GRAN SASSO SCIENCE INSTITUTE
SCHOOL OF ADVANCED STUDIES
Scuola Universitaria Superiore

Institute of Space Sciences

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IEEC

MATINS: A new finite-volume code for the magneto thermal evolution neutron stars in 3 dimensions

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MAGNESIA

erc

Multimessenger Continous GW 12-July-2023

Image credits: NASA’s Goddard Space Flight Center/Chris Smith

MATINS

MAgneto-**T**hermal evolution
of **I**solated **N**eutron **S**tars

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Motivation

Magneto-thermal evolutionary models are able to account for the phenomenological diversity of different classes of neutron stars and link them within a unified evolutionary path

The Hall instability is expected to give rise to 3D modes, even for axisymmetric initial conditions

The 3D evolution leads to the formation of hot-spots on the stellar surface that can account for the pulsed fraction observed in some sources (see e.g. Igoshev+2021)

Magneto-Thermal Evolution

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \left\{ \eta \nabla \times (e^{\nu}\mathbf{B}) + \left[\frac{c e^{-\nu}}{4\pi e n_e} \nabla \times (e^{\nu}\mathbf{B}) \right] \times (e^{\nu}\mathbf{B}) \right\}$$

$$\underbrace{\int_V c_v \frac{\partial (e^{\nu} T)}{\partial t} dV}_{\text{Variation of temperature within a volume}} + \underbrace{\int_{\partial V} e^{2\nu} \mathbf{F} \cdot d\mathbf{A}}_{\text{Flux of heat through the volume boundary}} = \underbrace{\int_V e^{2\nu} \dot{\epsilon}\, dV}_{\text{Sources (sinks): heat generated (lost) within the volume}}$$

Variation of temperature within a volume

Flux of heat through the volume boundary

Sources (sinks): heat generated (lost) within the volume

Magneto-Thermal Evolution

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \left\{ \eta \nabla \times (e^{\nu}\mathbf{B}) + \left[\frac{c e^{-\nu}}{4\pi e n_e} \nabla \times (e^{\nu}\mathbf{B}) \right] \times (e^{\nu}\mathbf{B}) \right\}$$

Coupling

$$\int_V c_v \frac{\partial (e^{\nu} T)}{\partial t} dV + \int_{\partial V} e^{2\nu} \mathbf{F} \cdot d\mathbf{A} = \int_V e^{2\nu} \dot{\epsilon} \, dV$$

Variation of temperature within a volume

Flux of heat through the volume boundary

Sources (sinks): heat generated (lost) within the volume

Heat Flux

$$e^{\nu(r)}\mathbf{F} = -\, k_{\perp}\left[\nabla \tilde{T} + (\omega_B \tau_0)^2 (\mathbf{b} \cdot \nabla \tilde{T})\mathbf{b} + (\omega_B \tau_0)(\mathbf{b} \times \nabla \tilde{T})\right]$$

Parallel to the temperature gradient

Parallel to the magnetic field

Hall term (orthogonal to the temperature gradient and the magnetic field)

$$\frac{k_{\parallel}}{k_{\perp}} \simeq 1 + (\omega_B \tau_0)^2$$

Heat Diffusion Equation

$$\int_V c_v \frac{\partial(e^{\nu}T)}{\partial t} dV + \int_{\partial V} e^{2\nu}\mathbf{F}\cdot d\mathbf{A} = \int_V e^{2\nu}\dot{\epsilon}\, dV$$

Discretization

$$V^{i,j,k}\frac{c_{v;i,j,k}}{\Delta t}(\tilde{T}^{n+1}_{i,j,k} - \tilde{T}^{n}_{i,j,k}) + \Phi_{i,j,k}(e^{2\nu_i}\mathbf{F}_{i,j,k}) = V^{i,j,k}e^{2\nu_i}\dot{\epsilon}_{i,j,k}$$

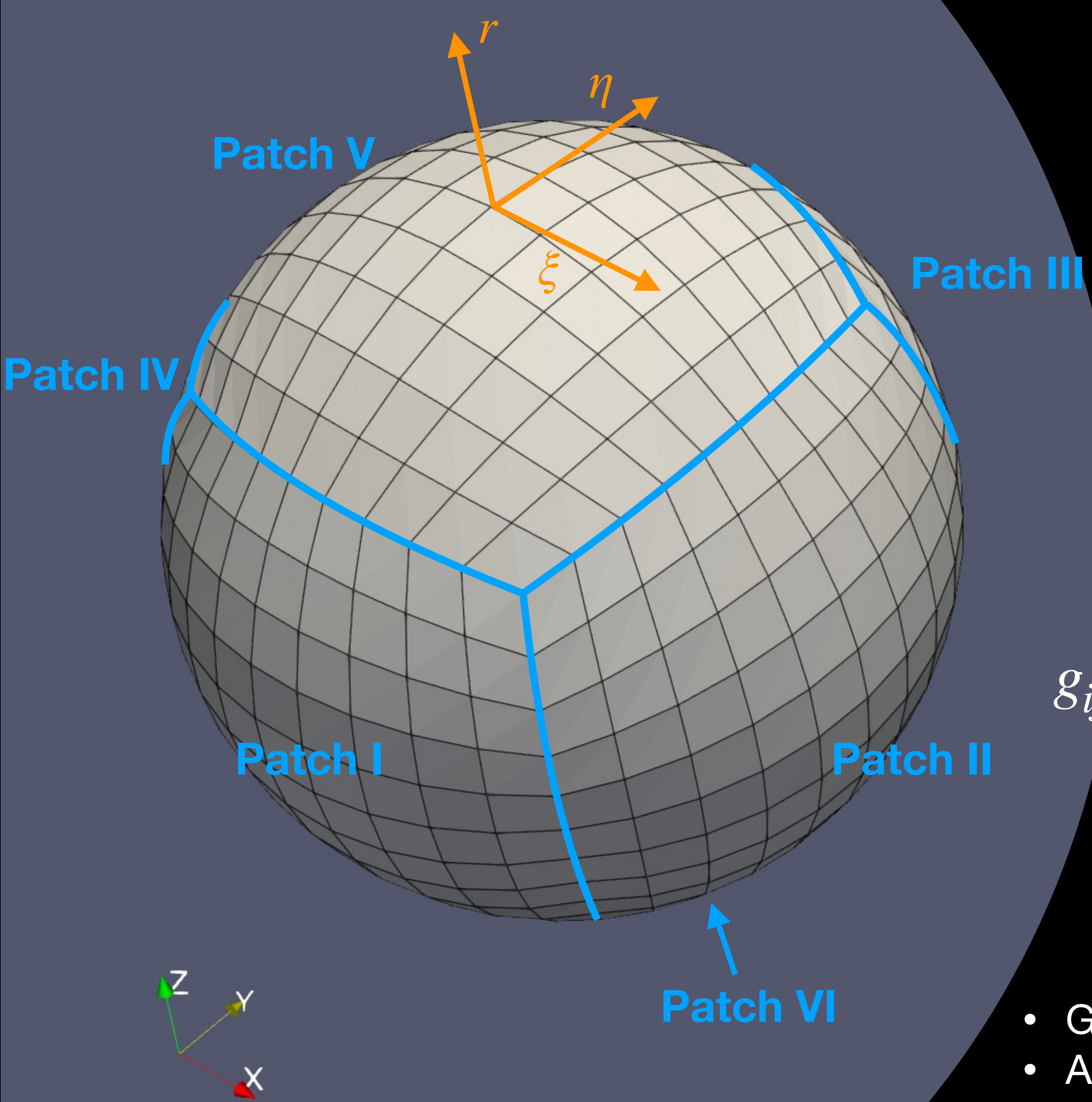
Implicit Scheme

$$\hat{M}^s_l\ \tilde{T}^{n+1}_s = v(\tilde{T}^n_l) \quad \longrightarrow \quad \tilde{T}^{n+1}_s = (\hat{M}^{-1})^s_l\ v(\tilde{T}^n_\alpha)$$

LAPACK library

Grid: Cubed Sphere

(Ronchi+ 1996)



Desirable Features:

- Radial Coordinate (r)
- Non-singular

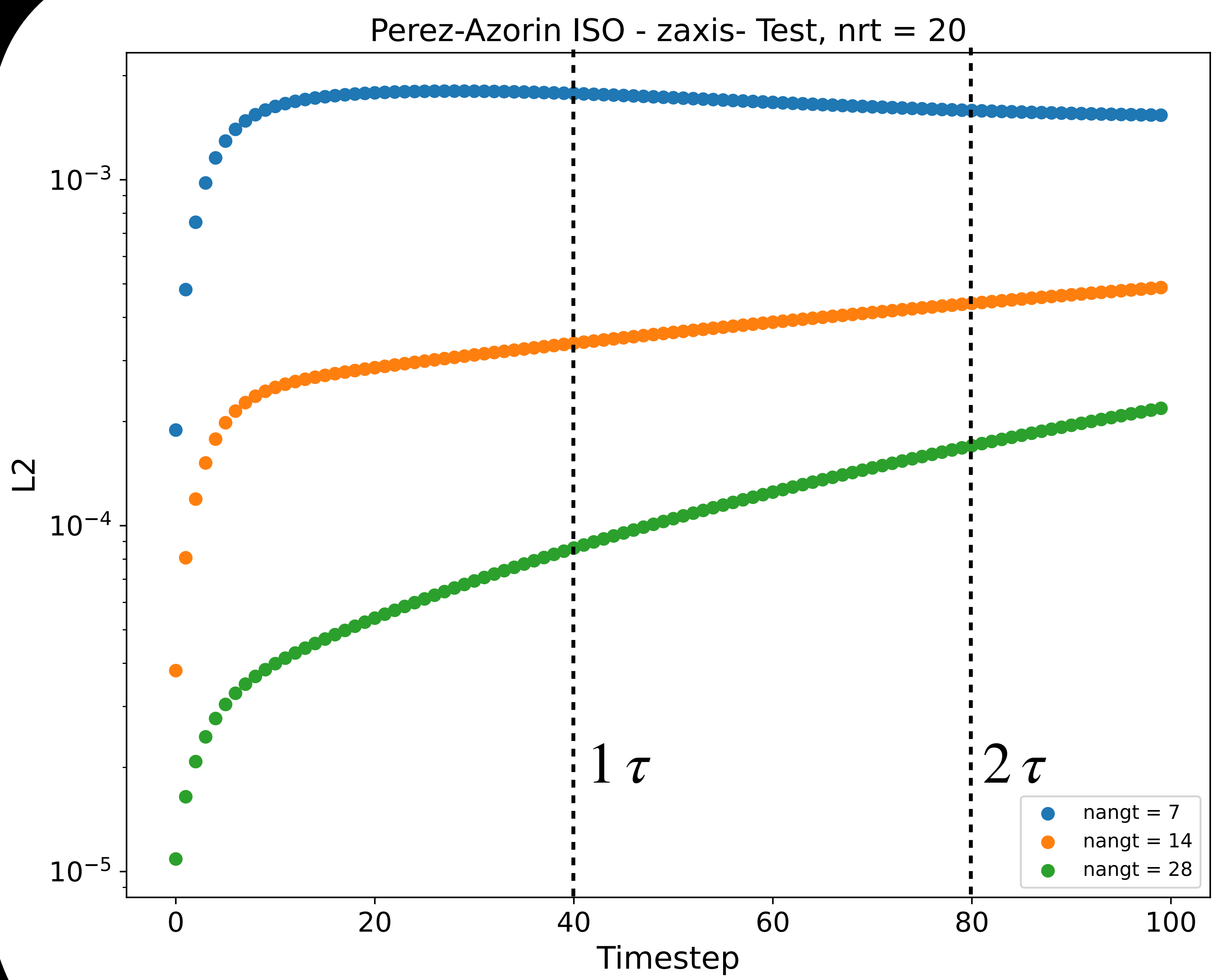
Non-orthogonal coordinate system

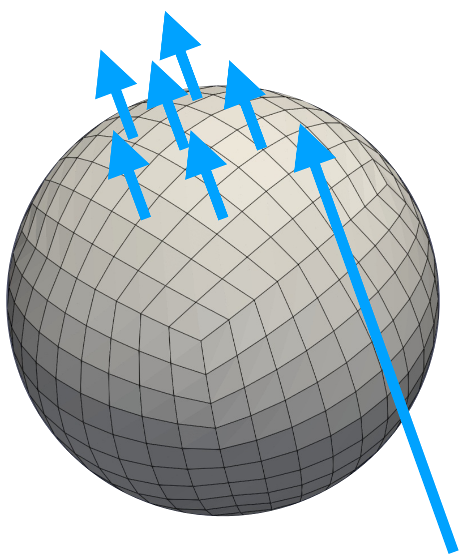
$$g_{ij} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -\frac{X(\xi)Y(\eta)}{C(\xi)D(\eta)} \\ 0 & -\frac{X(\xi)Y(\eta)}{C(\xi)D(\eta)} & 1 \end{pmatrix}$$

Already used in:

- GR codes (e.g. Fragile+2008)
- Atmospheric codes (eg. GEOS-Chem; http://acmg.seas.harvard.edu/geos/)
- MHD codes (eg. Koldoba+2002)

Perez-Azorin+2006 Anisotropic test



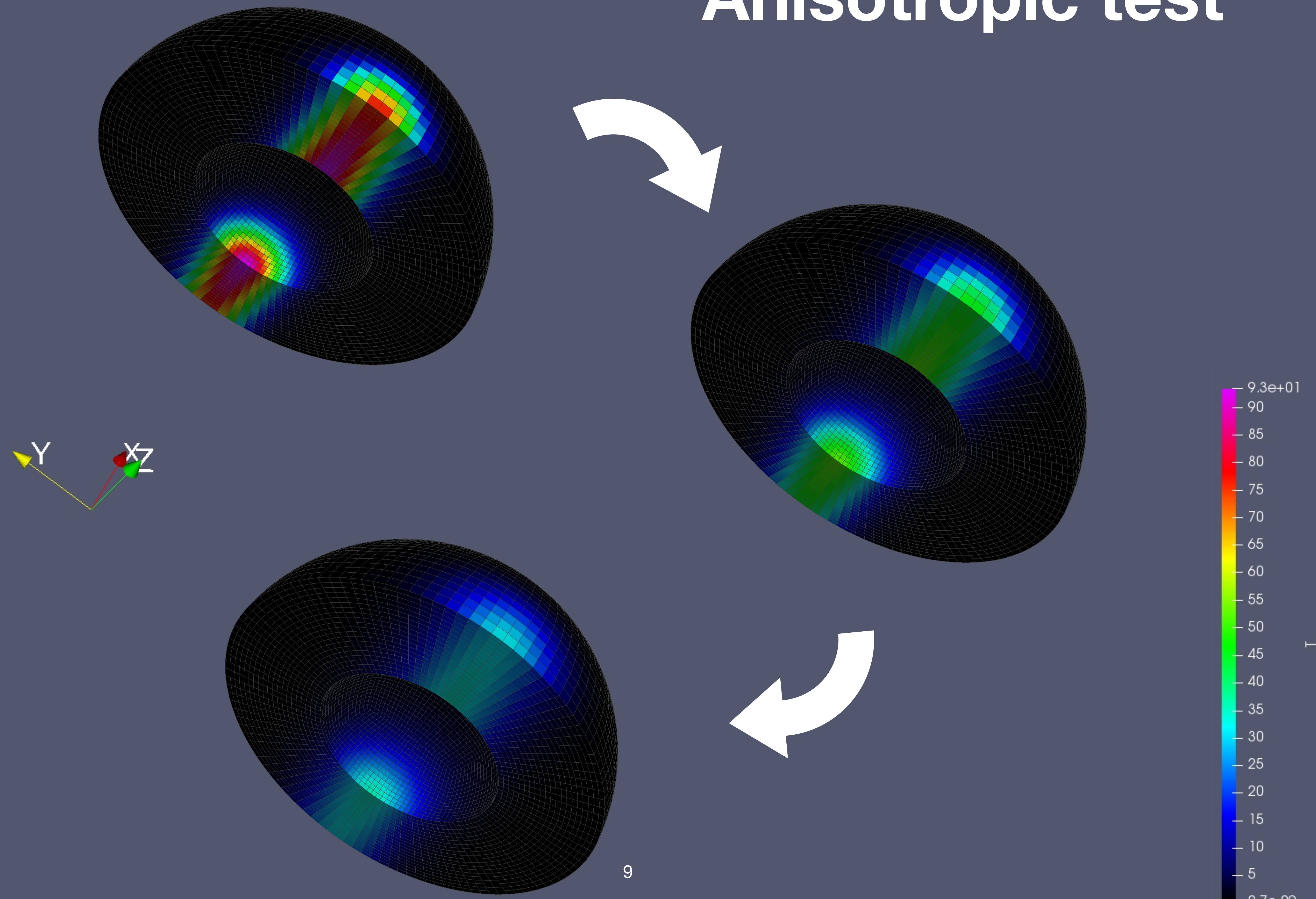


$$T(r,\theta,t) = T_0\left(\frac{t_0}{t}\right)^{3/2}\exp\left[-\frac{r^2}{4k_\perp t}f(\theta,\omega_B\tau_0)\right]$$

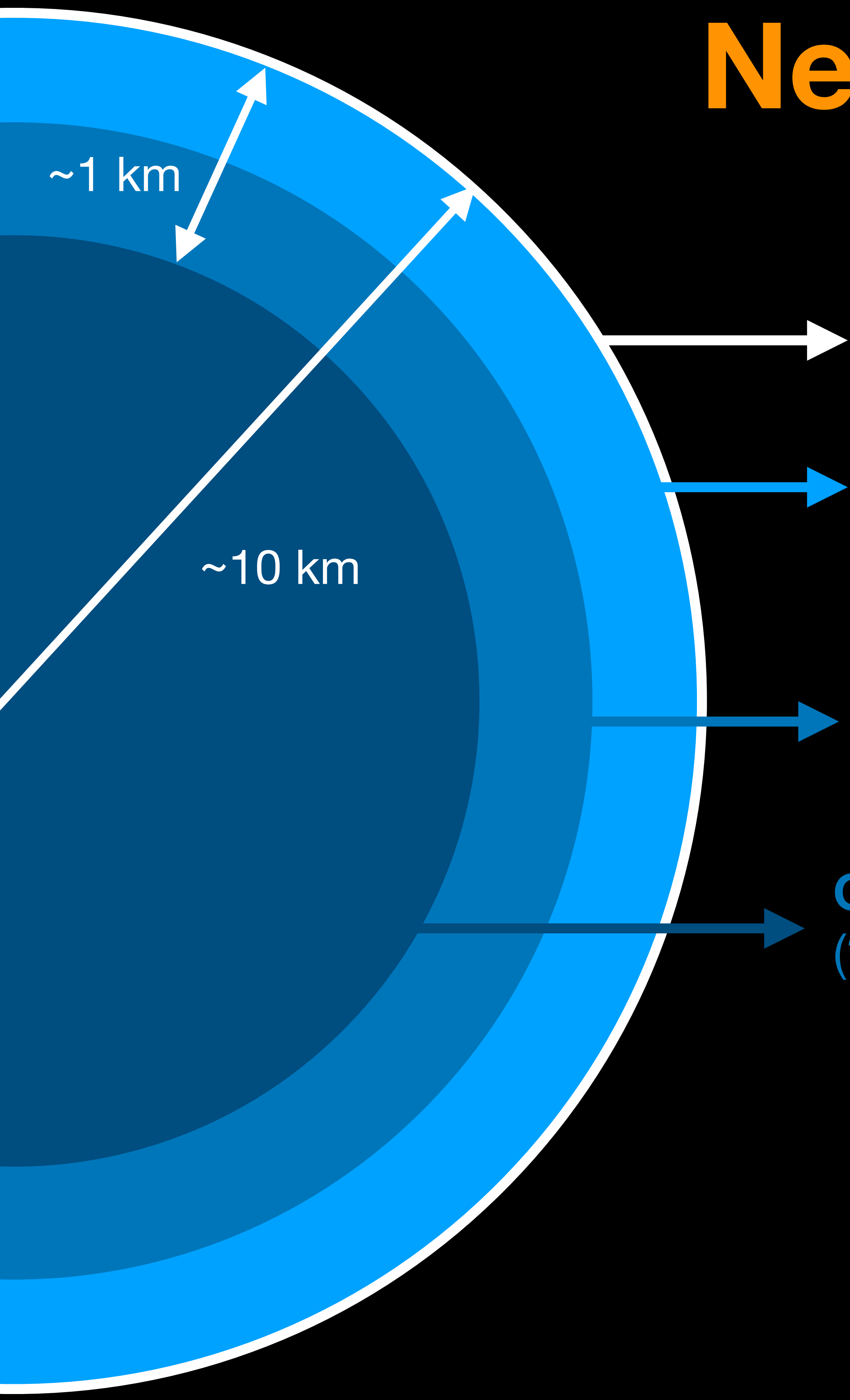
$$f(\theta,\omega_B\tau_0) = \sin^2\theta + \frac{\cos^2\theta}{1+(\omega_B\tau_0)^2}$$

$$L2 = \frac{\sum_i^{\text{all cells}}(T_{num;i} - T_{anlt;i})^2}{\sum_i^{\text{all cells}}(T_{anlt;i})^2}$$

Perez-Azorin+2006
Anisotropic test



Neutron Star Structure



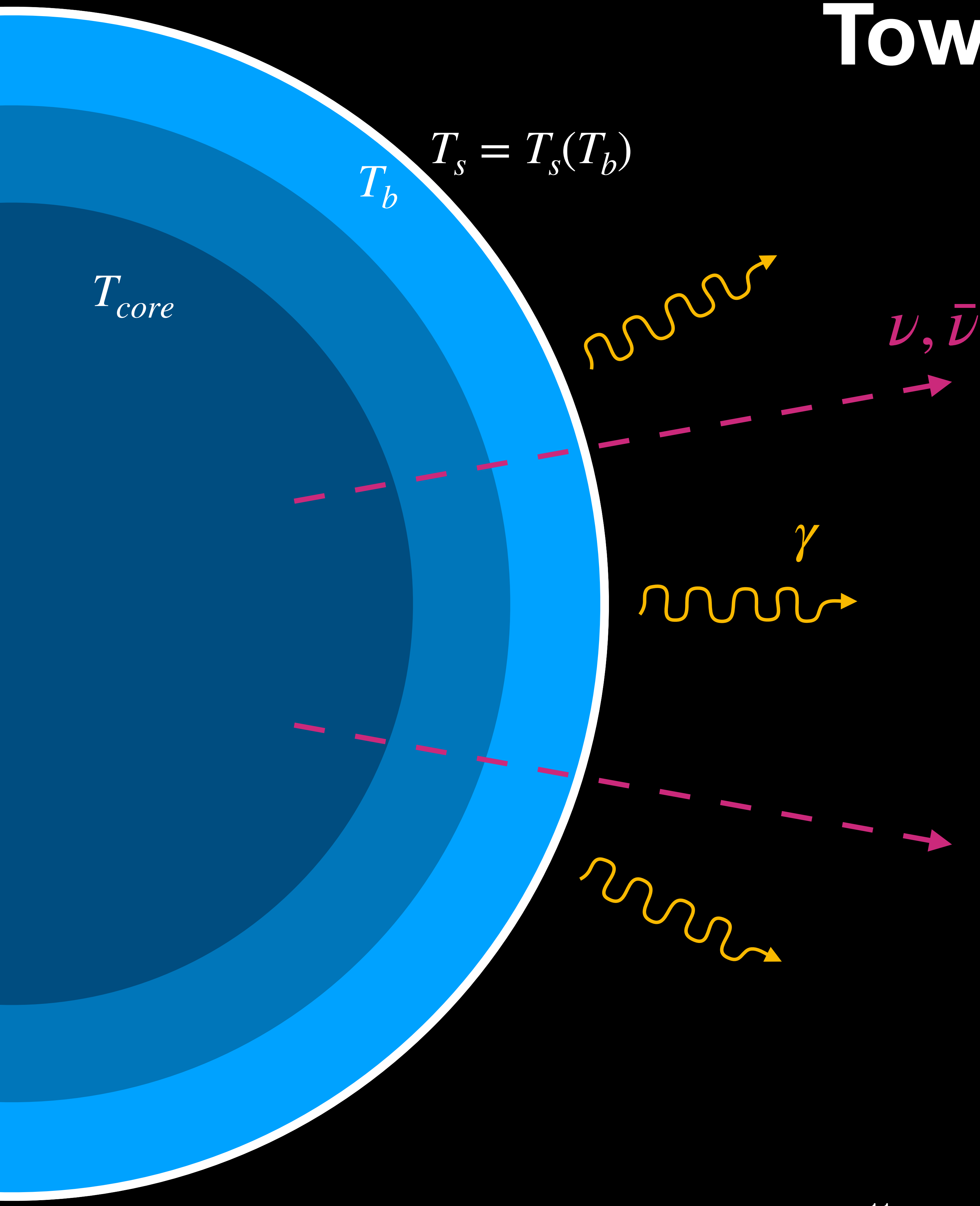
Ocean: ~ 1-100 m, Coulomb liquid, light or heavy nuclei

Outer Crust: Coulomb lattice of heavy nuclei + relativistic degenerate electrons

Inner Crust: neutrons start to drip out from nuclei

Core: neutron and proton superfluids, hyperons (?), quark-gluon plasma (?), pion condensate (?)

