Cosmological implications of the Higgs vacuum metastability during inflation

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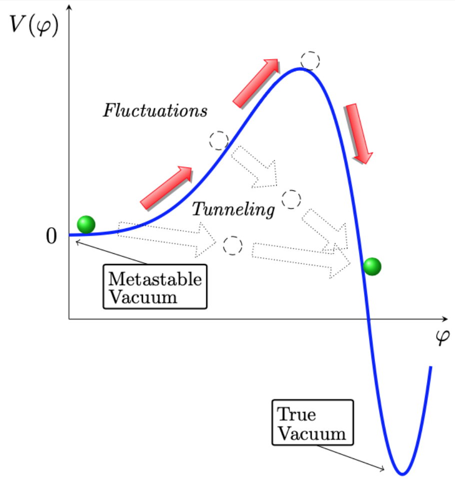
The Standard Model strikes back

Measurements of SM parameters (m_h, m_t) place us in a metastable electroweak vacuum: $V_{\mathrm{H}}(h, \mu, R) = \frac{\xi(\mu)}{2} R h^2 + \frac{\lambda(\mu)}{4} h^4 + \ldots$



Markkanen *et al*, "Cosmological Aspects of Higgs Vacuum Metastability", 2018.

A new hope: constraining physics from vacuum decay

- Low decay rate Γ today, but higher rates in the early Universe.
- Decay expands at c with singularity within $\rightarrow$ true vacuum bubbles:

$$d\langle\mathcal{N}\rangle = \Gamma d\mathcal{V} \Rightarrow \langle\mathcal{N}\rangle = \int_{\text{past}} d^4x\sqrt{-g}\Gamma(x)$$

- Universe still in metastable vacuum $\rightarrow$ no bubbles in past light-cone:

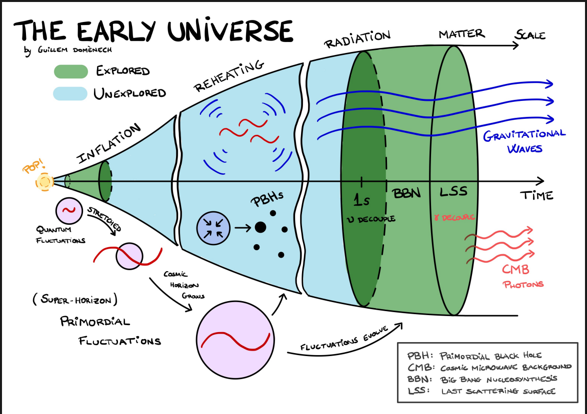
$$\langle\mathcal{N}\rangle \lesssim 1$$

Vacuum bubbles expectation value (during inflation)

$$\langle\mathcal{N}\rangle = \frac{4\pi}{3}\int_0^{N_{\text{start}}} dN \left(\frac{a_{\text{inf}}\left(\eta_0 - \eta\left(N\right)\right)}{e^N}\right)^3 \frac{\Gamma(N)}{H(N)} \leq 1$$

Inflation: a success story

- Period of exponential expansion of the universe before the HBB.
- Originally proposed to solve the cosmological problems at the time.
- Success: links the origin of LSS to the initial quantum fluctuations.
- Evidence: CMB anisotropies.



G. Dom'enech, "The unexplored early universe", 2022, domenechcosmo.netlify.app.

Interlude: overview of computation

1. Calculate ΔV_{H} and plug it in $\Gamma \approx \left(\frac{R}{12}\right)^2 e^{-\frac{384\pi^2 \Delta V_{\mathrm{H}}}{R^2}}$.

2. Cosmological quantities according to the inflationary model; for Starobinsky inflation $V_{\mathrm{I}}(\phi) = \frac{3M^2 M_P^4}{4}\left(1 - e^{-\sqrt{\frac{2}{3}}\frac{\phi}{M_P}}\right)^2$.

3. Complete calculation of $\langle \mathcal{N} \rangle$ imposing the condition $\langle \mathcal{N} \rangle \leq 1$.

$$\langle \mathcal{N} \rangle = \frac{4\pi}{3} \int_0^{N_{\mathrm{start}}} dN \left(\frac{a_{\mathrm{inf}} \left(\eta_0 - \eta\left(N\right)\right)}{e^N} \right)^3 \frac{\Gamma(N)}{H(N)} \leq 1$$

4. Result: constraints on $\xi \geq \xi_{\langle \mathcal{N} \rangle = 1}$ and cosmological implications from the time of predominant bubble nucleation.

The return of the ξ

$$S=\int d^4x\sqrt{-g_J}\left[\frac{M_P^2}{2}\left(1-\frac{\xi h^2}{M_P^2}\right)R_J+\frac{1}{12M^2}R_J^2+\frac{1}{2}g_J^{\mu\nu}\partial_\mu h\partial_\nu h-\frac{\lambda}{4}h^4\right]$$

$$\Rightarrow \ldots \Rightarrow \mathcal{L}\approx\frac{M_P^2}{2}R+\frac{1}{2}\partial_\mu\tilde{\phi}\partial^\mu\tilde{\phi}+\frac{1}{2}\partial_\mu\rho\partial^\mu\rho-\tilde{U}(\tilde{\phi},\rho)$$

$$\tilde{U}=V_I+\frac{m_{\text{eff}}^2}{2}\rho^2+\frac{\lambda_{\text{eff}}}{4}\rho^4+\ldots$$

