Multi-Messenger Continuous Gravitational Wave Workshop

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Universal relations to measure neutron star properties from targeted r-mode searches

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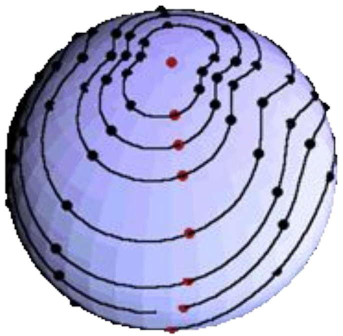
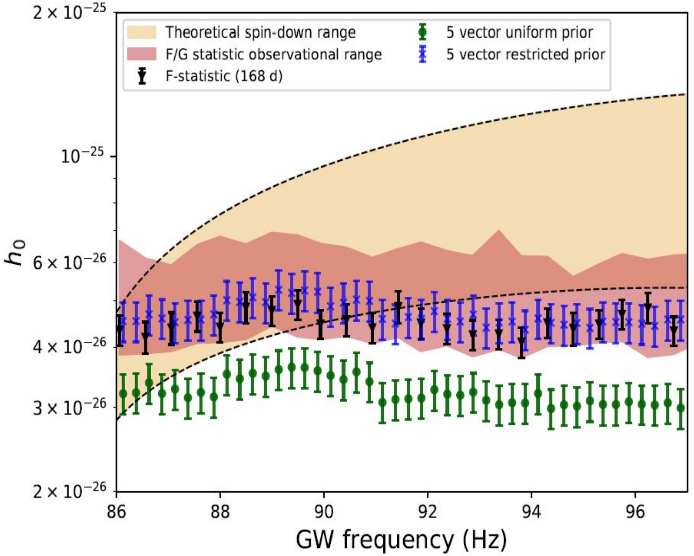
R-modes as source of CGW

- ➢ Toroidal mode of fluid oscillation in NS for which the restoring force is the Coriolis force.
- ➢ Unstable in all rotating stars due to the CFS mechanism although dissipation mechanisms can damp and saturate the oscillations.
- ➢ Spindown of young pulsars leading to continuous GW emissions.



Courtesy : C. Hanna and B. Owen



- ➢ GW Searches using LVK data :
  - Crab pulsar (Rajbhandari et al. 2021)
  - PSR J0537-6910 with n ≈ 7 (Fesik & Papa 2020, Abbott et al. 2021)
- ➢ No detection of GW but upper limits on r-mode amplitude were obtained.

R-modes frequency and Universal Relations

- Newtonian limit ; $k = f_{GW} / f_{rot} = 2m / l(l+1)$
- Correction due to GR and rapid rotation:

$$f_{GW} / f_{rot} = A - B\,(f_{rot}/f_K)^2$$

- For slowly rotating stars, effect of the term B is negligible.

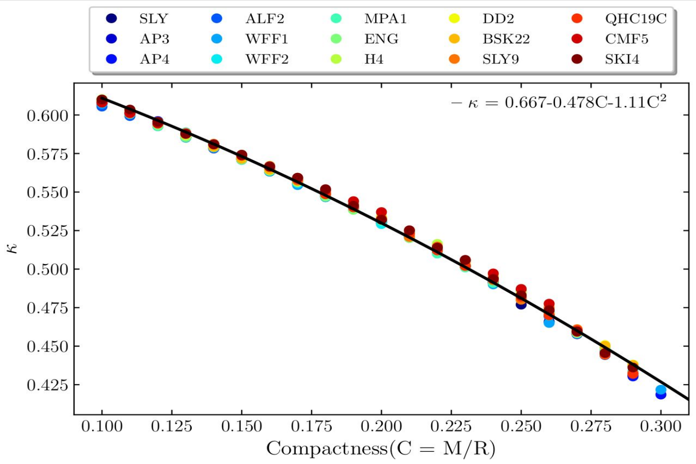
$$k = 0.627 + 0.079C - 2.25C^2$$

Idrisy et al., PRD 91,024001(2015)

$$k = 0.667 - 0.478C - 1.11C^2$$

Ghosh et al ApJ 944, 53 (2023)

