General Relativistic Simulations of Compact Binary Mergers

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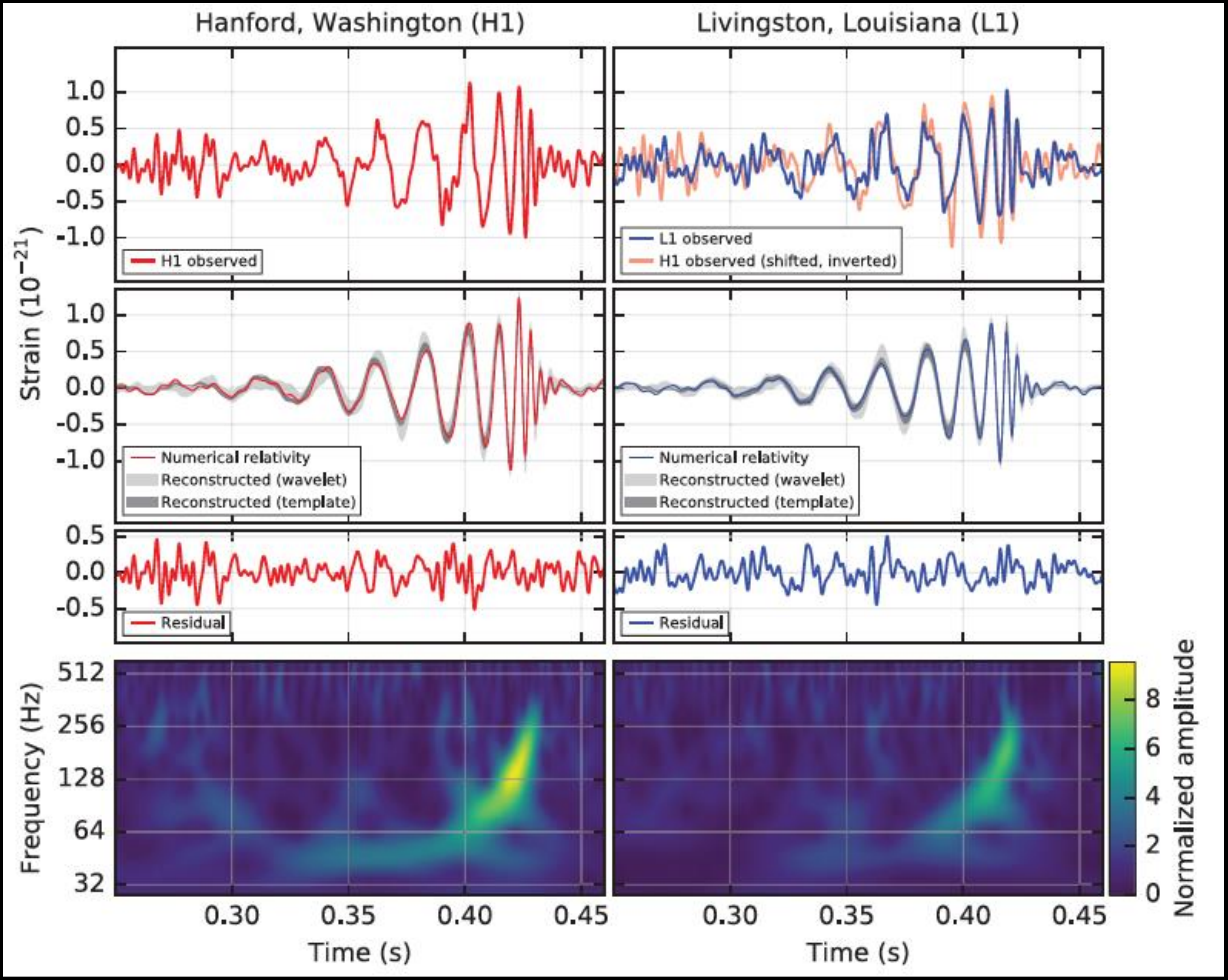
Texel 2023

1. Motivation

Predict gravitational wave signal



Signal

=

Theory

+

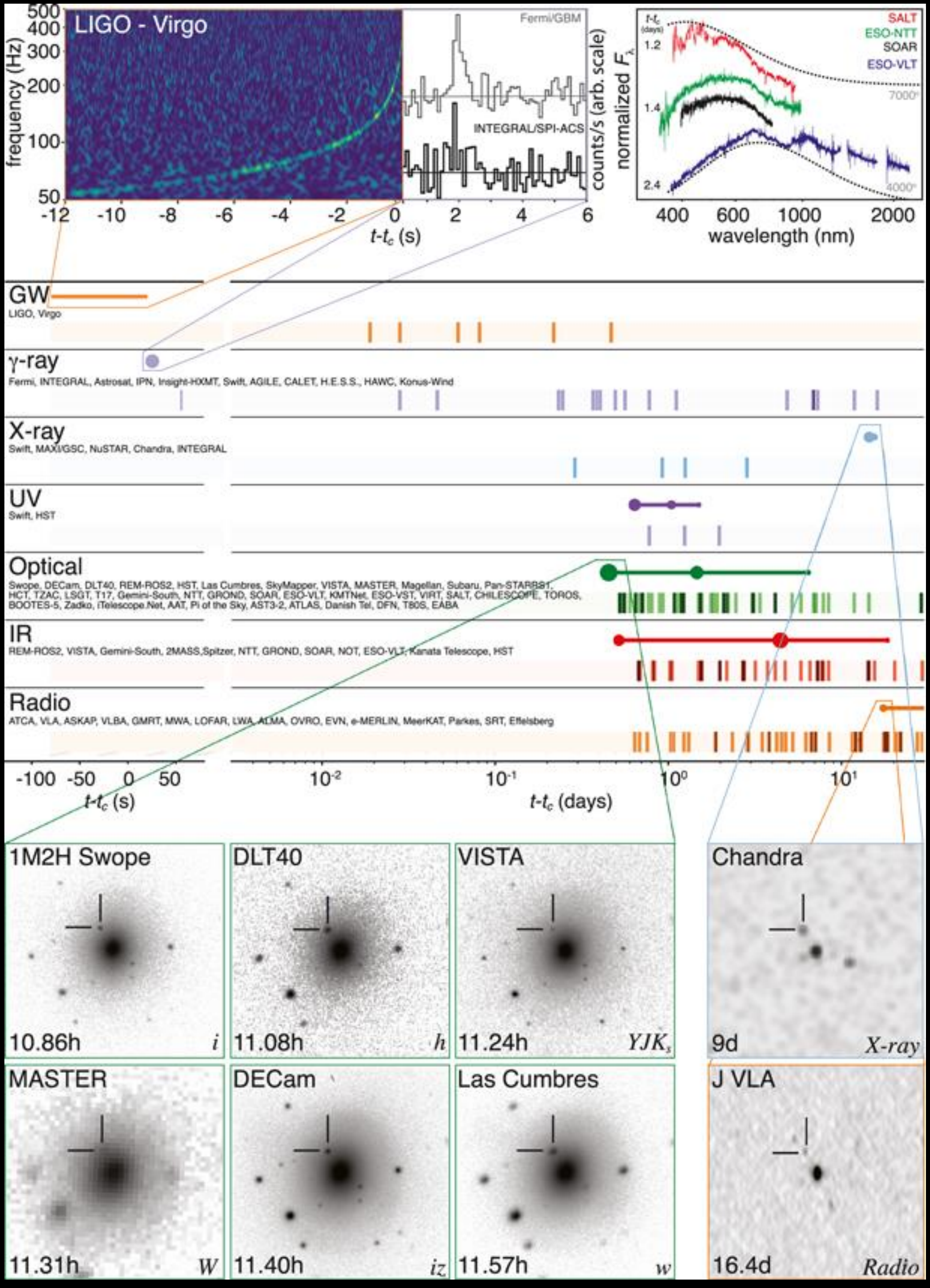
Noise

Image: LIGO Collaboration, PRL

Objective of waveform generation

- Waveforms should *cover the relevant parameter space* (high dimensional)
- Waveforms should be *accurate enough to avoid biases / signal losses*
  - For detection, waveforms should avoid excessive loss of SNR (typically 97% overlap is required for template banks)
  - For parameter estimation, the numerical error should be smaller than the statistical error due to detector noise.
    - This becomes **harder** as the signal becomes louder!
    - Current waveforms are insufficient for LISA data analysis (and maybe for the best upcoming LIGO events?)

Merger dynamics and post-merger signals



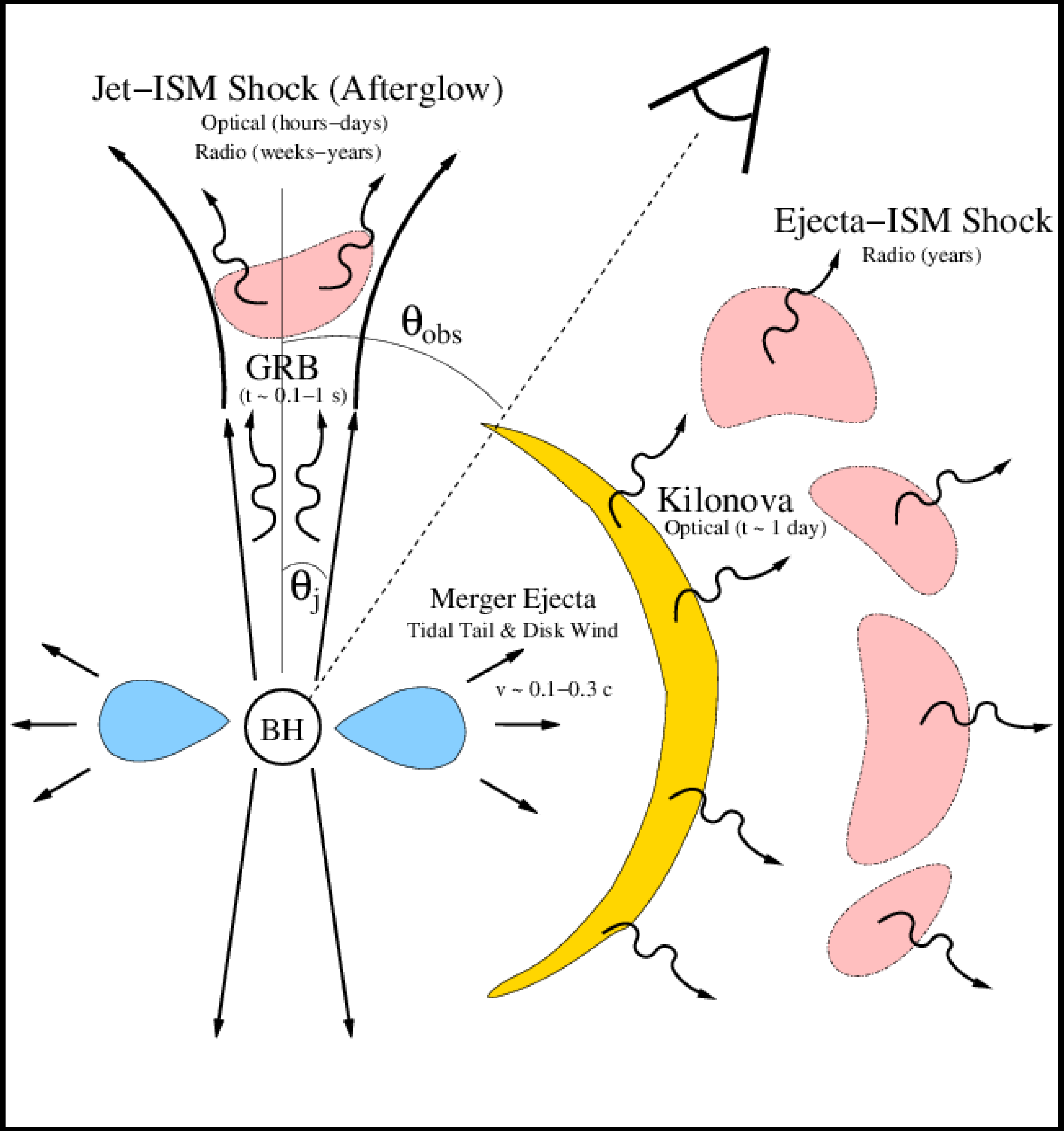


Image: Metzger & Berger 2012

Other applications:

