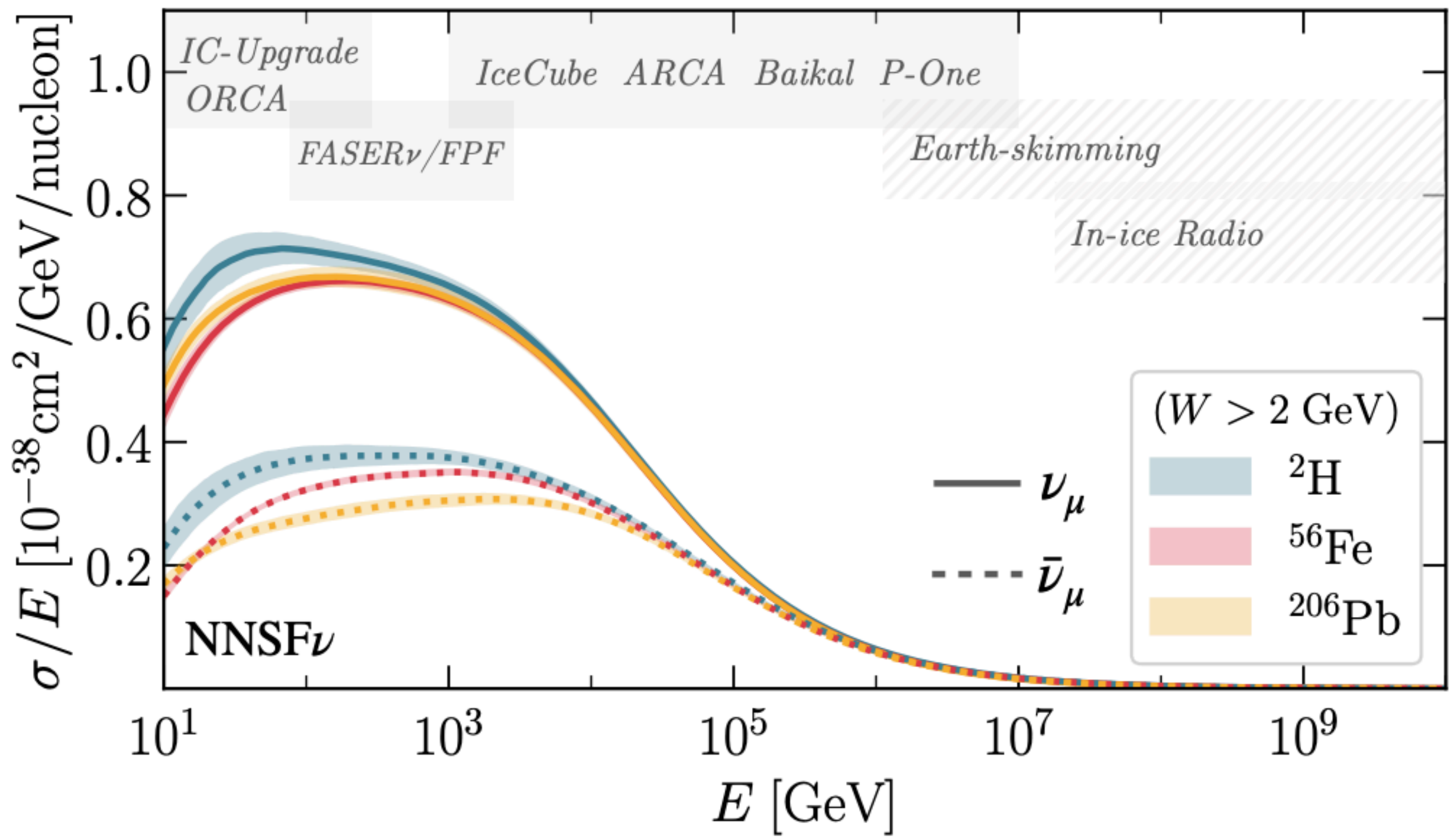
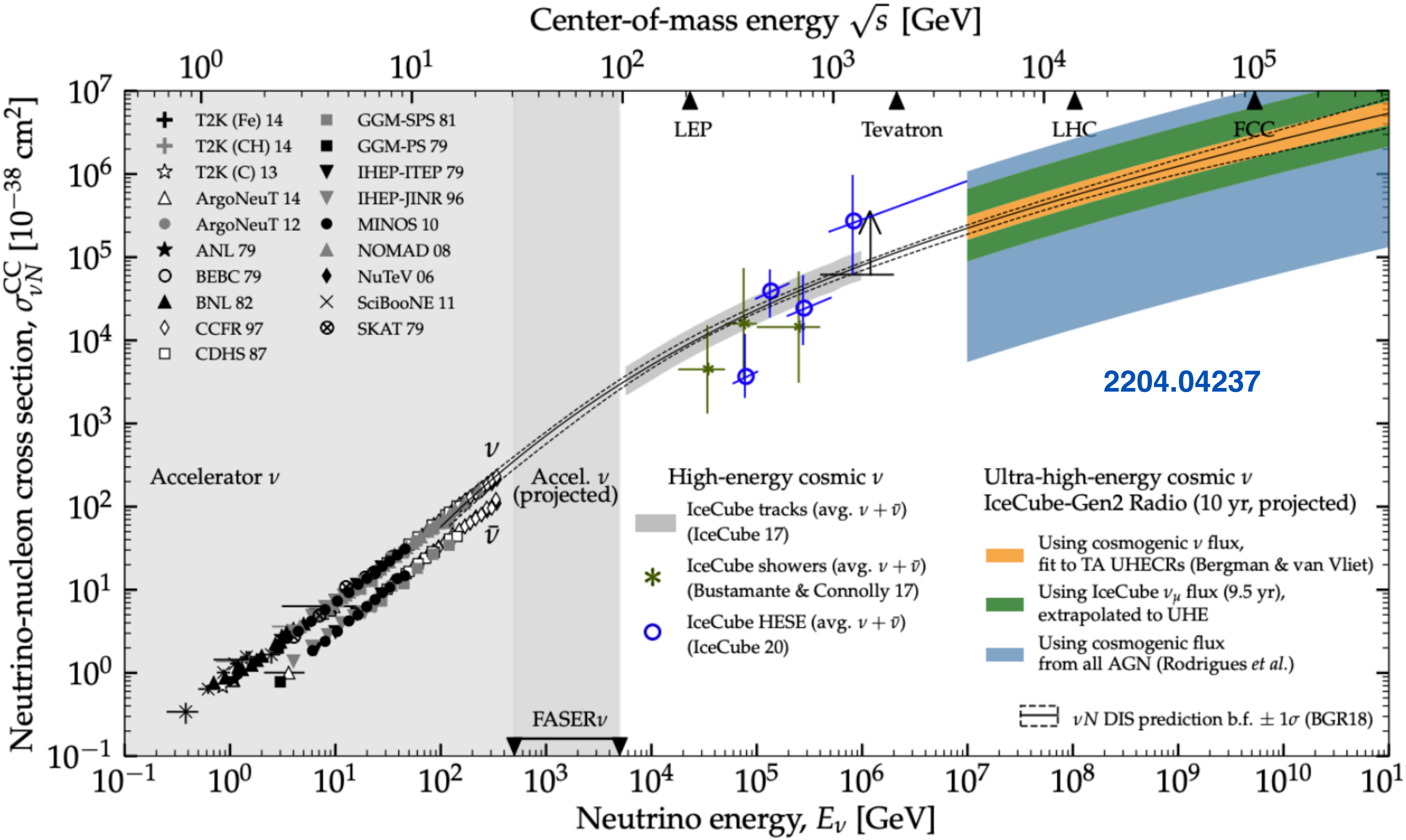
Neutrino Structure Functions from GeV to TeV energies

Juan Rojo, VU Amsterdam & Nikhef



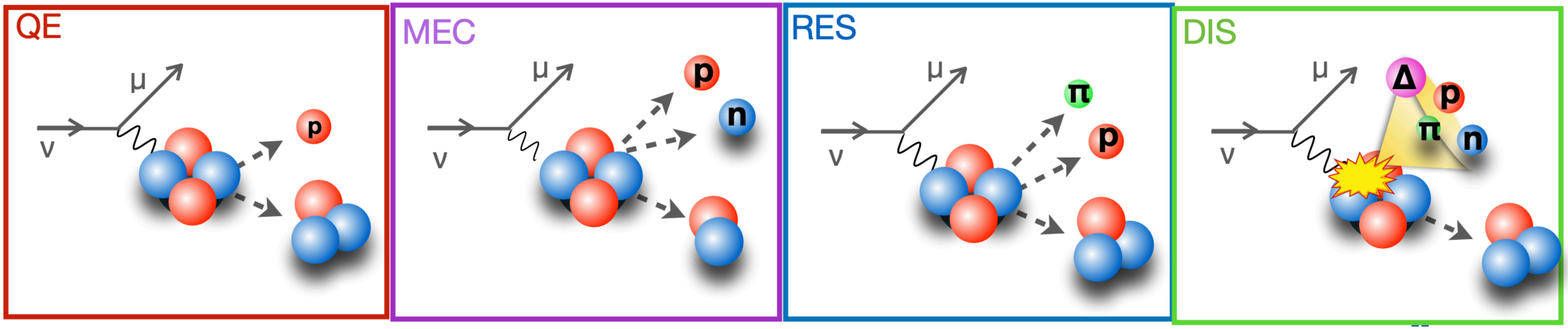
KM3NET @ Nikhef group meeting, 24.03.2023

The neutrino cross-section landscape



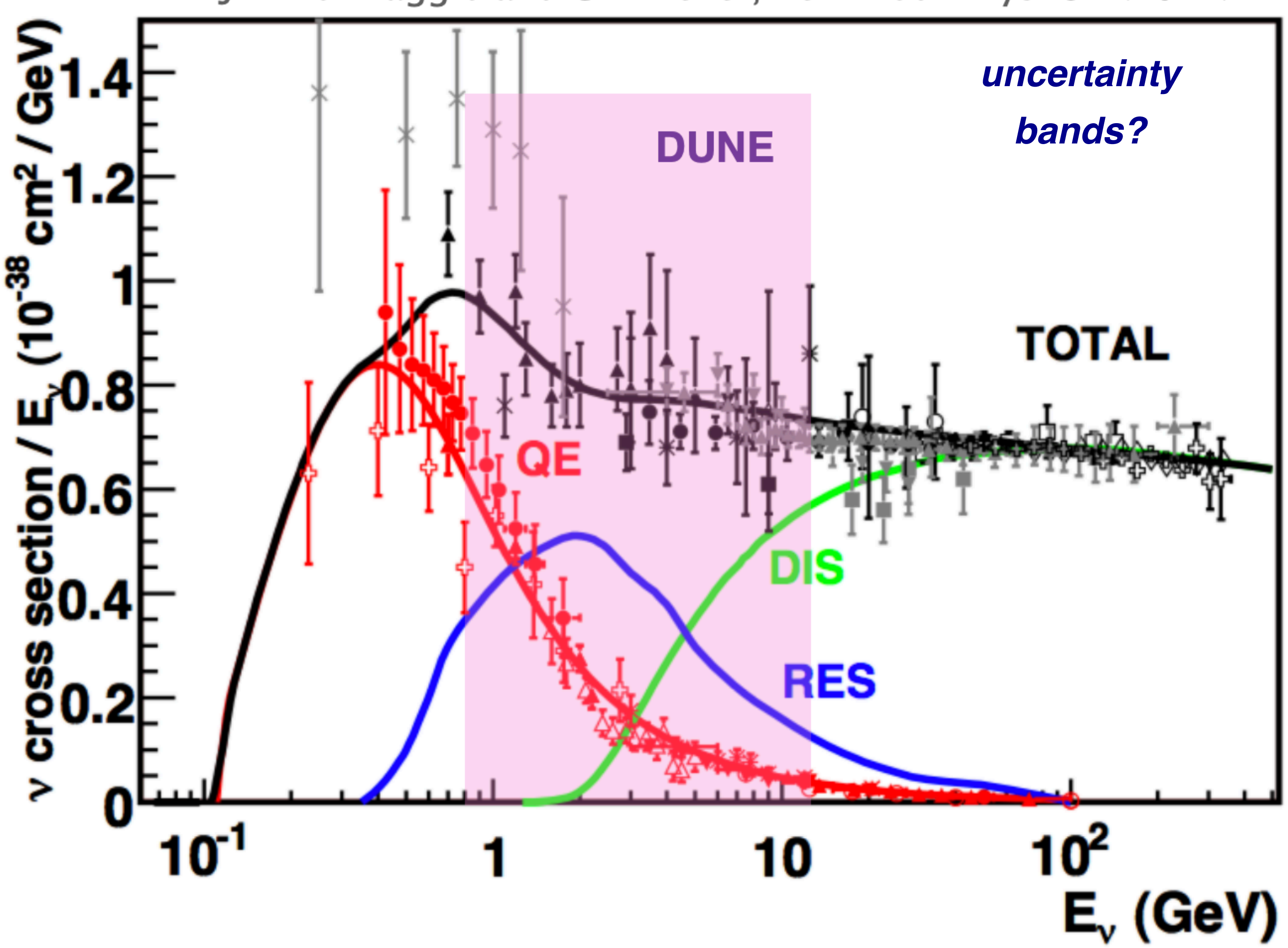
Depending on the neutrino energy, **different interaction mechanisms** dominate the neutrino-nucleus cross-section