Universal relation (compactness range)	κ	A
(Idrisy et al. 2015)(0.11-0.31)	0.614-0.433	1.39-1.57
Tabulated EOS(38)(0.11-0.31)	0.601-0.412	1.40-1.59
Tabulated EOS(38)(0.10-0.35)	0.608-0.364	1.39-1.64
Non-parametric EOS(40)(0.11-0.31)	0.596-0.415	1.39-1.59
Non-parametric EOS(40)(0.10-0.35)	0.604-0.371	1.40-1.63



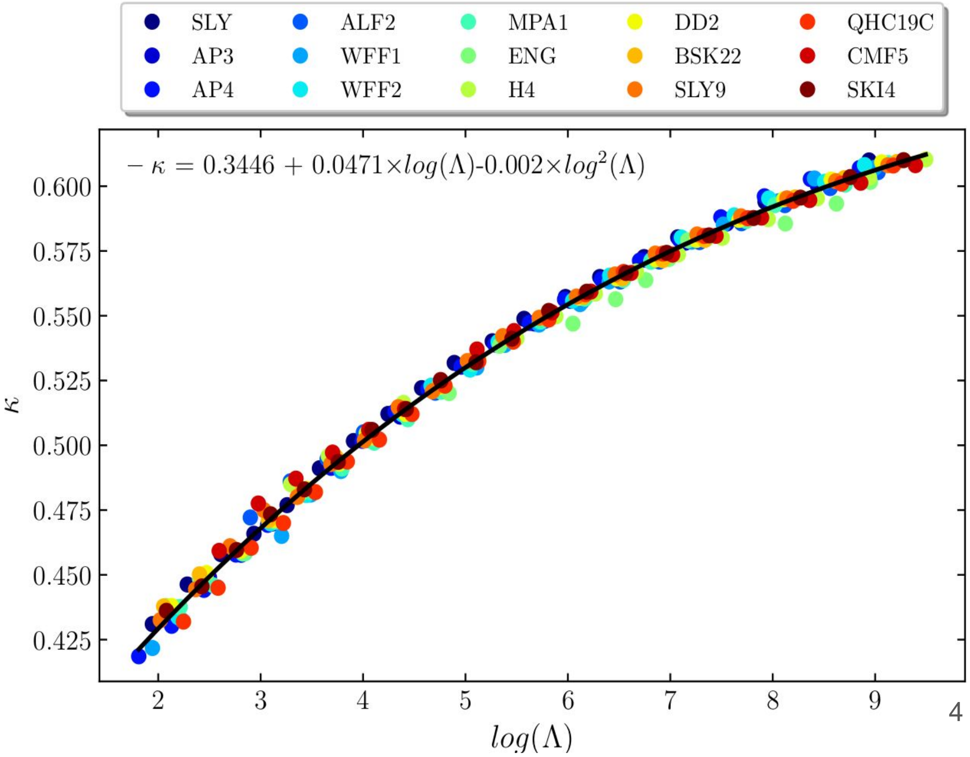
- range of A : **1.39 < A < 1.59** **PSR J0537** : 86 - 99 Hz (86-97 Hz)

Universal relations (Tabulated EoS)

Ghosh et al ApJ 944, 53 (2023)

$$k = 0.3446 + 0.0471\log(\Lambda) - 0.002\log^2(\Lambda)$$

- Universal relation between r-mode frequency and dimensionless tidal deformability.
- These can be used to estimate the effects of dynamical tides from r-mode excitation during binary NS inspirals. (**Gupta et al. arxiv: 2205.01182, Ma et al. PRD 103, 063020 (2021)**)



Distance measurement of GW sources

Distance estimation from binary mergers

- Inspiralling binaries are "Standard Siren"
- GW amplitude

$$h_0 = \frac{4\pi^2 G^{5/3}}{c^4}(f_{GW}\mathcal{M})^{5/3}\frac{1}{f_{GW}d}$$

- From the orbital decay energy

$$\dot{f}_{GW} = \frac{96}{5}\pi^{8/3}\left(\frac{G\mathcal{M}}{c^3}\right)^{5/3} f_{GW}^{11/3}$$

- Eliminate $\mathcal{M}$ to get luminosity distance

$$d = \frac{5c}{24\pi^2}\frac{1}{h_0}\frac{\dot{f}_{GW}}{f_{GW}^3}$$

- Measurement of H_0 if location can be fixed by independent method.

