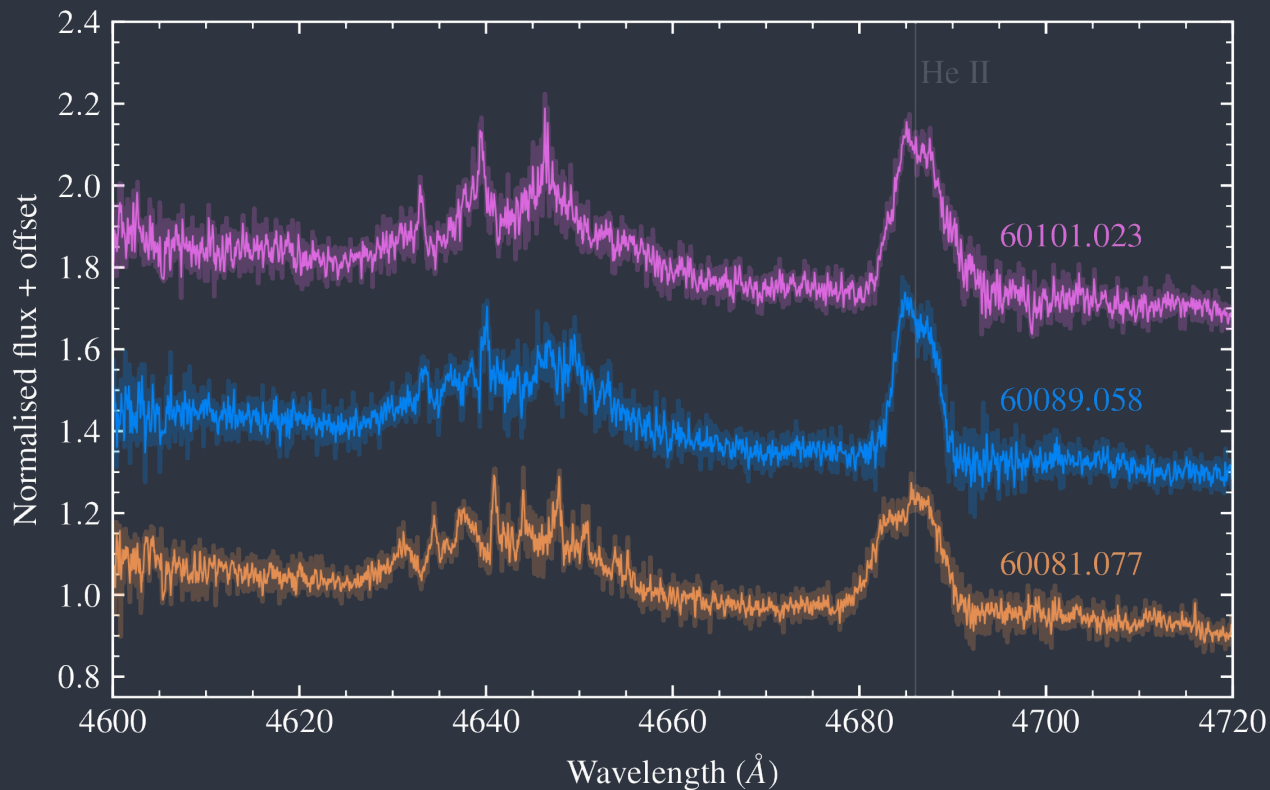
# Sco X-1 is hard!

High mass accretion rate:

- Conceals the donor from direct observation
- Leads to complex, time-variable disk structure
- Highly-broadened disk features that can't be centroided easily

## What are some of the next steps?

# PEGS+: data secured!

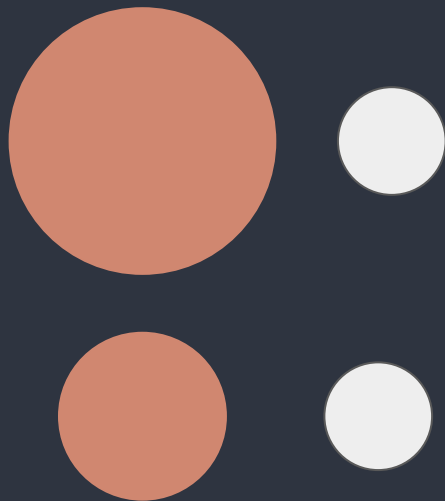


2 years of coverage with SALT/HRS secured - roughly one spectrum per week when visible.