The neutrino cross-section landscape

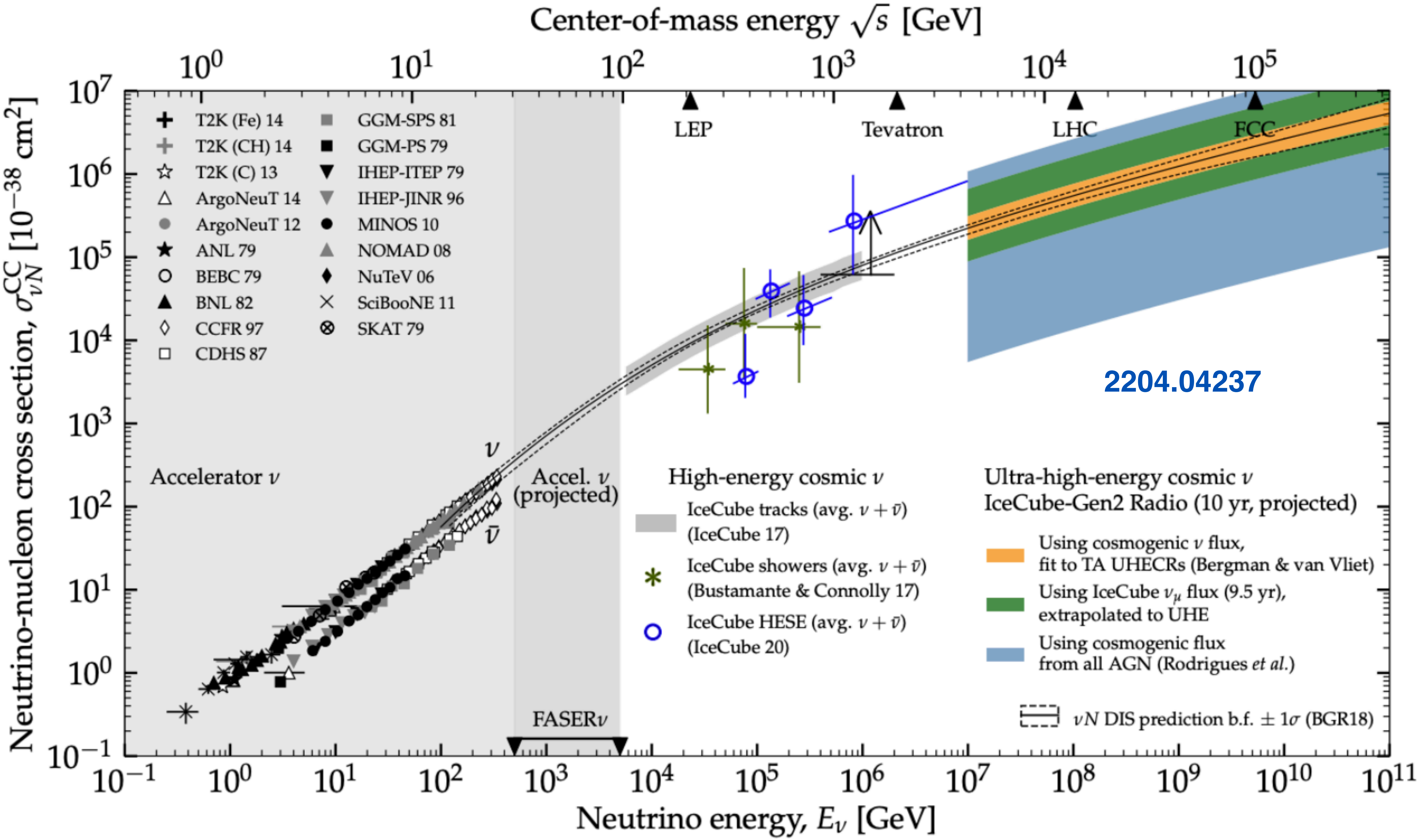


- For energies > 5 GeV, **inelastic scattering** dominates the inclusive cross-section
- Common misconception: inelastic scattering does **not** coincide with **deep-inelastic scattering** (DIS) where pQCD can be applied!
- How robust is our **theoretical understanding** of neutrino inelastic scattering interactions?

J.A. Formaggio and G.P. Zeller, Rev. Mod. Phys. 84 (2012)



The neutrino cross-section landscape

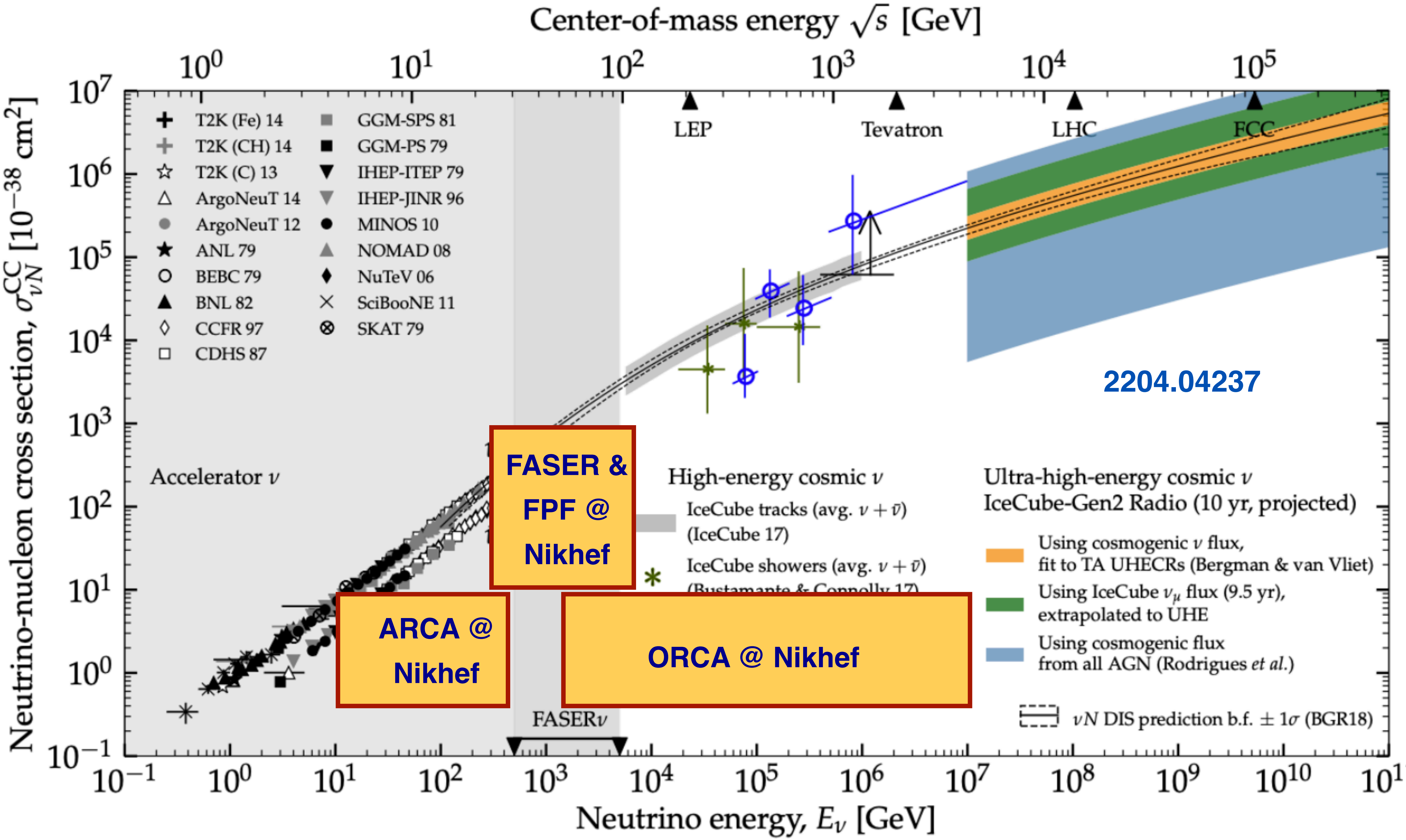


Inelastic scattering (SIS + DIS)

Deep-inelastic scattering (>95%)

sizable kinematic region where **Shallow Inelastic Scattering** (SIS) cannot be neglected