Schutz B. F., Nature 323, 310 (1986)

Distance estimation from CGW emission

- CGW are not so "Standard Siren"
- GW amplitude

$$h_0 = \sqrt{\frac{512\pi^7}{5}}\frac{G}{c^5}(\alpha M R^3 \tilde{J})\frac{1}{d}f_{GW}^3$$

- Assuming Spin-down is dominated by r-mode emission

$$\dot{f}_{GW} = -\frac{4096\pi^7}{25}\frac{G}{c^7}\frac{M^2R^6\tilde{J}^2}{I}\alpha^2 f_{GW}^7$$

- Eliminate $\alpha M R^3 \tilde{J}$ to get luminosity distance

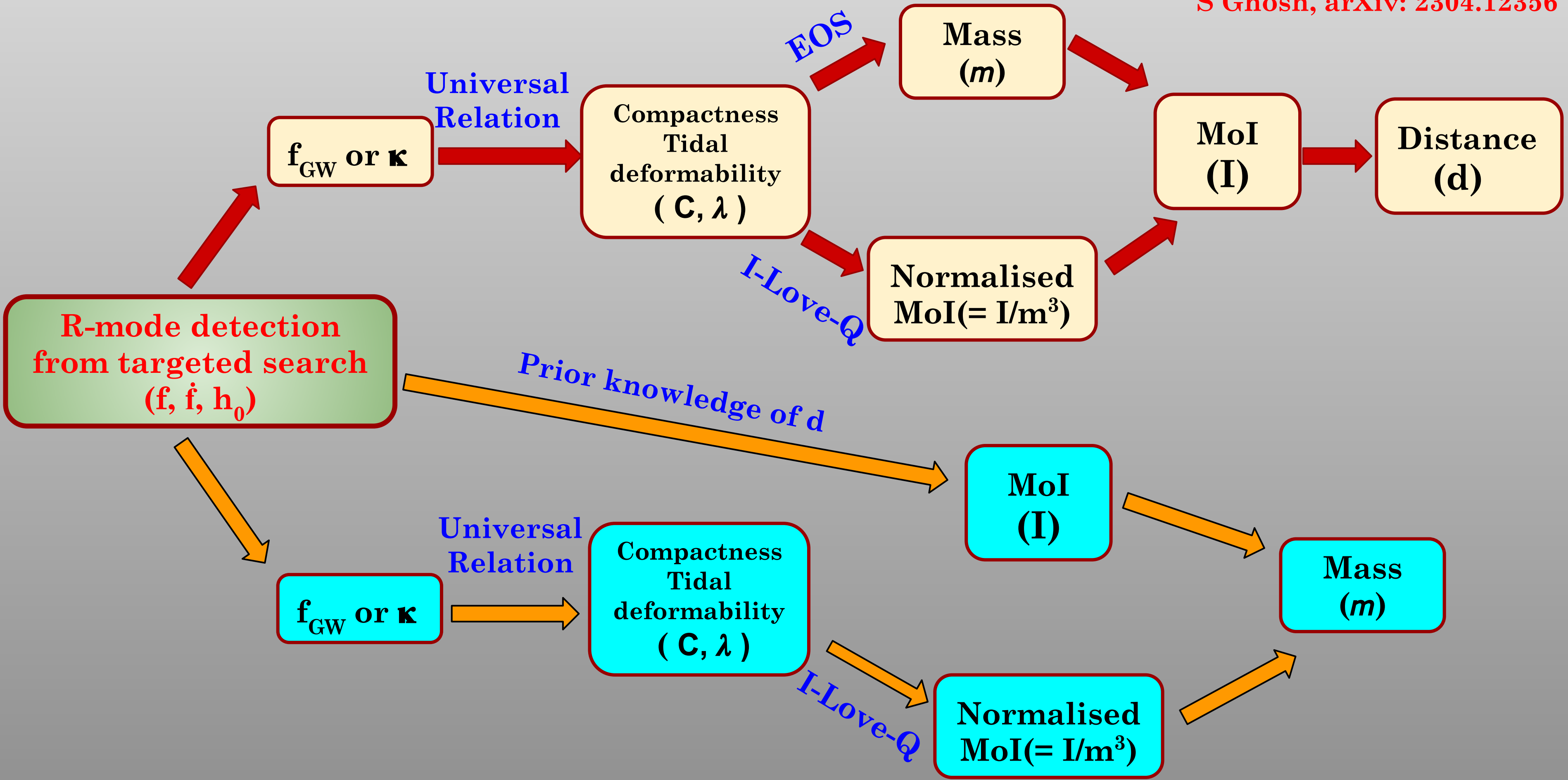
$$\frac{\sqrt{I}}{d} = h_0\sqrt{\frac{f_{GW}}{\dot{f}_{GW}}}\sqrt{\frac{8c^3}{45G}}$$