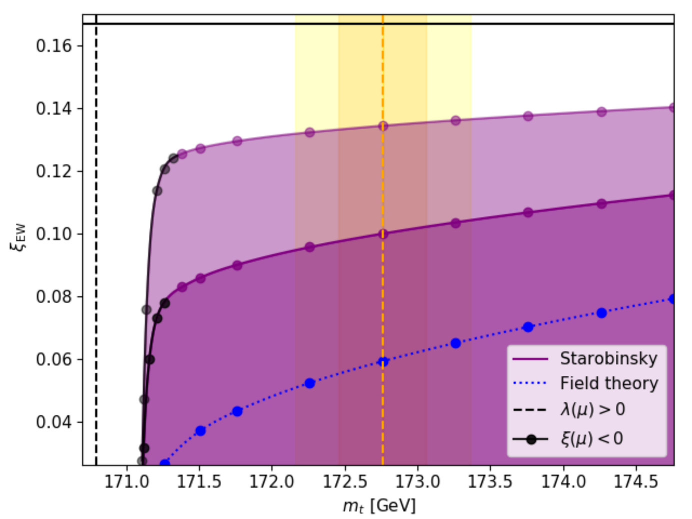
$$V_{\mathrm{I}}(\tilde{\phi})=\frac{3M^2M_P^4}{4}\left(1-e^{-\sqrt{\frac{2}{3}}\frac{\tilde{\phi}}{M_P}}\right)^2,$$

$$m_{\text{eff}}^2=\xi R+3M^2M_P^2\Xi\left(1-e^{-\sqrt{\frac{2}{3}}\frac{\tilde{\phi}}{M_P}}\right)e^{-\sqrt{\frac{2}{3}}\frac{\tilde{\phi}}{M_P}}+\frac{\Xi}{M_P^2}\partial_\mu\tilde{\phi}\partial^\mu\tilde{\phi},$$

$$\lambda_{\text{eff}}=\lambda+3M^2\Xi^2e^{-2\sqrt{\frac{2}{3}}\frac{\tilde{\phi}}{M_P}}+\frac{4\left[\xi R+\Delta m_1^2\right]\Xi^2}{M_P^2}+\frac{4\Xi^3}{M_P^4}\partial_\mu\tilde{\phi}\partial^\mu\tilde{\phi}.$$

A parameter space odyssey

Lower bounds on the Higgs curvature coupling for varying top quark mass during Starobinsky inflation.



The last slide

- Minimal model of the early universe: SM + Starobinsky inflation (observationally favoured) from modification of gravity $R + R^2$.

- Vacuum decay constraints on the Higgs-curvature coupling, with state-of-the-art $V_{\rm eff}^{\rm RGI}$ (3-loop couplings, 1-loop dS corrections):

$$\xi_{\rm EW} \gtrsim 0.1 > 0.06\,,$$

  give stricter ξ-bounds from extra negative terms in $V_{\rm H}^{\rm RGI}$.

- Bubble nucleation in the last moments of inflation: breakdown of dS approximations and necessity to consider the dynamics of reheating.

- Possibly hints against eternal inflation.

Additional slides

RG improved effective Higgs potential in de Sitter

- Minkowski terms to 3-loops, curvature corrections in dS at 1-loop:

$$V_{\rm H}(h,\mu,R) = \frac{\xi}{2}Rh^2 + \frac{\lambda}{4}h^4 + \frac{\alpha}{144}R^2 + \Delta V_{\rm loops}\,,$$

where the loop contribution can be parametrized as

$$\Delta V_{\rm loops} = \frac{1}{64\pi^2}\sum_{i=1}^{31}\left\{n_i\mathcal{M}_i^4\left[\log\left(\frac{|\mathcal{M}_i^2|}{\mu^2}\right) - d_i\right] + \frac{n_i' R^2}{144}\log\left(\frac{|\mathcal{M}_i^2|}{\mu^2}\right)\right\}$$

- RGI: choose $\mu = \mu_*(h,R)$ such that $\Delta V_{\rm loops}(h,\mu_*,R) = 0 \rightarrow$

RGI effective Higgs potential

$$V_{\rm H}^{\rm RGI}(h,R) = \frac{\xi(\mu_*(h,R))}{2}Rh^2 + \frac{\lambda(\mu_*(h,R))}{4}h^4 + \frac{\alpha(\mu_*(h,R))}{144}R^2$$

Markkanen *et al*, "The 1-loop effective potential for the Standard Model in curved spacetime", 2018.

Numerical solution for vacuum decay during inflation

Solve the system of coupled differential equations beyond slow-roll:

$$
\begin{aligned}
\frac{d^2\phi}{dN^2} &= \frac{V_{\rm I}(\phi)}{M_P^2 H^2}\left(\frac{d\phi}{dN} - M_P^2 \frac{V_{\rm I}'\phi)}{V_{\rm I}(\phi)}\right) \\
\frac{d\tilde{\eta}}{dN} &= -\tilde{\eta}(N) - \frac{1}{a_{\rm inf} H(N)} \\
\frac{d\langle\mathcal{N}\rangle}{dN} &= \gamma(N) = \frac{4\pi}{3}\left[a_{\rm inf}\left(\frac{3.21 e^{-N}}{a_0 H_0} - \tilde{\eta}(N)\right)\right]^3 \frac{\Gamma(N)}{H(N)}
\end{aligned}
$$

where $\tilde{\eta} = e^{-N}\eta$ with η: conformal time and

$$
H^2 = \frac{V_{\rm I}(\phi)}{3M_P^2}\left[1 - \frac{1}{6M_P^2}\left(\frac{d\phi}{dN}\right)^2\right]^{-1},
$$

$$
R = 6\left(\frac{\dot{a}^2}{a^2} + \frac{\ddot{a}}{a}\right) = 12H^2\left[1 - \frac{1}{4M_P^2}\left(\frac{d\phi}{dN}\right)^2\right].
$$