Properties of a multi-template search for a transient signal

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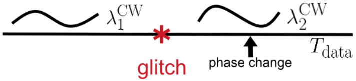
CONTINUOUS GRAVITATIONAL WAVES

Transient CW-signals

- finite duration: from days to months.
- breaking mechanism: glitches – sudden spin-up



  - Glitch is followed by a relaxation phase
  - CW emission during the relaxation phase
  - CW signal's phase parameters change after the glitch: $\lambda_1^{\mathrm{CW}} \neq \lambda_2^{\mathrm{CW}}$
- glitching sources:
  - observed: Vela, Crab, the fastest young pulsar PSR J0537-6910 (Antonopoulou et al., 2018);
  - expected: the newly born pulsars according to the *r-mode emission* theory (Andersson, 1998)

Transient CW-signal in the data

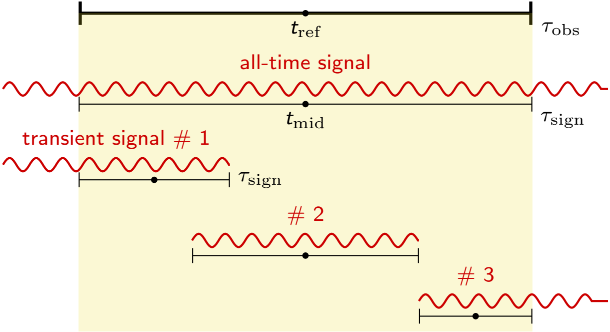


Figure: Scheme of various overlaps of a signal with the observing data.

In the absence of the glitch information, we search for a transient signal with unknown start time and duration in the available data.

2$\mathcal{F}$-statistics when the signal is always ON

Single template perfectly-matched search

$\mathcal{F}$-statistic when the signal is ON for the entire duration of the observation (the data) (Jaranowski et al., 1998):

$$2\mathcal{F}_0 = \sum_{i=1}^{4} (\tilde{z}_i)^2 \quad (1)$$

- $2\mathcal{F}_0 \sim \chi^2_4\left(\rho^2_0\right)$ – $\mathcal{F}$-statistic follows a chi-square distribution with four degrees of freedom with a non-centrality parameter ρ^2_0.
- ρ^2_0 is the signal-to-noise ratio (SNR) that is proportional to $T_{\rm data} h^2_0$.

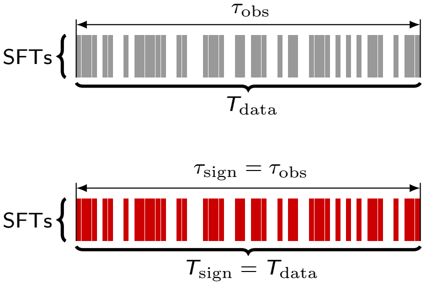


Figure: Schematic of the signal ON for the entire duration of the observation.

2$\mathcal{F}$-statistics of a transient signal

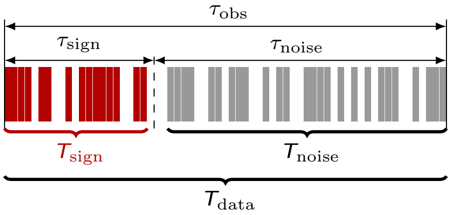
2$\mathcal{F}$-statistics of a transient signal in single template perfectly-matched search

$$2\mathcal{F}_{\mathrm{TS}} = \sum_{i=1}^{4} \left(\sqrt{\kappa}\tilde{Z}_i^{\mathrm{sign}} + \sqrt{1-\kappa}\,\tilde{Z}_i^{\mathrm{noise}} \right)^2 \tag{2}$$

- $\kappa = \frac{T_{\mathrm{sign}}}{T_{\mathrm{data}}}$ is signal's fraction in the data
- $2\mathcal{F}_{\mathrm{TS}} \sim \chi_4^2\,(\rho_{\mathrm{TS}}^2)$

$$\rho_{\mathrm{TS}}^2 \sim \kappa^2\, T_{\mathrm{data}}\, h_0^2 \tag{3}$$

(In agreement with (Prix et al., 2011).)



For derivation of Eq. 2 see our upcoming paper *"Long-duration transient signals: a practical detection method"* (Fesik and Papa, 2023)

Eq. 2 tells us how to simulate the output of an $\mathcal{F}$-statistic search in the presence of a transient signal.