- Distance is always degenerate with moment of inertia.

Sieniawska & Jones, MNRAS 509, 5179 (2022)

Measuring NS parameters from a r-mode detection

S Ghosh, arXiv: 2304.12356



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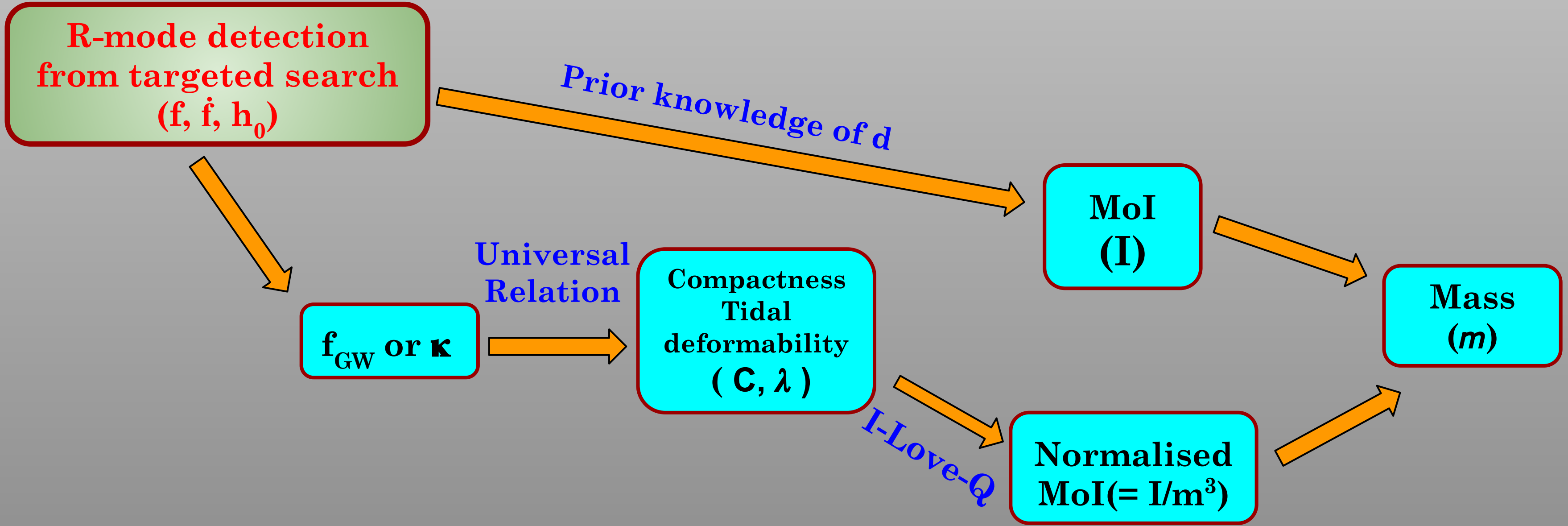