The neutrino cross-section landscape



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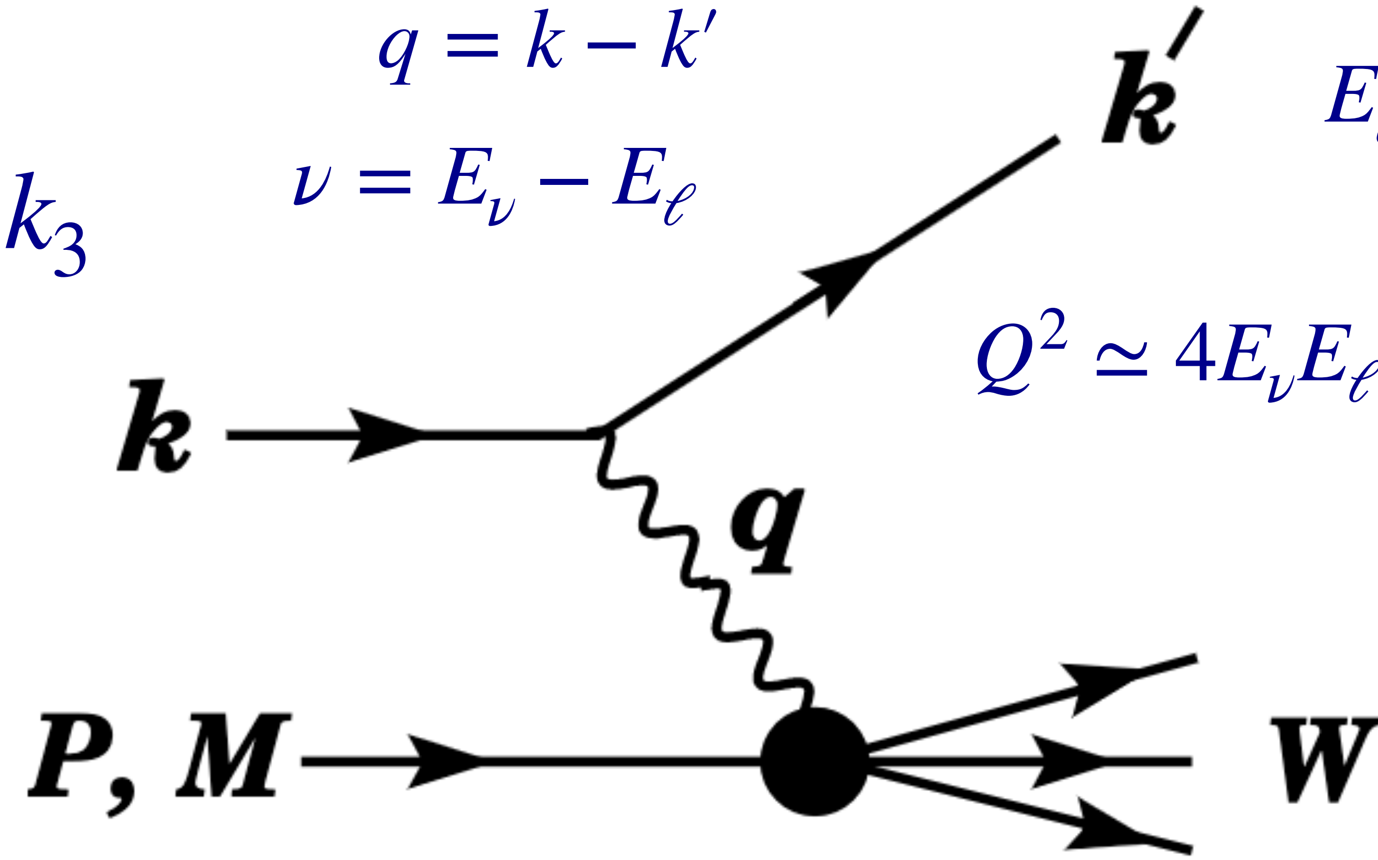
Neutrino Inelastic Structure Functions

$$q = k - k'$$

$$E_\ell = k'_0$$

$$E_\nu = k_0 = k_3$$

$$\nu = E_\nu - E_\ell$$



$$Q^2 \simeq 4E_\nu E_\ell \sin^2(\theta/2)$$

$$y = \frac{\nu}{E_\nu} = 1 - \frac{E_\ell}{E_\nu}$$

$$E_h = E_\nu - E_\ell$$

$$x = \frac{Q^2}{2m_N\nu}$$

$$W^2 = Q^2\frac{(1-x)}{x} = m_N^2 + 2m_N\nu - Q^2$$

- For neutrino experiments the **flux and flavour of the incoming neutrinos depends on the energy**: one can either take it from existing calculation or constrain it from the data (self-calibration)

- Here we focus on **charged-current inclusive scattering**, with a single charged lepton in final state

Neutrino Inelastic Structure Functions

- Assume that we can access the **outgoing charged lepton energy**, the **lepton scattering angle**, and the **total hadronic energy or invariant mass of the hadronic final state**

$$\left(E_\ell,\ \theta,\ W^2\right) \quad \text{or} \quad \left(E_\ell,\ \theta,\ E_h\right)$$

- Then we can reconstruct **Bjorken-*x*, momentum transfer square**, and **incoming neutrino energy**

$$\left(x,\ Q^2,\ E_\nu\right) \quad \text{or} \quad \left(x,\ Q^2,\ y\right)$$

by using the following equations

$$E_h = E_\nu - E_\ell \longrightarrow \text{fixes neutrino energy}$$

$$Q^2 \simeq 4E_\nu E_\ell \sin^2(\theta/2) \longrightarrow \text{fixes four-momentum transfer}$$

$$x = \frac{Q^2}{2m_N(E_\nu - E_\ell)} \longrightarrow \text{fixes Bjorken-}x$$

nb ideally we'd like to over-constrain the kinematics by measuring more variables than unknowns

Neutrino Inelastic Structure Functions

- Given the kinematics of an event, for **inelastic scattering** interaction probability will be proportional to the **double-differential cross-section**

$$\frac{d^2\sigma^{\nu A}(x,Q^2,y)}{dxdy} = \frac{G_F^2 s/2\pi}{\left(1+Q^2/m_W^2\right)^2}\left[(1-y)F_2^{\nu A}(x,Q^2) + y^2 x F_1^{\nu A}(x,Q^2) + y\left(1-\frac{y}{2}\right)xF_3^{\nu A}(x,Q^2)\right]$$

$$\frac{d^2\sigma^{\nu A}(x,Q^2,y)}{dxdy} = \frac{G_F^2 s/4\pi}{\left(1+Q^2/m_W^2\right)^2}\left[Y_+F_2^{\nu A}(x,Q^2) - y^2 F_L^{\nu A}(x,Q^2) + Y_- xF_3^{\nu A}(x,Q^2)\right]$$

- Traditionally neutrino measurements are presented at the level of individual structure functions, but this requires extra assumptions: cleaner to measure directly the reduced cross-section

- The number of events in a given bin will be given by

$$N_{\rm ev}(x\in[x_{\rm min},x_{\rm max}], Q^2\in[Q^2_{\rm min},Q^2_{\rm max}, E_\nu\in[E_{\nu,\rm min},E_{\nu,\rm max}]) \propto \int_{x_{\rm min}}^{x_{\rm max}} dx \int_{Q^2_{\rm min}}^{Q^2_{\rm max}} dQ^2 \int_{E_{\nu,\rm min}}^{E_{\nu,\rm max}} dE_\nu \frac{d^2\sigma(x,Q^2,E_\nu)}{dxdy} f(E_\nu)$$

experiment-dependent factor

scattering cross-section

incoming neutrino flux

- Accurate modelling of the structure functions (differential cross-sections) key for neutrino phenomenology

Neutrino Inelastic Structure Functions

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- For $Q > 2$ GeV, neutrino structure functions can be expressed in terms of **hard-scattering coefficient functions** and **proton/nuclear parton distributions**

$$F_i^{\nu A}(x,Q^2) = \sum_{j=q,\bar{q},g}\int_x^1 \frac{dz}{z} C_{i,j}^{\nu N}(z,\alpha_s(Q^2)) f_j^{(A)}\left(\frac{x}{z},Q^2\right), \quad i=2,3,L$$

Compute in pQCD (up to N3LO)

Parametrise and extract from data

e.g. at LO for H target

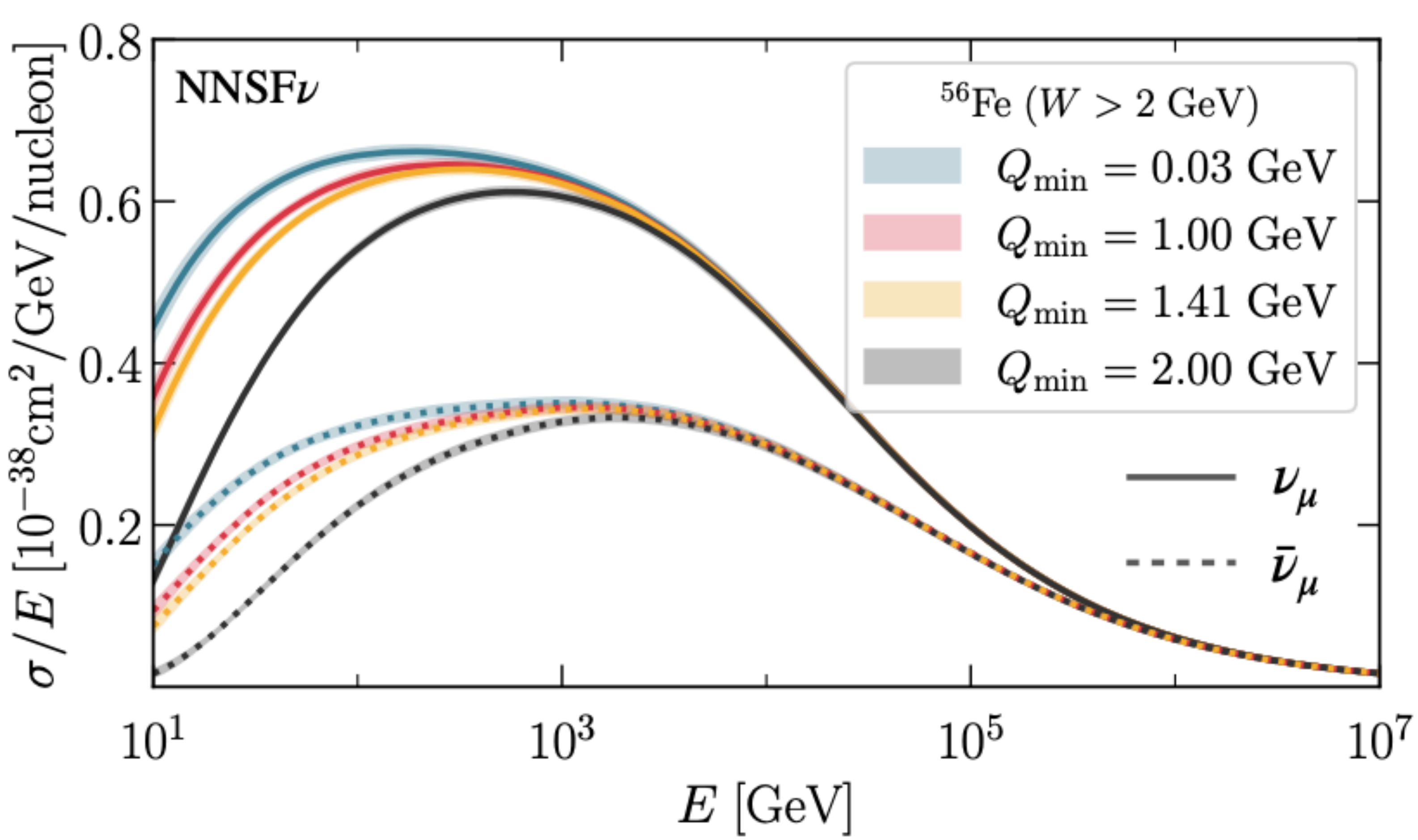
$$\begin{aligned}
F_2^{\nu p}(x,Q^2) &= 2x\,(f_{\bar{u}} + f_d + f_s + f_{\bar{c}})\,(x,Q^2)\,,\\
F_2^{\bar{\nu} p}(x,Q^2) &= 2x\,(f_u + f_{\bar{d}} + f_{\bar{s}} + f_c)\,(x,Q^2)\,,\\
xF_3^{\nu p}(x,Q^2) &= 2x\,(-f_{\bar{u}} + f_d + f_s - f_{\bar{c}})\,(x,Q^2)\,,\\
xF_3^{\bar{\nu} p}(x,Q^2) &= 2x\,(f_u - f_{\bar{d}} - f_{\bar{s}} + f_c)\,(x,Q^2)\,,
\end{aligned}$$

- At lower Q values, **QCD factorisation breaks down** and need alternative approach