Template search for a CW-signal

The signal is always ON in the data

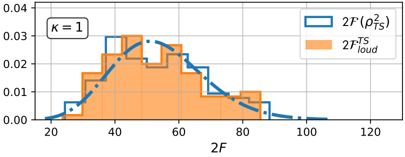


Figure: The distribution of the perfectly-matched $2\mathcal{F}({\rho^2}_{\mathrm{TS}})$ (blue) and the loudest $2\mathcal{F}_{\mathrm{loud}}$ (orange) from a template search for a transient signal in the Gaussian noise with SNR ${\rho^2}_{\mathrm{TS}}(\kappa) \approx 50$ (blue dot-dashed) for different realisations of $\kappa = T_{\mathrm{sign}}/T_{\mathrm{data}}$ in the data with gaps (taken O1 LIGO run).

The mismatch between the signal waveform λ_{sign} and template waveform $\lambda_{\mathrm{sign}} \pm \Delta\lambda$ is

$$m\,(\Delta\lambda; \lambda_{\mathrm{sign}}) = \frac{\rho^2(0) - \rho^2(\Delta\lambda)}{\rho^2\,(0)}. \quad (4)$$

the loss in signal-to-noise ρ^2 due to the mismatch m between the signal and template waveforms does not exceed some predetermined value.

When signal is always ON in the data, we are able to set up a template grid with predetermined average mismatch.

Template search in the presence of a transient signal

The signal is NOT always ON in the data

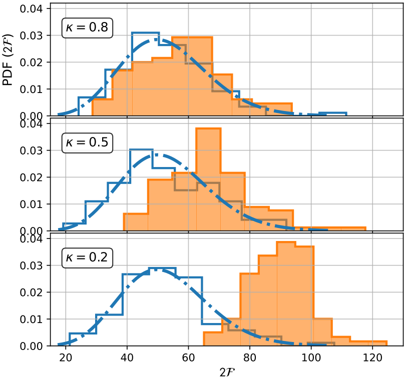


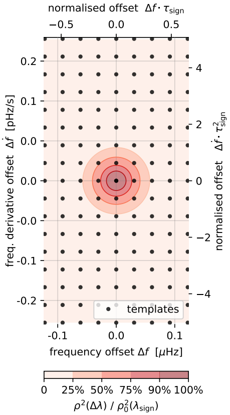
Figure: The distribution of the perfectly-matched $2\mathcal{F}({\rho^2}_{\mathrm{TS}})$ (blue) and the loudest $2\mathcal{F}_{\mathrm{loud}}$ (orange) from a template search for a transient signal.

In the presence of a transient signal in a multi-template search, the expected loudest $2\mathcal{F}$-value from the searches is systematically *larger* than the expected value from a signal-template perfectly-matched search.

SNR-reduction profile of a transient signal

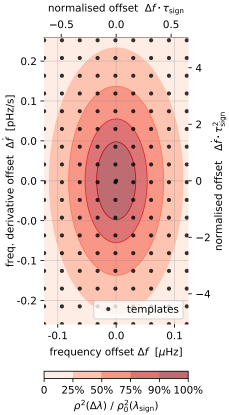
all-time signal

$T_{\mathrm{sign}} = T_{\mathrm{data}}$



transient signal

$T_{\mathrm{sign}} < T_{\mathrm{data}}$



- The SNR-reduction profile of a transient signal does not change with the increased search duration
- The template resolution scales as:

$$\delta f \sim \frac{1}{\tau_{\mathrm{obs}}}, \quad \delta \dot{f} \sim \frac{1}{{\tau_{\mathrm{obs}}}^2}, \quad \delta \ddot{f} \sim \frac{1}{{\tau_{\mathrm{obs}}}^3} \quad (5)$$

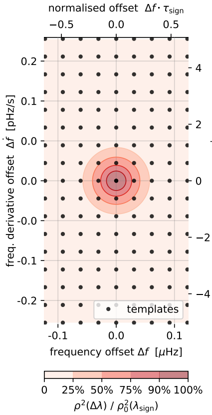
- Within a given offset in frequency $\Delta f^{(i)}$, the number of templates scales as:

$$N^{f}_{\mathrm{templ}} \sim \frac{1}{\kappa_0}, \quad N^{\dot{f}}_{\mathrm{templ}} \sim \frac{1}{\kappa_0^2}, \quad N^{\ddot{f}}_{\mathrm{templ}} \sim \frac{1}{\kappa_0^3} \quad (6)$$

SNR-reduction profile of a transient signal

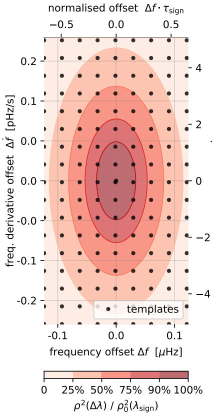
all-time signal

$T_{\rm sign} = T_{\rm data}$



transient signal

$T_{\rm sign} < T_{\rm data}$



- Within a given offset in frequency $\Delta f^{(i)}$, the number of templates scales as:

$$N^{f}_{\rm templ} \sim \frac{1}{\kappa_0}, \quad N^{\dot{f}}_{\rm templ} \sim \frac{1}{\kappa_0^2}, \quad N^{\ddot{f}}_{\rm templ} \sim \frac{1}{\kappa_0^3} \tag{7}$$