Towards a realistic simulation



Computational domain:

- Crust ✓
- Core (1 zone) ✓
- Envelope ✗

Microphysics

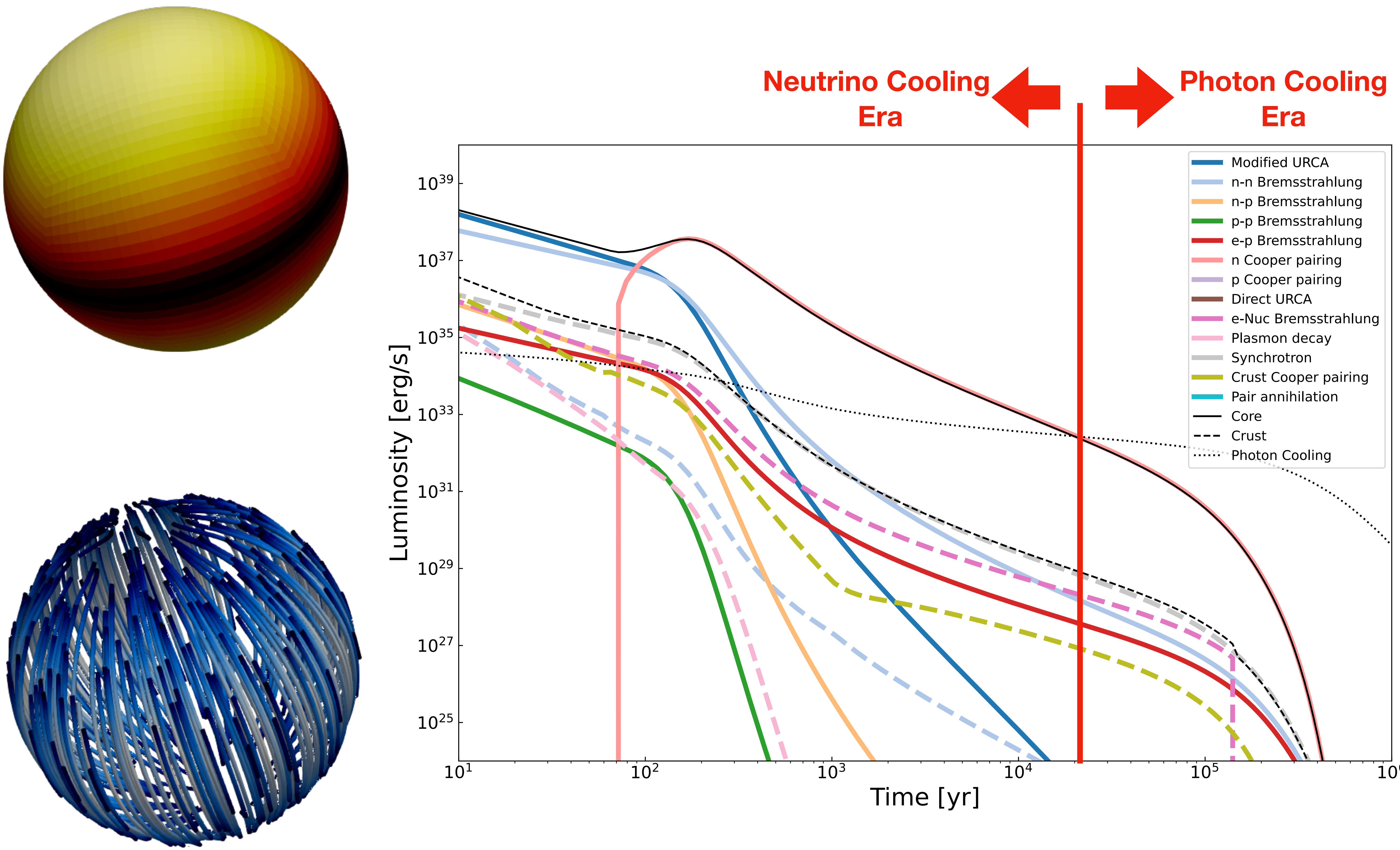
- Analytical
- Tabulated (Pothekin code)

Internal Boundary

$$\frac{dT_{core}}{dt} = \left\langle \frac{e^{2\nu}\dot{\epsilon}_\nu(T_{core})}{c_v(T_{core})} \right\rangle$$

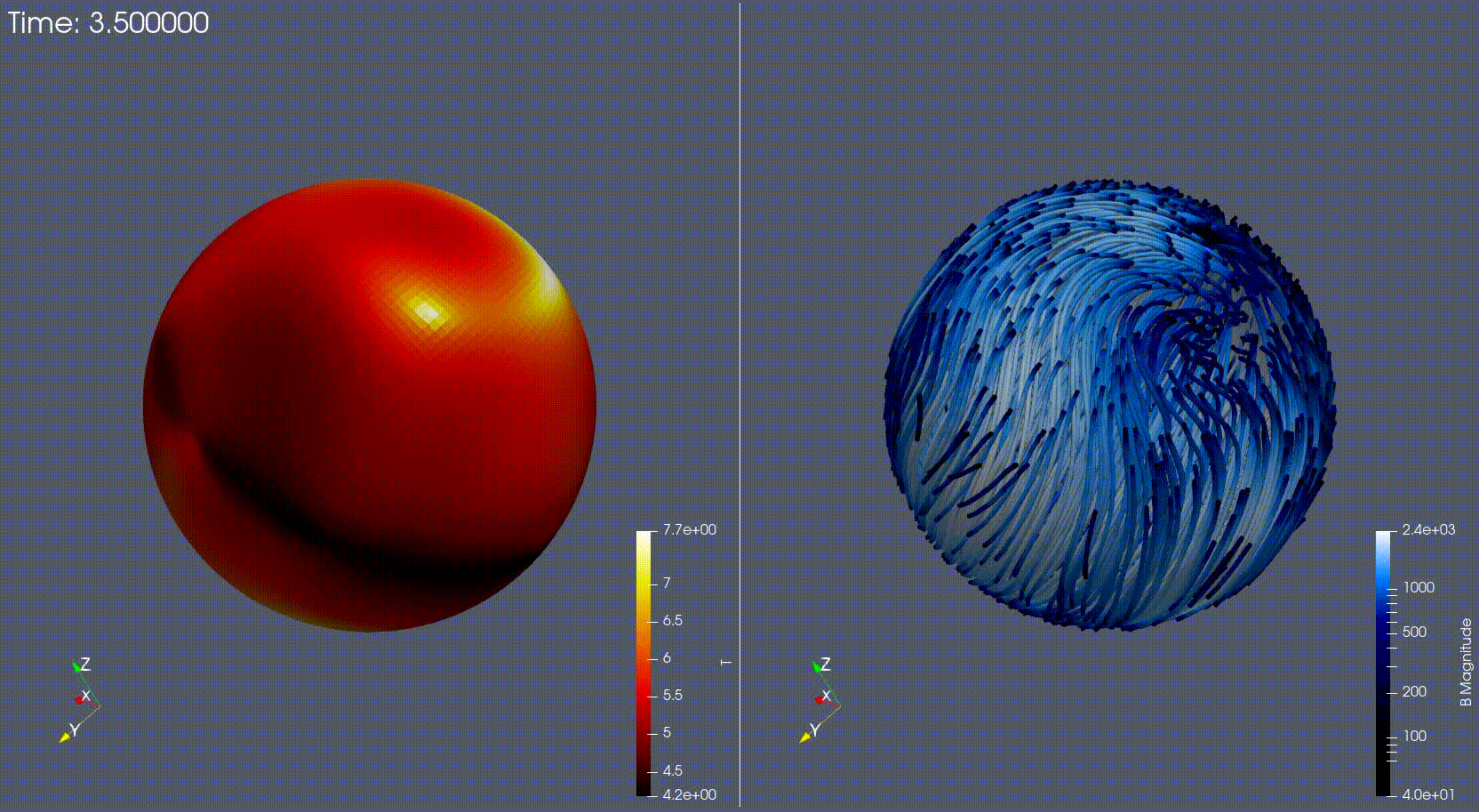
External Boundary

$$F_b = F_r \propto T_s^4$$

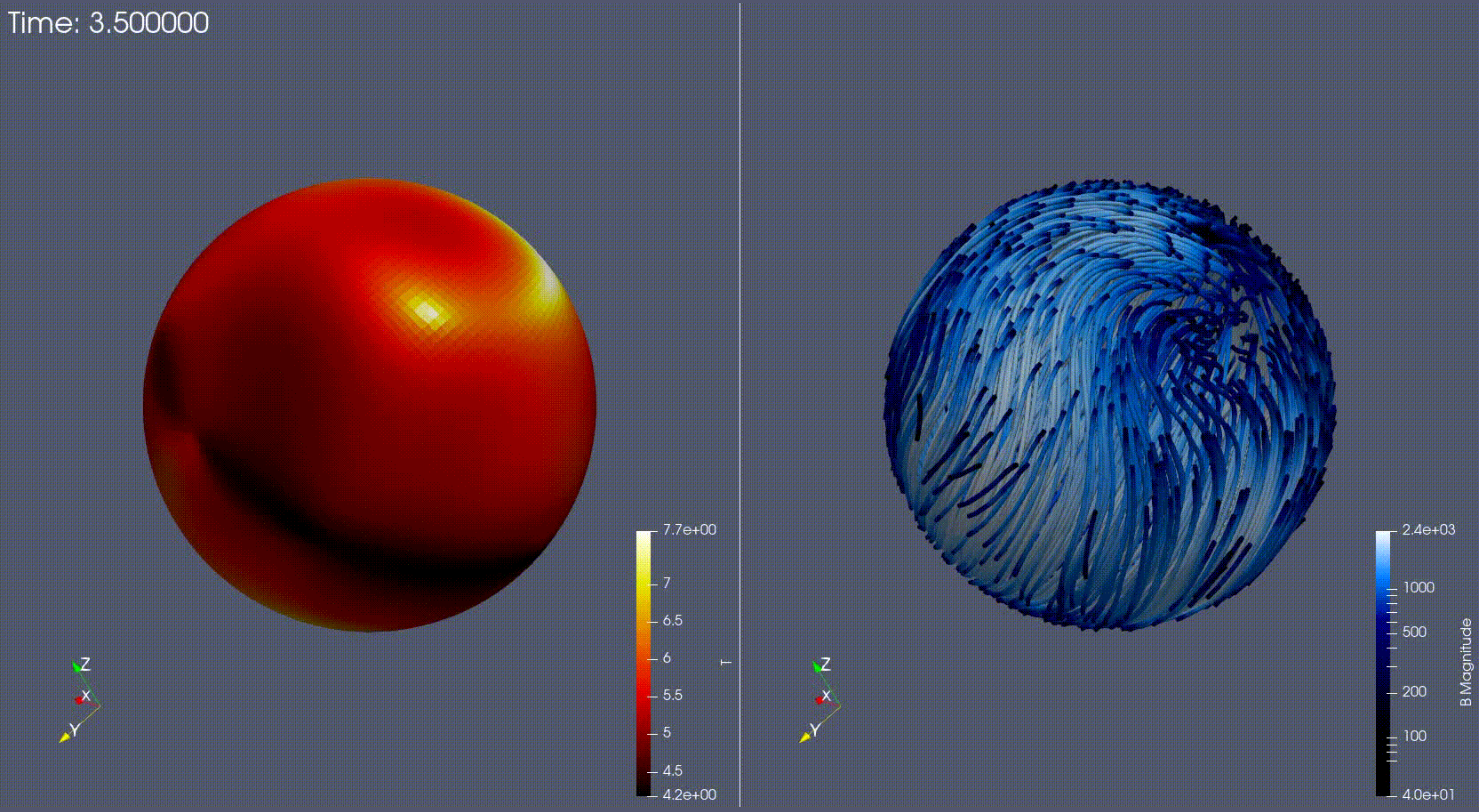


Neutrino Cooling Era
Photon Cooling Era
Luminosity [erg/s]
Time [yr]
Modified URCA
n-n Bremsstrahlung
n-p Bremsstrahlung
p-p Bremsstrahlung
e-p Bremsstrahlung
n Cooper pairing
p Cooper pairing
Direct URCA
e-Nuc Bremsstrahlung
Plasmon decay
Synchrotron
Crust Cooper pairing
Pair annihilation
Core
Crust
Photon Cooling

Non-Axisymmetric Run

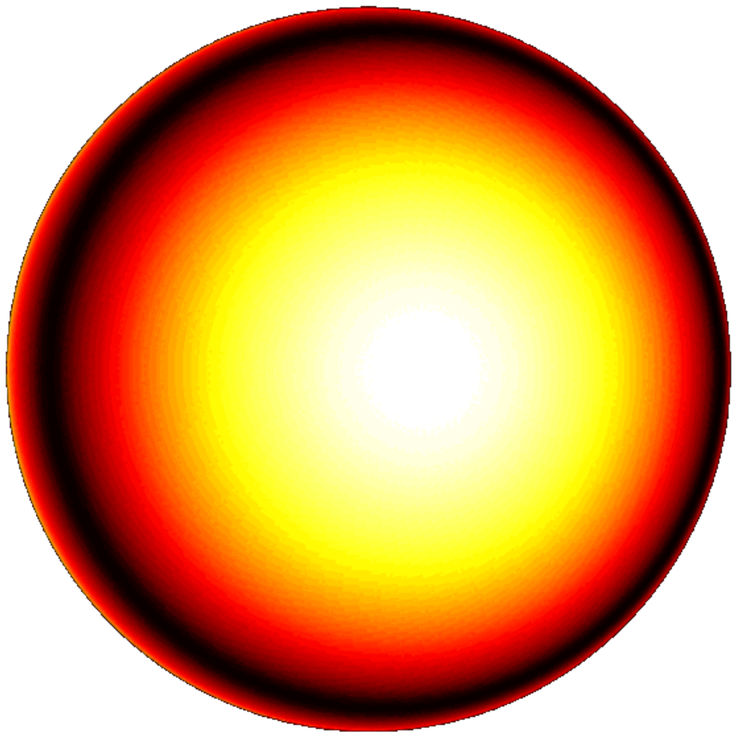


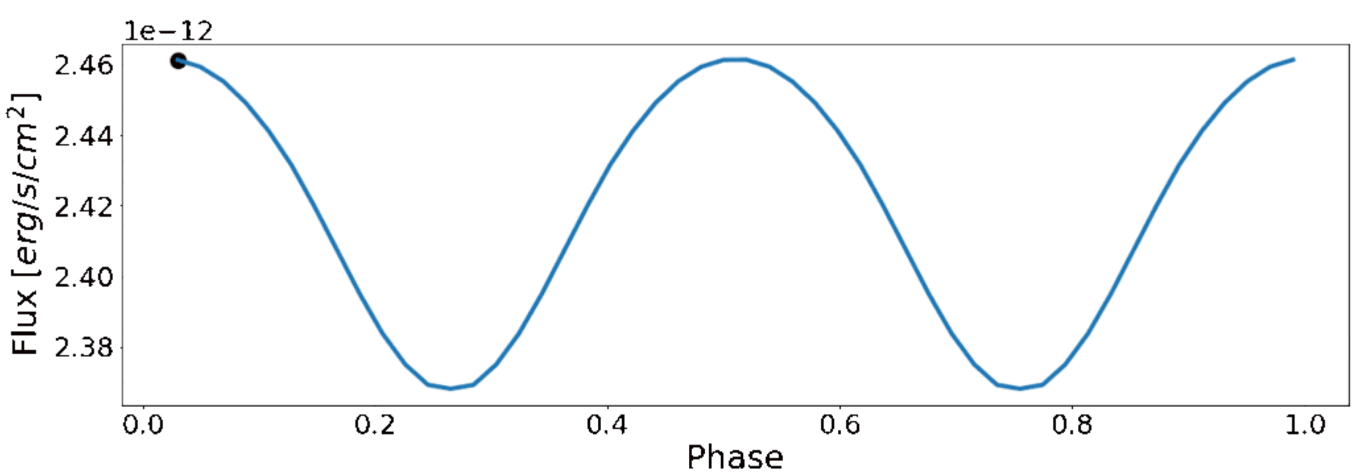
Non-Axisymmetric Run

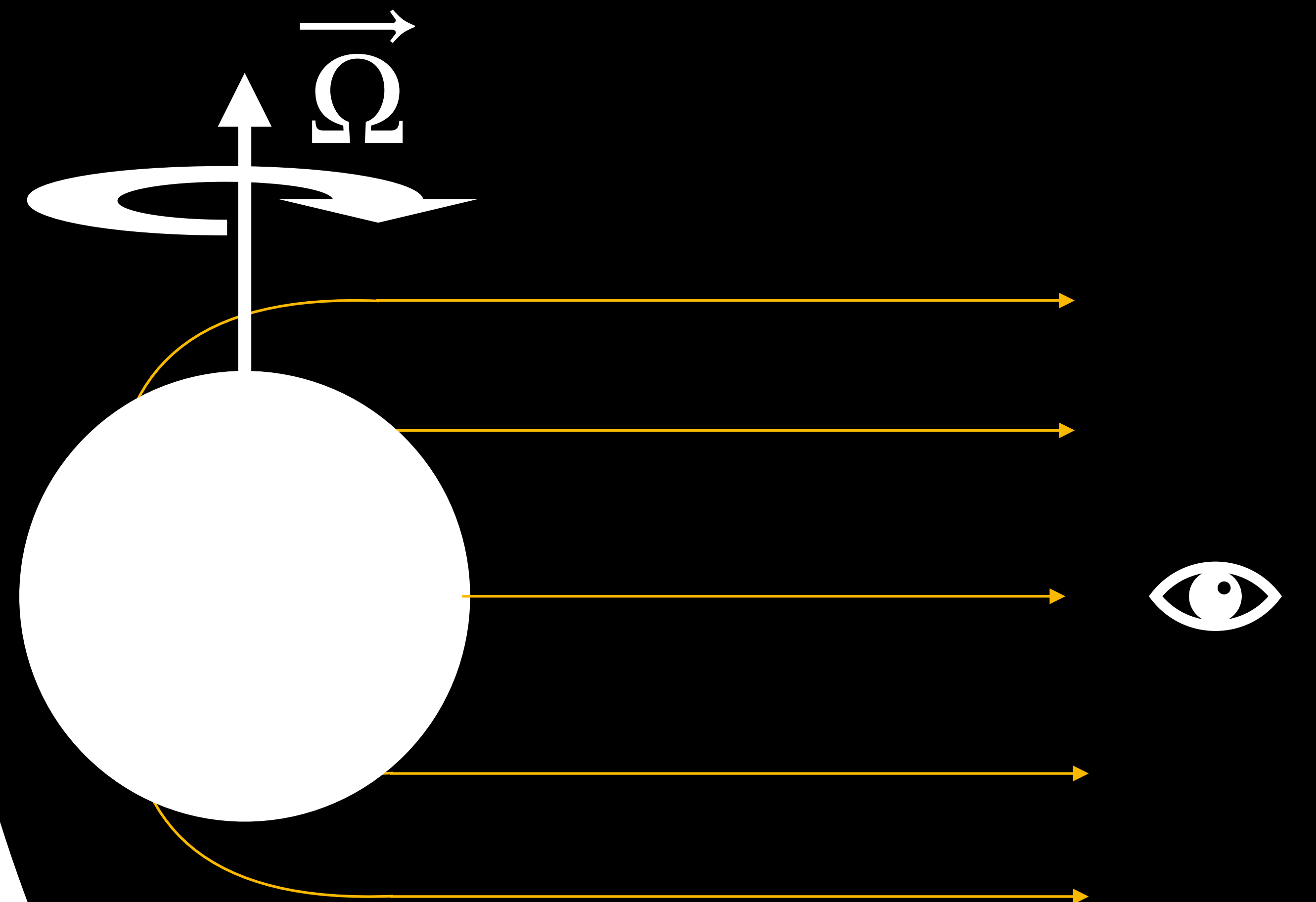


In collaboration with Prof. Rosalba Perna (Stoney Brook University)

Developing a **ray-tracing code** to model pulsating thermal emission from magnetars







Ascenzi et al. in prep.