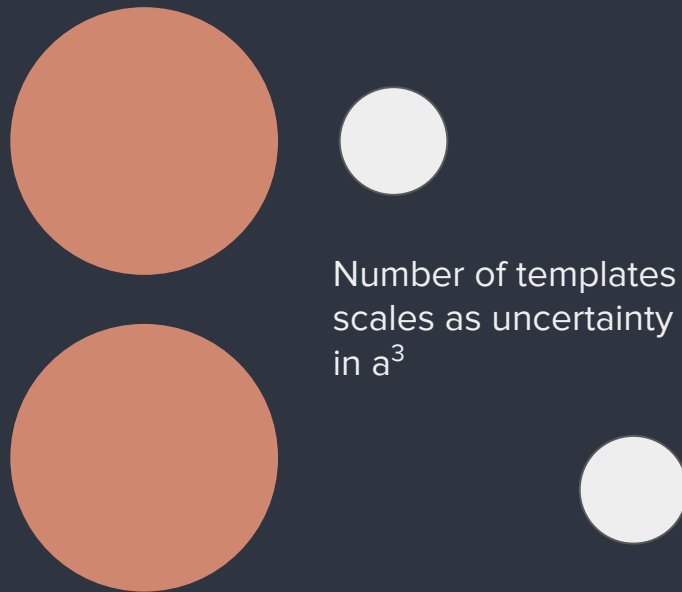
Preserve  $\sim 30$ s ephemeris uncertainty throughout O4, with potential further reductions in covariance.

# Remaining dominant uncertainties: binary parameters

Binary mass ratio  $q$   
(sets  $K_1$  directly)



Semi-major axis  $a$



Number of templates  
scales as uncertainty  
in  $a^3$

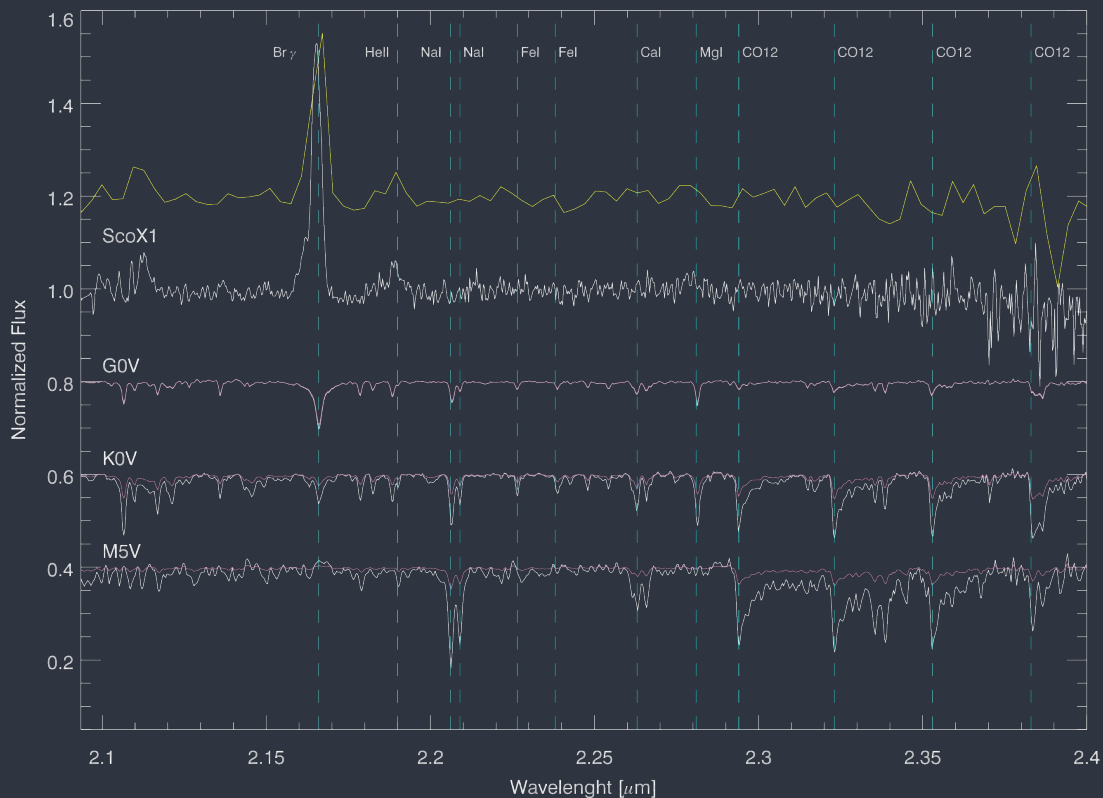
# K1 via Doppler tomography: challenging!