Supernova explosions

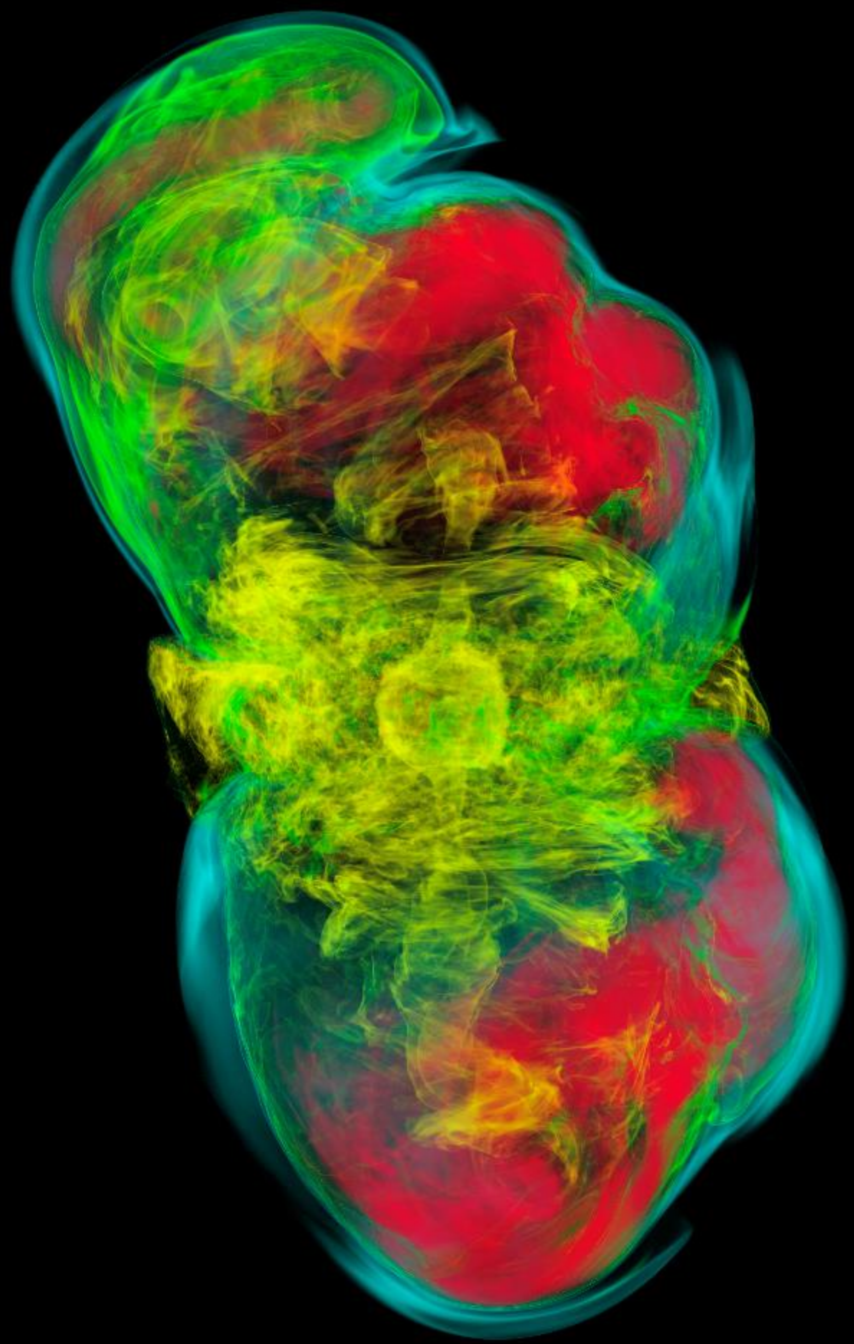


Image: P. Moesta

Accretion in time-dependent spacetimes

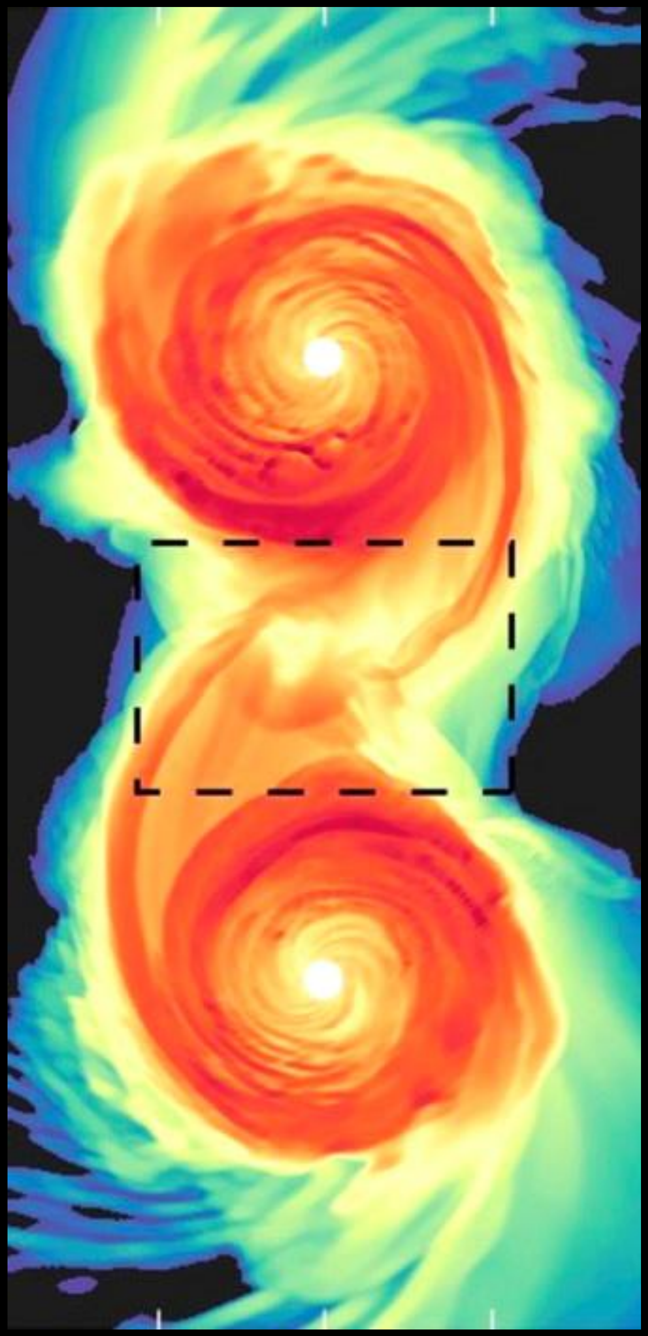


Image: Bowen et al. 2017

Accretion in BBH systems

Overview of lectures

- Numerical relativity for BBH systems: methods and challenges
- Results and uses of BBH simulations
- Physics of BNS/BHNS mergers
- EM counterparts to BNS/BHNS mergers
- Magnetic fields and neutrinos in BNS/BHNS mergers
- Numerical methods for BNS/BHNS mergers

2. Numerical Relativity : Challenges and Formalism

Numerical relativity

Einstein's equation :

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

Spacetime curvature

Matter

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R^\lambda_\lambda g_{\mu\nu} \qquad R_{\mu\nu} = R^\lambda_{\mu\lambda\nu}$$

$$R^\mu_{\alpha\beta\gamma} = (\Gamma^\mu_{\alpha\gamma,\beta} - \Gamma^\mu_{\alpha\beta,\gamma} + \Gamma^\mu_{\sigma\beta}\Gamma^\sigma_{\gamma\alpha} - \Gamma^\mu_{\sigma\gamma}\Gamma^\sigma_{\beta\alpha})$$

$$\Gamma^\alpha_{\beta\gamma} = \frac{1}{2} g^{\alpha\sigma}(g_{\beta\sigma,\gamma} + g_{\gamma\sigma,\beta} - g_{\beta\gamma,\sigma})$$

Numerical relativity problem:

Transform these equations in ***evolution equations*** which are stable and well-posed.
Make sure that any ***constraint*** is satisfied at all times.

First step: go back to a 3+1 formulation of the problem, so that "time evolution" is a meaningful concept…

3+1 (ADM) formalism

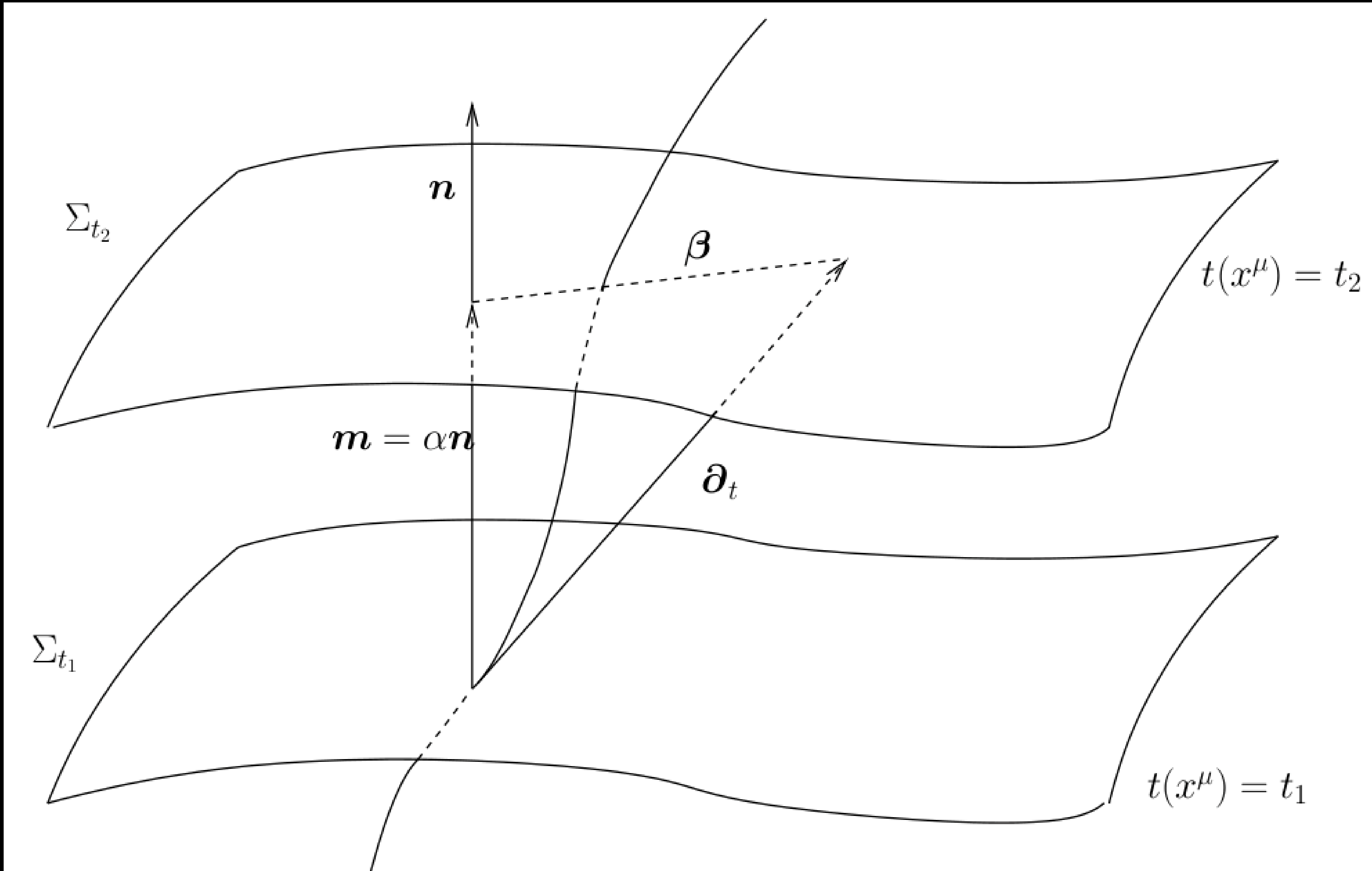


Image: Cardoso et al.