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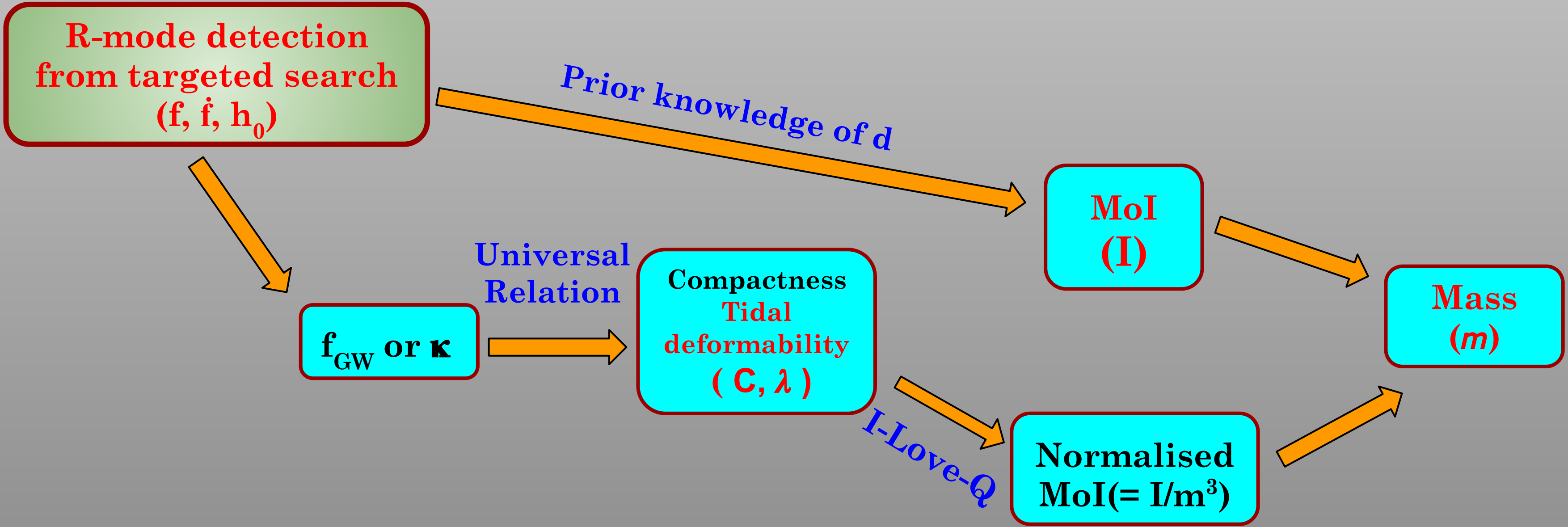


Signal Model and Fisher matrix

Jaranowski & Królak, PRD 59, 063003 (1999)

- Upto second order in frequency derivatives, the phase is given by

$$\psi = \psi_0 + 2\pi\left[ft + \frac{1}{2}\dot{f}t^2 + \frac{1}{6}\ddot{f}t^3\right]$$

- The Fisher Covariance matrix is given by the inverse of the below matrix

$$\Gamma = \frac{g^{-1}}{\rho^2} \text{ where } g_{ij} = \left\langle \frac{\partial\psi}{\partial f^{(i)}}\frac{\partial\psi}{\partial f^{(j)}}\right\rangle - \left\langle \frac{\partial\psi}{\partial f^{(i)}}\right\rangle\left\langle \frac{\partial\psi}{\partial f^{(j)}}\right\rangle$$

- ϱ^2 is the signal-to-noise ratio(SNR) assuming optimal match between the true signal and the best-fit template

$$\rho^2 = \int_0^\infty \frac{4|\tilde{h}(f)|^2}{S_n(f)}df$$

- The error for any physical quantity can be calculated from the Fisher matrix information. For example,