Centre-of-symmetry search in the Bowen disk: where is the centre of the disk in velocity space?



Hard to do robustly, especially as the disk is so tenuous anyway.

# In search of the elusive donor star



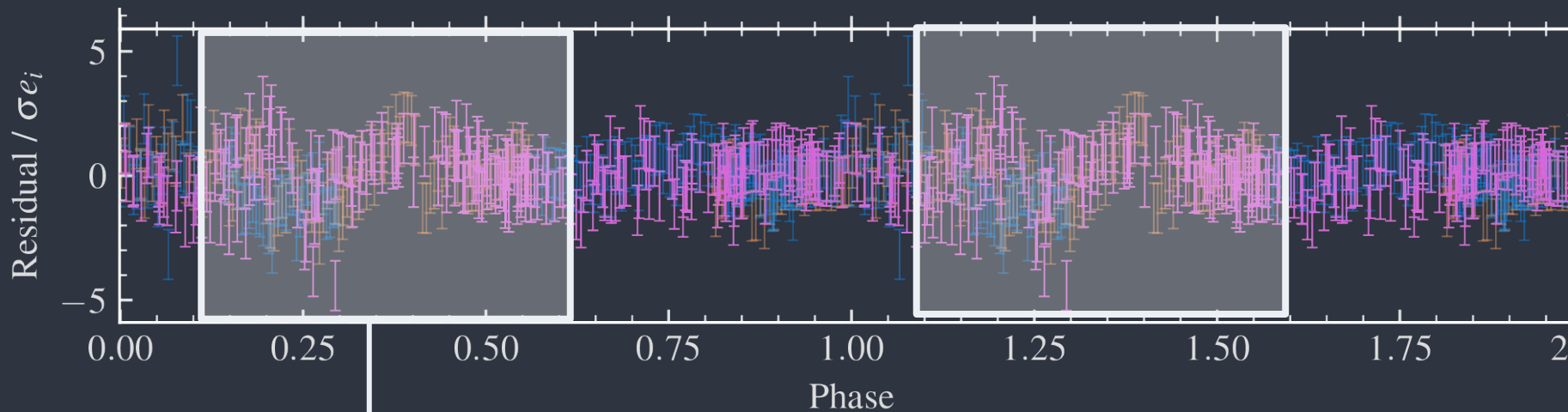
NIR spectroscopy of Sco X-1 in search of donor features.

**Mata-Sanchez+ (2015)**

Can we push further to lower limits with NIR spectroscopy, or some other approach required?



# Hints of Roche lobe geometry?



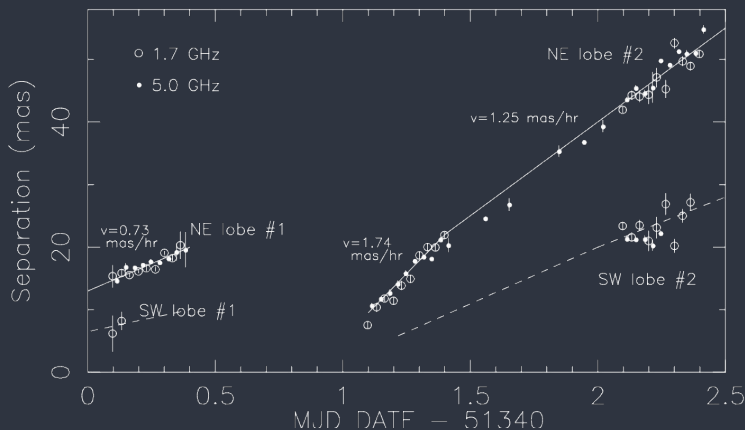
Potentially constraining on donor properties - but need more data and **robust modelling workflows** to validate this!

Is this just a quirk?

# Inclination and position angle

Now have independent measurements from independent techniques: but need to be cautious in interpreting these - disk/jet truly perpendicular?

**Radio VLBI (Fomalont+2001)**  
**(44 +/- 6 deg) - most robust**



Light curve modelling  
**(Cherepashchuk+2021)** gives lower values (25-34 deg), although beware heavy model dependencies.

Potential new constraints from X-ray observations - already have a polarisation angle detection **(Long+2023)** consistent with the VLBI position angle

Hopefully with next-gen detectors + other binary constraints we can just marginalise over this.

# Tutorial: build your own ephemeris!

[github.com/tkillestein/mmcw\\_pegs](https://github.com/tkillestein/mmcw_pegs)

Real VLT data! Simplified likelihood!  
Sensible initial params!