The role of the low-Q region

inclusive neutrino cross-section receives **sizeable contributions from $Q < 2$ GeV region**, where structure functions cannot be evaluated in the pQCD framework

$$\sigma(E_\nu) = \int_{Q^2_{\rm min}}^{2m_N E_\nu} dQ^2 \left[\int_{Q^2/(2m_N y E_\nu)}^{1} dx \, \frac{d^2\sigma}{dx dQ^2}(x, Q^2, E_\nu) \right]$$



- Even for neutrino energies of hundreds of GeV, **sizeable contribution from low-Q region**
- Perturbative QCD not reliable, what to do?
- So far, most calculations used in phenomenology rely on **phenomenological models**

The Bodek-Yang model

The **Bodek-Yang model** is popular to describe **inelastic neutrino DIS** structure functions

based on **effective leading-order PDFs** (GRV98LO) supplemented to phenomenological scaling variables and *K*-factors to improve agreement with data

$$f_i^{\rm LO}(x,Q^2) \;\rightarrow\; f_i^{\rm LO,BY}(\xi,Q^2) \qquad \xi = \frac{2x(Q^2+m_f^2+B)}{Q^2\left[1+\sqrt{1+(2m_N x)^2/Q^2}\right]+2Ax}$$

Limitations of the BY model of neutrino structure functions:

- Obsolete PDF parametrisation that **ignores constraints from the last 25 years**
- Neglects **higher-order QCD corrections** (can be up to 100%)
- Cannot be used above 100 TeV: **not an option for UHE neutrinos**
- Does not provide **uncertainty estimate**, difficult to assess its accuracy and precision
- Cannot be systematically improvable e.g. by new data

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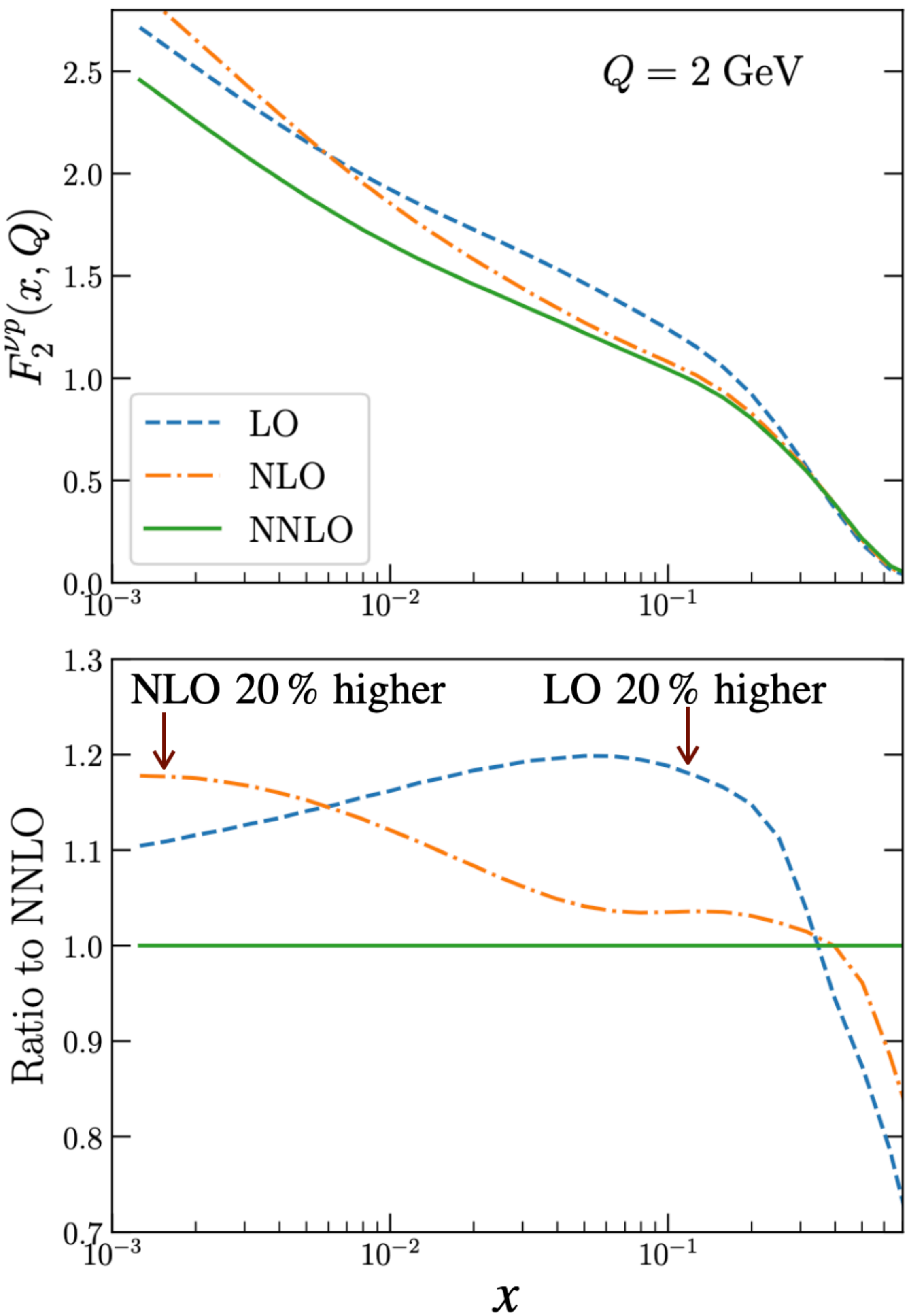

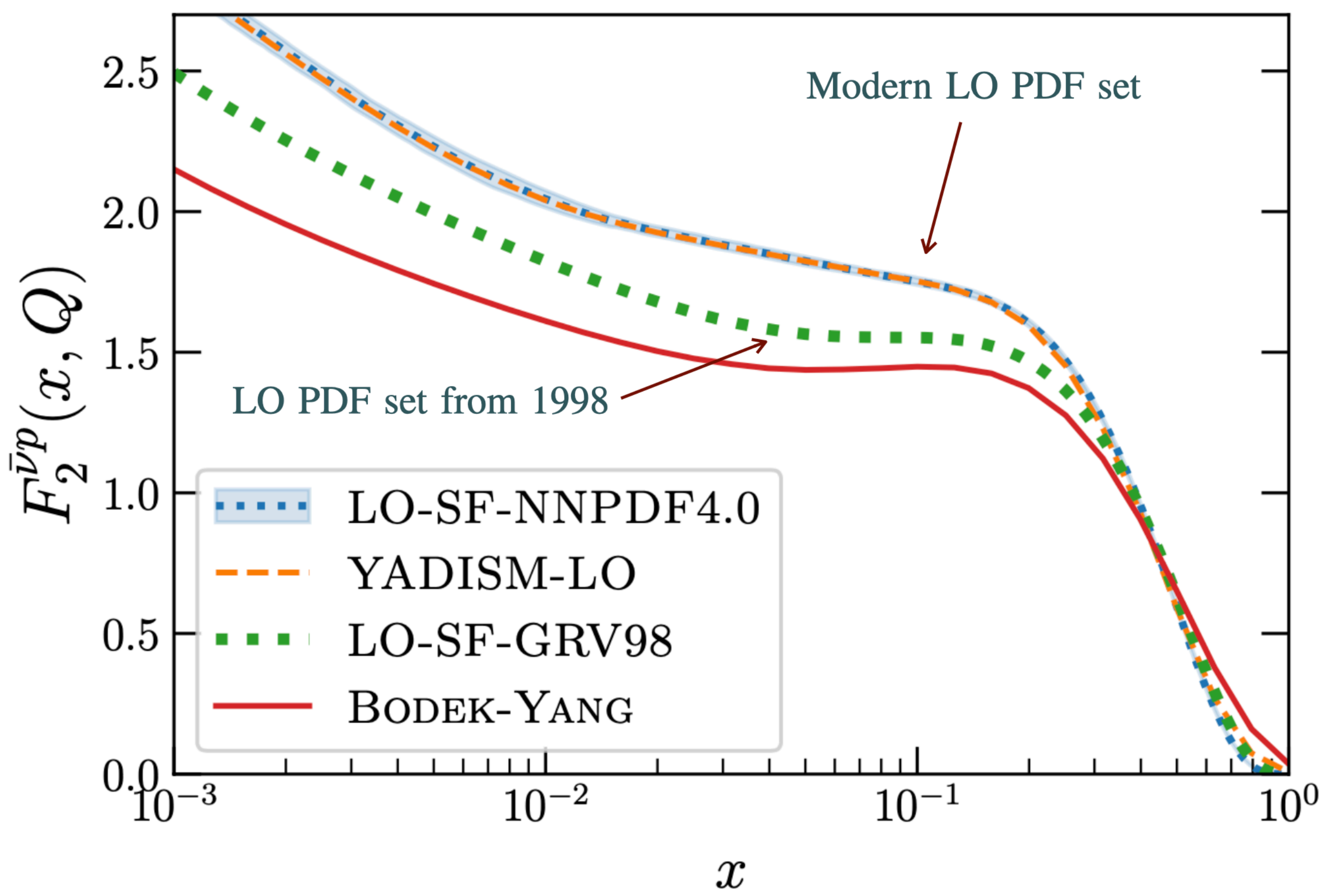

Improved models of neutrino-nucleon interactions essential for FPF physics (as well as IceCube, KM3NET, …)

The NNSFν approach

Motivation: realise the first determination of neutrino structure functions **valid from photoproduction $Q = 0$ all the way to $Q = 100$ TeV**, enabling calculation of inclusive inelastic cross-sections **for neutrinos from 5 GeV to 10^{12} GeV energies**

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Why?

- Avoid the need to rely on **phenomenological model calculations**
- Unique, consistent calculation can be used in **all neutrino experiments** sensitive to inelastic scattering
- Provide **robust estimate of uncertainties**, systematically improvable with new data and better theory
- Account for state-of-the-art results on **proton and nuclear structure** for DIS structure functions

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How?