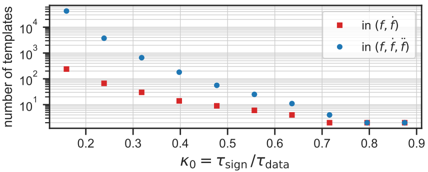


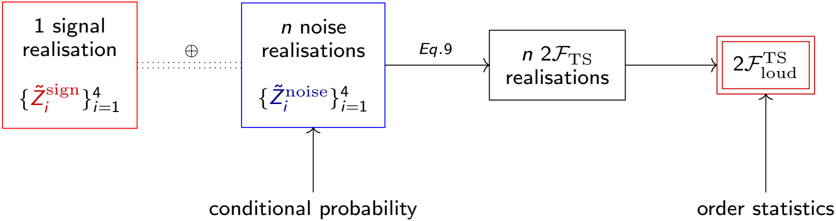
Figure: Number of templates with $\geq 90\%$ overlap in $f, \dot{f}$ and in $f, \dot{f}, \ddot{f}$ for transient signals of different duration span $\kappa_0 = \tau_{\rm sign}/\tau_{\rm obs}$.

Simulating transient statistics

$2\mathcal{F}$-statistics of a transient signal:

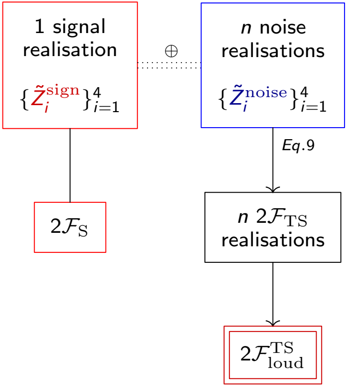
$$2\mathcal{F}_{\mathrm{TS}} = \sum_{i=1}^{4} \left(\sqrt{\kappa} \tilde{Z}_i^{\mathrm{sign}} + \sqrt{1-\kappa}\, \tilde{Z}_i^{\mathrm{noise}} \right)^2 \sim \chi_4^2 \left(\rho_{\mathrm{TS}}^2 \right) \tag{8}$$

Scheme of one loudest $2\mathcal{F}_{\mathrm{TS}}$ simulation:



$2\mathcal{F}$-statistics of the loudest

$$2\mathcal{F}_{\mathrm{TS}} = \sum_{i=1}^{4} \left(\kappa \tilde{Z}_i^{\mathrm{sign}} + (1-\kappa)\, \tilde{Z}_i^{\mathrm{noise}} \right)^2 \sim \chi_4^2 \left(\rho_{\mathrm{TS}}^2 \right) \tag{9}$$



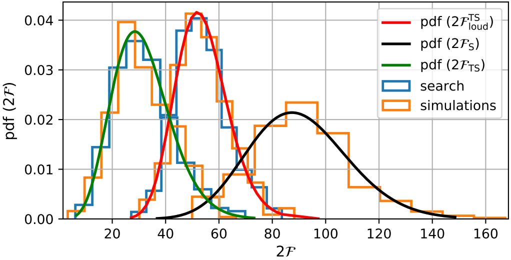


Figure: Comparing $2\mathcal{F}$-statistics of searches and simulations for a transient signal of duration $\kappa = 0.29\,(\kappa_0 = 0.25)$.

Expected $2\mathcal{F}$ of a transient signal in a perfectly-matched search

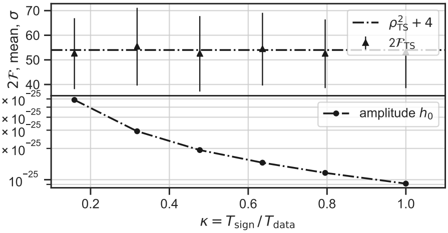


Figure: Searches in Gaussian noise spanning $\tau_{\rm obs} \approx 123$ days, with gaps as in the O1 LIGO run. The data contain a transient signal with a certain κ and a $\rho^2{}_{\rm TS}(\kappa) \approx 50$. The lower plot shows the signal amplitudes h_0; the upper plot shows the average $2\mathcal{F}_{\rm TS}$ from the single-template search. The bars indicate 1 standard deviation.