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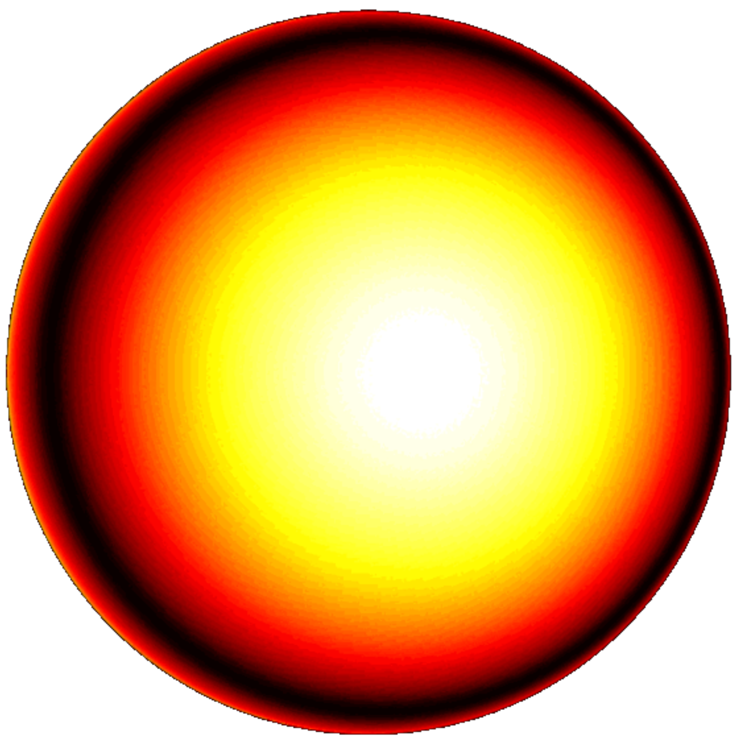
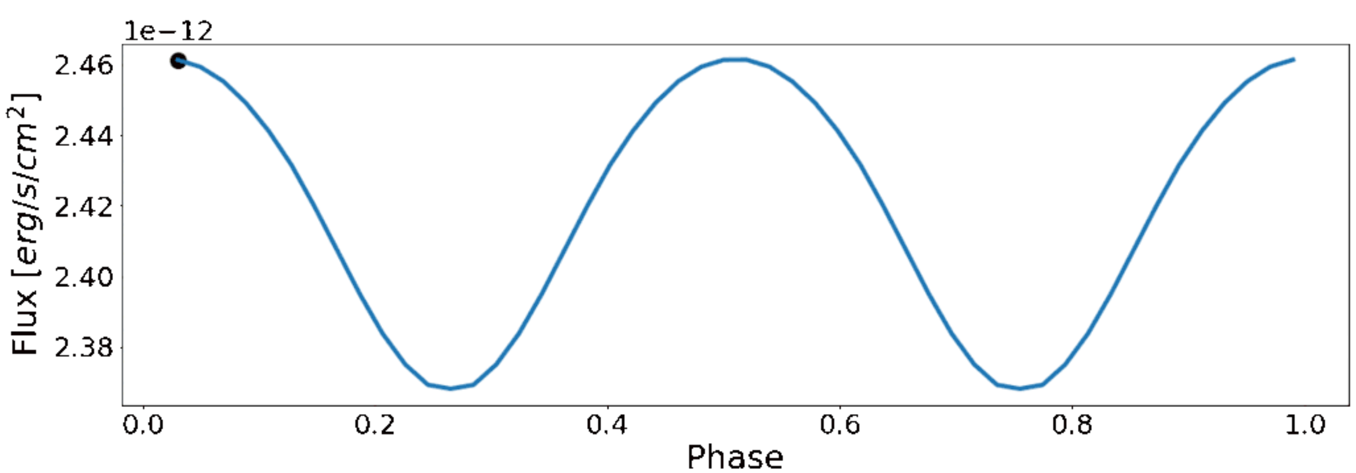
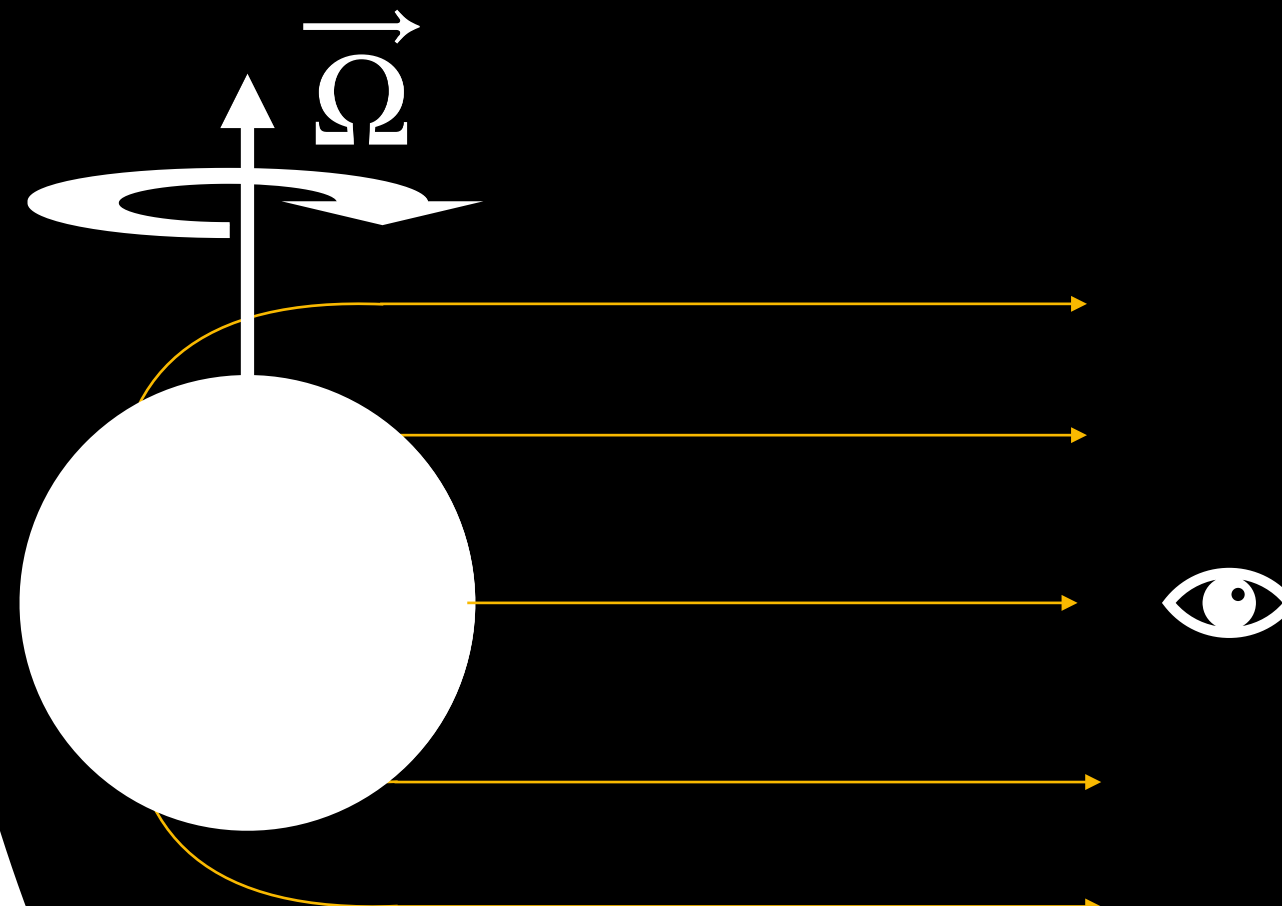
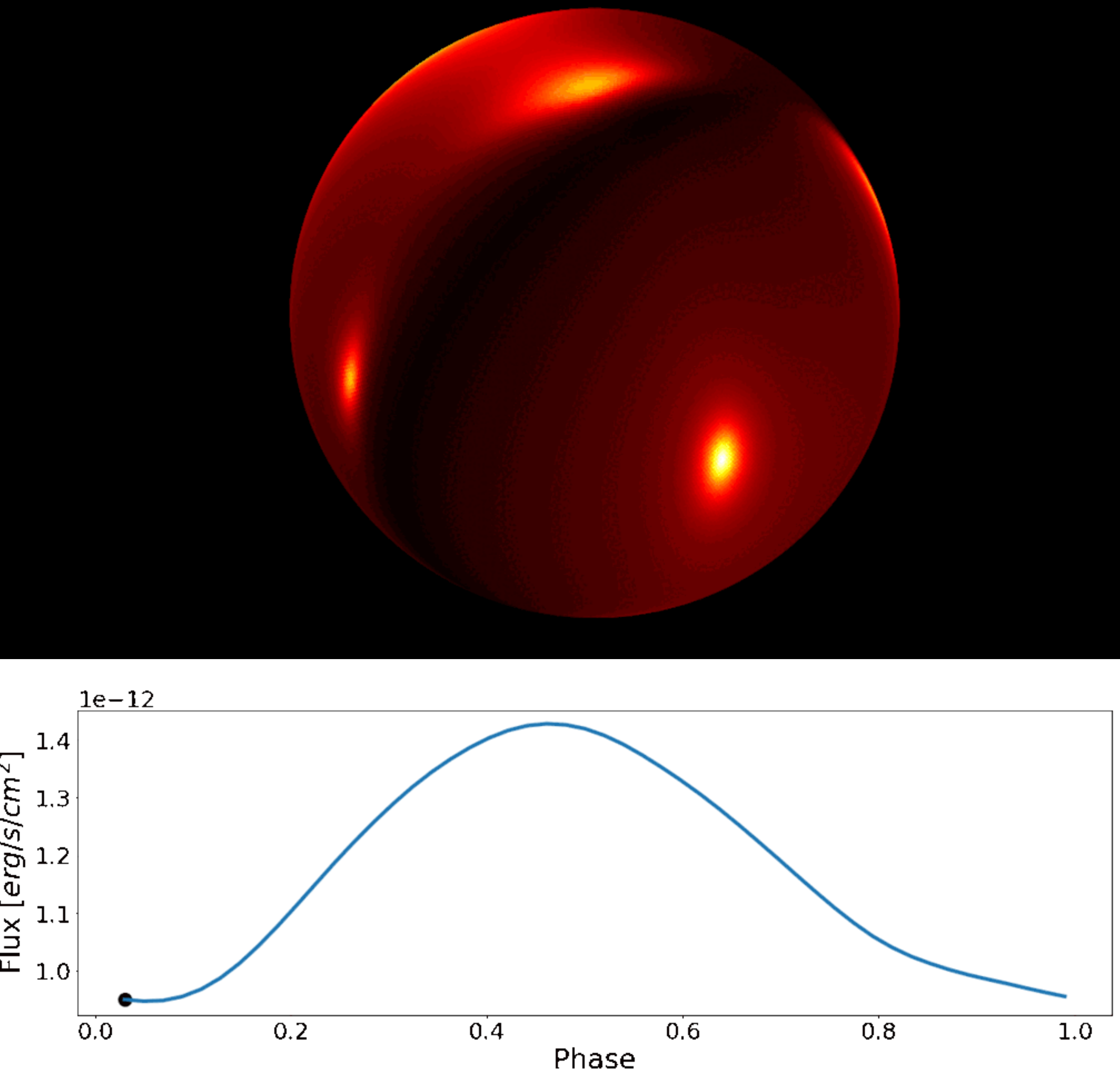




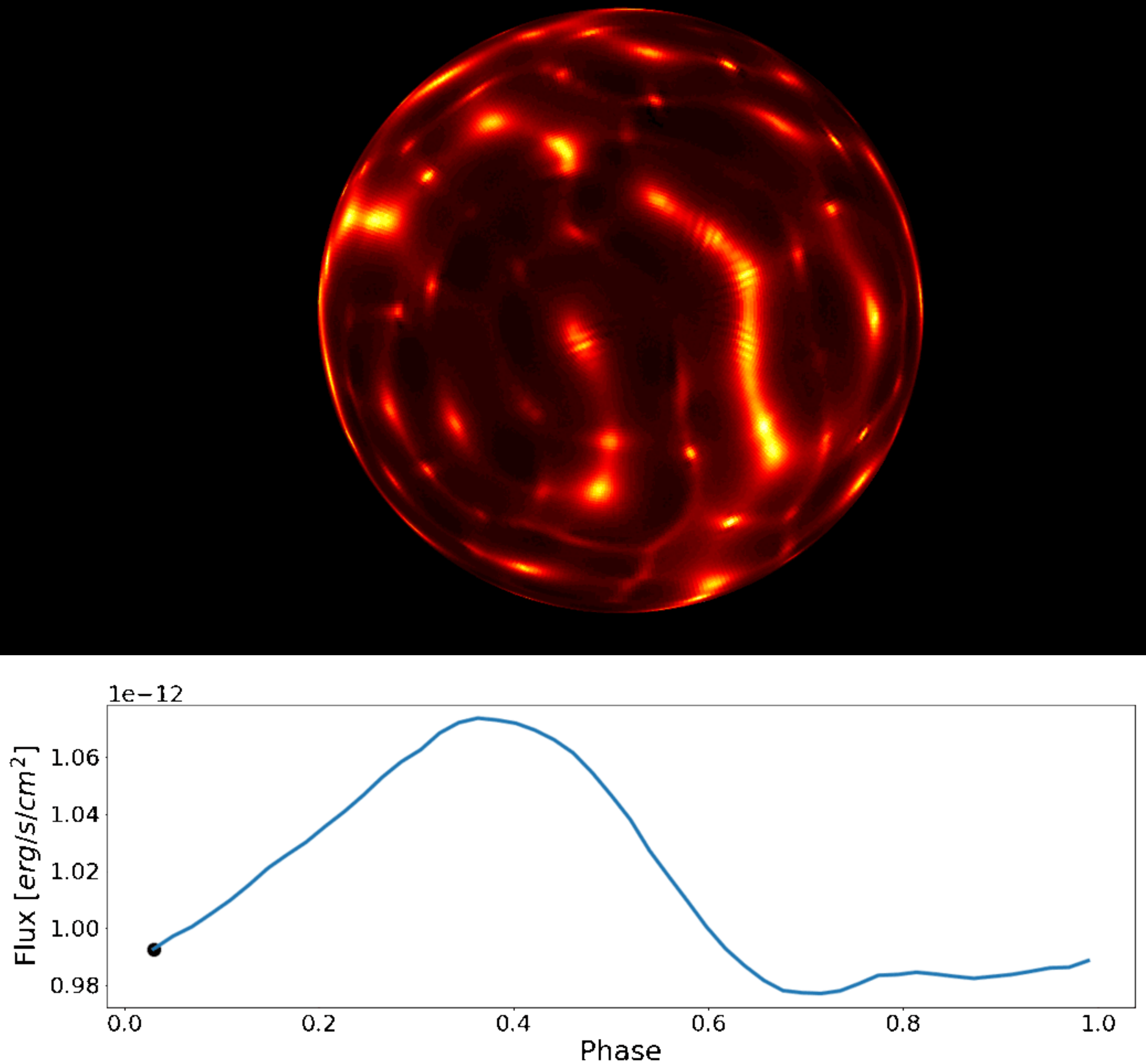


Ascenzi et al. in prep.

Pulsed Fraction $\sim 20\,\%$



Pulsed Fraction $\sim 4\,\%$



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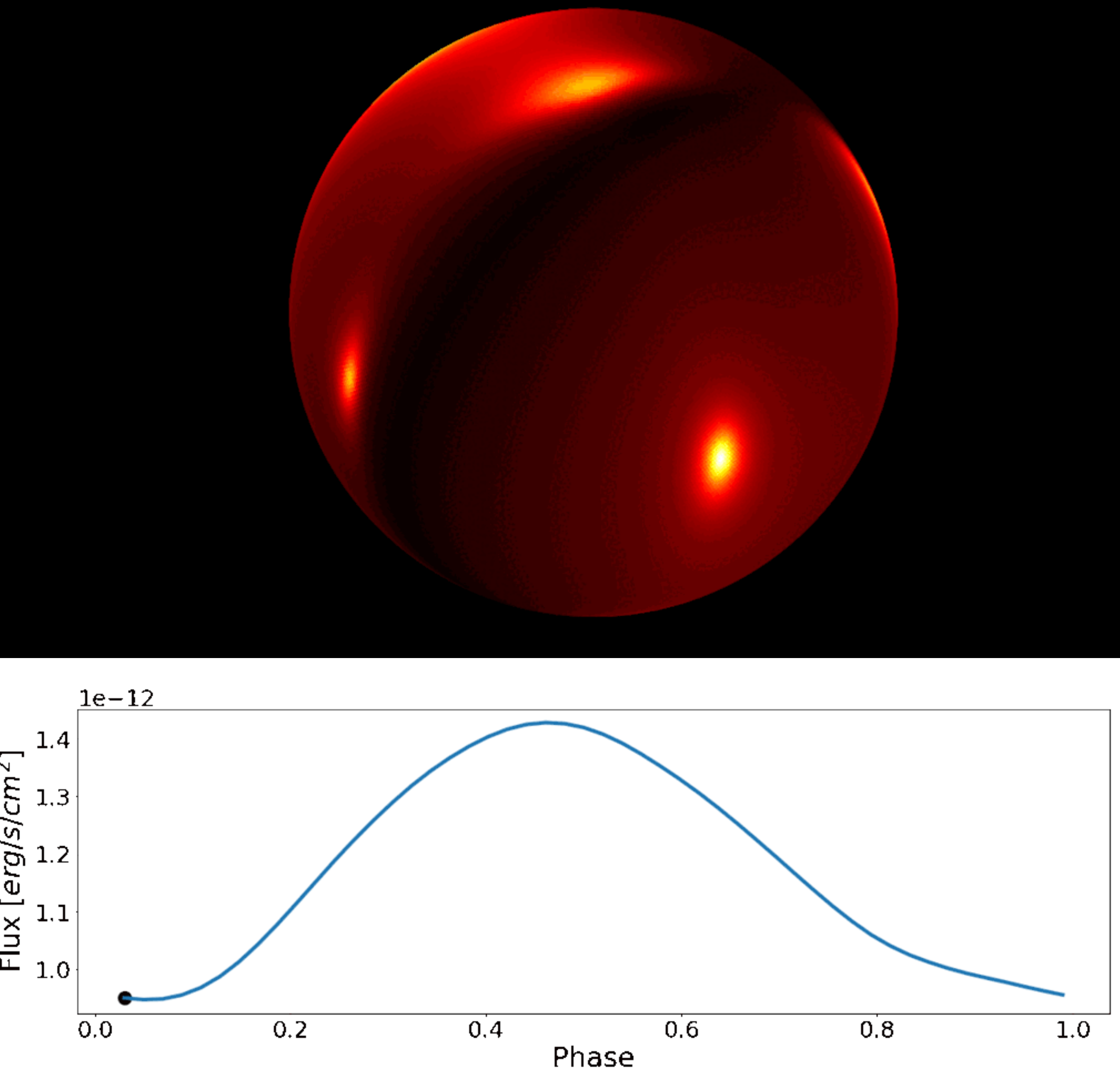

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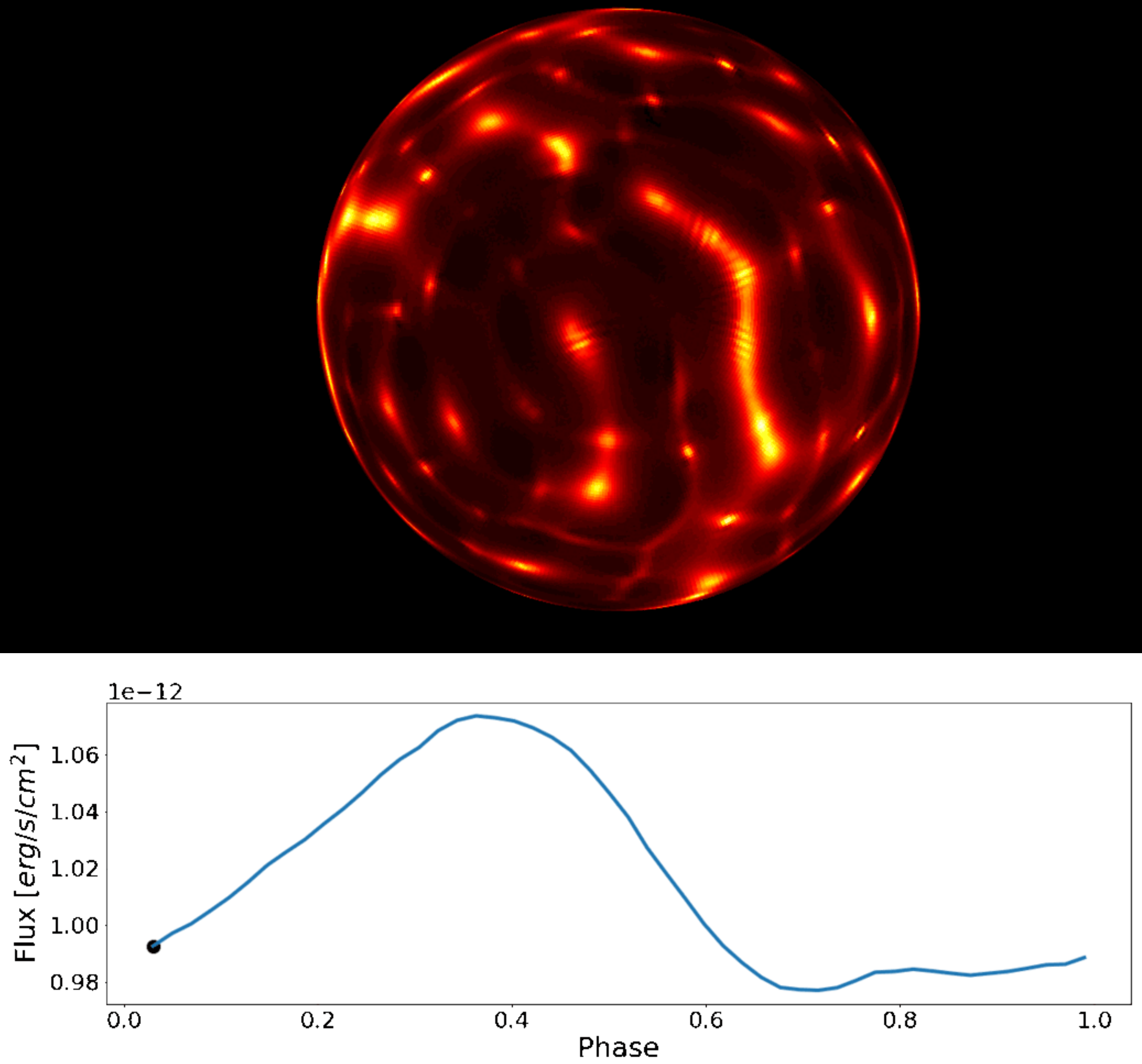


What we achieved so far…

MATINS
MAgneto-Thermal evolution of Isolated Neutron Stars

- Working magnetic (see Dehman+2022) and thermal (Ascenzi+ in prep.) evolution codes in a cubed sphere grid
- Simplified microphysics described by analytical formula
- Detailed realistic microphysics from tabulated models (see Pothekin+2015 for a review)
- Coupling between the thermal and magnetic evolution
- Parallelized Code

Differences with PARODY code

(eg. Wood &Hollerbach2015, Gourgouliatos+2016, De Grandis+2020)

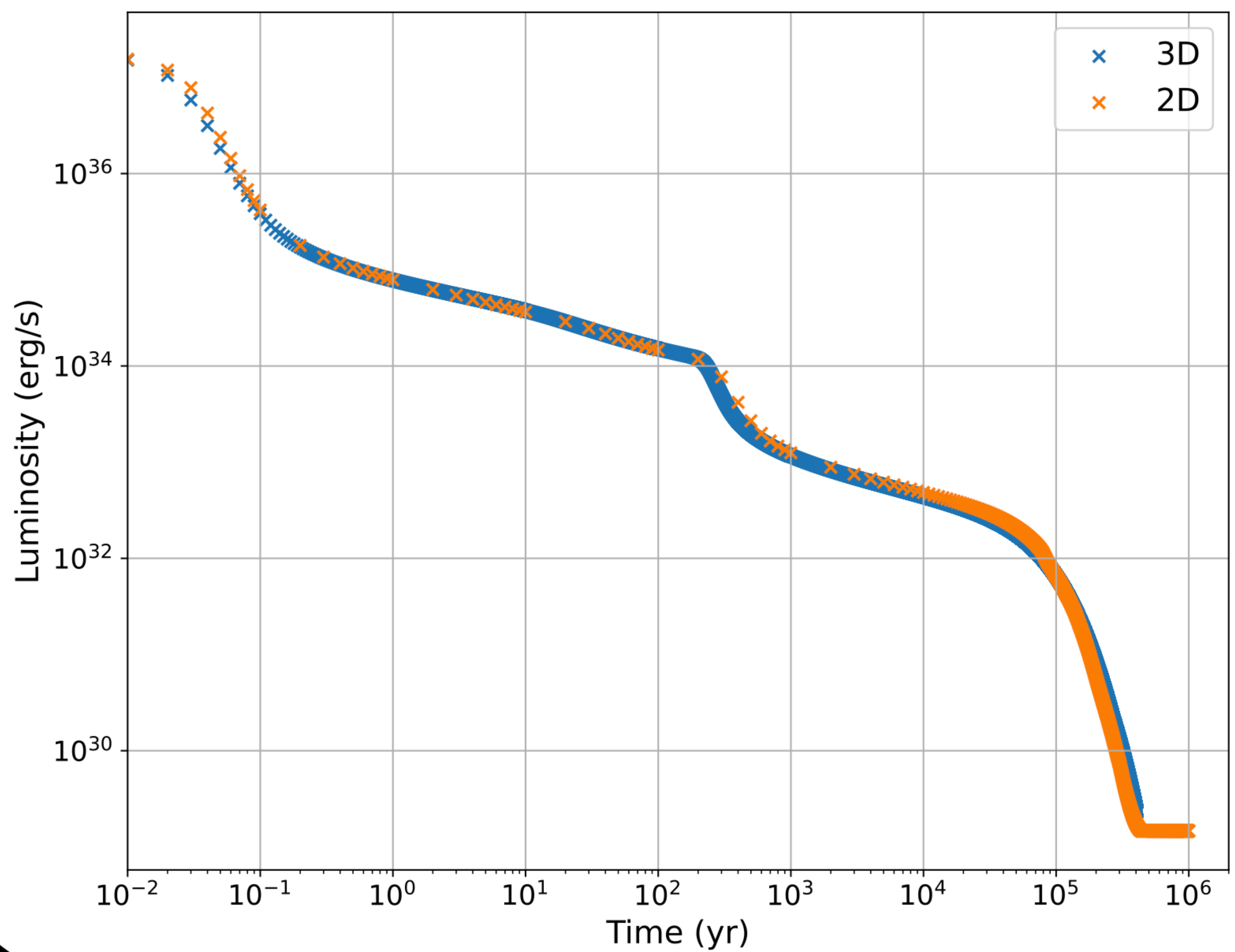
- Finite Volume Scheme vs Pseudo-Spectral Scheme
- Tabulated microphysics calculated with the public code by Alexander Potekhin (http://www.ioffe.ru/astro/conduct/)

Thank you for your attention!