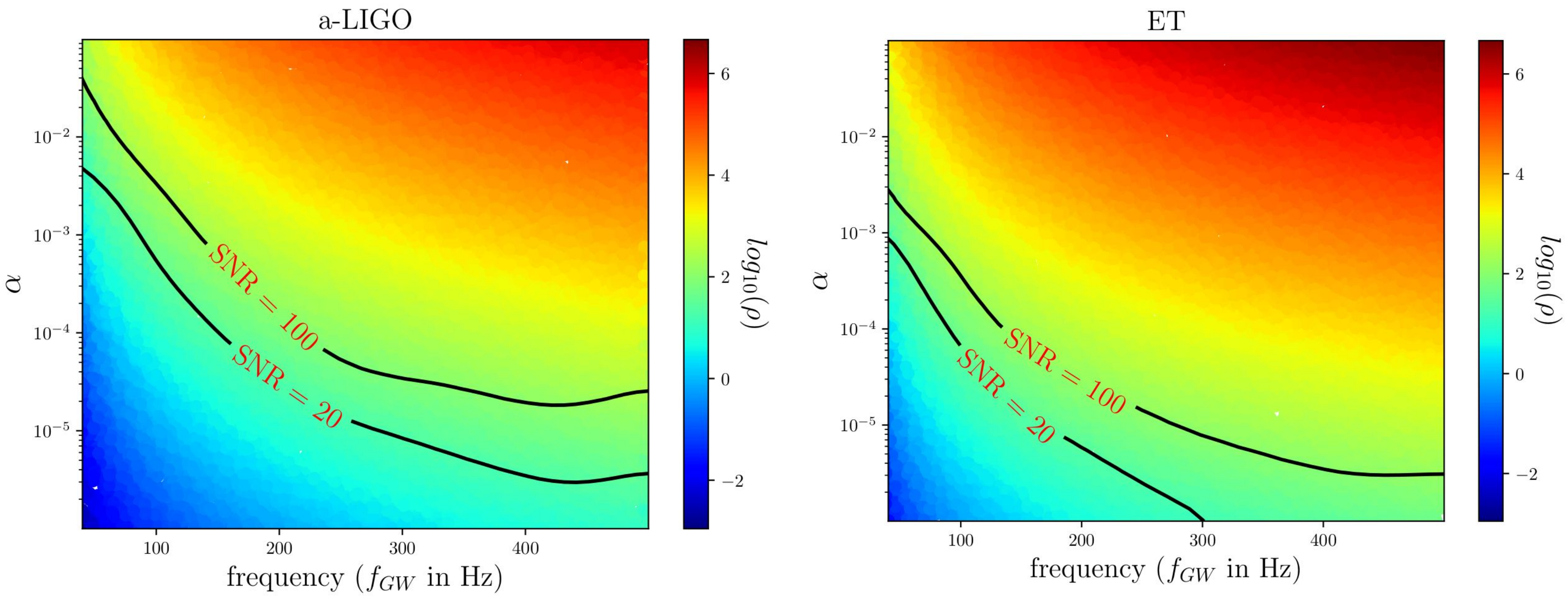
$$\sigma\left(\frac{d}{\sqrt{I}}\right)^2 = \frac{45G}{8c^3}\frac{1}{(\pi\rho h_0)^2}\left[\frac{75}{T}\frac{\dot{f}}{f^3} + \frac{1620}{T^4 f\dot{f}} + \frac{675}{T^3 f^2} + \pi^2\frac{\dot{f}}{f}\right]$$