- Constrain low-Q structure functions **directly from data** using deep learning models
- Ensure that **pQCD predictions for proton, nuclear structure functions** are reproduced in whole $(x$, high-Q, $A)$ region
- Deliver in terms of fast interpolation grids that can be interfaced to **neutrino event generators such as GENIE**
- Make the code **open-source** so that it can be easily updated when additional theory or experimental information becomes available

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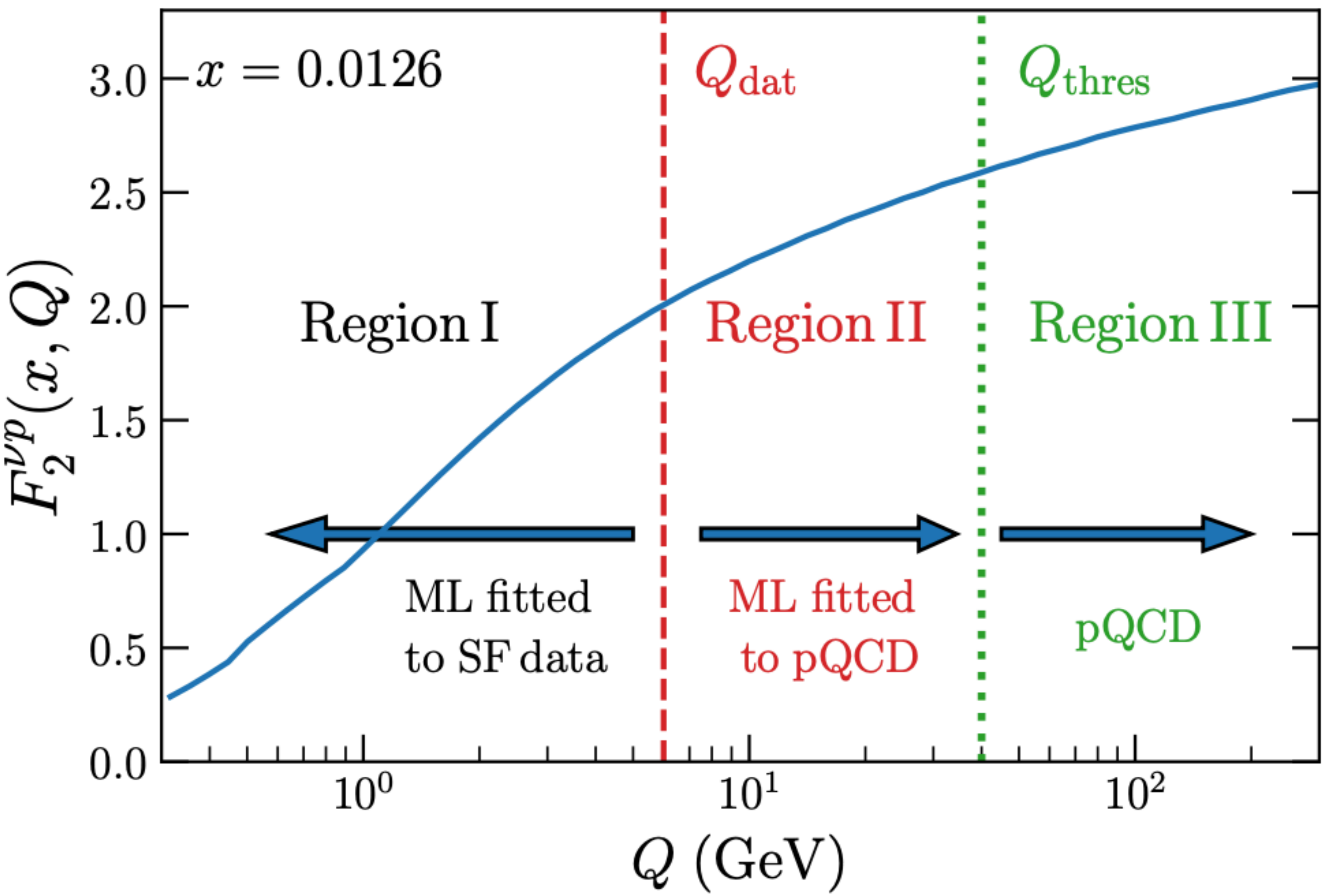
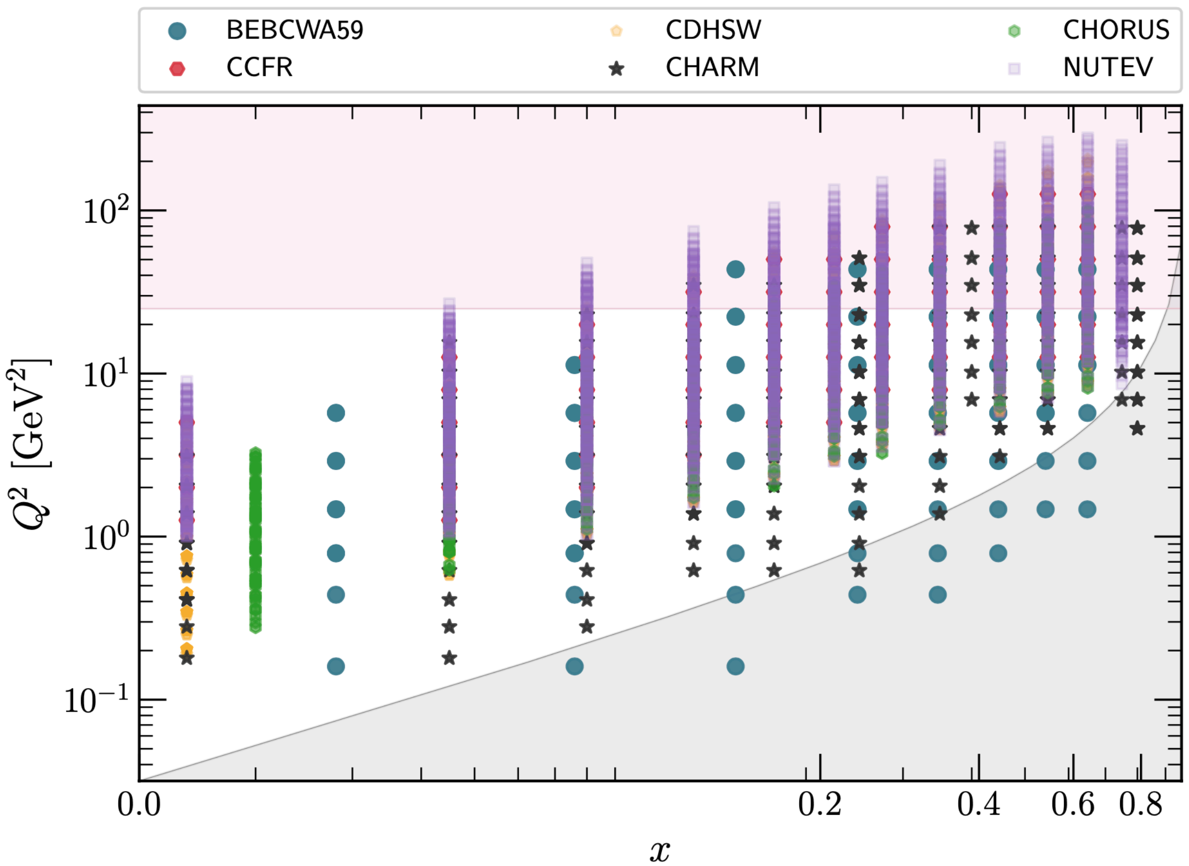
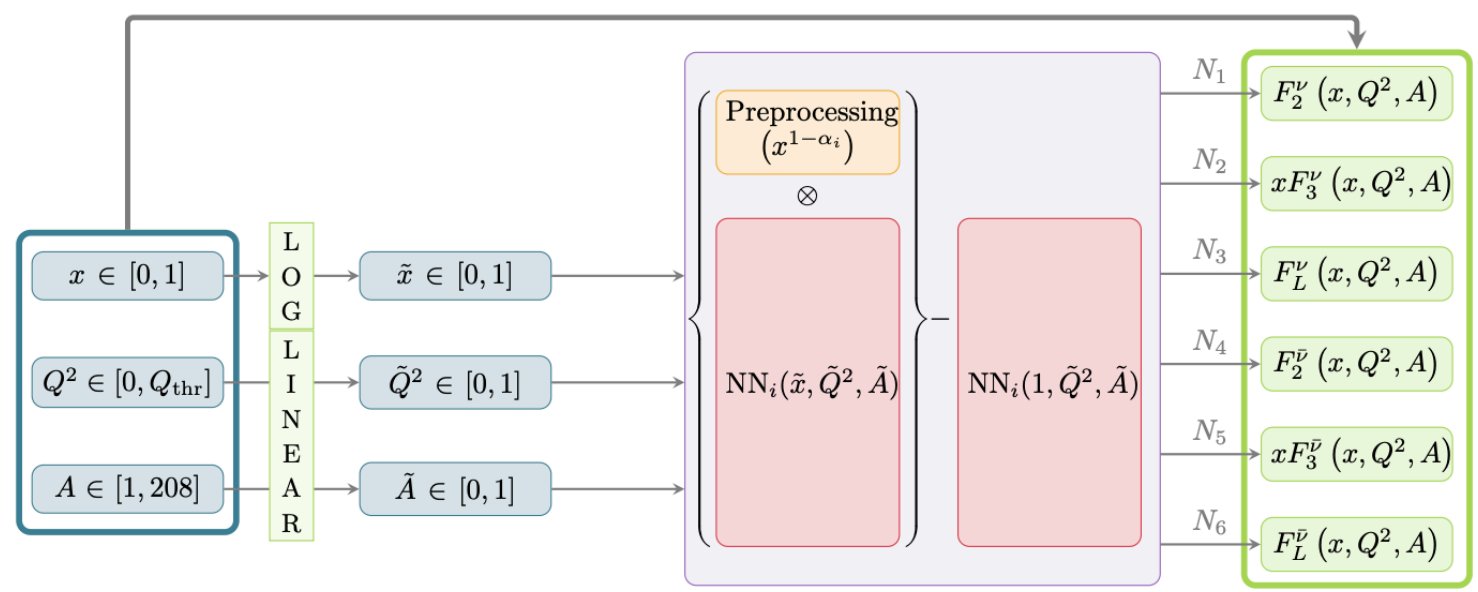


The NNSFν approach

- Use available data on neutrino-nucleus scattering to **parametrise and determine inelastic structure functions** by means of the NNPDF fitting methodology



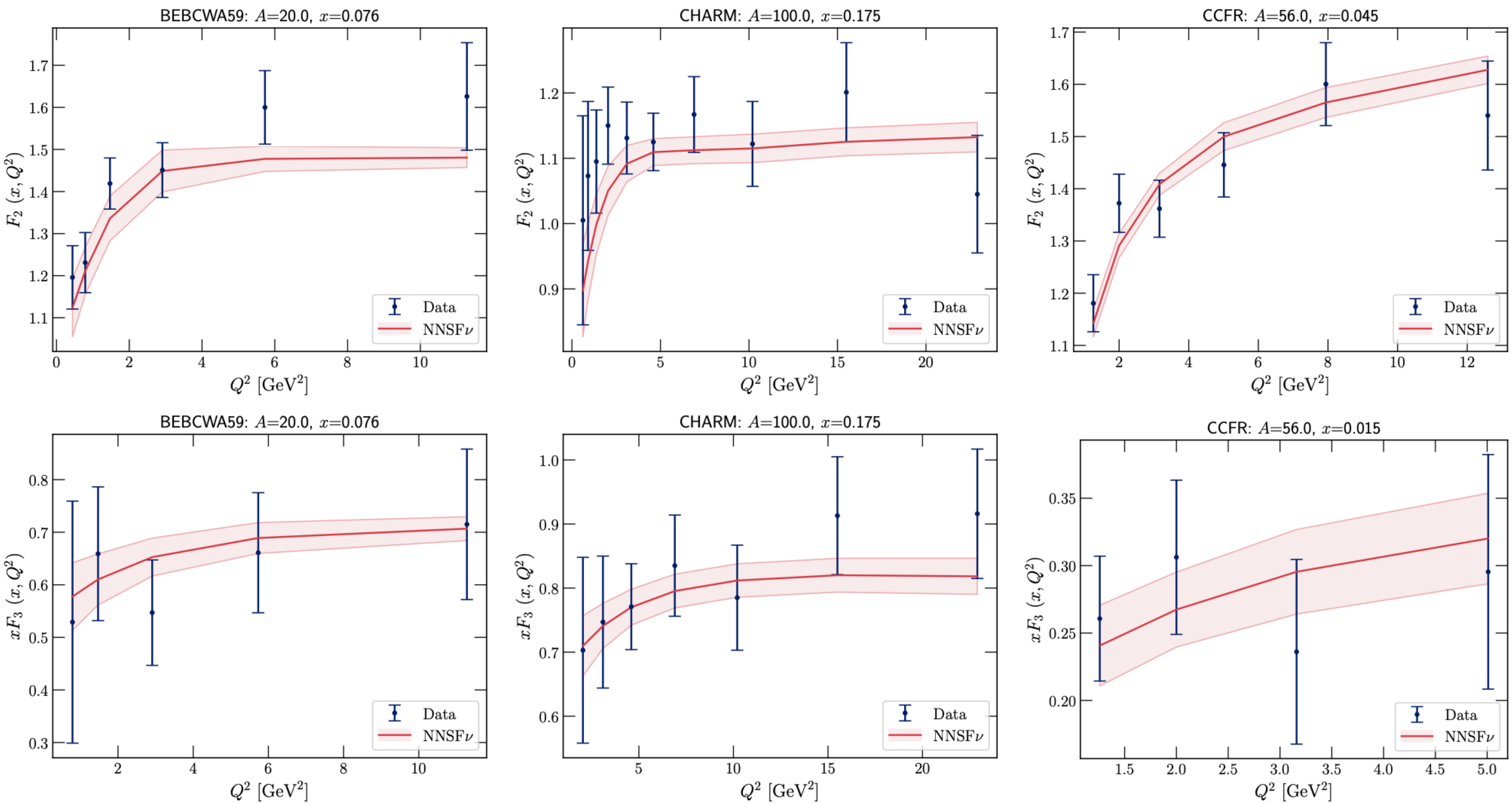


- This data-driven parametrisation is made to **converge to the pQCD calculation** for large enough Q^2 values as implemented with Lagrange multipliers