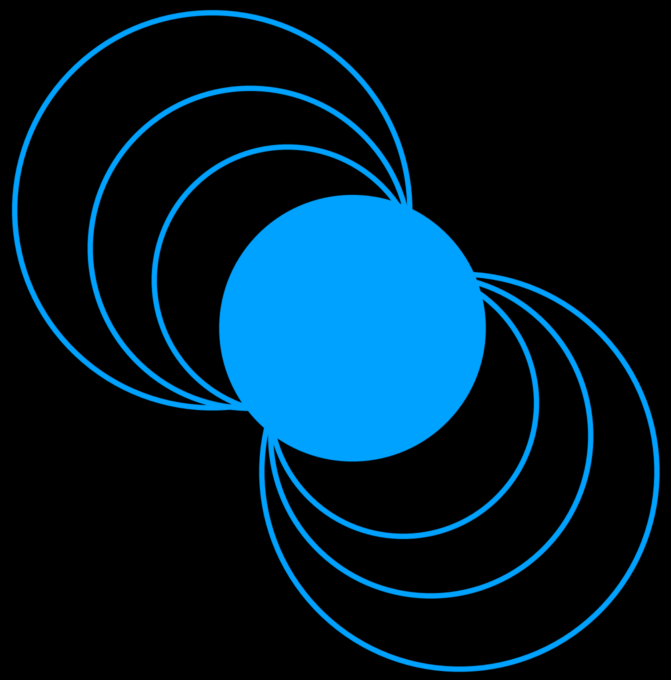
Backup-Slides

Comparison with 2D

- Dipolar crust confined magnetic field
- Tabulated microphysics from Pothekin code



Neutron Stars: an overview



- Generated after the gravitational collapse of the **core of a massive star** $M_{ZAMS} \sim 8 - 20/30\, M_{\odot}$
- **Compact objects**: 1-2 $M_{\odot}$ enclosed in a radius of 10-13 km
- **Fast Rotation**: O(1 ms - 10 s)
- **Strong Magnets**: O($10^8 - 10^{15}\, G$)

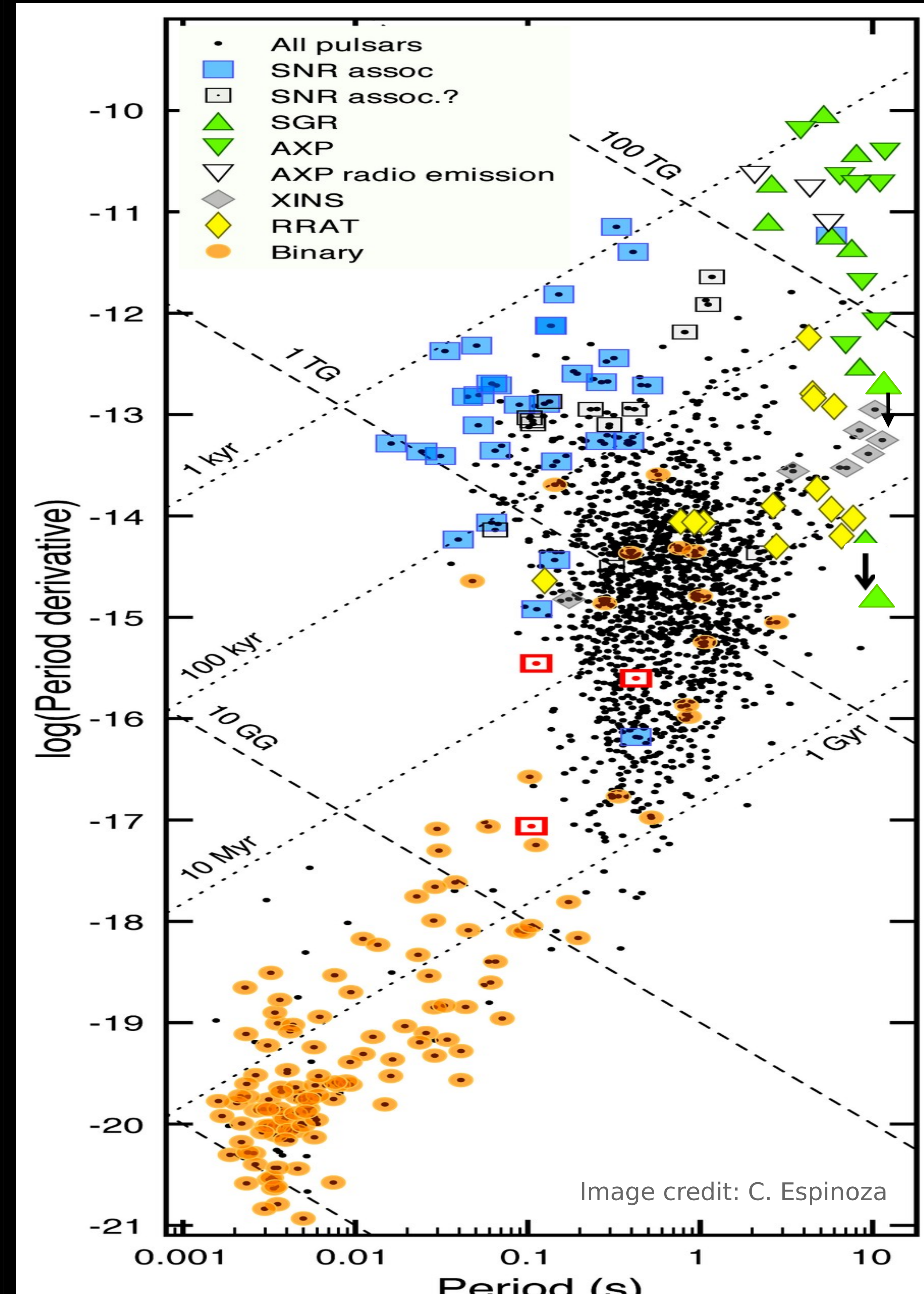
How do we see neutron stars?

Courtesy of Daniele Viganò

Ultimate sources of energy for the emission of NS:

1. Rotational energy to kinetic energy of magnetospheric plasma, which acquire energy and radiates from radio to gamma rays (standard pulsars)
2. **Residual heat (cooling)**: X-rays from the hot surface (10^5-10^6 K) if young
3. Accretion in binary systems
4. **Magnetic fields** (interior and magnetospheric)
5. Collision (binary neutron star mergers)

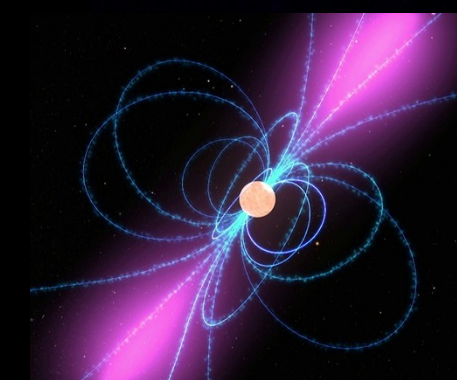
The neutron star zoo



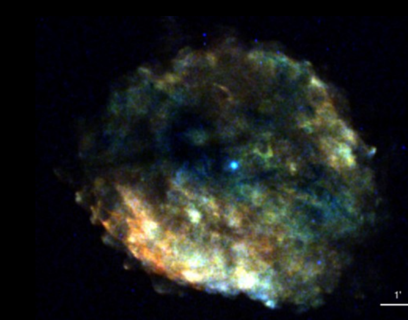
Magnetars: B-powered

XDINS: kT-powered

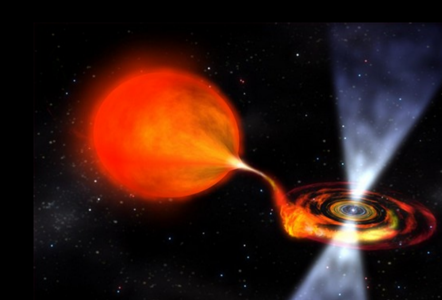
Pulsars: rotation-powered



CCOs: kT-powered



MSPs recycled in binaries: rotation-powered

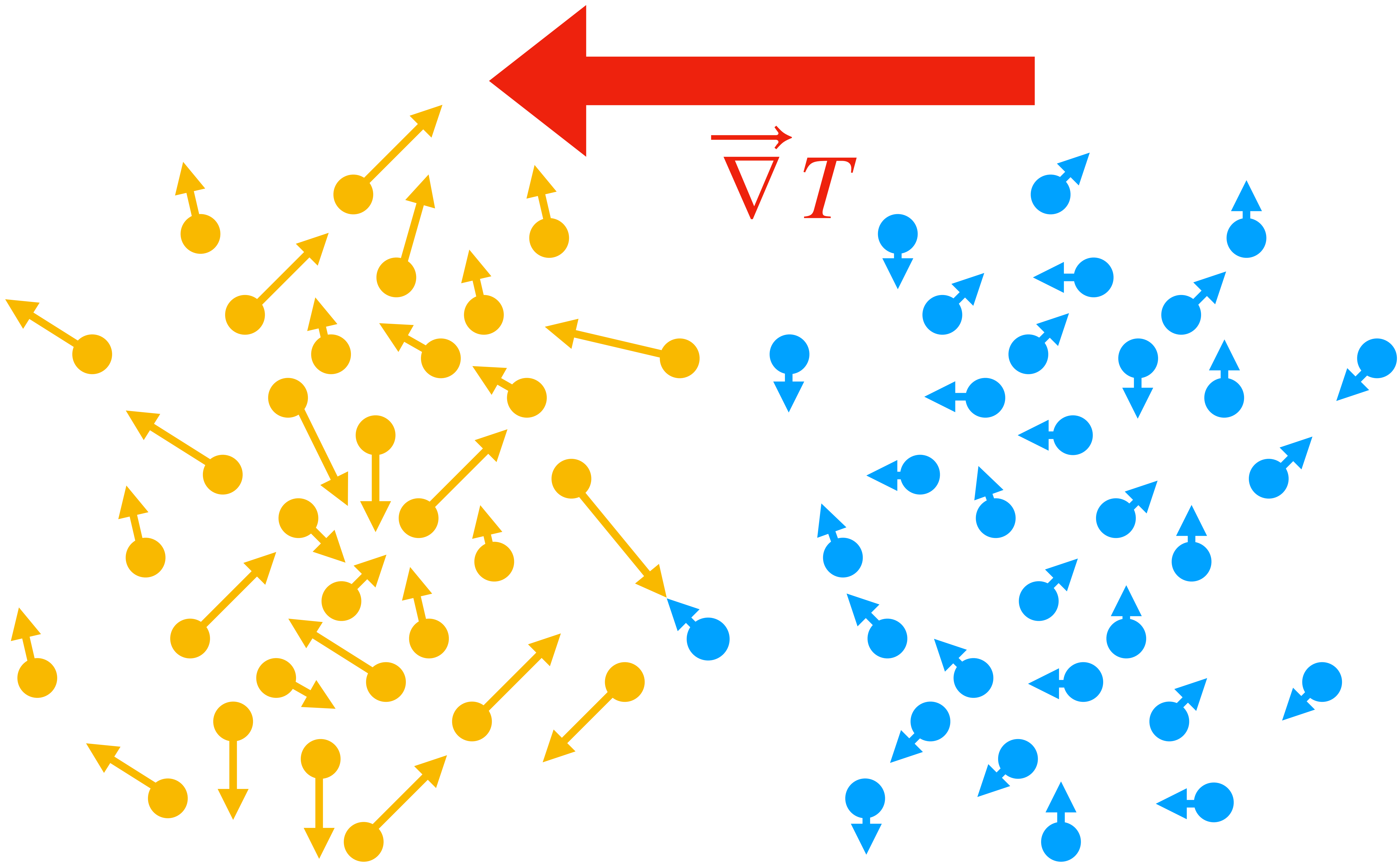


Isotropic Case

$$e^{\nu(r)}\mathbf{F} = -k_\perp \nabla \tilde{T}$$

$$\omega_B = 0$$

$$k_\perp = k_\parallel$$



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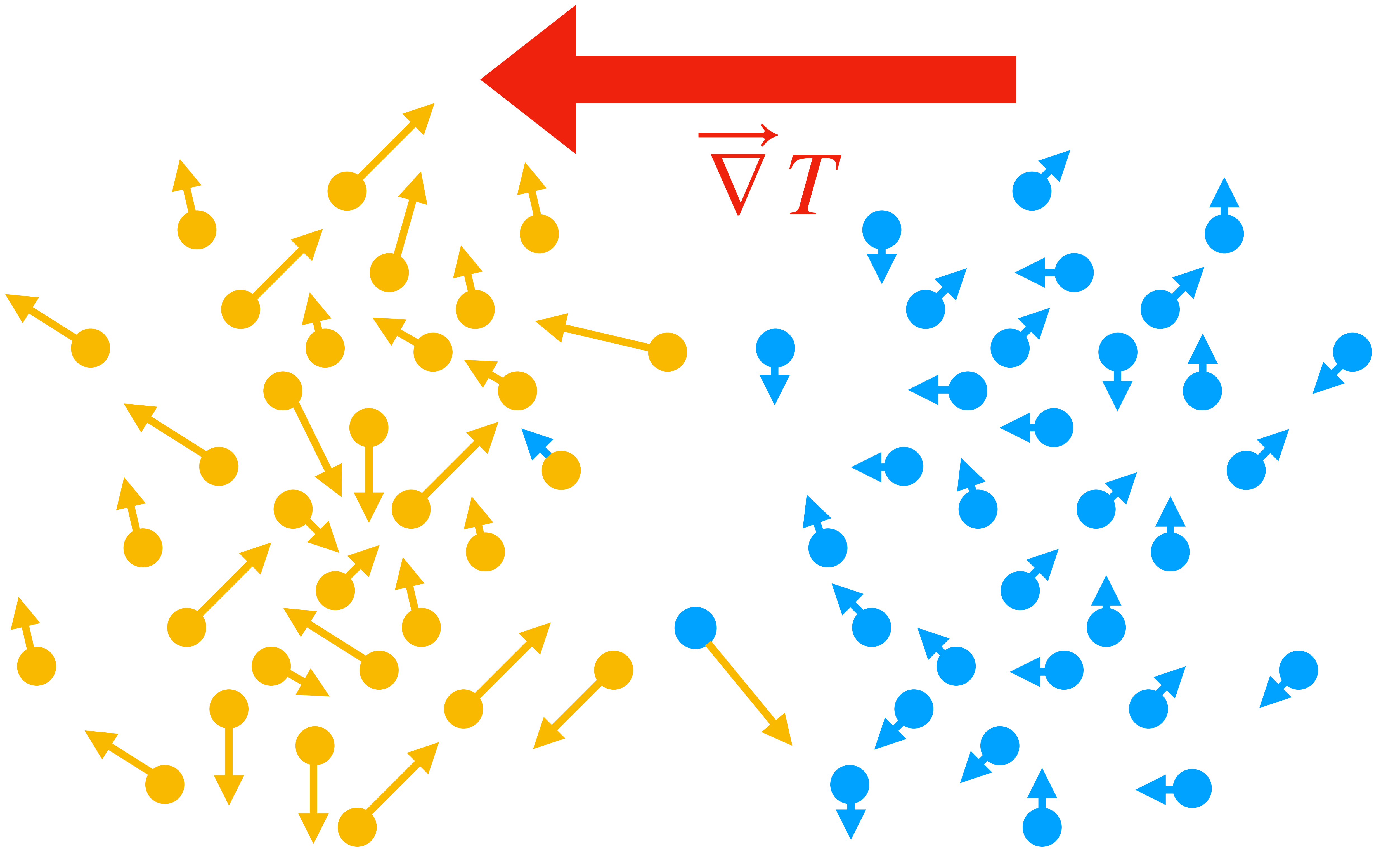
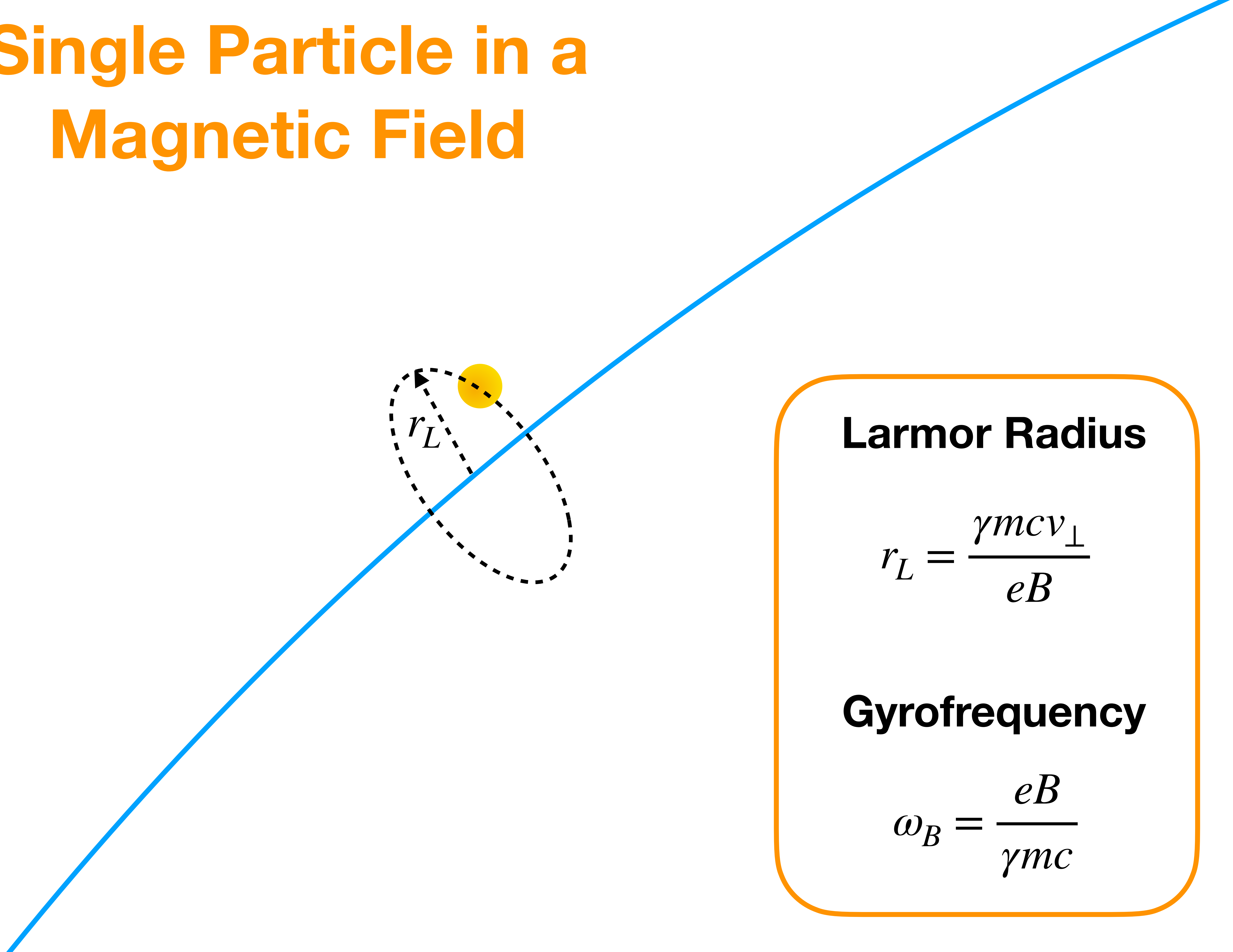


Single Particle in a Magnetic Field

Single Particle in a Magnetic Field



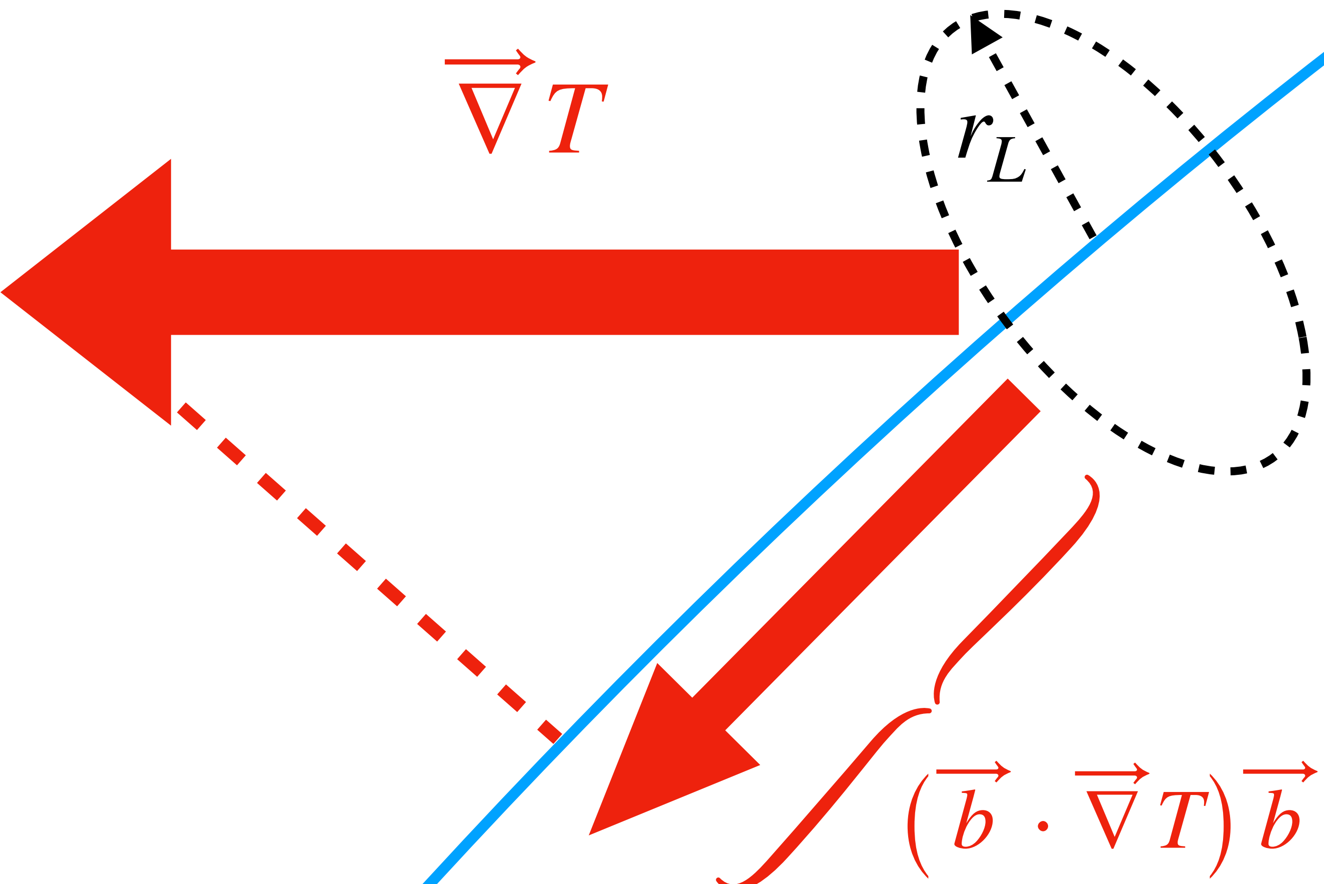
Larmor Radius

$$r_L = \frac{\gamma m c v_\perp}{eB}$$

Gyrofrequency

$$\omega_B = \frac{eB}{\gamma m c}$$

Heat Diffusion with Magnetic Field



Fourier Term

$$-k_\perp \nabla \tilde{T} = -\frac{k_\parallel}{1 + (\omega_B \tau_0)^2} \nabla \tilde{T}$$