3+1 (ADM) formalism

Define a **foliation** of spacetime using 3D hypersurfaces parametrized by a coordinate “t”

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = -\alpha^2 dt^2 + \gamma_{ij}(dx^i + \beta^i dt)(dx^j + \beta^j dt)$$

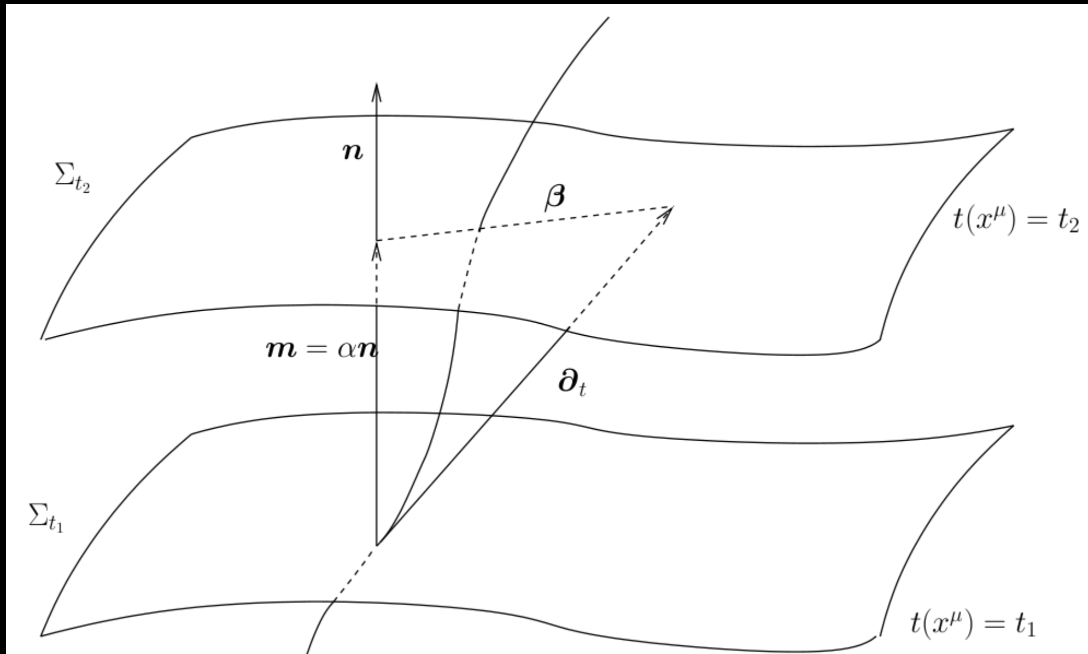


Image: Cardoso et al.

$$t^\mu = \alpha n^\mu + \beta^\mu$$

n = normal to slice

t = tangent to line of constant x^i

α = Lapse; β = Shift

With the **extrinsic curvature** $K_{ij} = -\mathcal{L}_n \gamma_{ij}$

$$\partial_t \gamma_{ij} = -2\alpha K_{ij} + \nabla_i \beta_j + \nabla_j \beta_i$$

$$\partial_t K_{ij} = \alpha(R_{ij} - 2K_{ik}K^k_j + KK_{ij} - \nabla_i\nabla_j\alpha + \beta^k\nabla_k K_{ij} + K_{ik}\nabla_j\beta^k + K_{kj}\nabla_i\beta^k) + \mathrm{Matter}$$

+ <u>Constraints</u>:

$$R + K^2 - K_{ij}K^{ij} = \mathrm{Matter}$$
$$\nabla_j(K^{ij} - \gamma^{ij}K) = \mathrm{Matter}$$

EM analogy

$$\partial_t \gamma_{ij} = -2\alpha K_{ij} + \nabla_i \beta_j + \nabla_j \beta_i$$
$$\partial_t K_{ij} = \alpha(R_{ij} - 2K_{ik}K^k_j + KK_{ij} - \nabla_i \nabla_j \alpha + \beta^k \nabla_k K_{ij} + K_{ik}\nabla_j \beta^k + K_{kj}\nabla_i \beta^k) + \mathrm{Matter}$$

$$\Longleftrightarrow$$

$$\partial_t B = -\nabla \times E$$
$$\partial_t E = \nabla \times B - J$$

$$R + K^2 - K_{ij}K^{ij} = \mathrm{Matter}$$
$$\nabla_j (K^{ij} - \gamma^{ij} K) = \mathrm{Matter}$$

$$\Longleftrightarrow$$

$$\nabla E = \rho$$
$$\nabla B = 0$$

3+1 formalism

- Issues with 3+1 formalism:
  - How do we choose the foliation? Lapse and shift have to be chosen on-the-fly [Gauge freedom]!
  - How do we stop the growth of the constraints from small numerical errors / bad initial data / boundary conditions?
  - How do we choose appropriate initial data?
  - How do we get a well-posed problem?
  - How do we deal with singularities?

Stable numerical relativity

- Solutions took decades to take shape…
- Multiple advances needed:
  - Use constraint damping
  - Deal with BH singularities (or conveniently ignore them) / make ``appropriate'' gauge choices
  - Express evolution equations in symmetric hyperbolic form (well-posed initial value problem with bounded evolution)

Constraint damping

- Gundlach et al.: to limit growth of constraints, add "damping" term to the equations

EM analogy (ideal MHD) :

$$\partial_t B = -\nabla \times E - \nabla\phi$$
$$\partial_t \phi = -\nabla . B - \gamma\phi$$

$$\partial_t^2 \phi - \Delta\phi = -\gamma \partial_t \phi$$

- Constraint-violating mode is now a damped wave moving at the speed of light!

- GR constraint damping is more complex algebraically… and the correct choice for the damping parameter is an unsolved problem

Hyperbolic equations and characteristics

Let's assume evolution equations for a *state vector* U

$$\partial_t U_a + A^{ib}_a \partial_i U_b = S_a$$

It is strongly hyperbolic if for all vectors w, $A^{ib}_a w_i$ is diagonalizable (has real eigenvalues)

- Advantages:
  - Eigenvectors of $A^{ib}_a w_i$ (characteristic modes) travel as a wave at a known speed (char. speed) across a surface with normal w_i
  - Growth and damping of the modes only due to source terms (and nonlinear interactions between modes)
  - Boundary conditions are applied solely on incoming modes
  - Communication between neighboring simulation "patches" can also use this characteristic structure
  - Analytical work provides conditions to avoid the formation of shocks

Hyperbolic forms of Einstein's equations

- Most commonly used formulation of Einstein's equations : Baumgarte-Shapiro-Shibata-Nakamura (BSSN)
- Developed as an improvement of the ADM formalism, mathematical justification only came later

$$
\begin{aligned}
\partial_0 \tilde{\gamma}_{ij} &= -2\alpha \tilde{A}_{ij}, & (1)\\
\partial_t \chi &= \frac{2}{3}\chi\left(\alpha K - \partial_a \beta^a\right) + \beta^i \partial_i \chi, & (2)\\
\partial_0 \tilde{A}_{ij} &= \chi\left(-D_i D_j \alpha + \alpha R_{ij}\right)^{TF} + \\
&\quad \alpha\left(K\tilde{A}_{ij} - 2\tilde{A}_{ik}\tilde{A}^k_j\right), & (3)\\
\partial_0 K &= -D^i D_i \alpha + \alpha\left(\tilde{A}_{ij}\tilde{A}^{ij} + \frac{1}{3}K^2\right), & (4)\\
\partial_t \tilde{\Gamma}^i &= \tilde{\gamma}^{jk}\partial_j\partial_k\beta^i + \frac{1}{3}\tilde{\gamma}^{ij}\partial_j\partial_k\beta^k + \beta^j\partial_j\tilde{\Gamma}^i - \\
&\quad \tilde{\Gamma}^j\partial_j\beta^i + \frac{2}{3}\tilde{\Gamma}^i\partial_j\beta^j - 2\tilde{A}^{ij}\partial_j\alpha + \\
&\quad 2\alpha\left(\tilde{\Gamma}^i{}_{jk}\tilde{A}^{jk} + 6\tilde{A}^{ij}\partial_j\phi - \frac{2}{3}\tilde{\gamma}^{ij}\partial_j K\right), & (5)
\end{aligned}
$$

$$
\begin{aligned}
\partial_0 \alpha &= -2\alpha K, & (10)\\
\partial_t \beta^a &= B^a, \quad \partial_t B^a = 3/4\,\partial_t\tilde{\Gamma}^a - \eta B^a. & (11)
\end{aligned}
$$

Images: Campanelli et al. 2005

<u>In its simplest form:</u>

24 coupled first order equations

Most widely used formulation of Einstein's equations

Hyperbolic forms of Einstein's equations

- Alternative: Generalized harmonic formulation
  - Explicitly developed to get a symmetric-hyperbolic formulation of GR

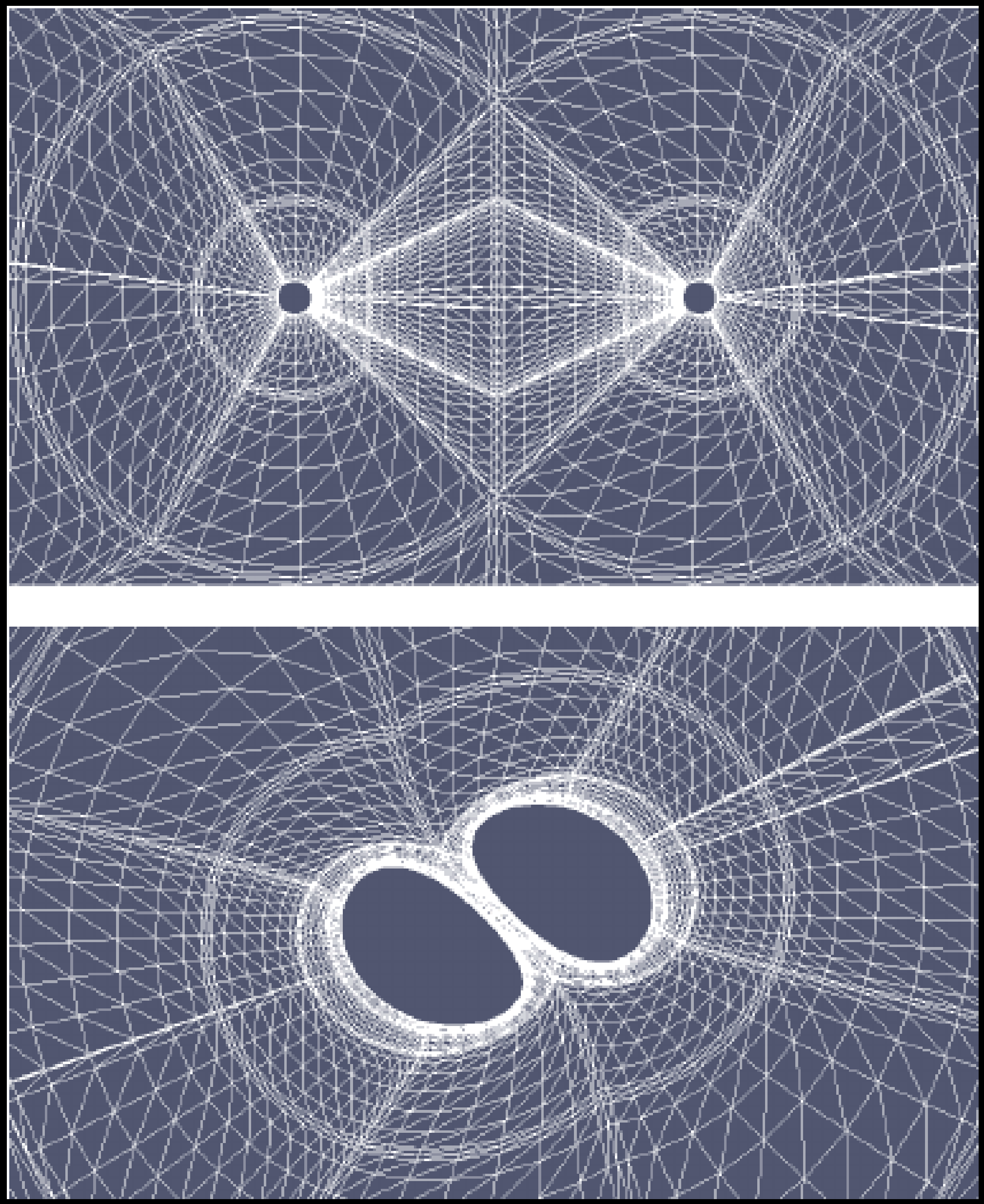
$$
\begin{aligned}
\partial_t\psi_{ab} \; &- (1+\gamma_1)N^k\partial_k\psi_{ab} = -N\Pi_{ab} - \gamma_1 N^i\Phi_{iab}, && (35)\\
\partial_t\Pi_{ab} \; &- N^k\partial_k\Pi_{ab} + Ng^{ki}\partial_k\Phi_{iab} - \gamma_1\gamma_2 N^k\partial_k\psi_{ab} \\
&= 2N\psi^{cd}\left(g^{ij}\Phi_{ica}\Phi_{jdb} - \Pi_{ca}\Pi_{db} - \psi^{ef}\Gamma_{ace}\Gamma_{bdf}\right) \\
&\quad - 2N\nabla_{(a}H_{b)} - \tfrac{1}{2}Nt^c t^d\Pi_{cd}\Pi_{ab} - Nt^c\Pi_{ci}g^{ij}\Phi_{jab} \\
&\quad + N\gamma_0\left[2\delta^c{}_{(a}t_{b)} - \psi_{ab}t^c\right](H_c+\Gamma_c) - \gamma_1\gamma_2 N^i\Phi_{iab}, && (36)\\
\partial_t\Phi_{iab} &- N^k\partial_k\Phi_{iab} + N\partial_i\Pi_{ab} - N\gamma_2\partial_i\psi_{ab} \\
&= \tfrac{1}{2}Nt^c t^d\Phi_{icd}\Pi_{ab} + Ng^{jk}t^c\Phi_{ijc}\Phi_{kab} - N\gamma_2\Phi_{iab}. && (37)
\end{aligned}
$$

$$
H_a(x,\psi) = \psi_{ab}\nabla_c\nabla^c x^b = -\Gamma_a
$$

Images: Lindblom et al. 2005

Solving the singularity issue

- Method 1 : Remove the BH from the grid
    - For a carefully chosen excision surface, no boundary condition is required (“nothing can escape a BH”)
    - Finding that surface can be tricky for moving BHs!
    - In practice: need all characteristic speeds on excision to point away from the computational domain