- In the neutrino energy region sensitive only to $Q >$ few GeV, **replace by pQCD calculation**

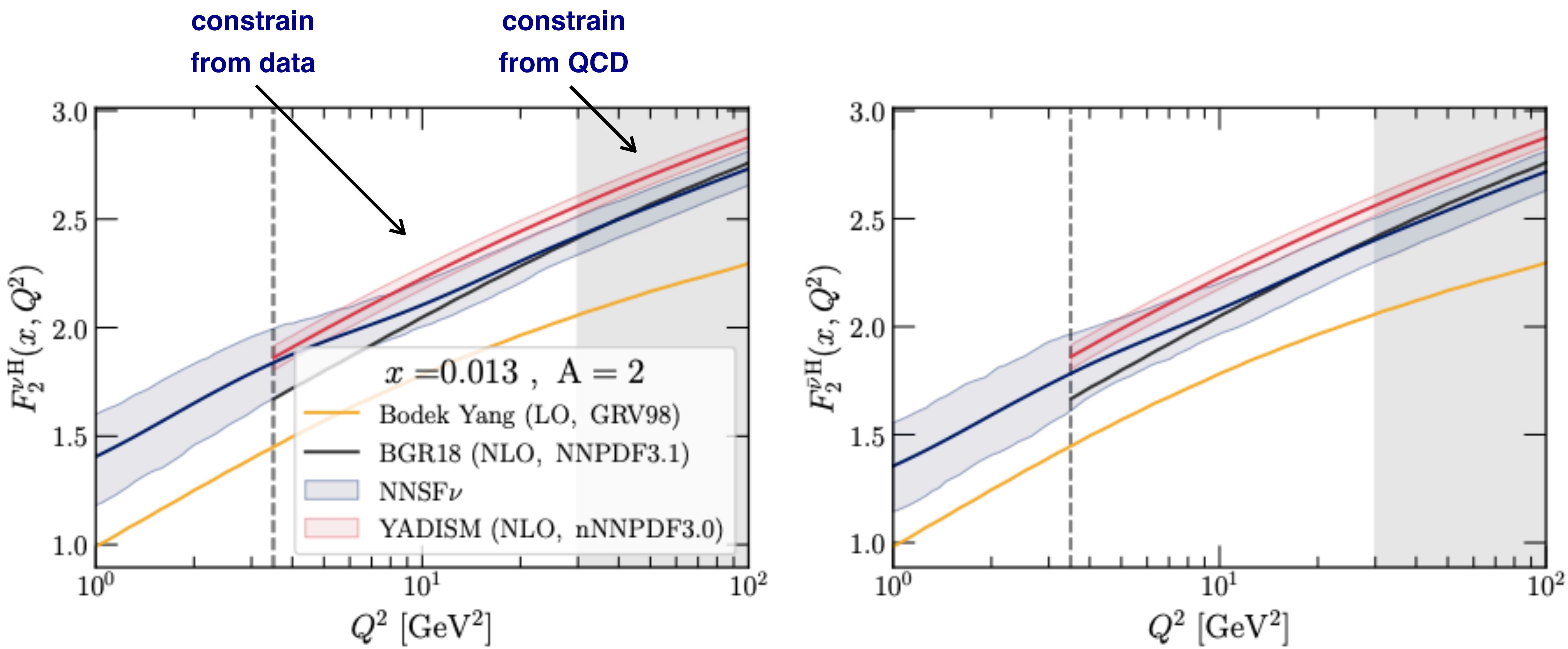
consistent determination of neutrino structure functions valid **for 12 orders of magnitude** from E_{nu} = few GeV up to $E_{nu}=10^{12}$ GeV

The NNSFν approach



Good description of available neutrino structure function data; strong preference to **fit differential cross-sections** rather than structure functions (cleaner), QCD constraints do not distort the fit

The NNSFν results



- **Smooth matching** between data-driven and pQCD regions, uncertainty estimate in whole energy range
- Structure functions and integrated cross-sections available via **user-friendly LHAPDF grids**
- For the first time, a **unique theory prediction** for neutrino inelastic scattering suitable for neutrinos with energies from a few GeV up to the multi-EeV region

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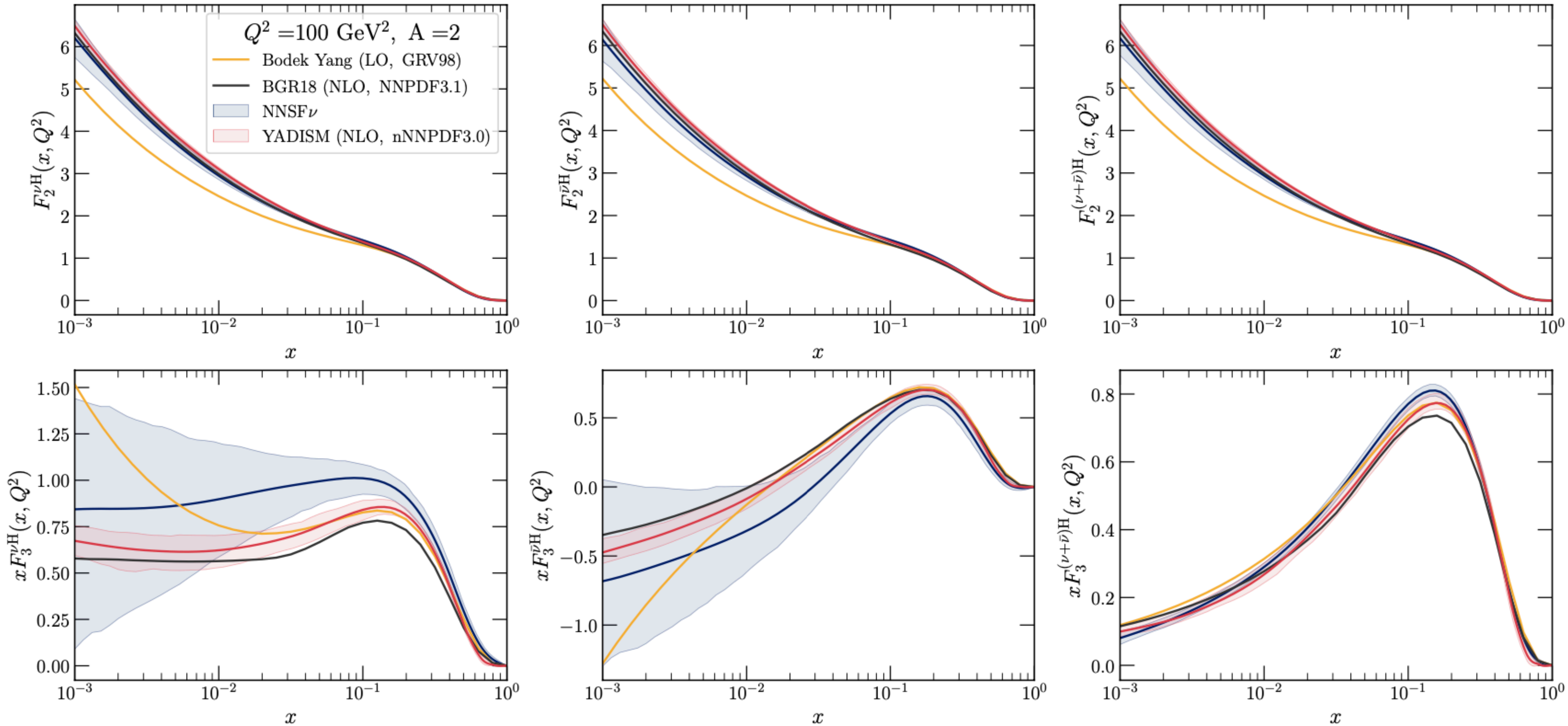

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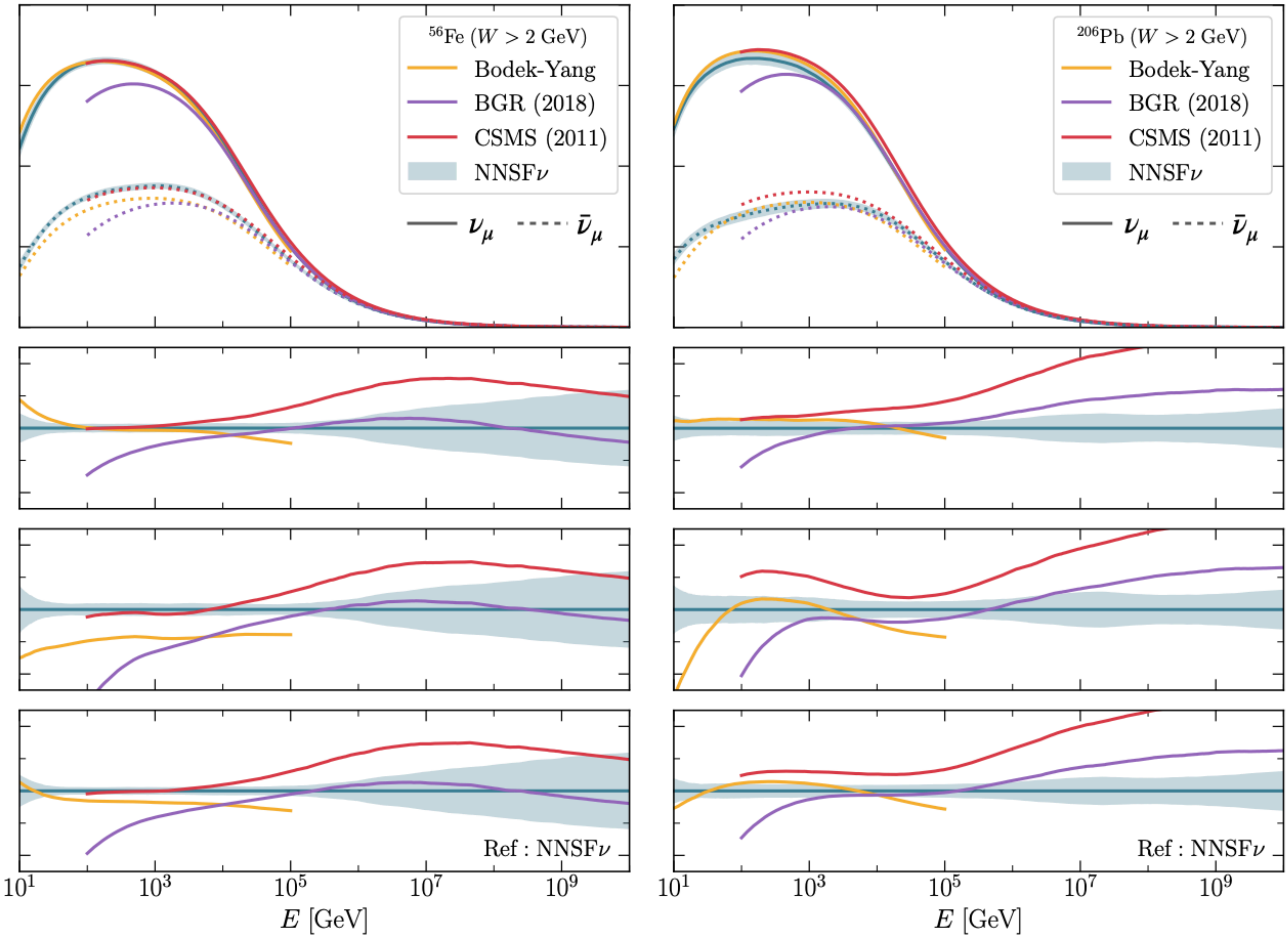