Parallel Term

$$-k_\perp (\omega_B \tau_0)^2 (\mathbf{b} \cdot \nabla \tilde{T})\mathbf{b} \simeq$$

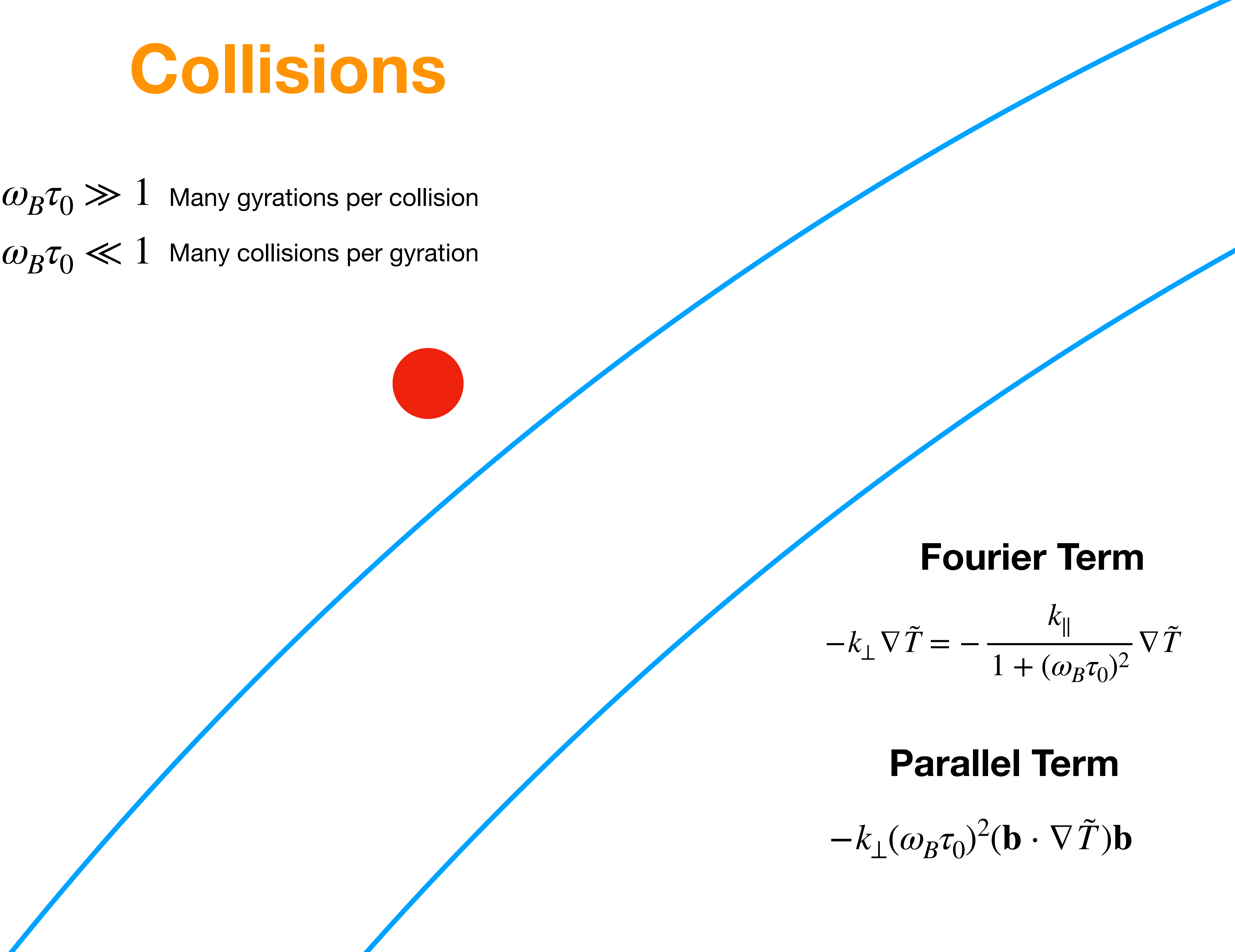
$$-k_\parallel (\mathbf{b} \cdot \nabla \tilde{T})\mathbf{b} = -k_\parallel (\nabla \tilde{T})_\parallel$$

$$e^{\nu(r)} \mathbf{F} = -\boxed{k_\perp \left[\nabla \tilde{T} + (\omega_B \tau_0)^2 (\mathbf{b} \cdot \nabla \tilde{T})\mathbf{b} \right.} \left. + (\omega_B \tau_0)(\mathbf{b} \times \nabla \tilde{T}) \right]$$

Collisions

$\omega_B \tau_0 \gg 1$ Many gyrations per collision

$\omega_B \tau_0 \ll 1$ Many collisions per gyration



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Collisions

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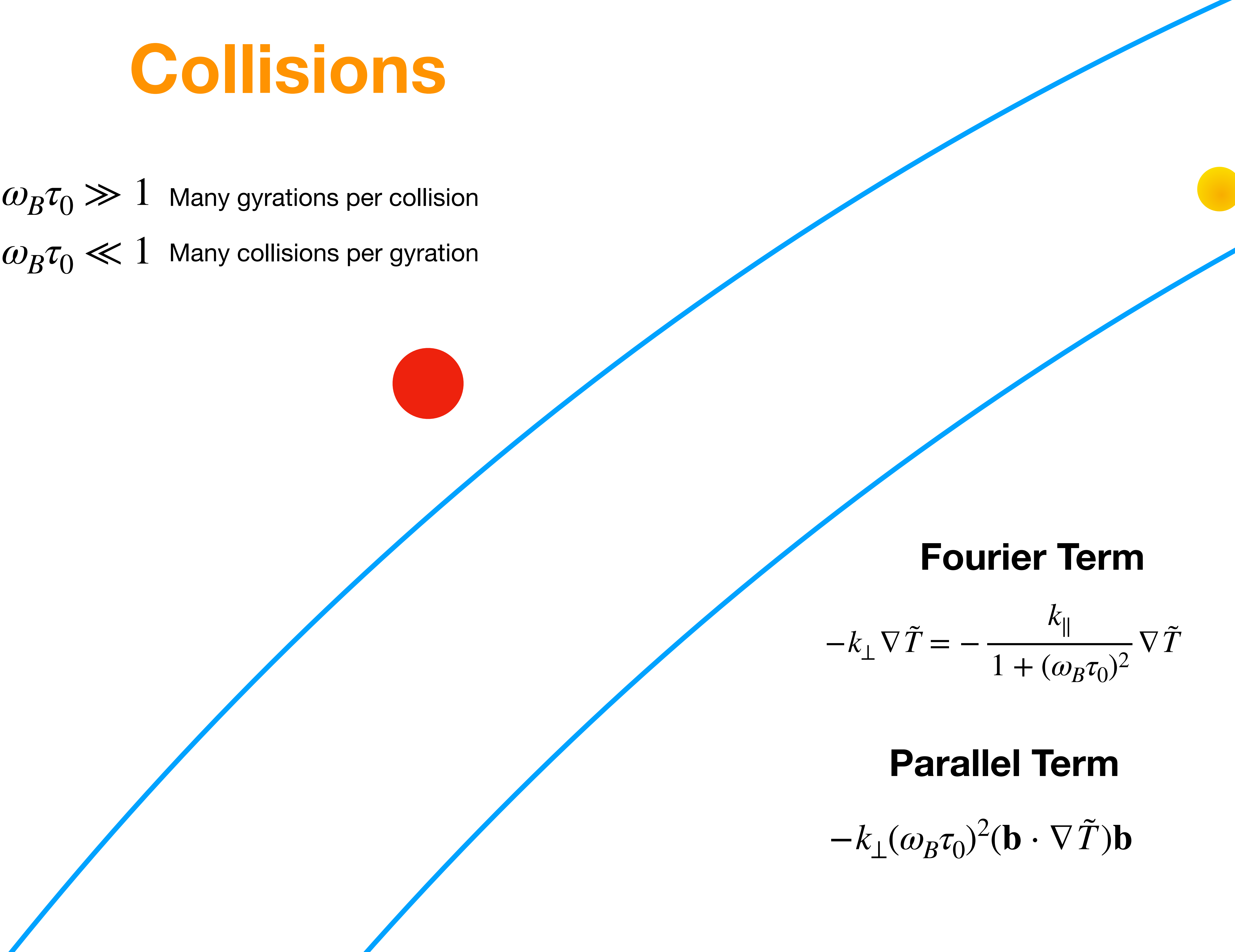


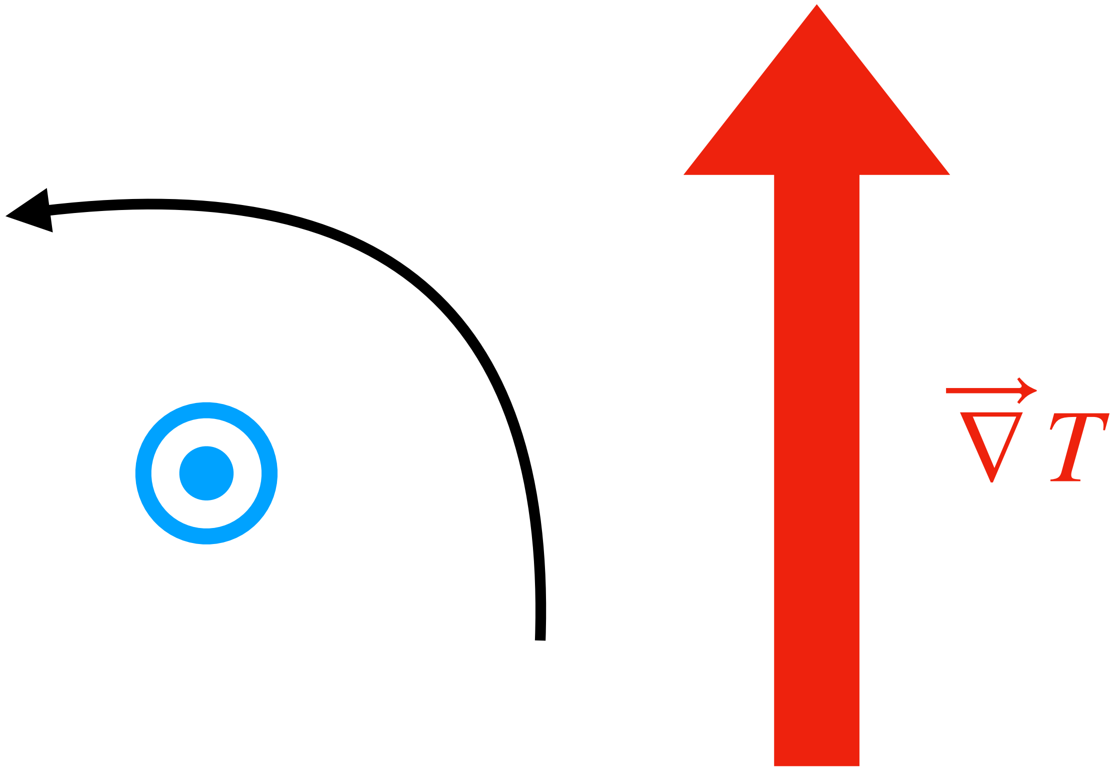
Fourier Term

$$-k_\perp \nabla \tilde{T} = -\frac{k_\parallel}{1 + (\omega_B \tau_0)^2} \nabla \tilde{T}$$

Parallel Term

$$-k_\perp (\omega_B \tau_0)^2 (\mathbf{b} \cdot \nabla \tilde{T}) \mathbf{b}$$

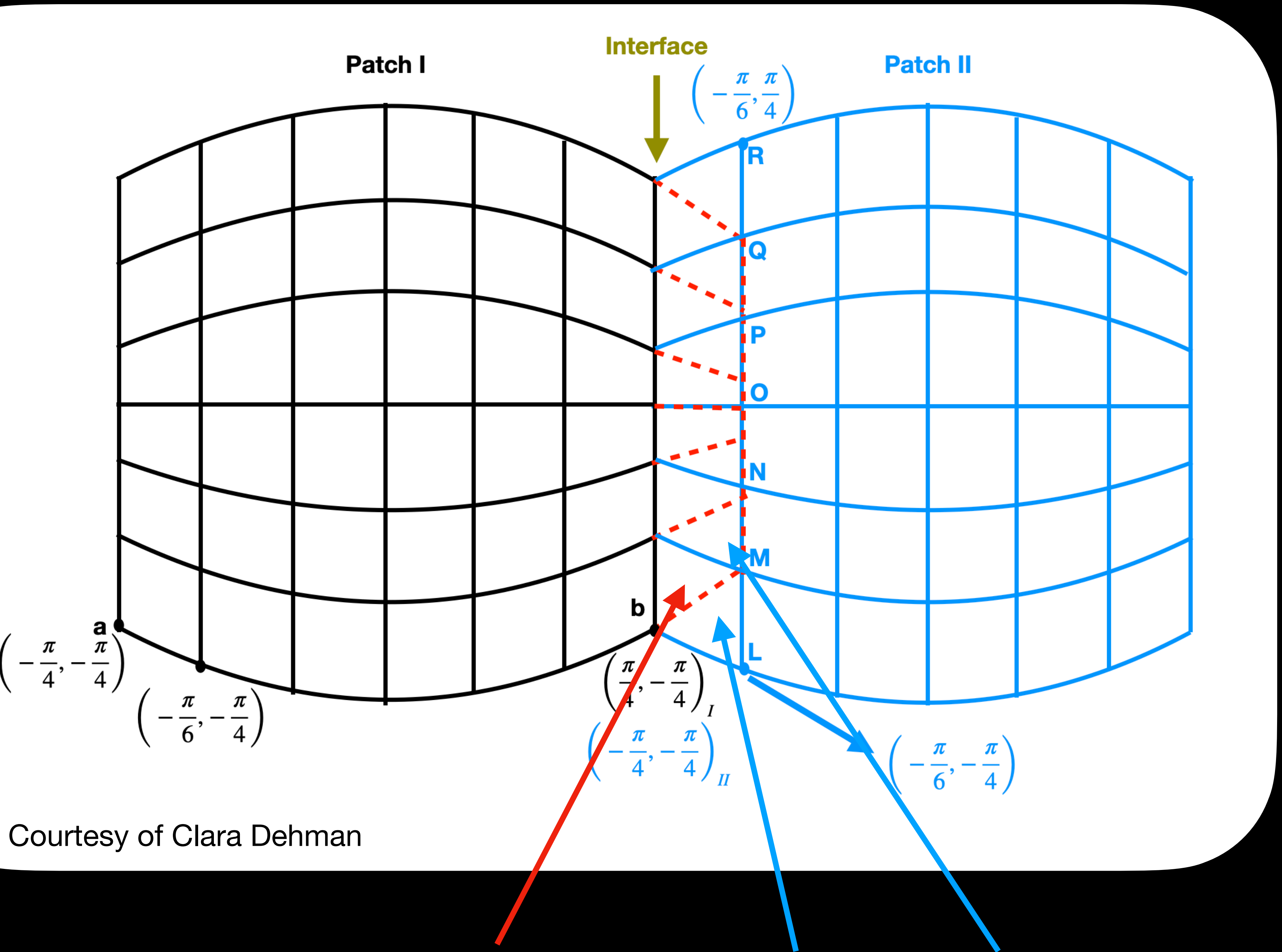
Hall Term



$$-k_\perp(\omega_B\tau_0)(\mathbf{b}\times\nabla\tilde{T})$$

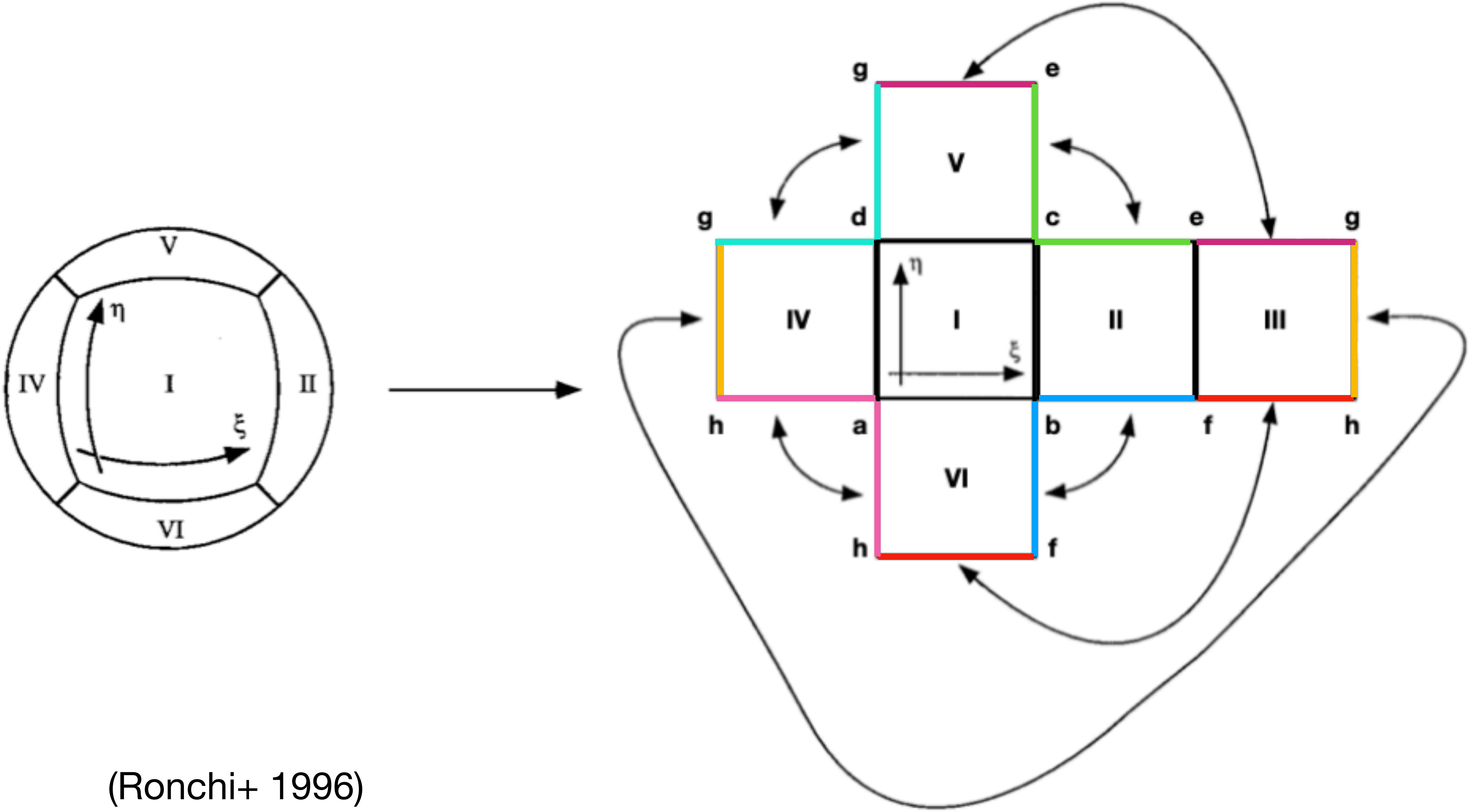
$$e^{\nu(r)}\mathbf{F} = -k_\perp\left[\nabla\tilde{T} + (\omega_B\tau_0)^2(\mathbf{b}\cdot\nabla\tilde{T})\mathbf{b} + (\omega_B\tau_0)(\mathbf{b}\times\nabla\tilde{T})\right]$$

Boundary Between Patches



$$T_{p,gh} = (1 - W_h)T_{pa,1} + W_h T_{pa,2}$$

Grid: Cubed Sphere, exploded view

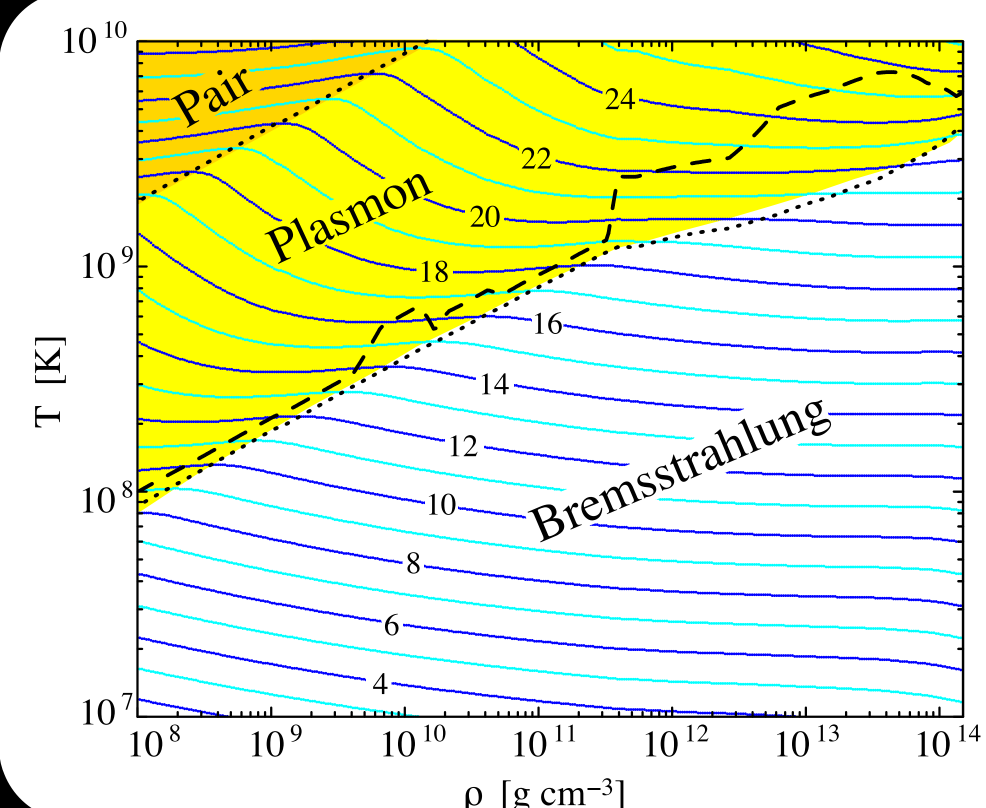


(Ronchi+ 1996)