Images: Hemberger et al. 2012

Solving the singularity issue

- Method 2 : Let the singularity move on your grid!

Use $\gamma_{ij} = \dfrac{\tilde{\gamma}_{ij}}{\chi}$ with non-singular, C^4 numerator and denominator

"Puncture" initial data : $\gamma_{ij} = \left(1 + u + \sum_i \dfrac{m_i}{2r_i}\right)^4 \delta_{ij}$ with finite u.

Find gauge conditions that keep the puncture well-behaved

$$\partial_n \alpha = -2\alpha K$$

$$\partial_n \beta^i = \frac{3}{4} B^i$$

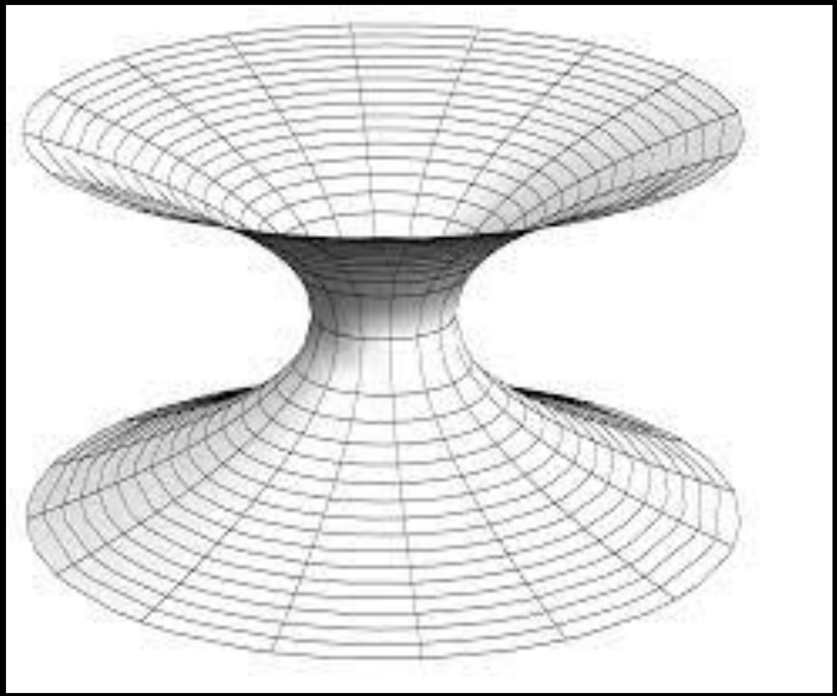
$$\partial_n B^i = S^i - \eta B^i$$

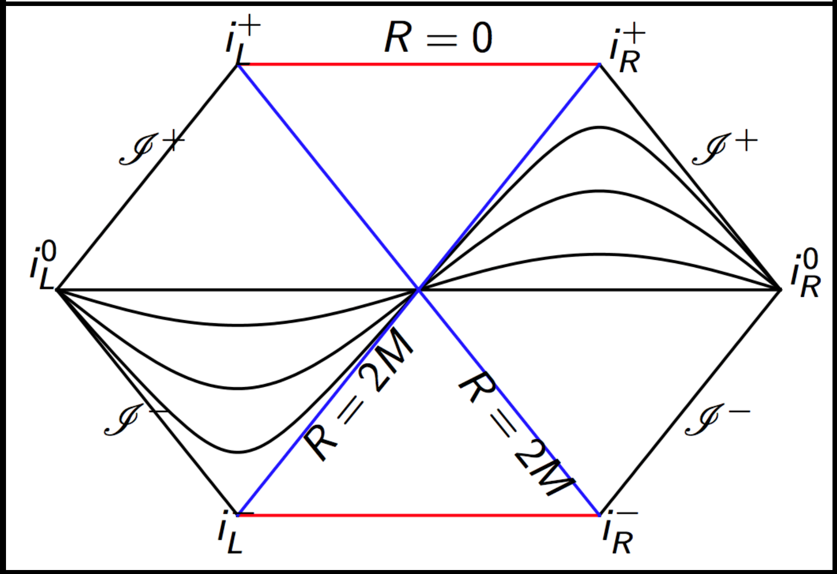
(1) Lapse condition is singularity avoiding
(2) Shift condition drives simulation towards "minimal distortion" solution
(3) Damping term avoids unphysical (gauge) oscillations of the shift