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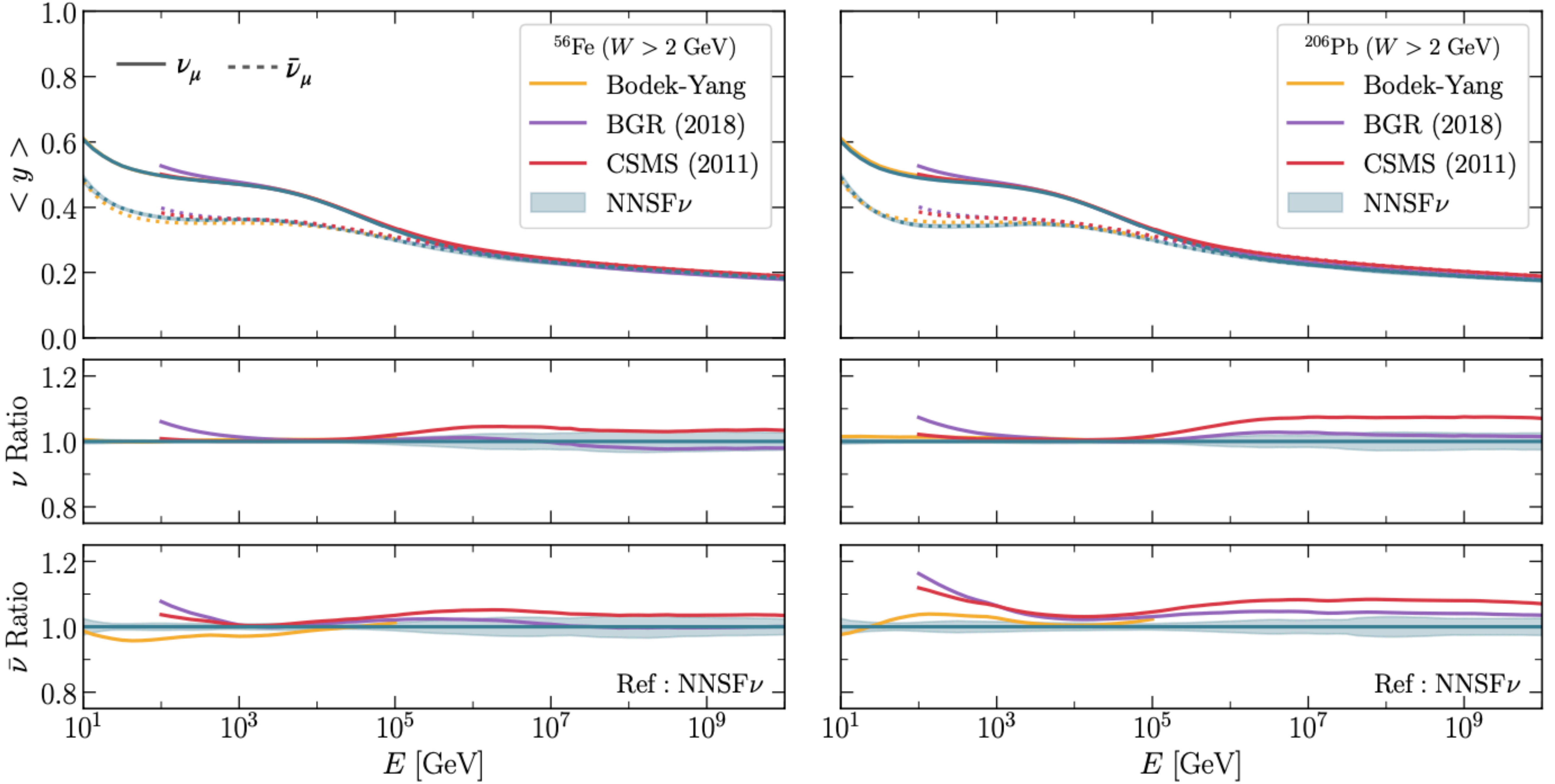
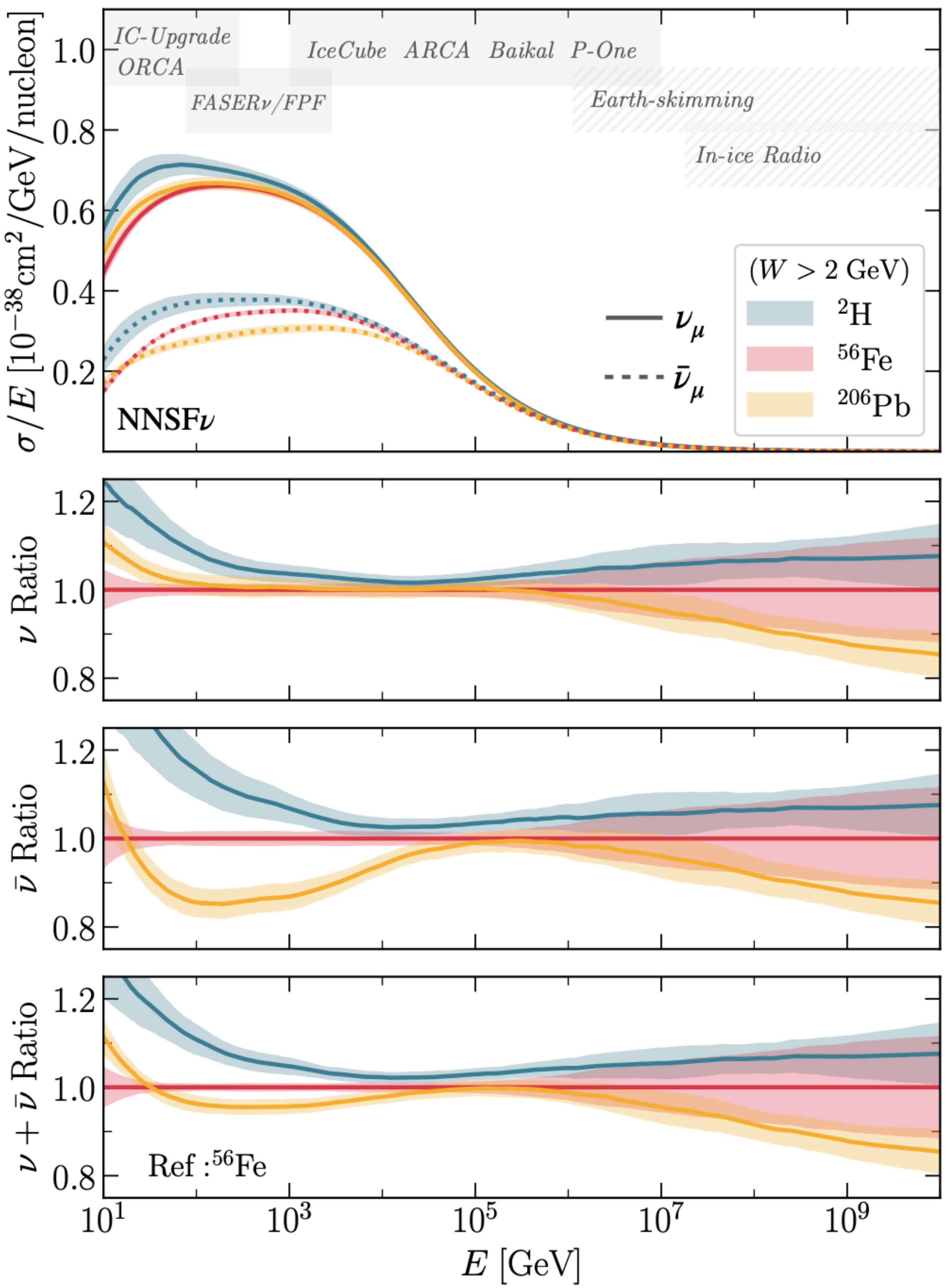
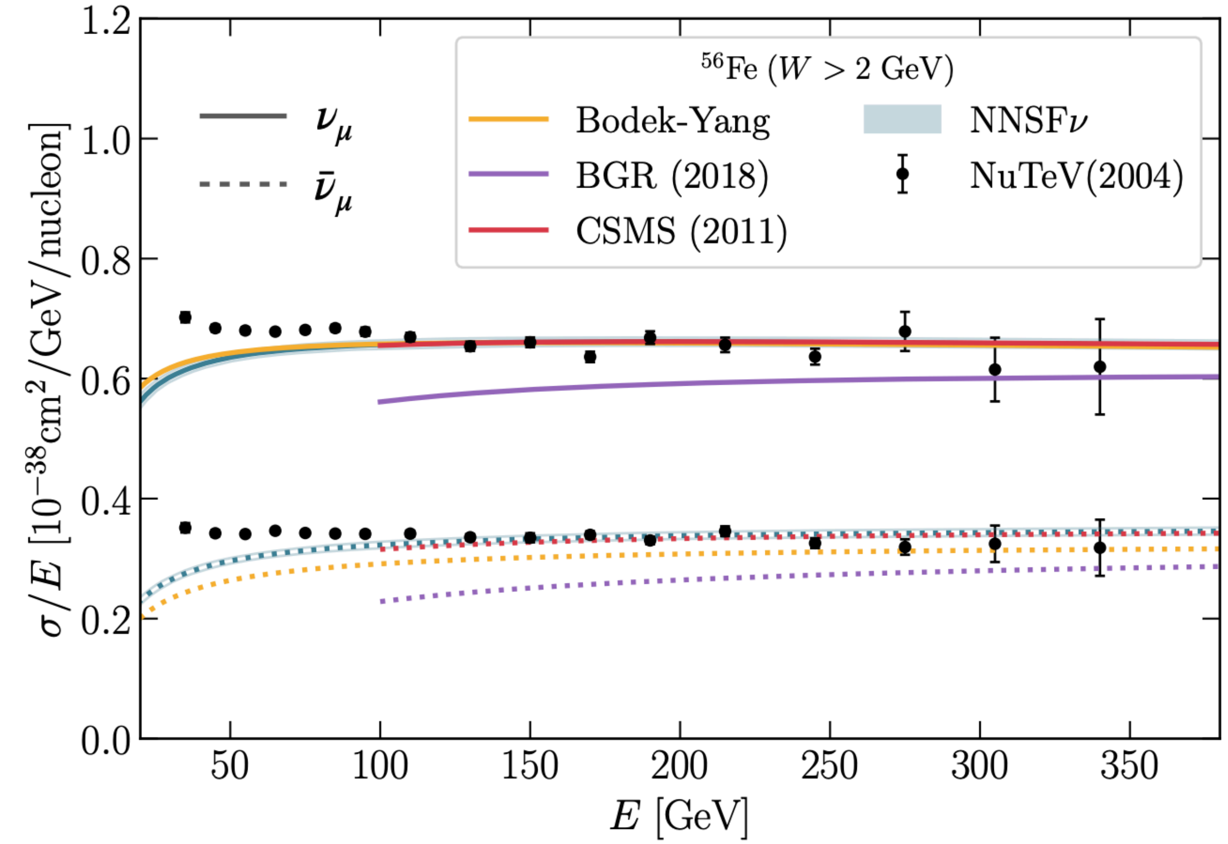