Neutrino Cooling

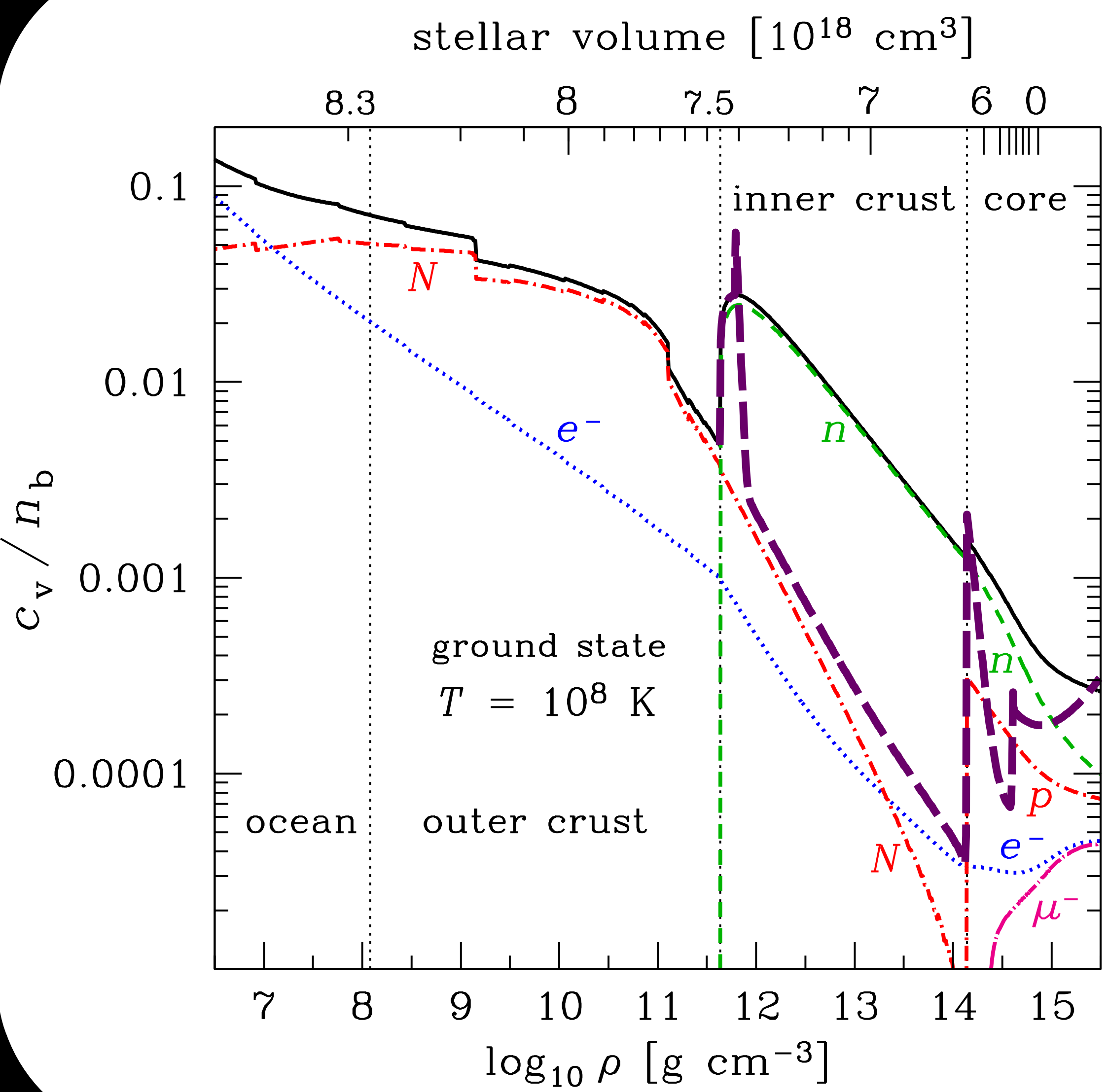
Table 1 Main neutrino emission processes[a]

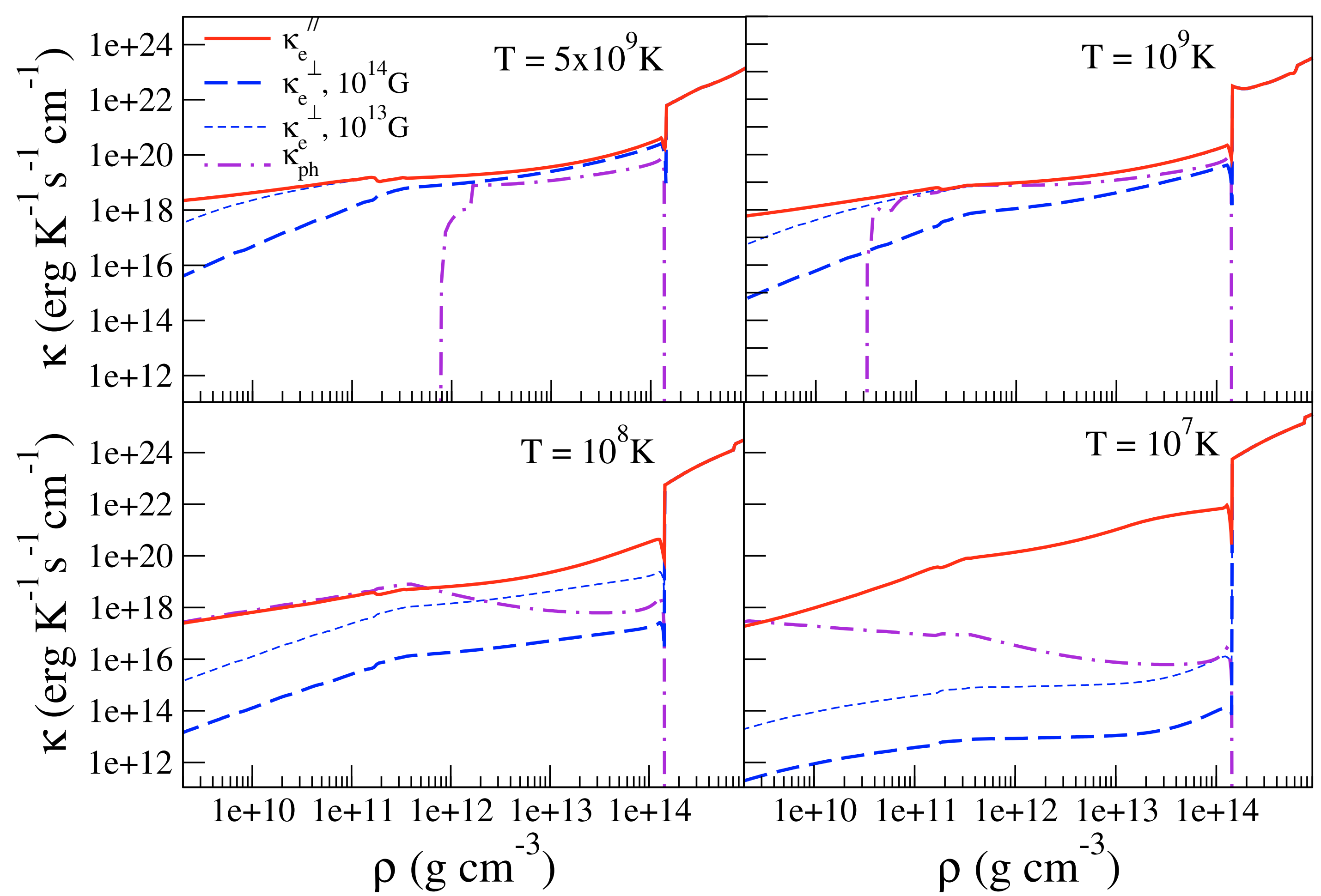
	Process / Control function	Symbolic notation[b]	Formulae for Q_ν and/or R
		In the crust	
1	Plasmon decay	$\Gamma \to \nu + \bar{\nu}$	Eqs. (15) – (32) of [1]
2	Electron-nucleus bremsstrahlung	$e^- + N \to e^- + N + \nu + \bar{\nu}$	Eqs. (6), (16) – (21) of [2]
3	Electron-positron annihilation	$e^- + e^+ \to \nu + \bar{\nu}$	Eq. (22) of [3]
4[c]	Electron synchrotron	$e^- \xrightarrow{B} e^- + \nu + \bar{\nu}$	Eq. (48) – (57) of [3]
		In the core	
1[d]	Direct Urca (Durca)	$n \to p + e^- + \bar{\nu}_e$, $p + e^- \to n + \nu_e$	Eq. (120) of [3]
	Magnetic modification[c]	$R_B^{(\mathrm{D})}$	Eqs. (247) – (250) of [3]
	Reduction factors[e]	$R_x^{(\mathrm{D})}$	Eqs. (199), (202)–(206) of [3]
2	Modified Urca (Murca) (neutron branch)	$n + n \to n + p + e^- + \bar{\nu}_e$, $n + p + e^- \to n + n + \nu_e$	Eq. (140) of [3]
	Reduction factors[e]	$R_x^{(\mathrm{M}n)}$	Appendix of [4]
3	Murca (proton branch)	$p + n \to p + p + e^- + \bar{\nu}_e$, $p + p + e^- \to p + n + \nu_e$	Eq. (142) of [3], corrected at $3p_{\mathrm{F}p} > p_{\mathrm{F}n} + p_{\mathrm{F}e}$ as per [4]
	Reduction factors[e]	$R_x^{(\mathrm{M}p)}$	Appendix (and Eq. (25)) of [4]
4	Baryon-baryon bremsstrahlung	$n + n \to n + n + \nu + \bar{\nu}$	Eq. (165) of [3]
		$n + p \to n + p + \nu + \bar{\nu}$	Eq. (166) of [3]
		$p + p \to p + p + \nu + \bar{\nu}$	Eq. (167) of [3]
	Reduction factors[e]	$R_x^{(nn)}$	Eqs. (221), (222), (228) of [3] and Eq. (60) of [4]
		$R_x^{(np)}$	Eq. (220), (229) of [3] and Eq. (54) of [4]
		$R_x^{(pp)}$	Eq. (221) of [3]
5[e]	Cooper pairing of baryons	$n + n \to [nn] + \nu + \bar{\nu}$, $p + p \to [pp] + \nu + \bar{\nu}$	Eqs. (236), (241) of [3], corrected as per [5] (Sect. 3.3)
6[c,e]	Electron-fluxoid bremsstrahlung	$e^- + f \to e^- + f + \nu + \bar{\nu}$	Eqs. (253), (263), (266) – (268) of [3]