Solving the singularity issue

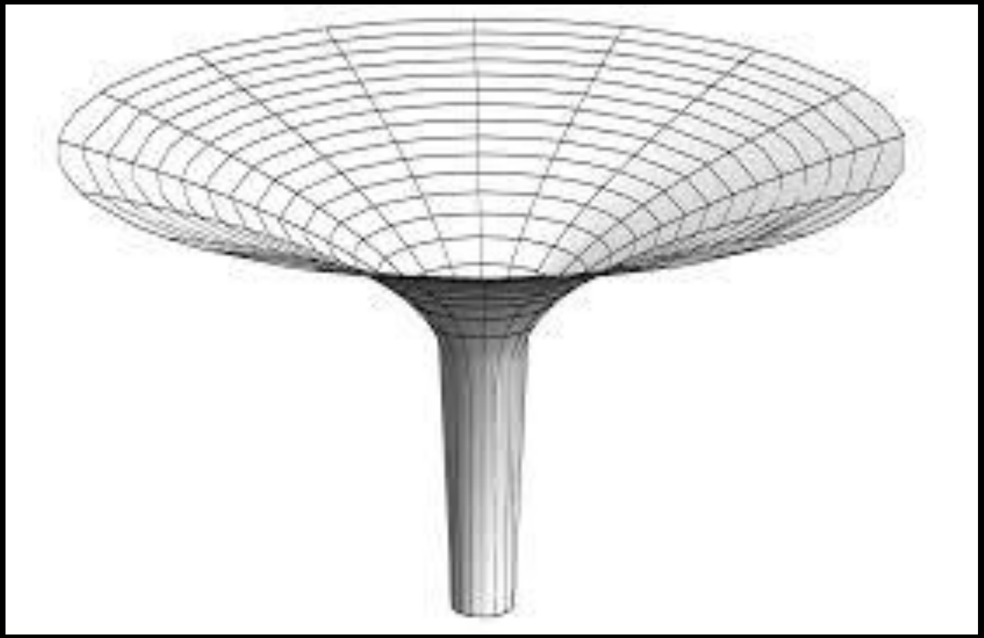
- Puncture / Trumpet evolution

Puncture

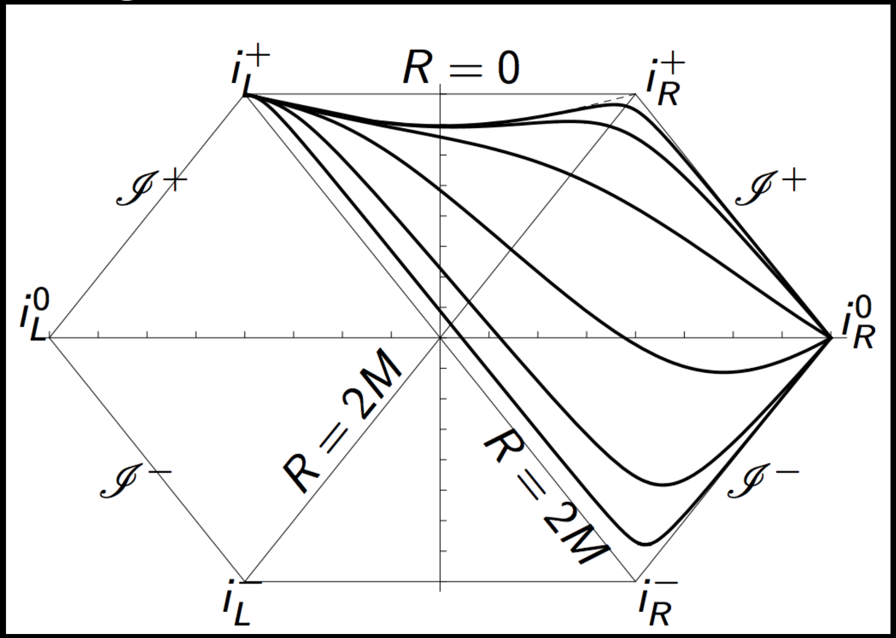




Trumpet



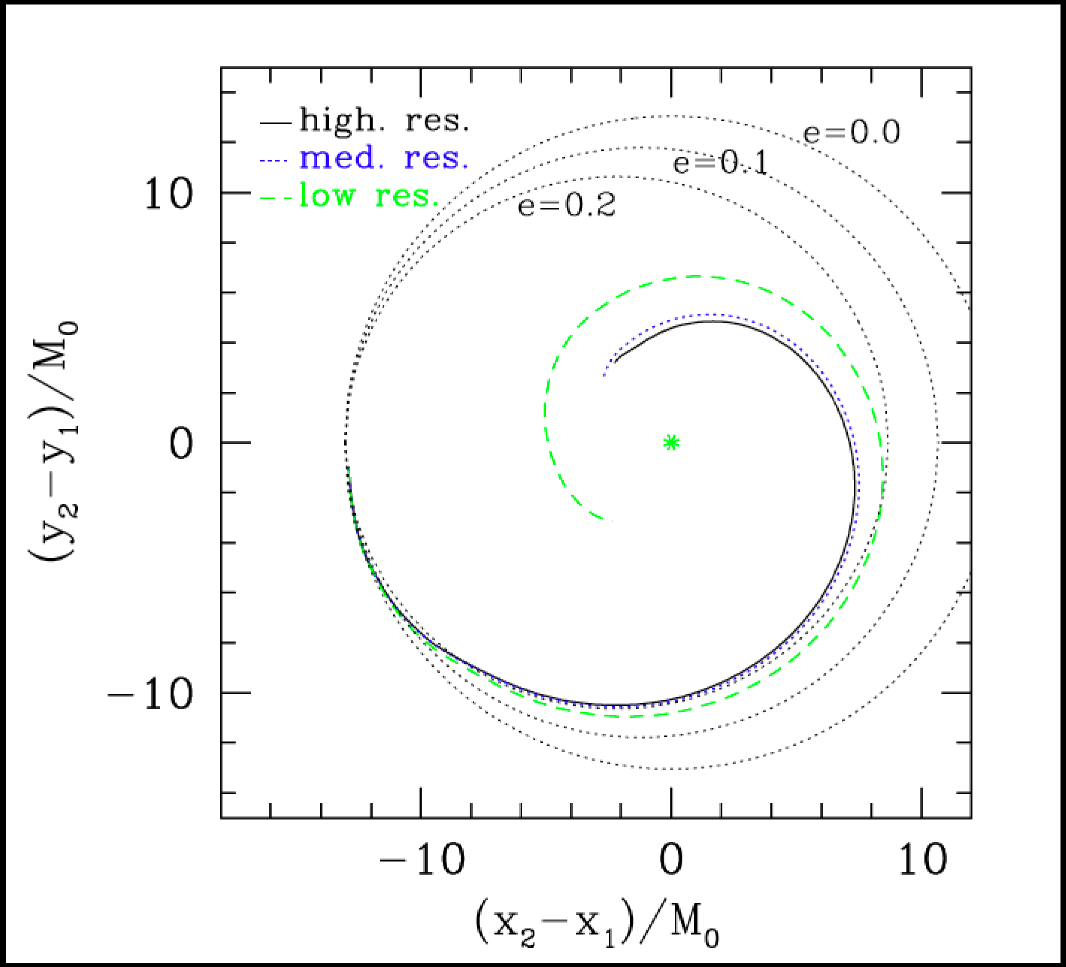
Images: Hannam et al. 2008

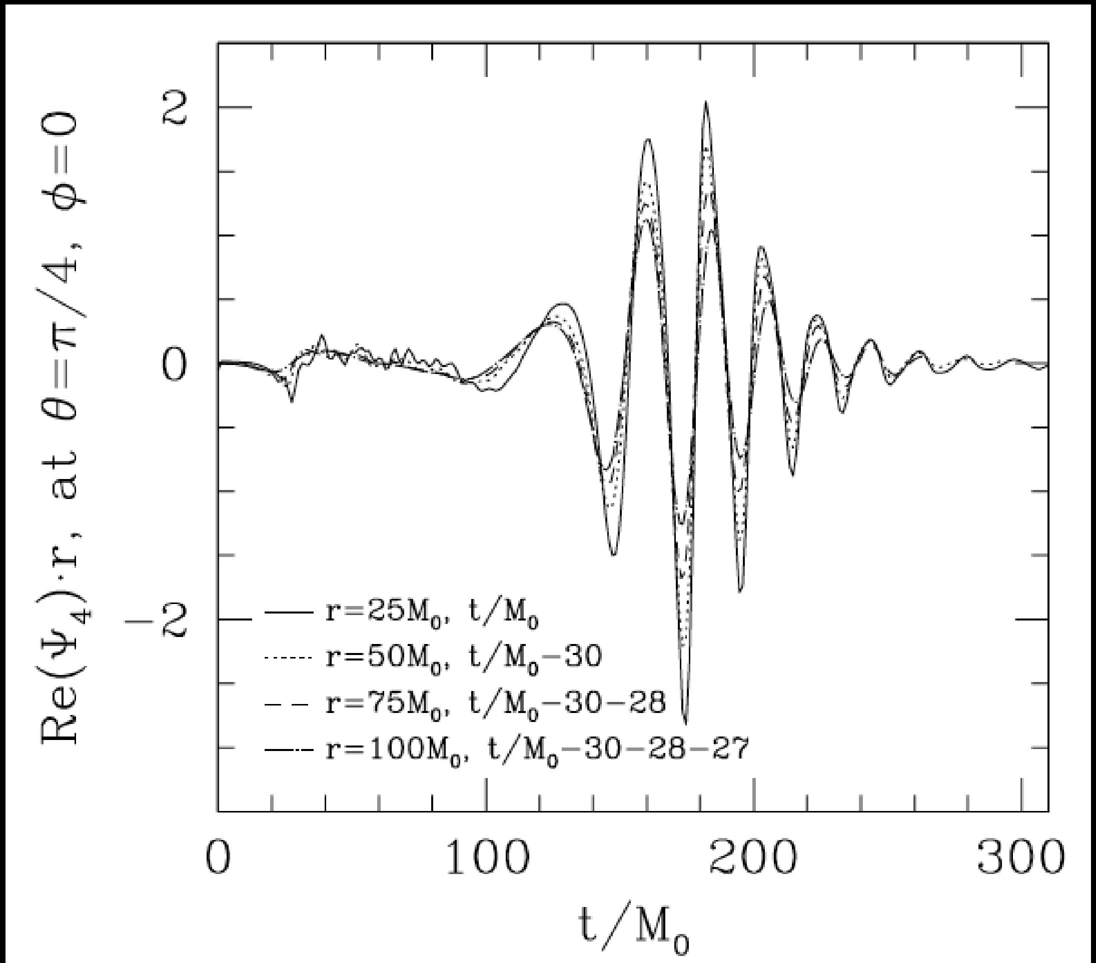


3. Numerical relativity : First Results

NR breakthrough

- First successful BBH merger : Pretorius 2005
  - Generalized harmonic formulation
  - “Dynamical excision” tracking the motion of the BHs

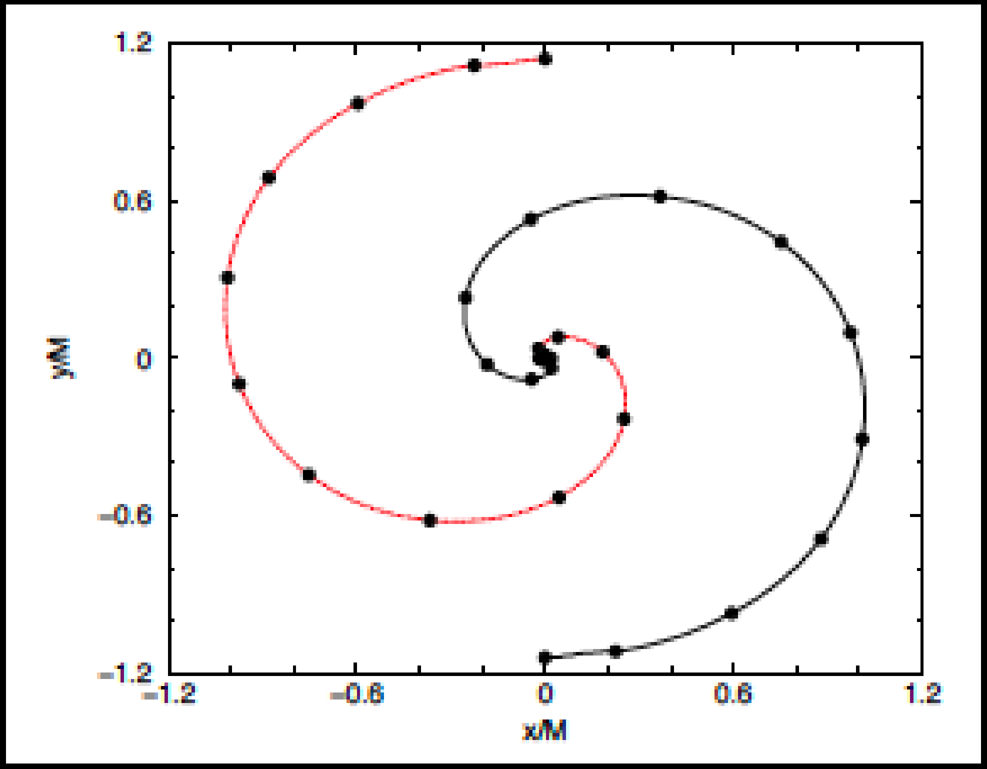


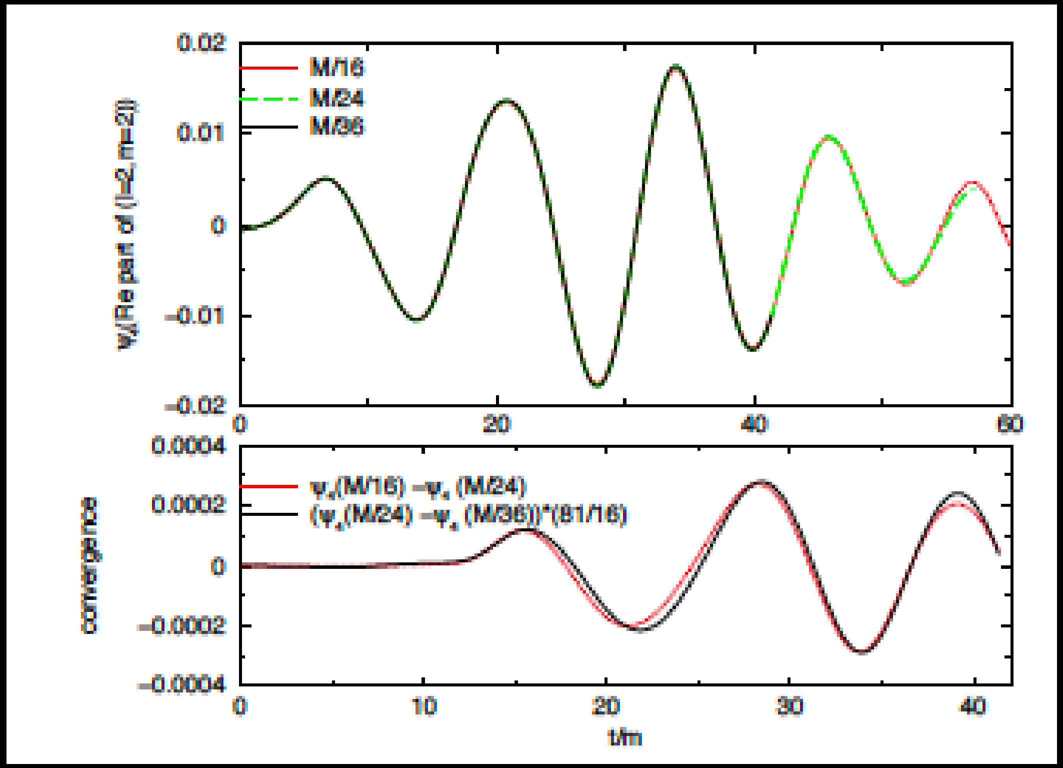


Images: Pretorius 2005

NR breakthrough

- Same year, two additional merger simulations are published using
  - BSSN formalism
  - Puncture initial data