- Can be used to evaluate any differential DIS variable, e.g. expectation value of **inelasticity**
- First calculation where nuclear effects are accounted for in a data-driven, model-independent manner

The NNSFν results

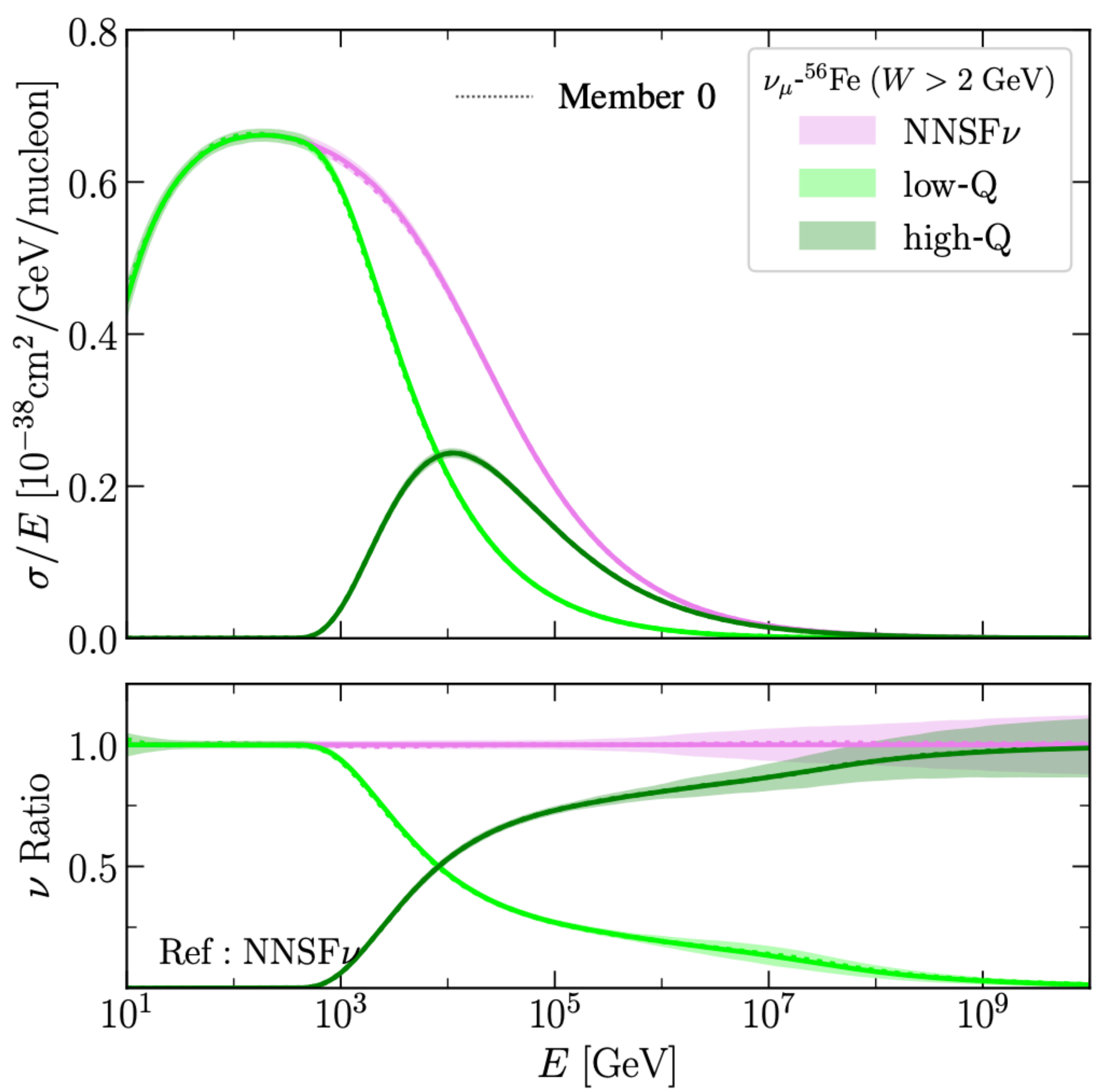




- Good agreement with available neutrino structure function and **cross-section data**
- Robust estimate of all relevant sources of experimental and theory **uncertainties**
- Model-independent determination of **nuclear corrections** to free-nucleon scattering

Using NNSFν for neutrino simulations

- The NNSFν structure functions are provided in terms of **fast LHAPDF interpolation grids**
- They can be readily used in **GENIE** by means of the **HEDIS** package (**official GENIE release)**
- Same GENIE/HEDIS interface: access other cross-section models like Bodek-Yang and BGR18
- Implementation in other neutrino event generators straightforward: no reason not to **adopt NNSFν in your neutrino scattering simulations**

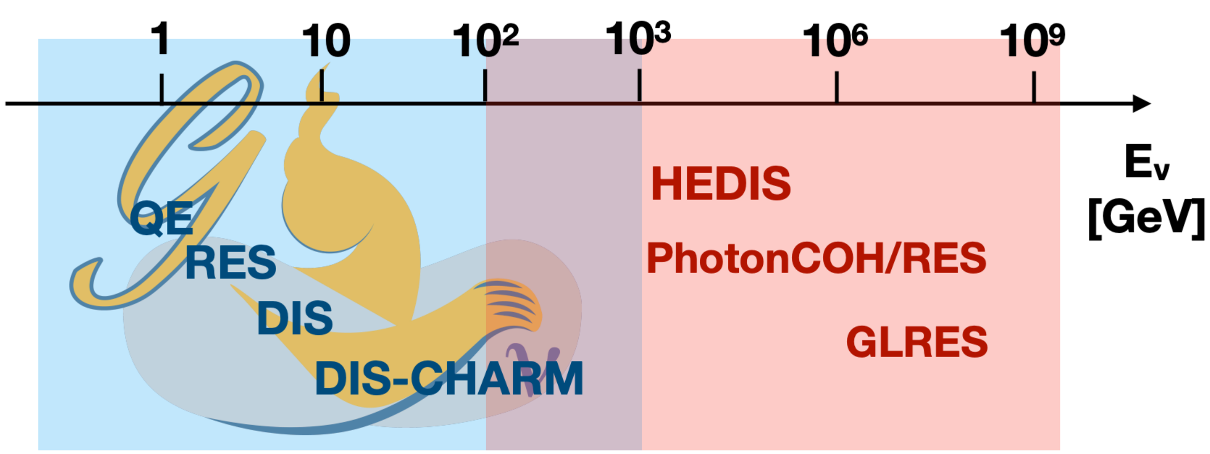


(Z, A) [target]	low-Q grid	high-Q grid
$(1, 2)$ [D]	NNSFnu_D_lowQ	NNSFnu_D_highQ
$(2, 4)$ [He]	NNSFnu_He_lowQ	NNSFnu_He_highQ
$(3, 6)$ [Li]	NNSFnu_Li_lowQ	NNSFnu_Li_highQ
$(4, 9)$ [Be]	NNSFnu_Be_lowQ	NNSFnu_Be_highQ
$(6, 12)$ [C]	NNSFnu_C_lowQ	NNSFnu_C_highQ
$(7, 14)$ [N]	NNSFnu_N_lowQ	NNSFnu_N_highQ
$(8, 16)$ [O]	NNSFnu_O_lowQ	NNSFnu_O_highQ
$(13, 27)$ [Al]	NNSFnu_Al_lowQ	NNSFnu_Al_highQ
$(15, 31)$ [Ea]	NNSFnu_Ea_lowQ	NNSFnu_Ea_highQ
$(20, 40)$ [Ca]	NNSFnu_Ca_lowQ	NNSFnu_Ca_highQ
$(26, 56)$ [Fe]	NNSFnu_Fe_lowQ	NNSFnu_Fe_highQ
$(29, 64)$ [Cu]	NNSFnu_Cu_lowQ	NNSFnu_Cu_highQ
$(47, 108)$ [Ag]	NNSFnu_Ag_lowQ	NNSFnu_Ag_highQ
$(50, 119)$ [Sn]	NNSFnu_Sn_lowQ	NNSFnu_Sn_highQ
$(54, 131)$ [Xe]	NNSFnu_Xe_lowQ	NNSFnu_Xe_highQ
$(74, 184)$ [W]	NNSFnu_W_lowQ	NNSFnu_W_highQ
$(79, 197)$ [Au]	NNSFnu_Au_lowQ	NNSFnu_Au_highQ
$(82, 208)$ [Pb]	NNSFnu_Pb_lowQ	NNSFnu_Pb_highQ

The HEDIS package

- Lead developer: **Alfonso Garcia Soto** (MIT & IFIC, former Nikhef postdoc)
- Original goal was to extend coverage of GENIE to **neutrino energies above 1 TeV**
- Current implementation, when combined with NNSFν, allows calculations of **inelastic scattering for all energies** from a few GeV to the multi-EeV regime

- Current status of GENIE in the high energy regime:
  - DIS based on Bodek-Yang model -> optimised for low Q^2.
  - Structure Function = C_{ij} LO⊗PDF LO (GRV98 Q^2[0.8,2·10^6]).
  - Contributions from heavy quarks are not included.



Targeted experiments

- KM3NeT:
  - Already using GENIE (both DIS and HEDIS) in its simulation framework (gSeaGen).
- IceCube(-Gen2):
  - Uses HEDIS as an auxiliary tool to crosscheck their simulation framework at HE.
- Neutrino facilities at LHC:
  - Data in 2021-2023 (FASERnu).
  - Overlapping region between DIS & HEDIS (~0.1-1TeV).
  - Not sure what simulation package they are using so far.
- Others: GVD-Baikal, P-ONE, GRAND, etc.

- New extension allows UHE interaction -> HEDIS
  - Newer PDFs with broader Q^2 phase space.
  - Structure Functions = C_{ij} NLO⊗PDF NLO.
  - Account for the heavy quark contributions.

Alfonso Garcia Soto, GENIE Users Meeting, Dec 2021

Summary and outlook

- The **NNSFν calculation of inelastic neutrino structure functions** and cross-sections reliable for full kinematic range of neutrino phenomenology, from a few GeV to multi-EeV

- It accounts for all **available data** and **state-of-the-art QCD theory constraints**

- Relevant for **many ongoing and future experiments**, from ORCA/ARCA and IceCube to FaserNu and the Forward Physics Facility @ HL-LHC

- **Available via the HEDIS module of GENIE,** ready to be used in the experiments.

- Next steps: interface to **parton showers** for exclusive event generation