Pothekin+2015

Microphysics





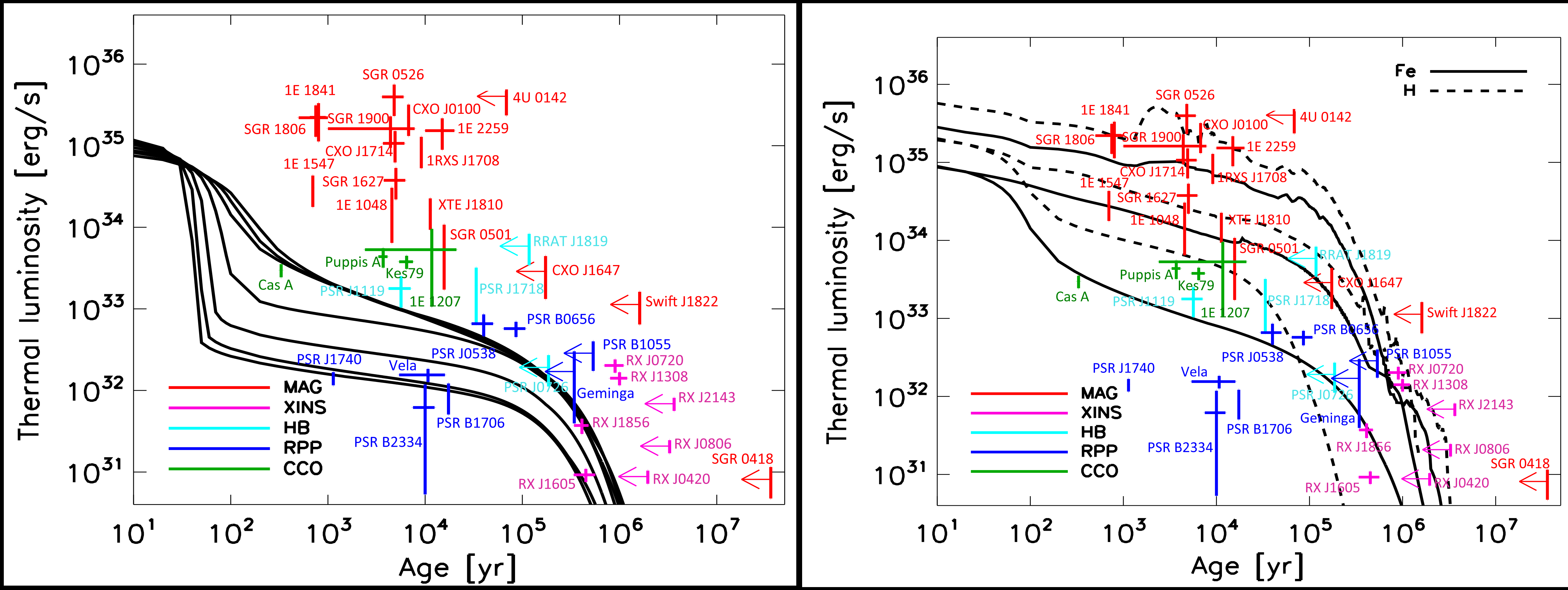
Pothekin+2015

Aguilera+2008

Cooling Curve

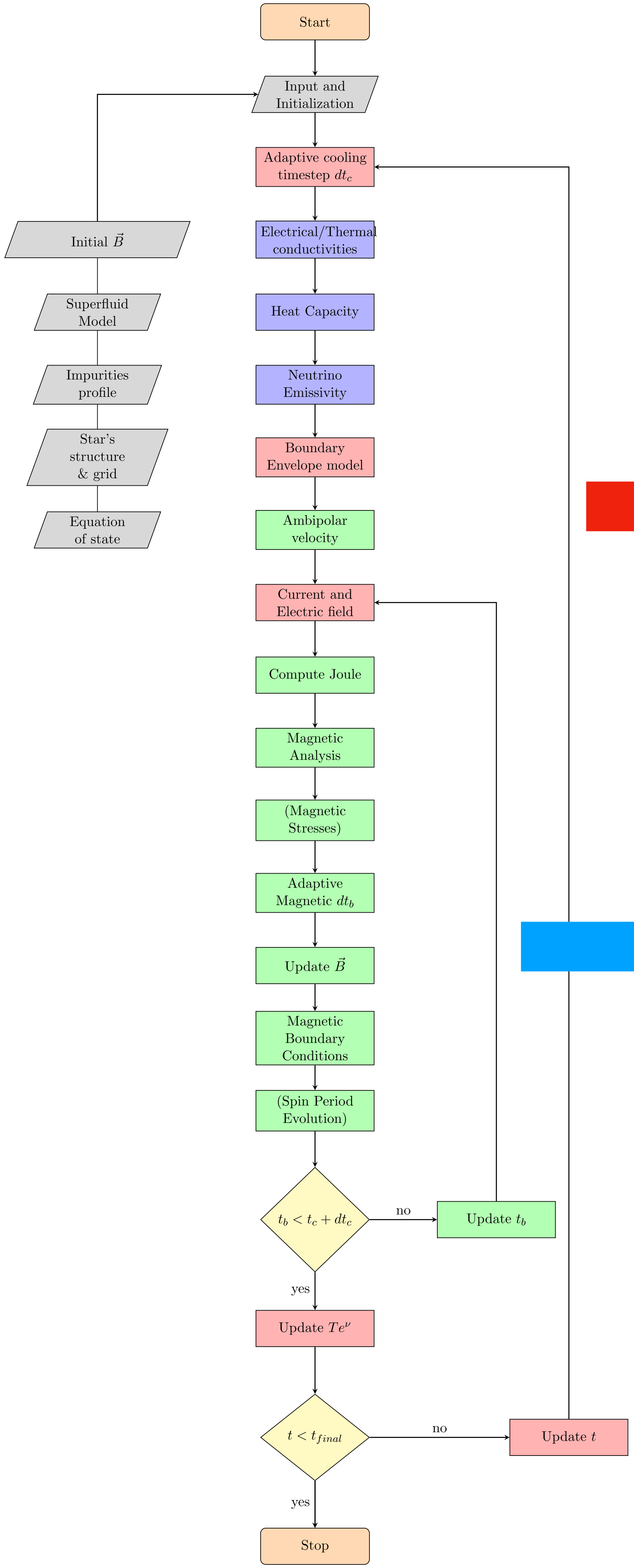
Non-Magnetized

Magnetized



Viganó+2013

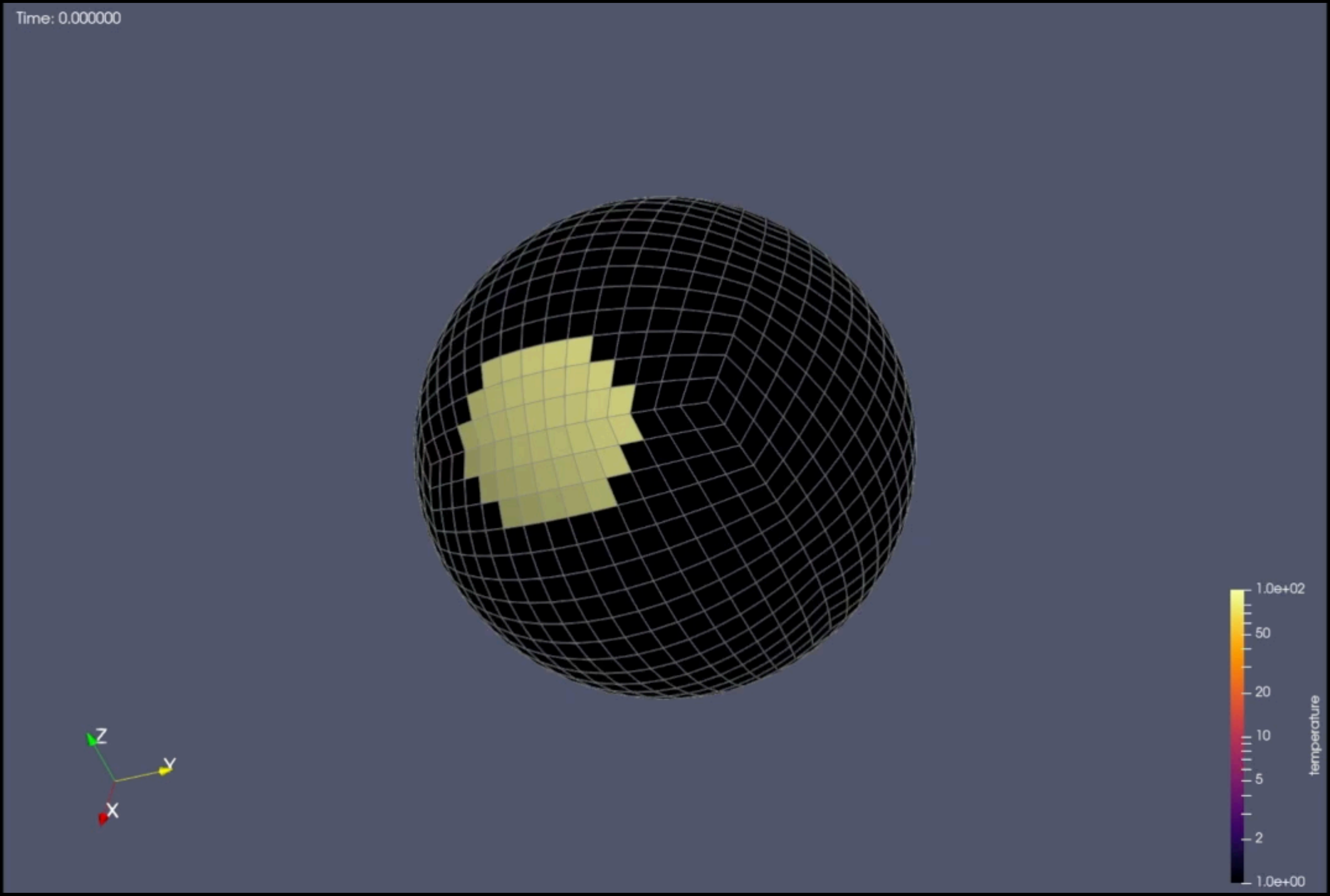
Code Workflow



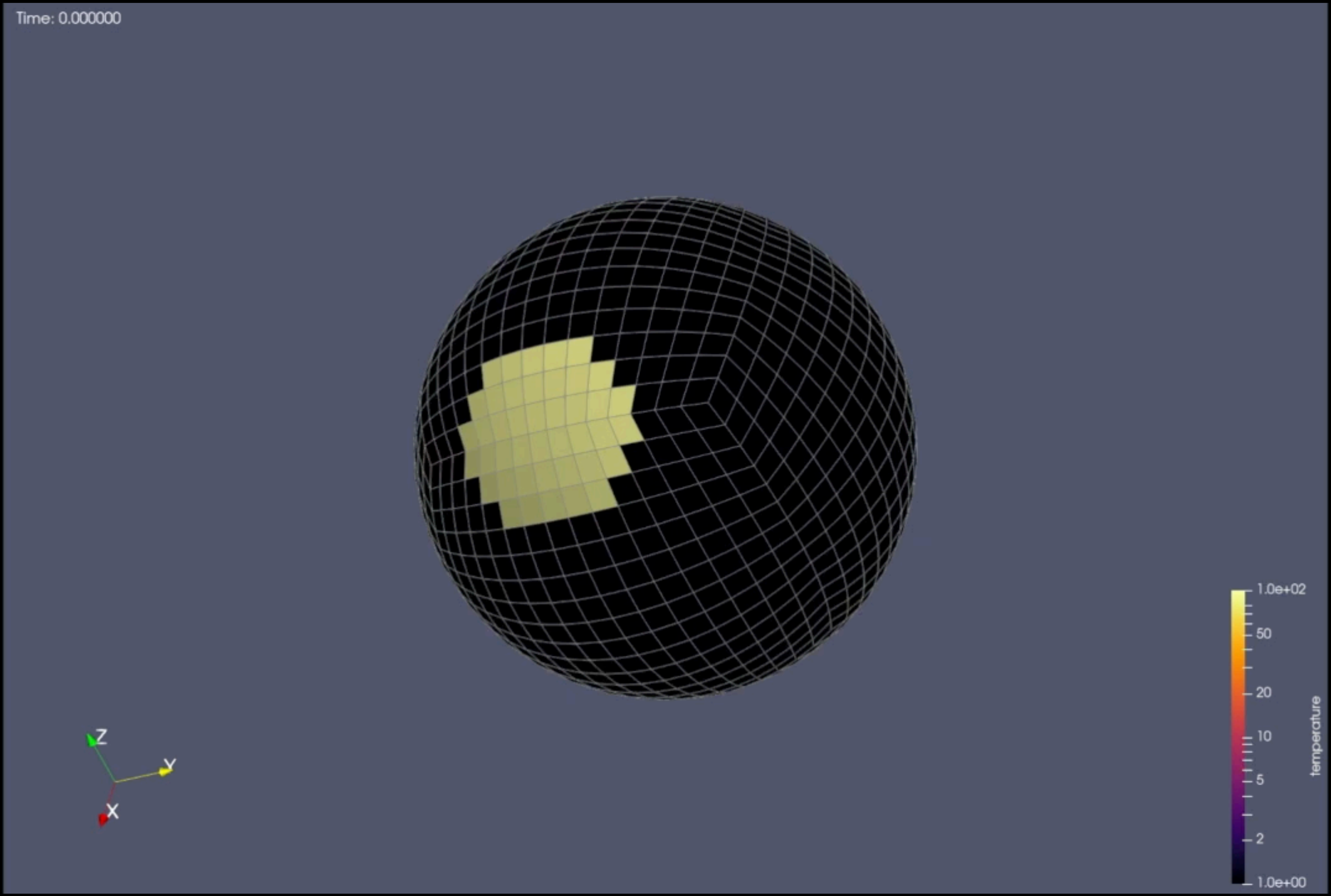
Thermal evolution loop

Magnetic evolution loop

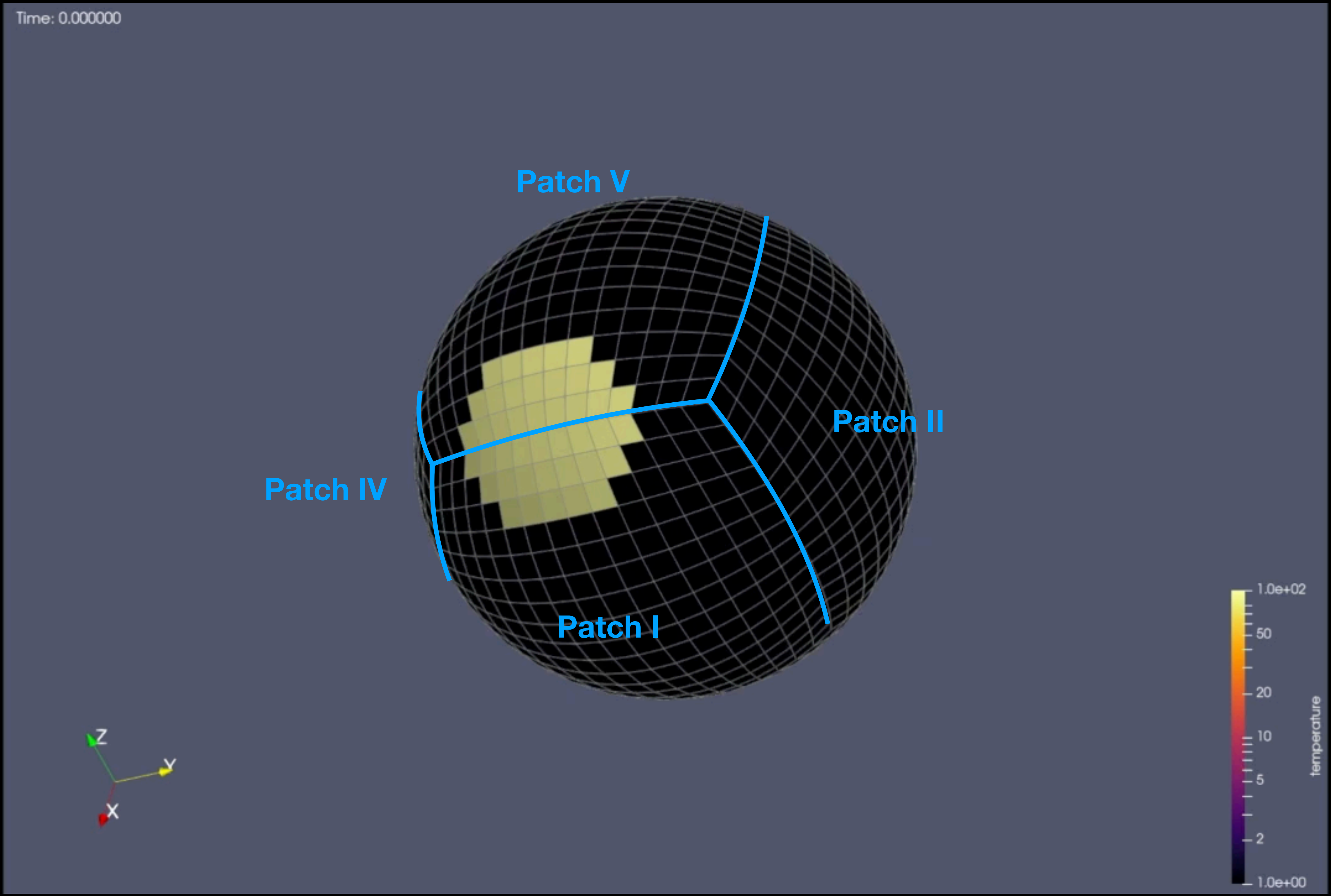
Hot Spot Test



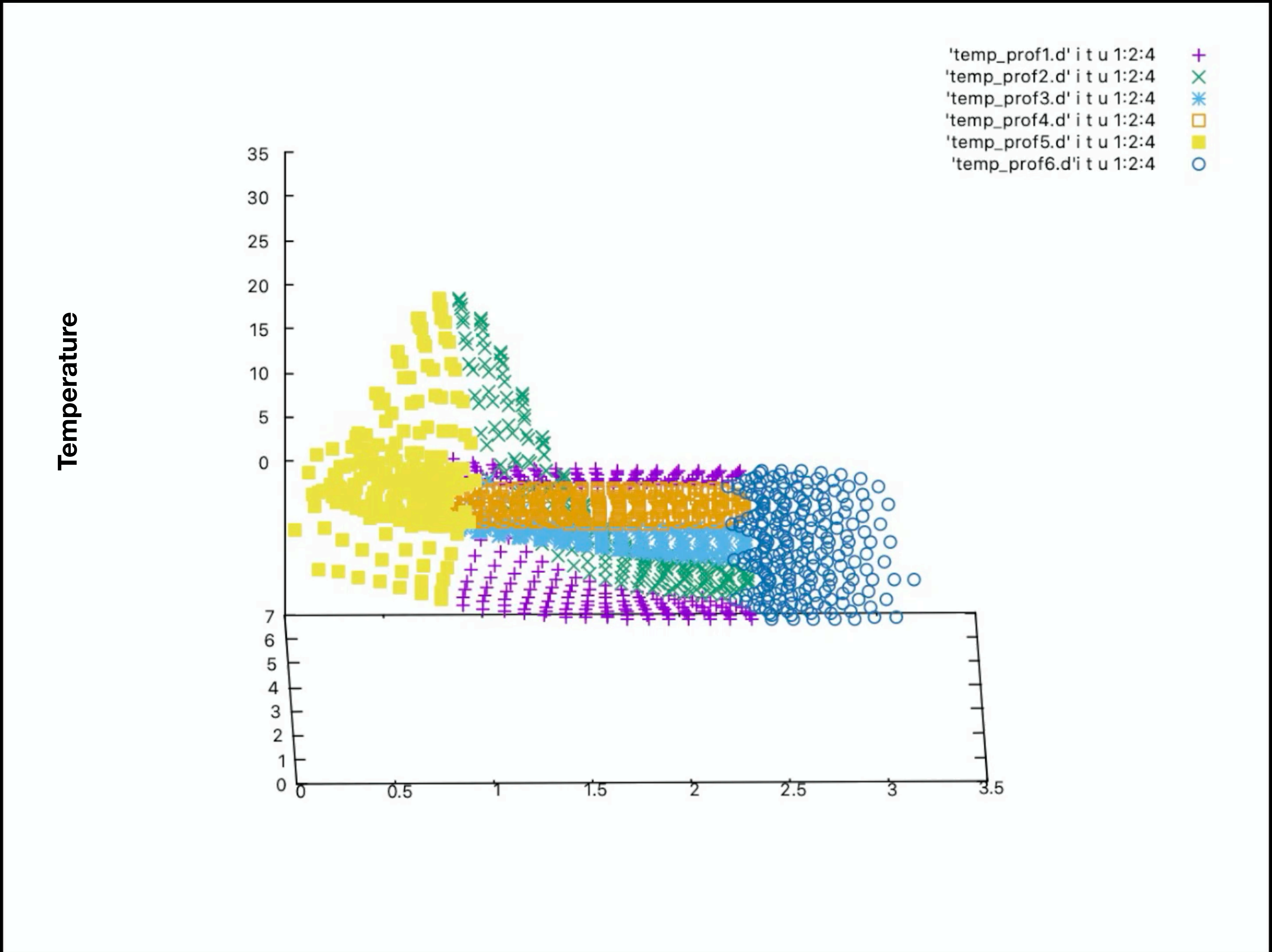
Hot Spot Test



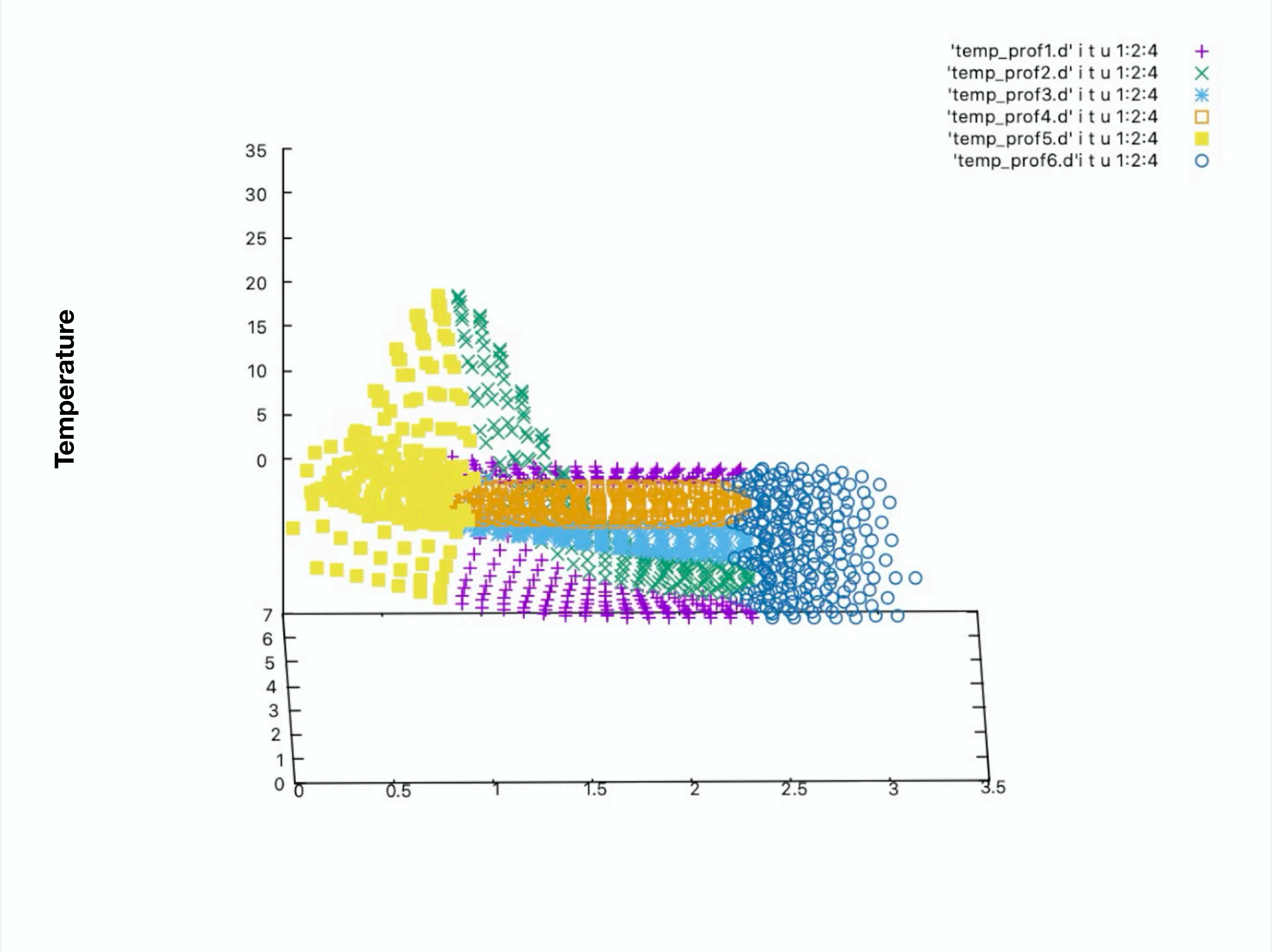
Hot Spot Test



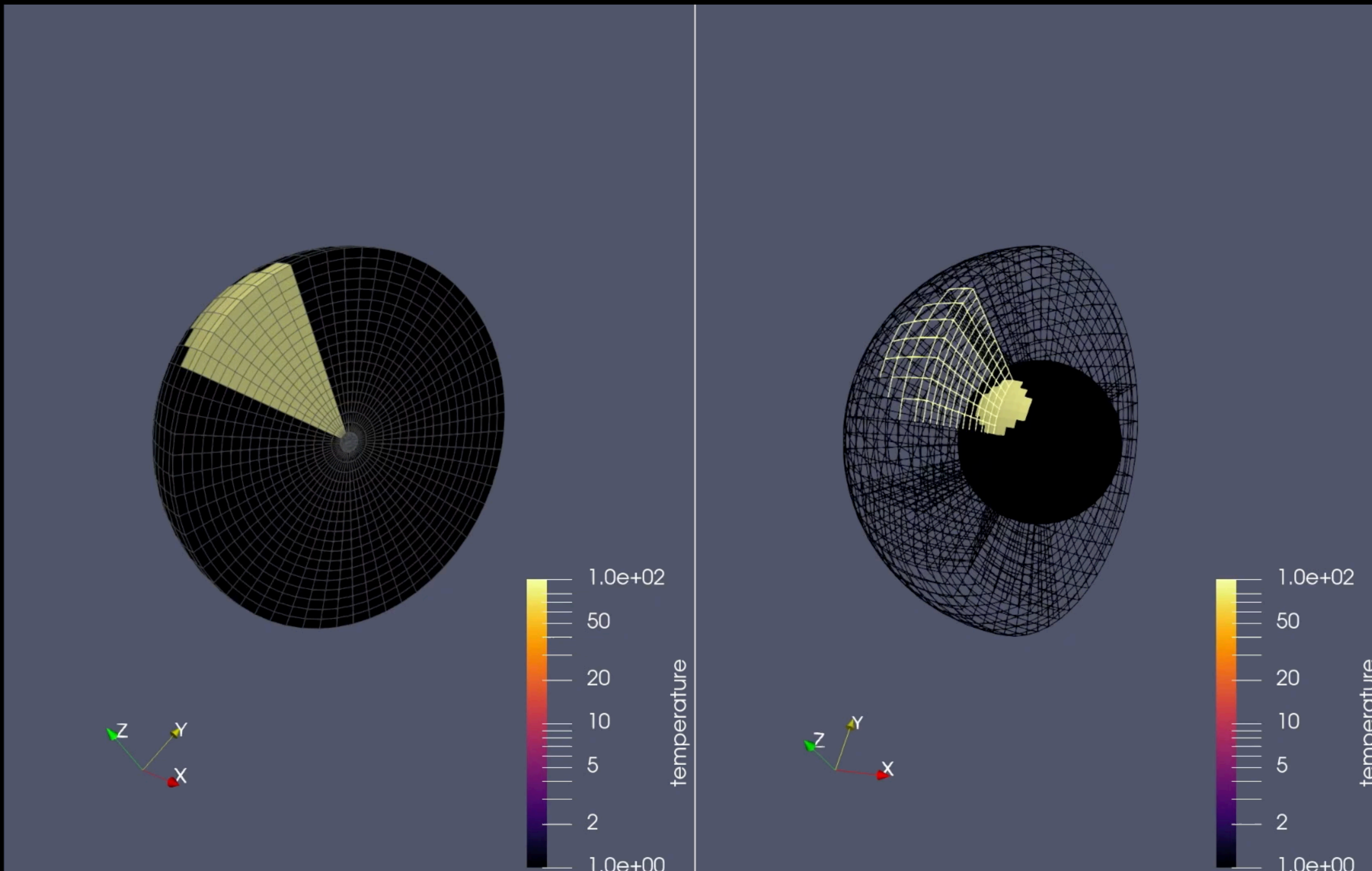
Hot Spot Test



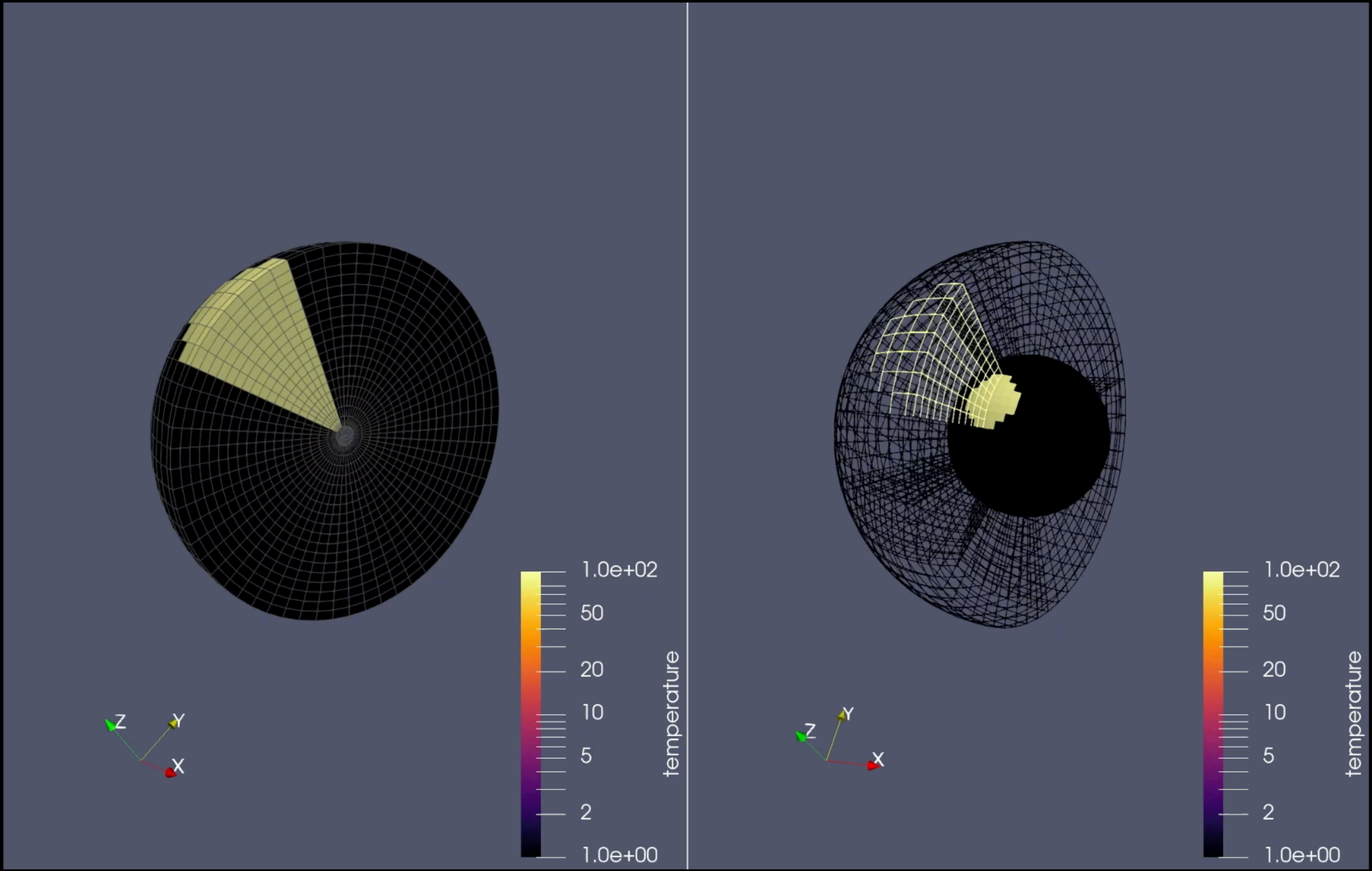
Hot Spot Test



Hot Spot Radial Profile



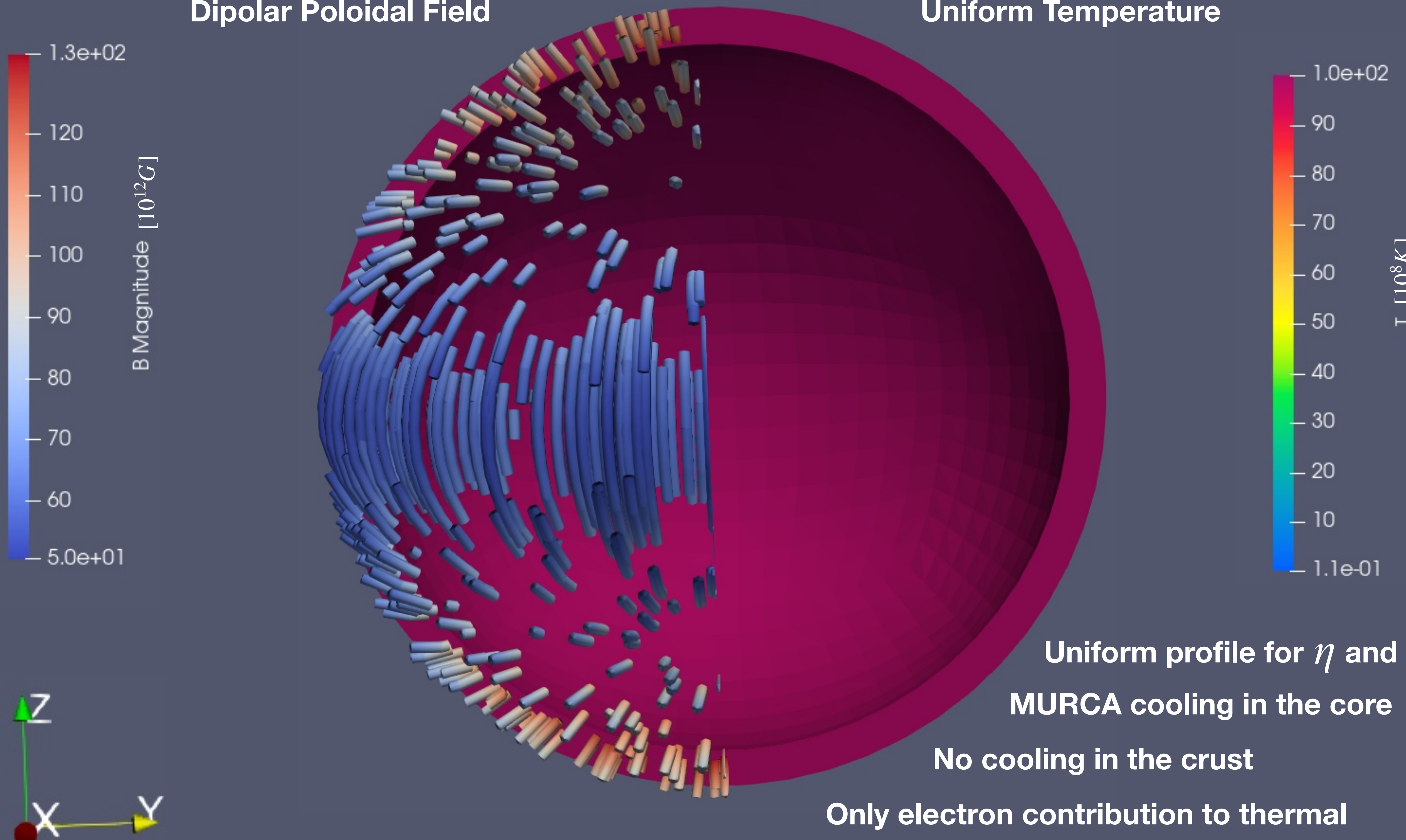
Hot Spot Radial Profile



A Semi-Realistic Model

Dipolar Poloidal Field

Uniform Temperature