Images: Campanelli et al. 2005

NR breakthrough

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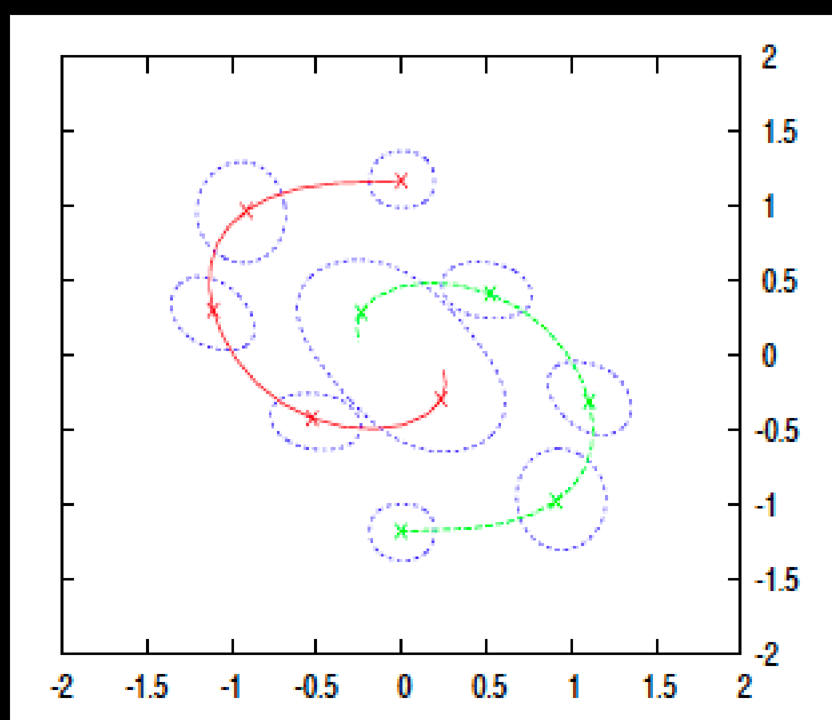
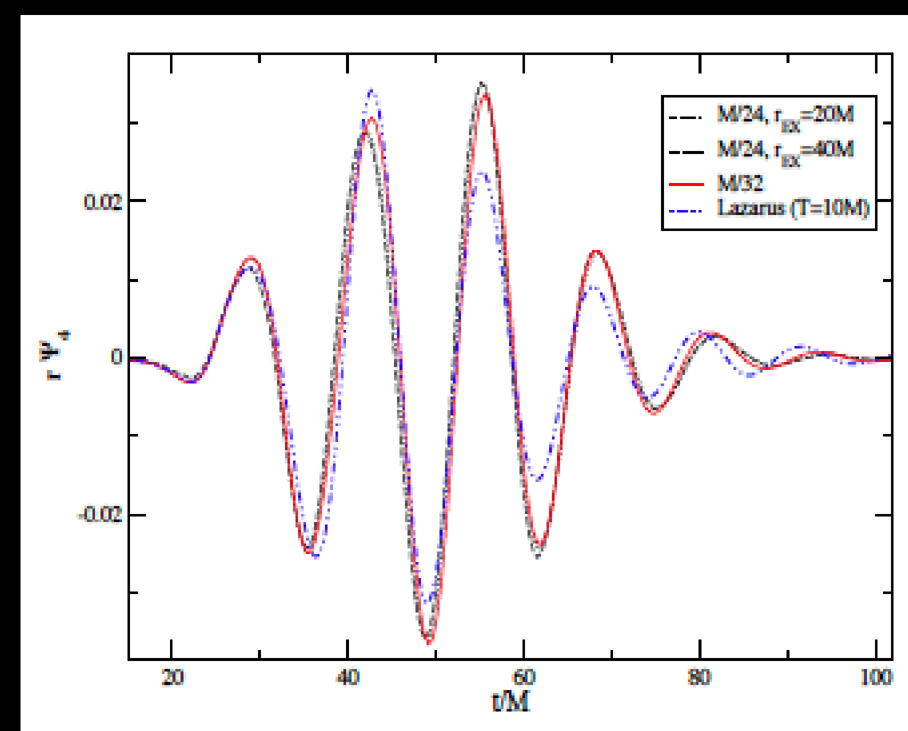
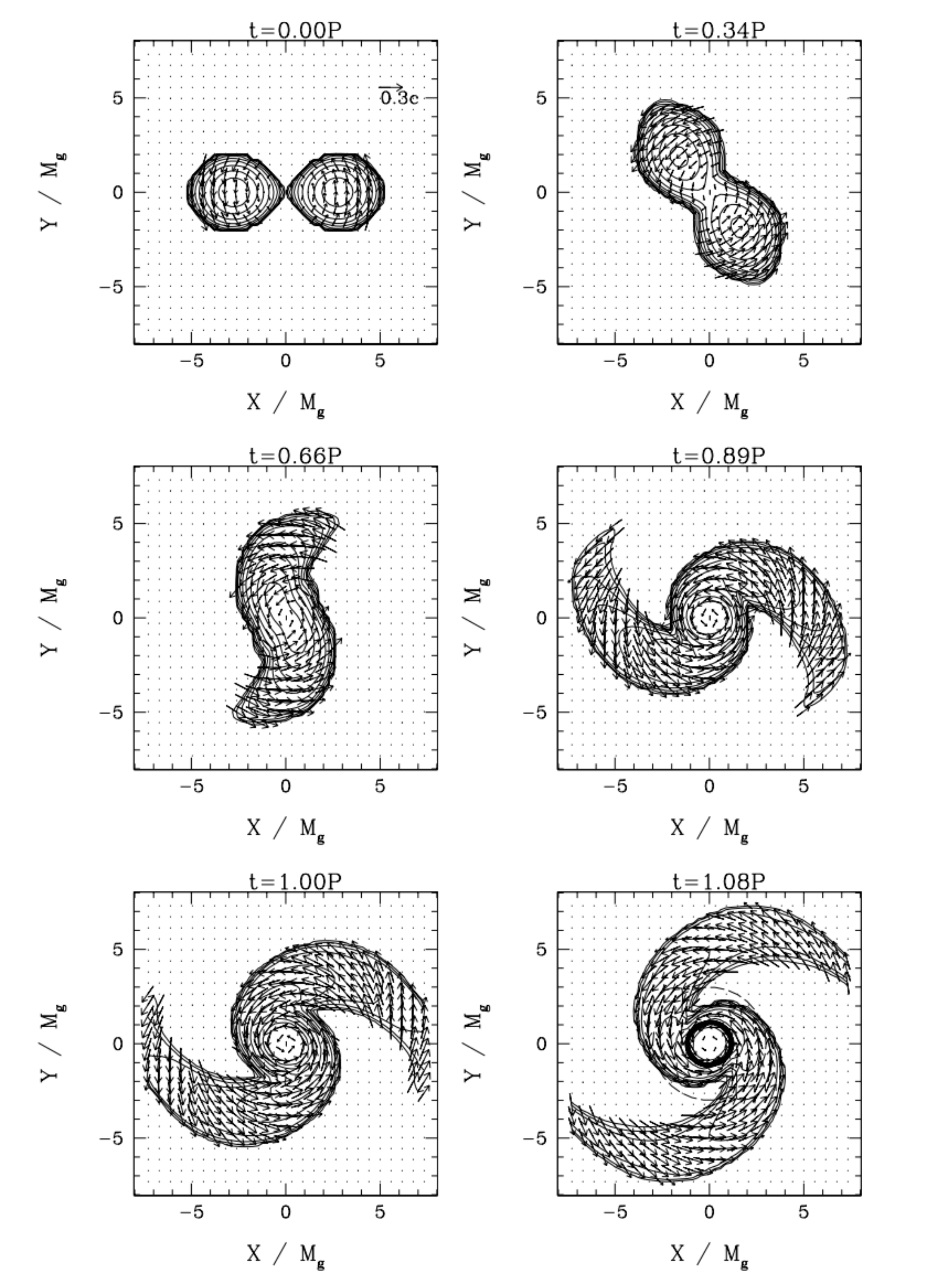




Images: Baker et al. 2005

NR pre-breakthrough



- The first successful merger simulations were for NS-NS systems (Shibata & Uryu 2000),… 5 years before the ``breakthrough''

4. Numerical relativity today: BBH simulations

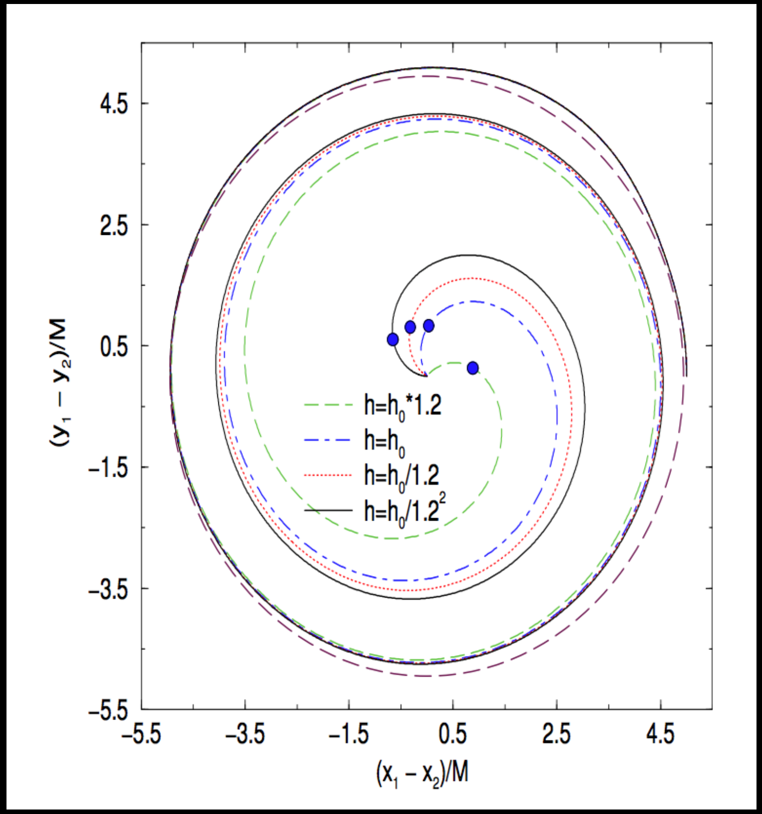
Modern evolution methods

- Finite difference codes
    - Most codes, incl. the widely used open source Einstein Toolkit
    - Typically use BSSN (except for early sims by Lehner et al.) and puncture
    - Nowadays, use high-order finite difference methods (4th-8th)
- Spectral methods (SpEC)
    - Generalized harmonic with excision
    - High-accuracy, low-robustness
    - Exponential convergence for smooth solutions (no neutron star)

NR Accomplishments

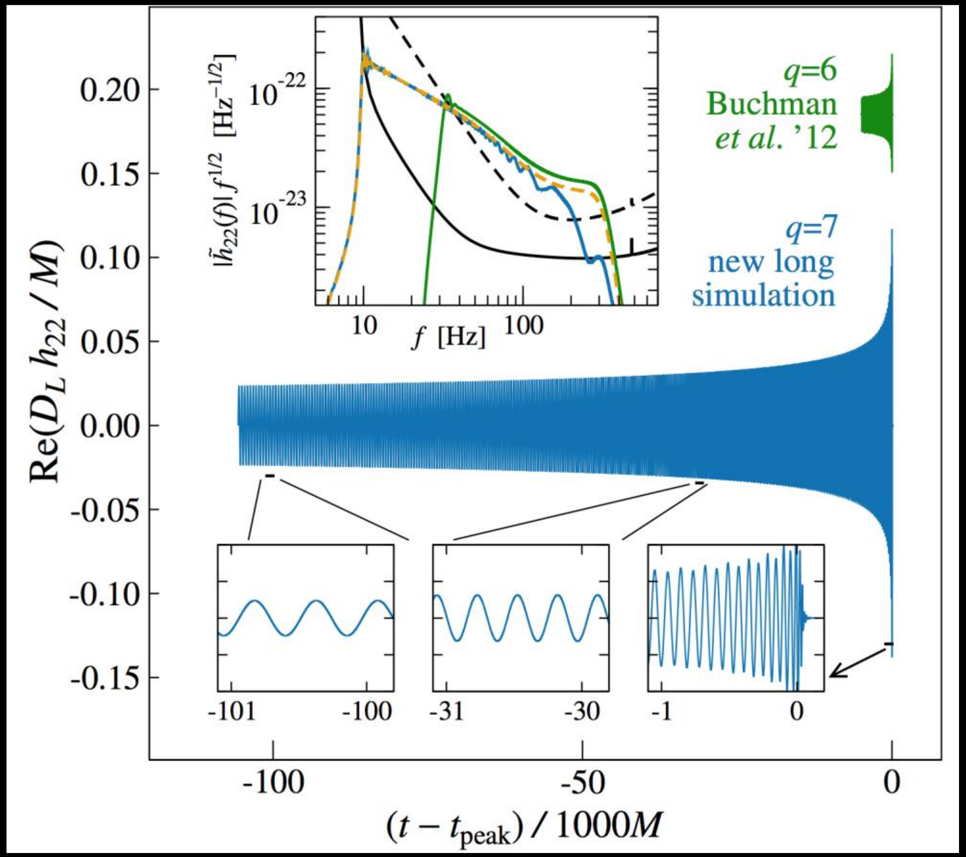
- Current codes can easily produce GW for mass ratios q~[1-10], BH spins a/M~[-0.9,0.9], tens of orbits, including precessing/eccentric systems
- Multiple numerical templates banks available with 100s of waveforms
- Extreme simulations push to q~100, a/M~0.9997, 175 orbits - usually with a trade-off on accuracy / length