S Ghosh, arXiv: 2304.12356

SNR Estimates

S Ghosh, arXiv: 2304.12356

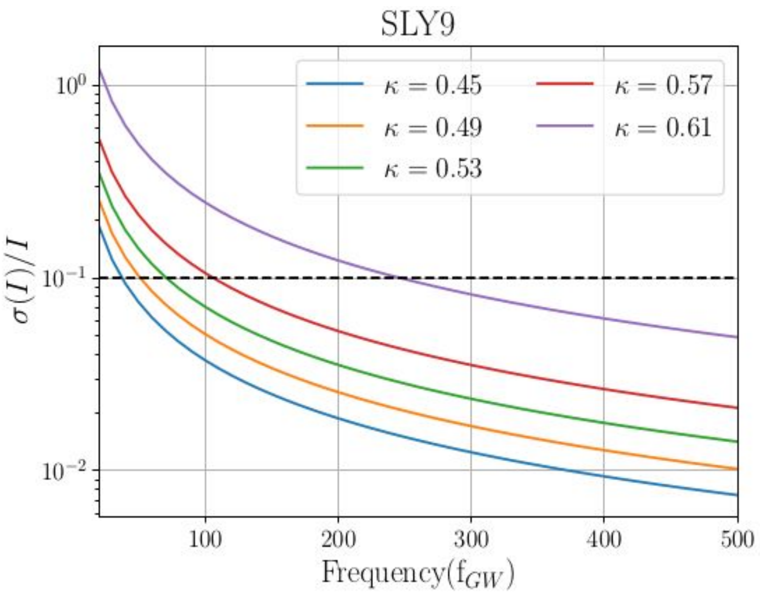
- A canonical NS at a distance of 1 kpc
- Observing period of 2 years with a-LIGO and ET sensitivity curves.

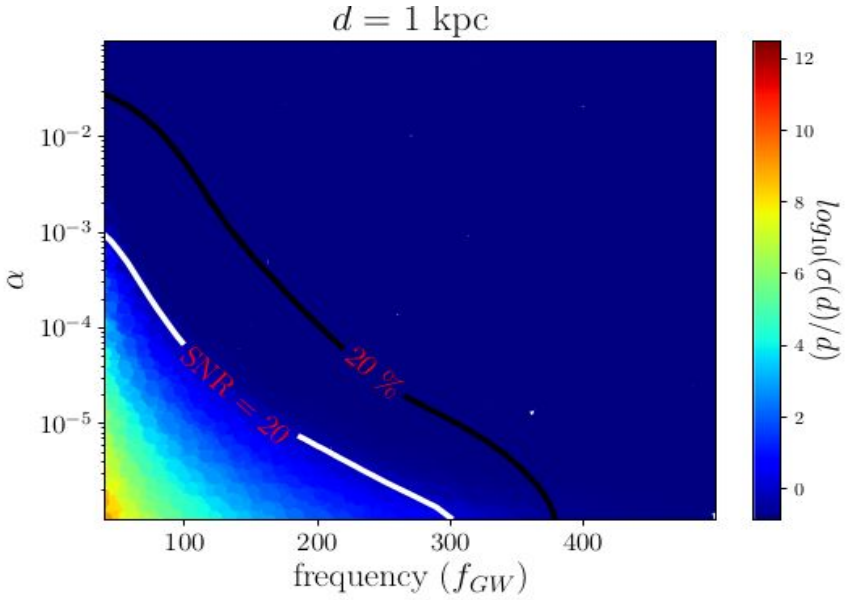


Error Estimations for MoI(I) and distance(d)

S Ghosh, arXiv: 2304.12356

- Error estimates does not change much with different EOSs.
- For most reasonable values of κ, MoI can be measured upto 10% accuracy for f_{GW} > 100 Hz.
- For detectable signals we can measure the distance up to an accuracy of 20% for f_{GW} > 200 Hz and α > 10^{-4} and these estimates scale accordingly with changing distance





Assumptions and Validations of the study

- **Entire spin-down is driven by CGW emission via r-modes - Not true for all pulsars** !!
  - Most promising candidate PSR J0537-6910 has $n \approx 7$.
  - Possible to re-do energy budget calculation given knowledge of n. (Lu et al. MNRAS, 521 (2023))

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- **Presence of solid crust (Yuri 2001), stratification (Andersson 2023), magnetic field (Luciano 2000) or superfluidity (Lindblom 2000) in the core that might affect the r-mode frequency.**
  - Effect negligible compared to GR and fast rotation corrections.

Summary and Future Works

- ➢ Universal relation between r-mode frequency and the NS compactness determines the search parameter space.
- ➢ The URs are updated with recent multi-messenger constraints on the NS EoS and consistent with their Newtonian limits.
- ➢ These universal relations along with the knowledge of EOS can be used to break the degeneracy between distance and moment of inertia.
- ➢ These universal relations with prior knowledge of distance from EM observation can give strict constraints on the dense matter EOS.
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Thanks for listening !!