Uniform profile for η and n_e

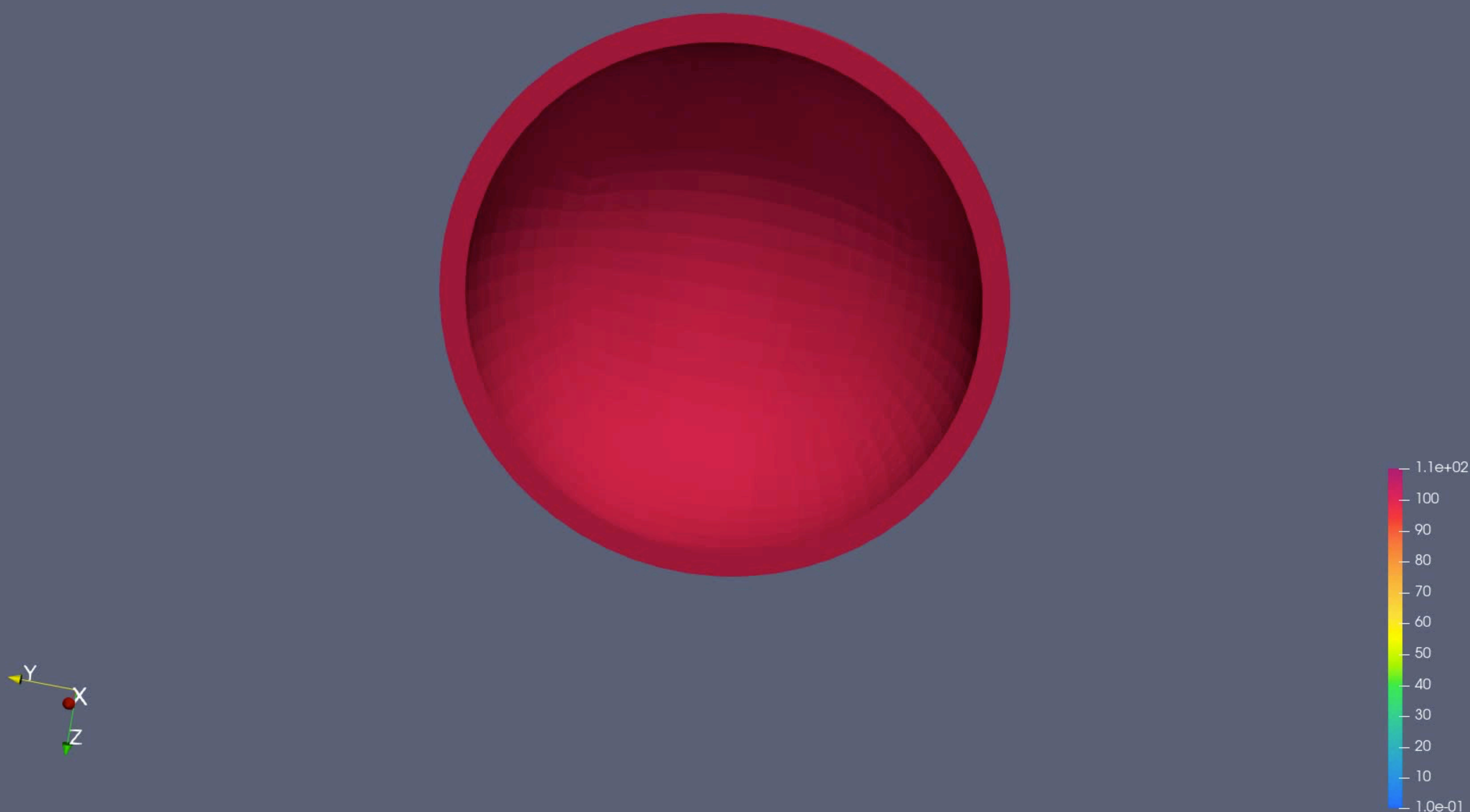
MURCA cooling in the core

No cooling in the crust

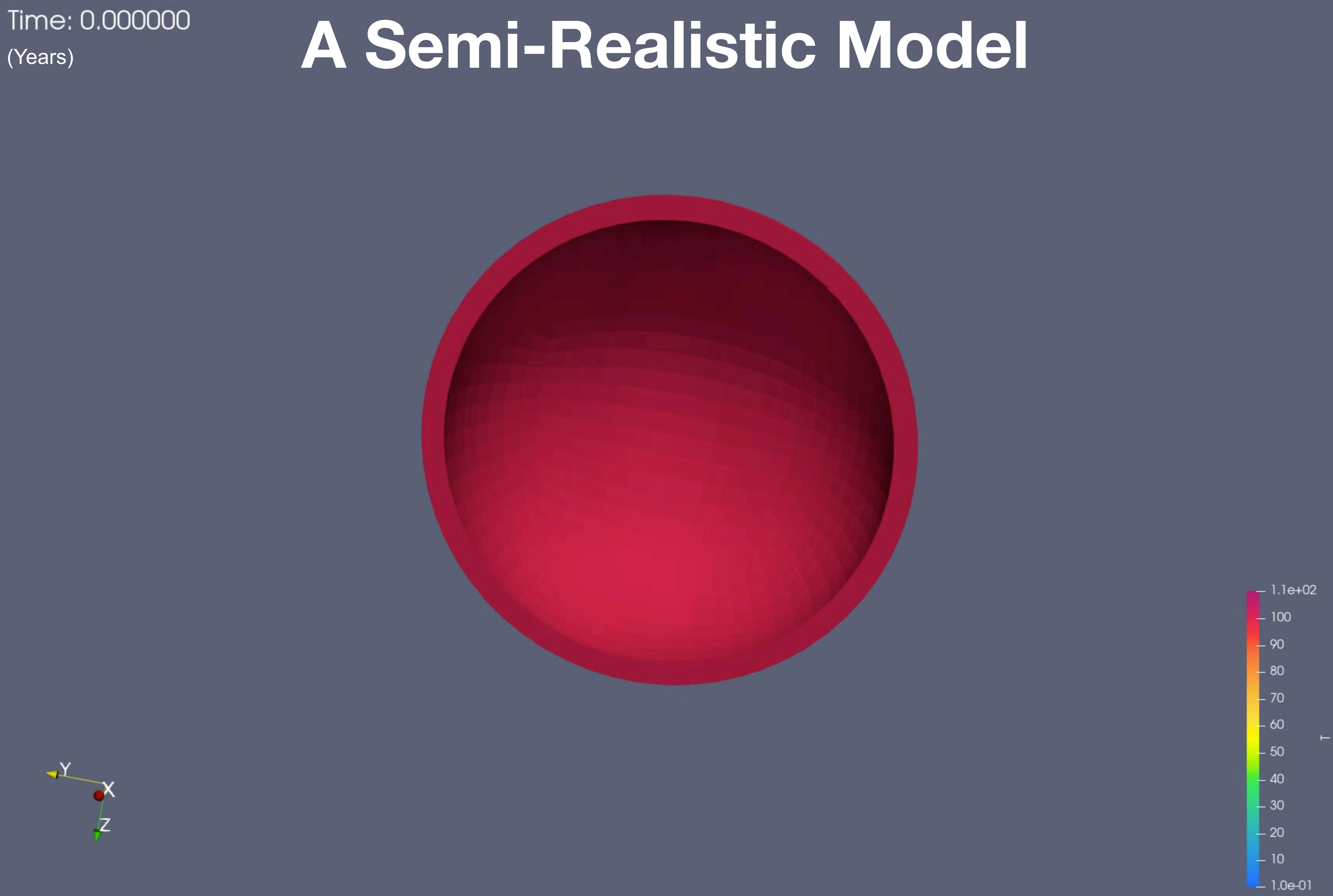
Only electron contribution to thermal conductivity and heat capacity

A Semi-Realistic Model

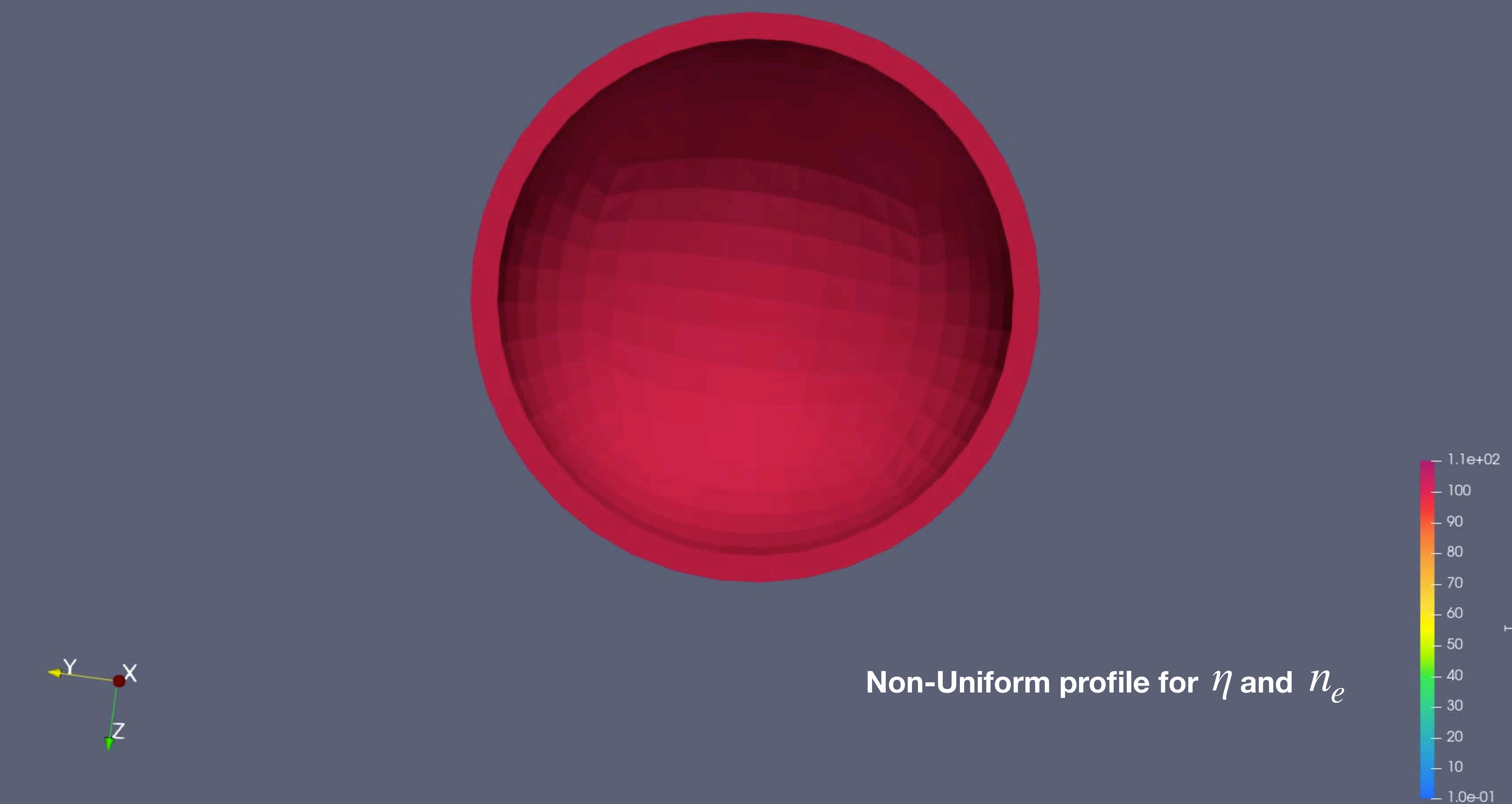
Time: 0.000000
(Years)



A Semi-Realistic Model



A Bit-More-Realistic Model



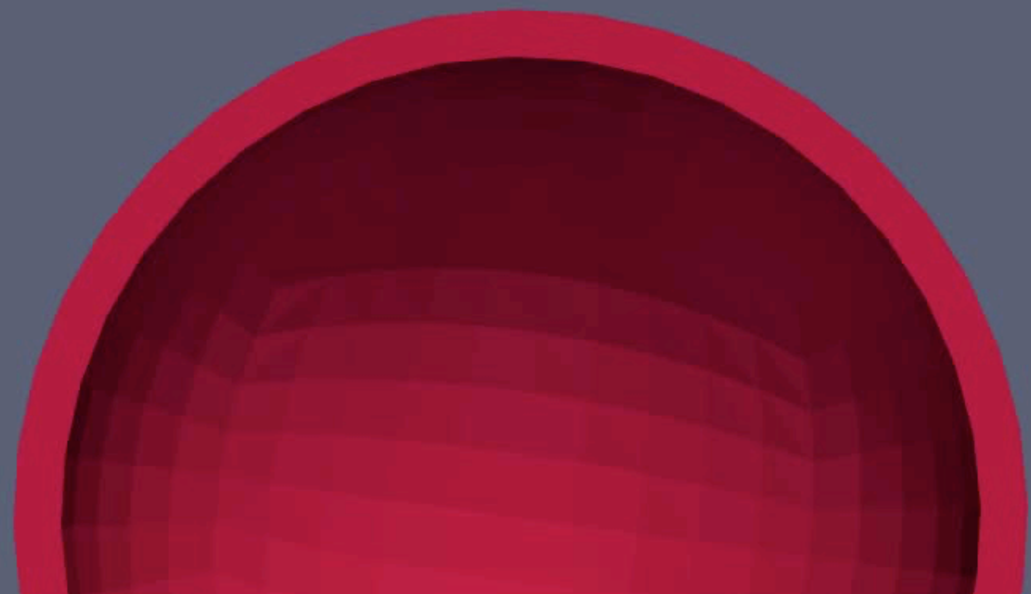
Non-Uniform profile for η and n_e

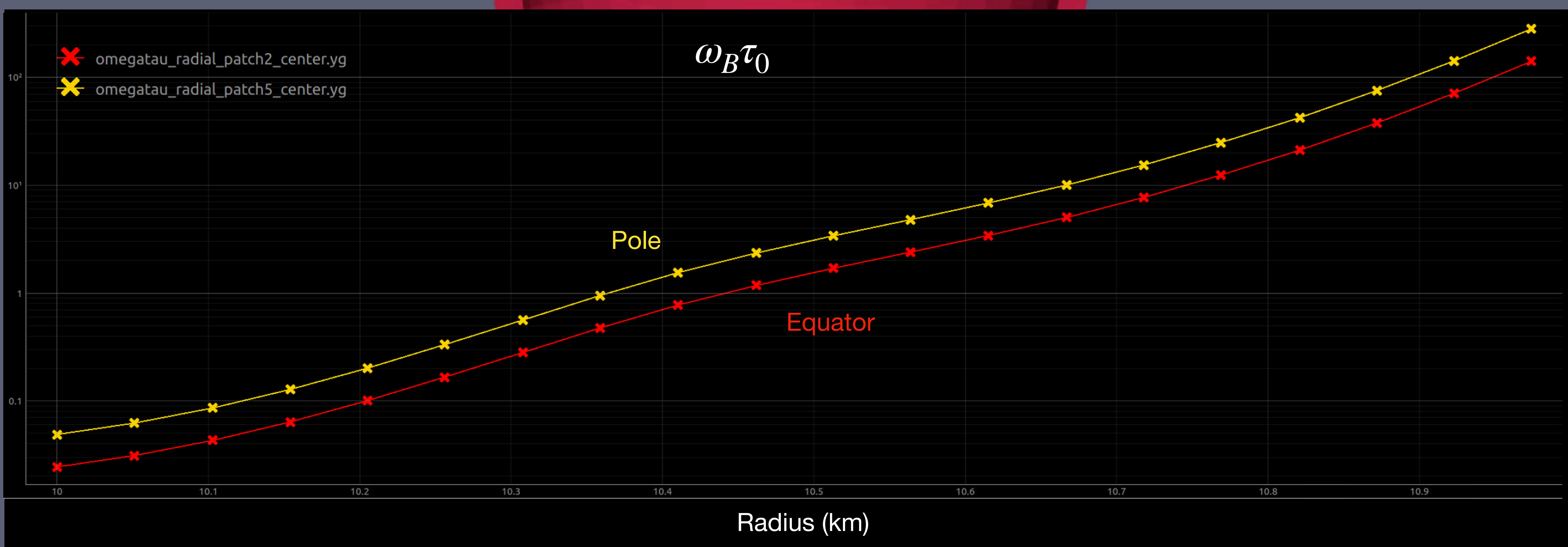
A Bit-More-Realistic Model

Non-Uniform profile for η and n_e

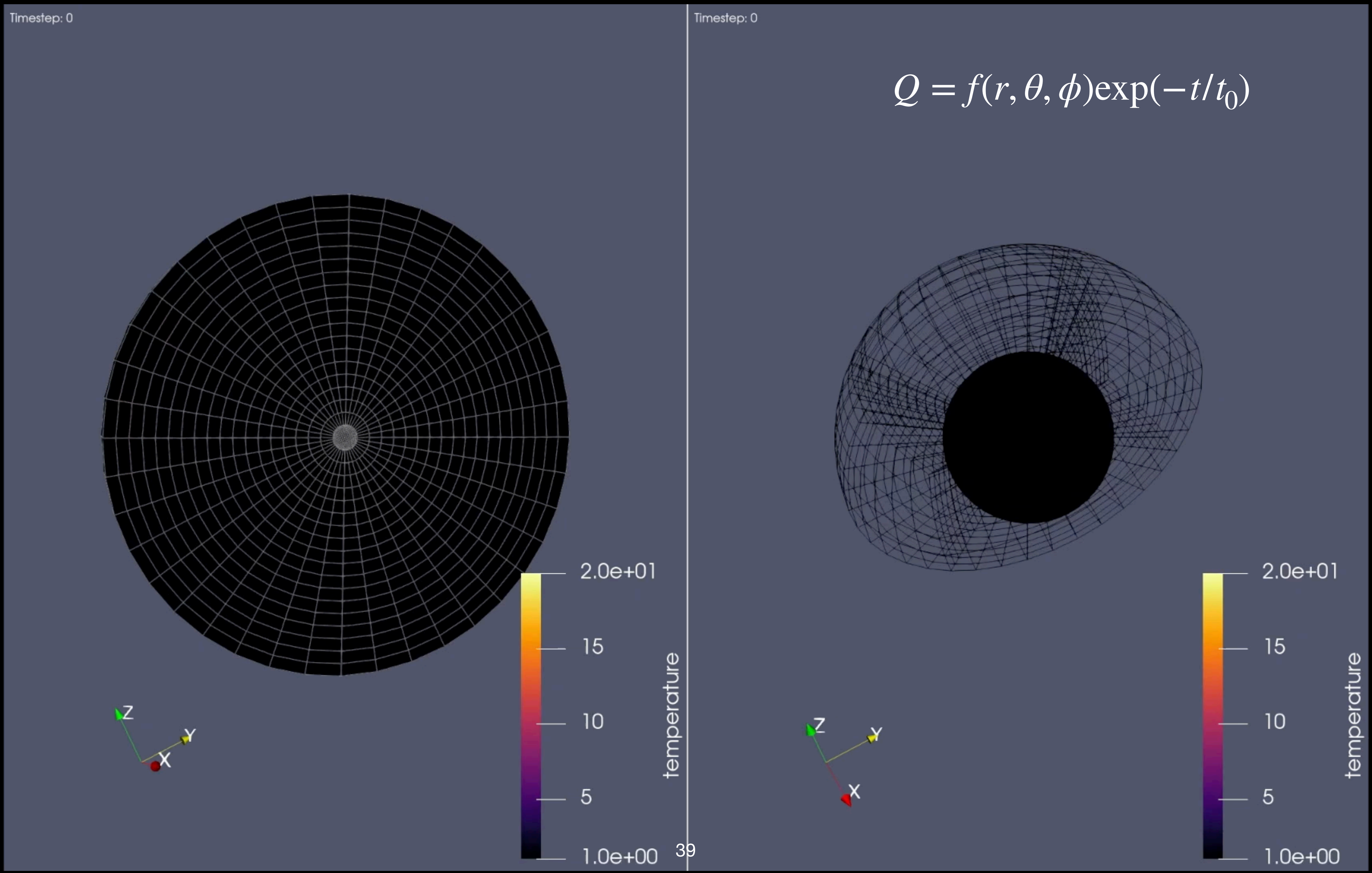
Time: 0.000000
(Years)

A Bit-More-Realistic Model

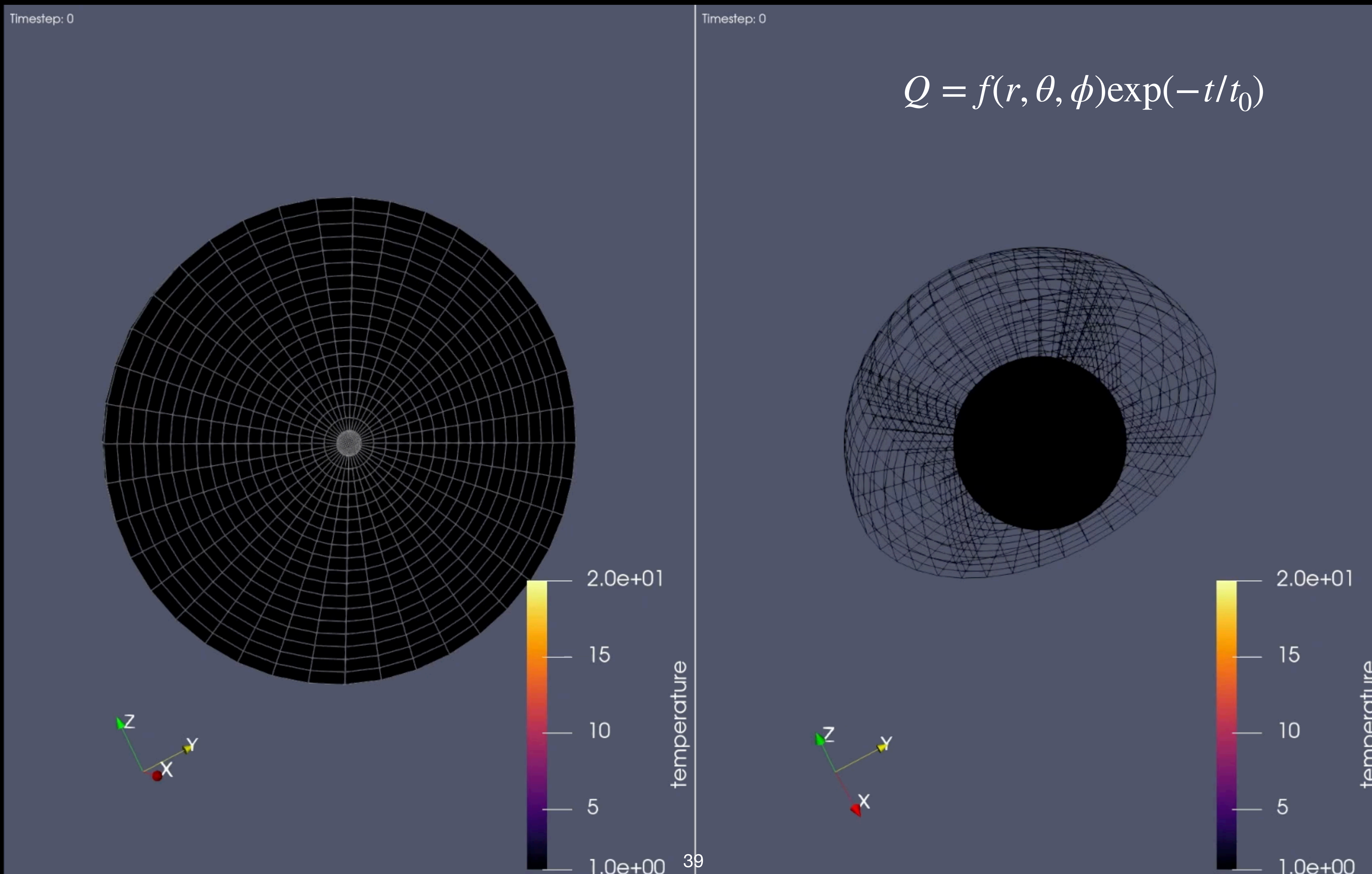




Source Term: Time dependent source



Source Term: Time dependent source



Perez-Azorin+2006 Anisotropic test