q=100 Merger



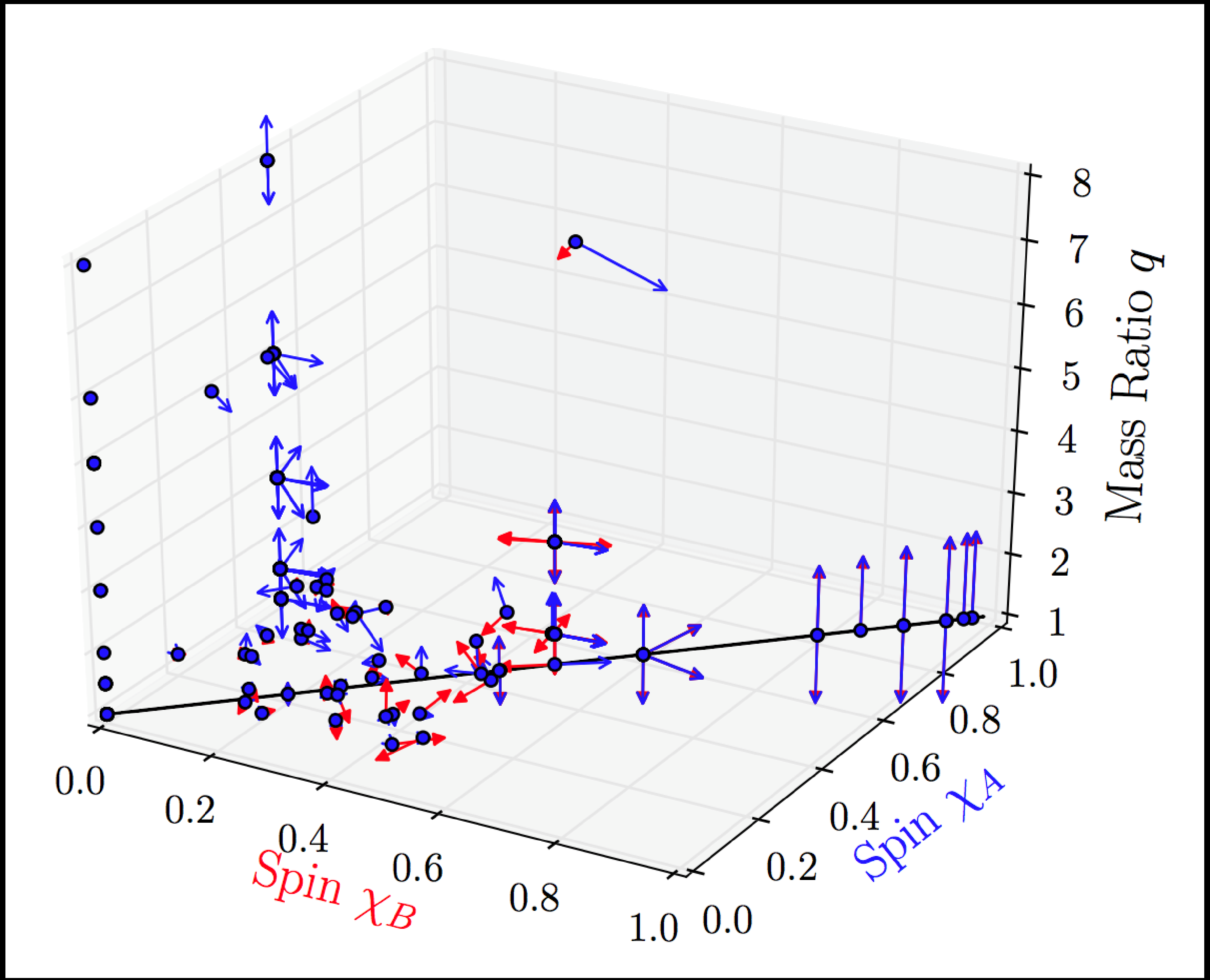
Lousto et al. 2011

175 orbits

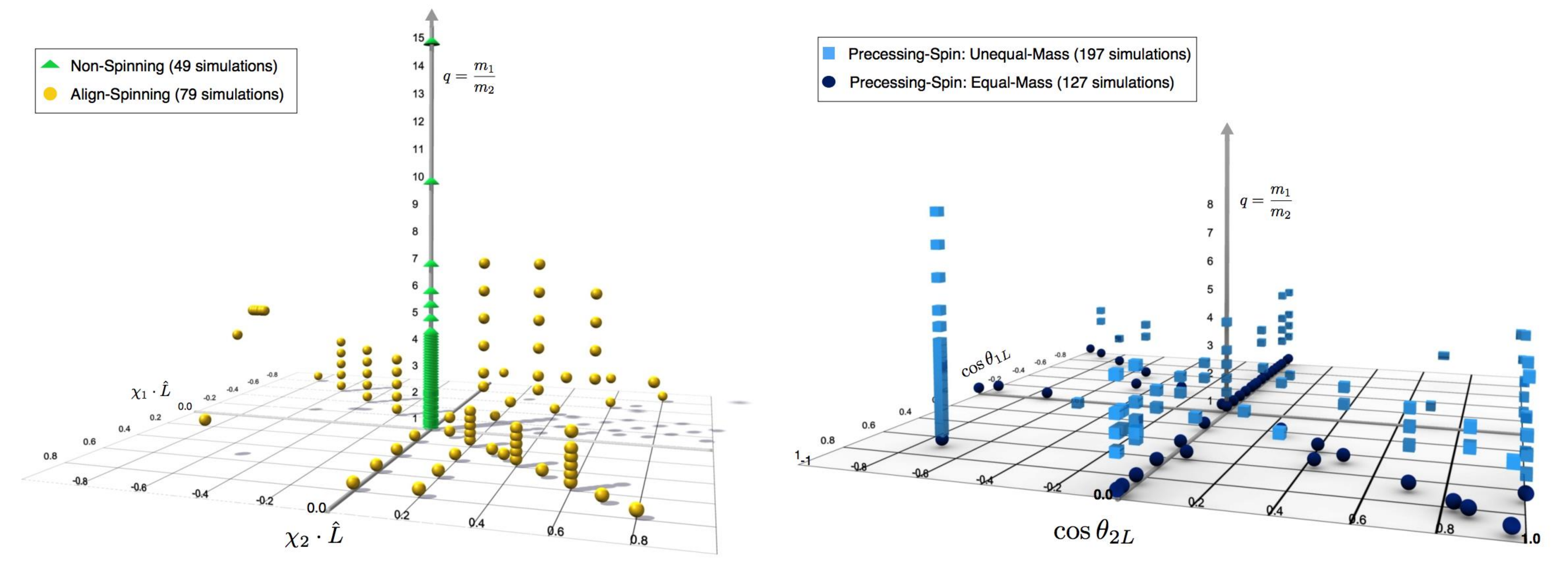


Szilagyi et al. 2015

Waveform catalogue [Mroue et al. 2013]



Waveform catalogue [Jani et al. 2016]



Waveform models and Simulations

Three main classes of waveform models today:

- Phenomenological models (Phenom)
  - Based on Post-Newtonian (PN) expansion
  - Fit corrections to PN of a chosen functional form to simulation results
- Effective-one-body (EOB) models
  - More advanced analytical model based on mapping of extreme mass ratio results to similar-mass binaries
  - Still include unknown terms that can be calibrated to simulation results
- Numerical relativity surrogates
  - More recent methods that solely use simulation data
  - Does not require analytical assumption, but limited by length / parameter space coverage of simulations

Other NR results for BH-BH

- Before LIGO, numerical relativity already had some success in astrophysics
- Kicks:
  - Gravitational waves carry energy, angular momentum, and linear momentum
  - Loss of momentum adds to 0 over a circular orbit… but not over a plunging orbit
  - Mergers can leave the remnant with velocities up to ~5000km/s (and commonly a few 100s km/s)

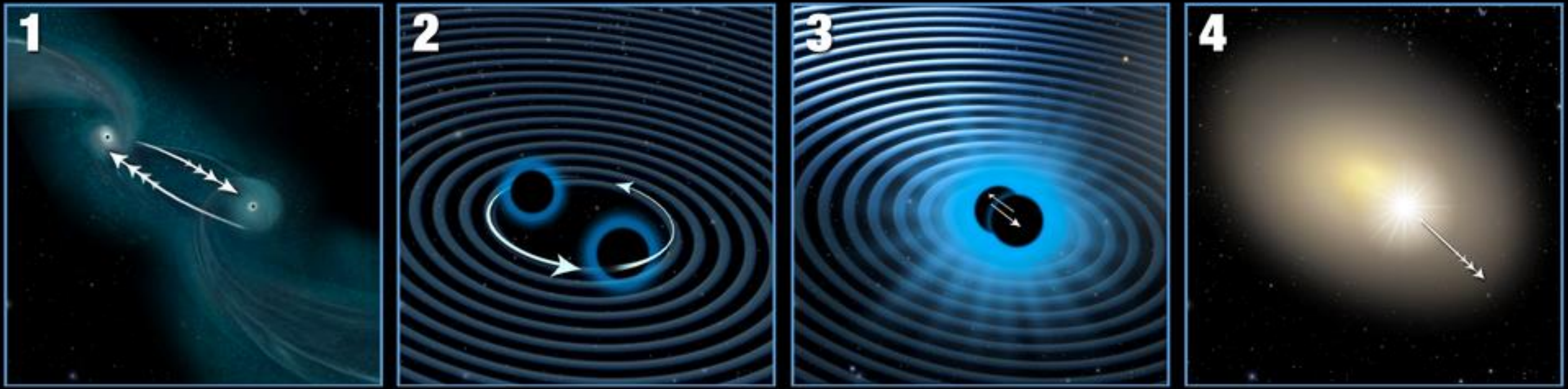


Image: NASA

Other NR results for BBH

- Before LIGO, numerical relativity already had some success in astrophysics

- Post-merger properties:

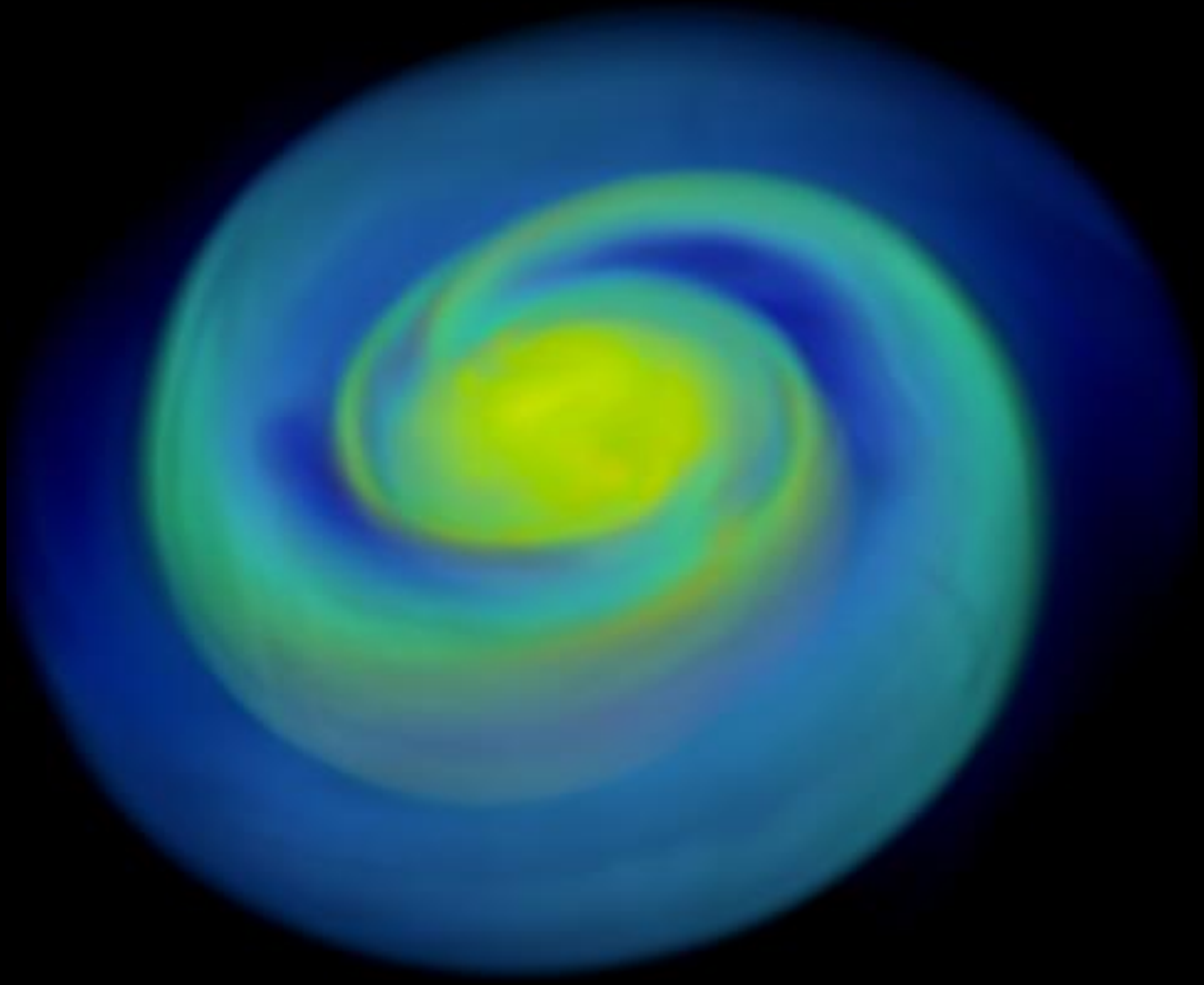
    - Numerical relativity predicts the mass and spin of the post-merger BH