Expected $2\mathcal{F}_{\rm TS}$ from the perfectly-matched searches:

$$\mathrm{E}\left[2\mathcal{F}_{\rm TS}\right] = \rho^2_{\rm TS} + 4 \qquad (10)$$

Relation between SNR of a transient signal and its amplitude:

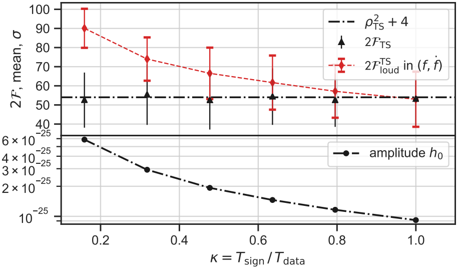
$$\rho^2_{\rm TS} \approx \kappa^2 \rho_0^2 \Rightarrow \rho^2_{\rm TS} \sim \kappa^2 h_0^2 \qquad (11)$$

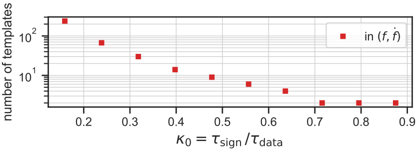
Amplitude of a signal scales as following:

$$h_0(\rho^2_{\rm TS} = \mathrm{const}) \sim 1/\kappa \qquad (12)$$

If the signal is half the duration, its amplitude must be twice as large.

Number n of noise realisations





Number n of noise realisations

The long-duration transient

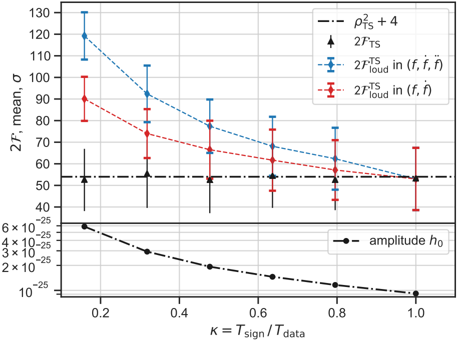


Figure: Search results from perfectly-matched single-template (green) and multi-template with recovered loudest $2\mathcal{F}_{\rm loud}$ in $(f, \dot{f})$ and $(f, \dot{f}, \ddot{f})$ for transient signals. The amplitude of signals is fixed (bottom plot).

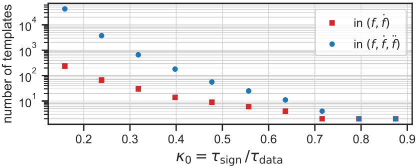


Figure: Number of templates with $\geq 90\%$ overlap in $f, \dot{f}$ and in $f, \dot{f}, \ddot{f}$ for transient signals of different duration span $\kappa_0 = \tau_{\rm sign}/\tau_{\rm obs}$.

The loss of sensitivity in a templated search for a transient signal

The amplitude of a transient signal at fixed SNR

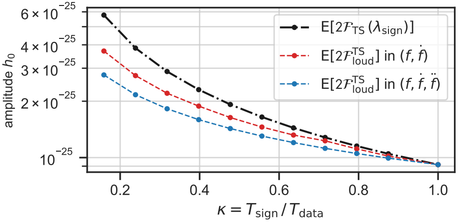


Figure: The amplitude at the same expected $2\mathcal{F}_{\mathrm{TS}}$ from perfectly-matched search (black) in O1 in the presence of a transient signal, and at the same expected loudest $2\mathcal{F}^{\mathrm{TS}}_{\mathrm{loud}}$ from multi-templates searches. Points are computed results, curves – approximations as functions of κ.

At fixed SNR, the amplitude of a transient signal scales as:

- κ^{-1}, when searched with a perfectly-matched template;
- $\kappa^{-0.8}$, when searched with a template grid in two dimensions $(f, \dot{f})$;
- $\kappa^{-0.6}$, when searched with a template grid in three dimensions $(f, \dot{f}, \ddot{f})$.

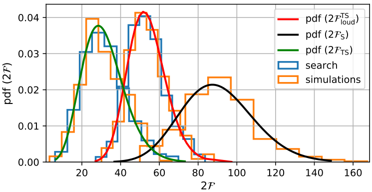
The loss of sensitivity is less prominent in a templated search for a transient signal.

Summary

- We derive the precise form for the $\mathcal{F}$-statistic and its optimal signal-to-noise ratio (SNR) when the signal is transient. This is relevant when characterizing a detection method.

$$2\mathcal{F}_{\mathrm{TS}} = \sum_{i=1}^{4} \left(\kappa \tilde{Z}_i^{\mathrm{sign}} + (1-\kappa)\, \tilde{Z}_i^{\mathrm{noise}} \right)^2 \sim \chi_4^2\left(\rho_{\mathrm{TS}}^2\right) \qquad (13)$$

- The statistics of the loudest search in the presence of a transient signal in a multi-template search result depends on the density of independent templates at the signal parameters.
- The knowledge of the statistics of the loudest allows to easily simulate the results of searches in data containing signals.



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Thank you for your attention!