    - Useful in cosmological simulations to estimate the evolution of the BH population

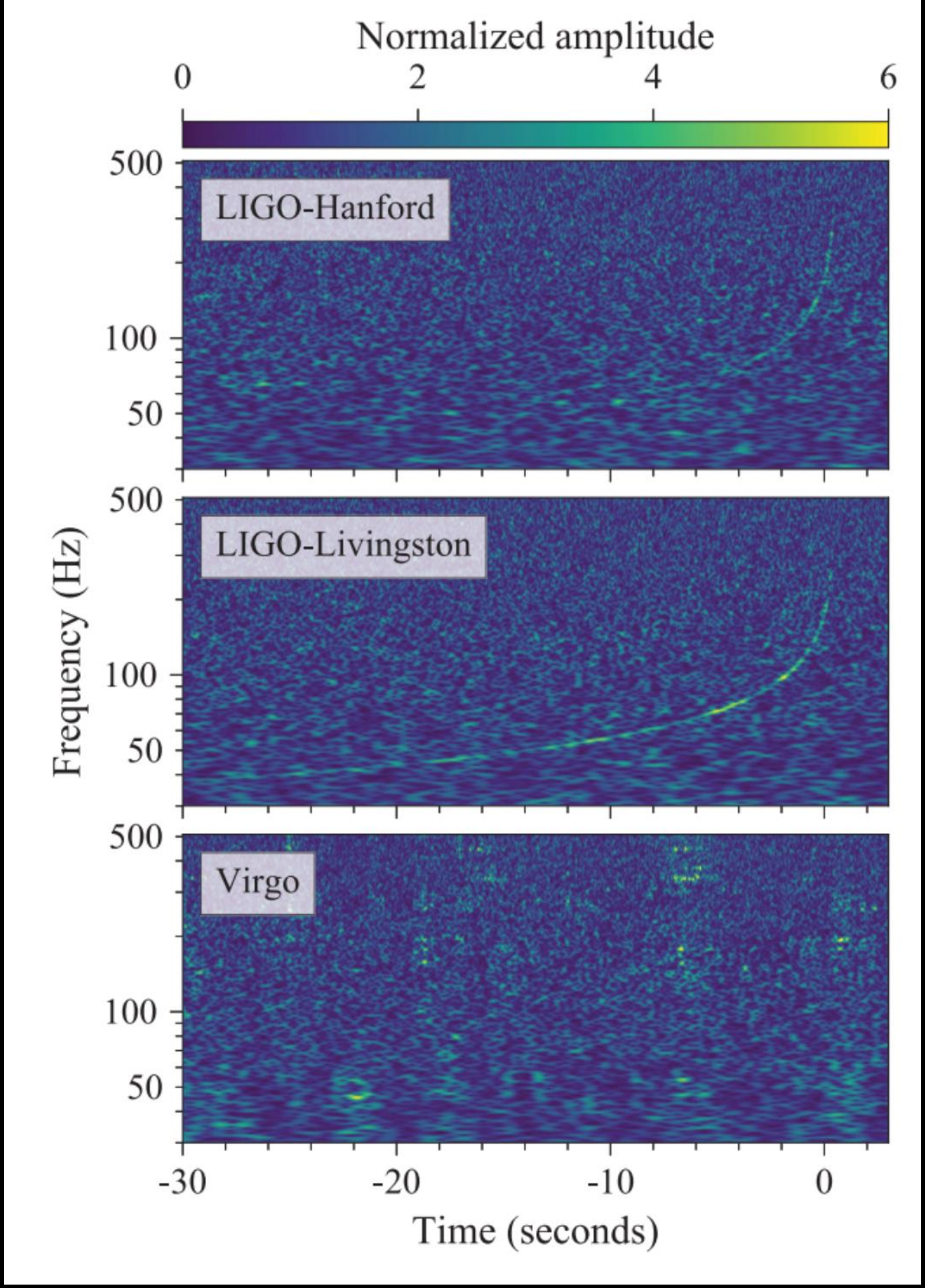
Numerical relativity for BH-BH - Conclusions

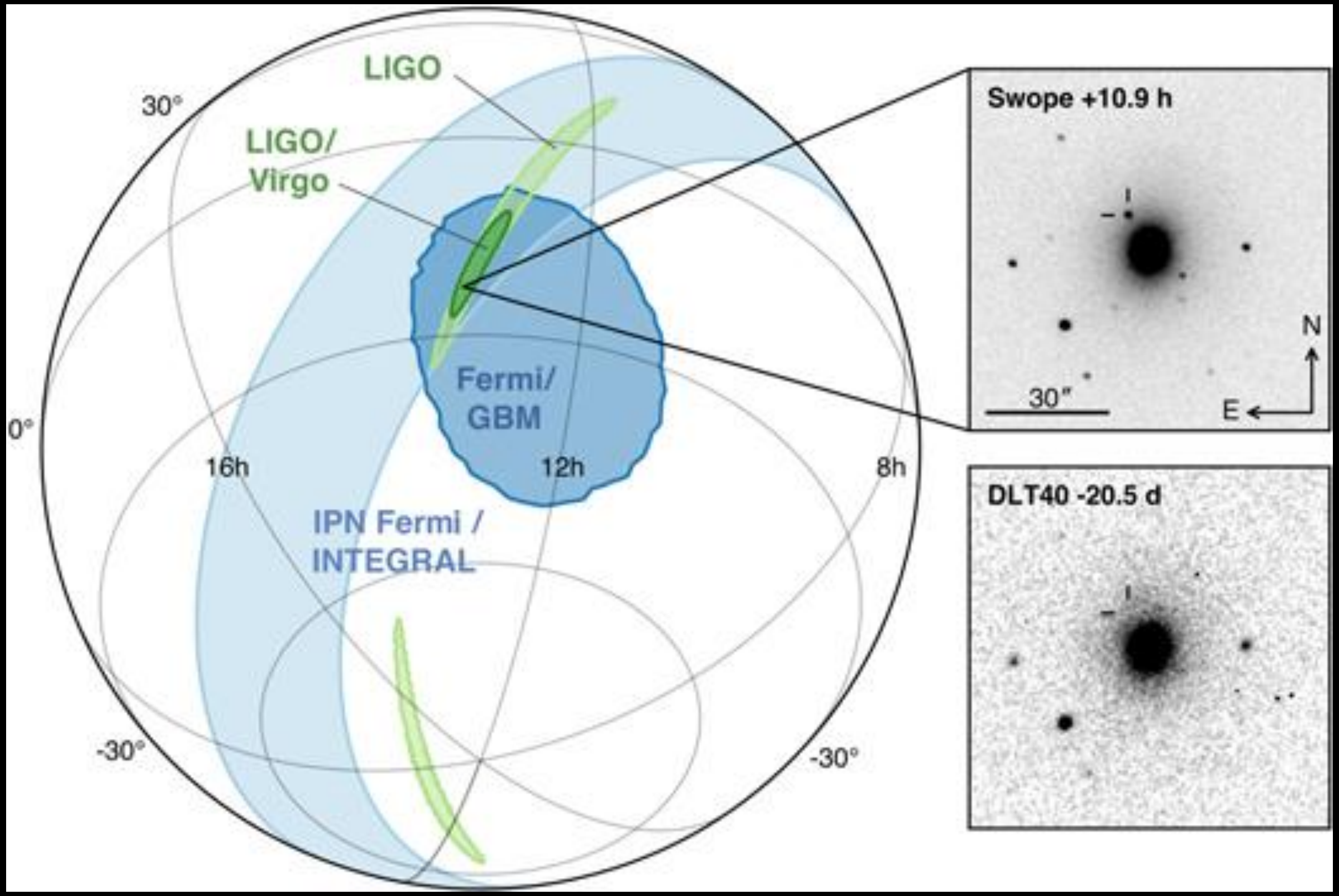
- Obtaining stable evolutions of Einstein's equations for binary systems is a complex problem that took decades to solve
  - Since 2005, BH-BH evolutions are possible
  - Since then, the accuracy, length, and parameter space coverage of the simulations has greatly improved
  - NR today plays a crucial role in the calibration of waveform templates used by LIGO!

5. Neutron Star Mergers: Overview



First NS-NS Detection





Images: LIGO Collaboration, PRL, ApJ

BH-NS Binaries

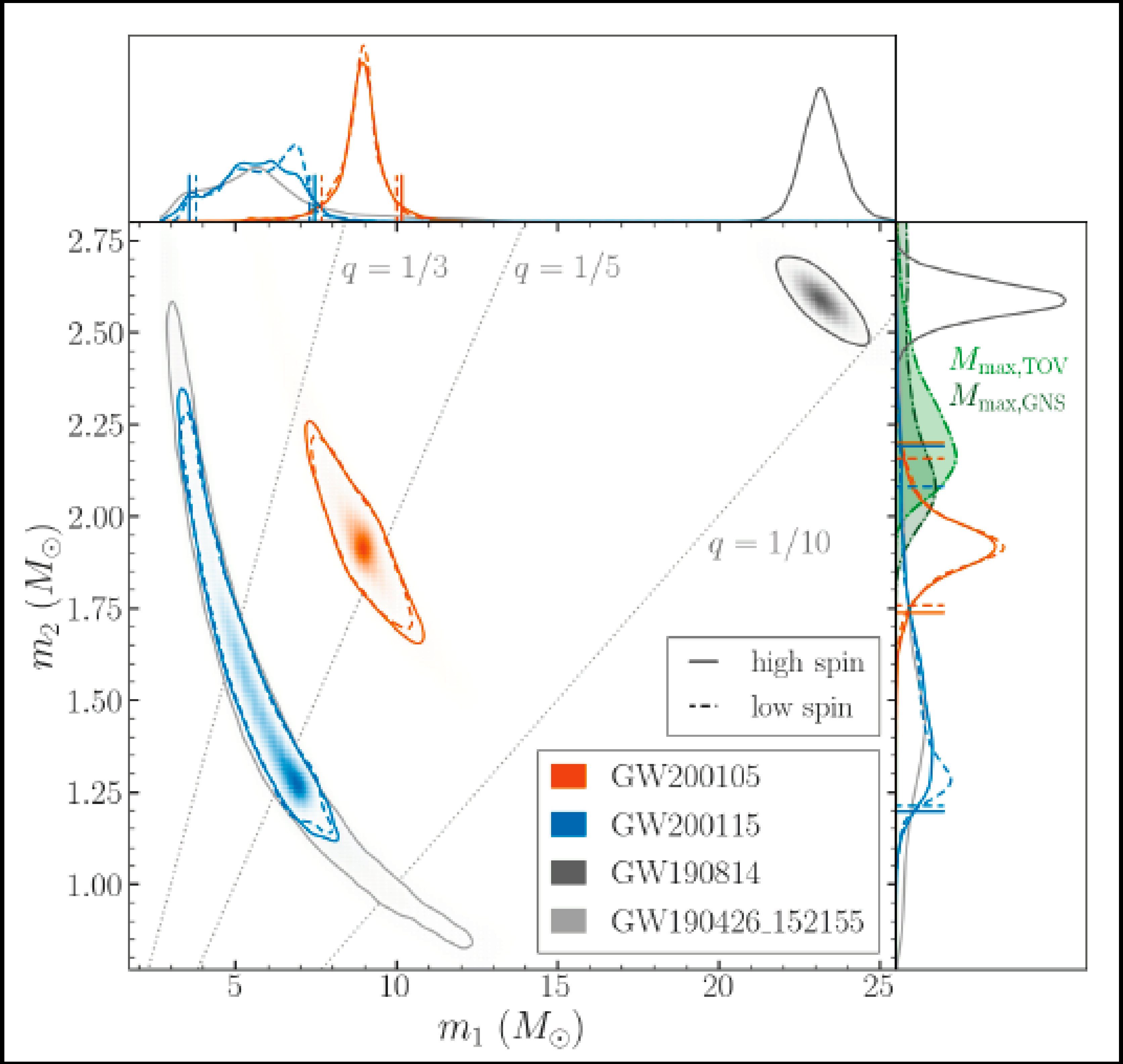


Image: LIGO/Virgo/KAGRA Collaboration

Neutron star mergers: Broad overview

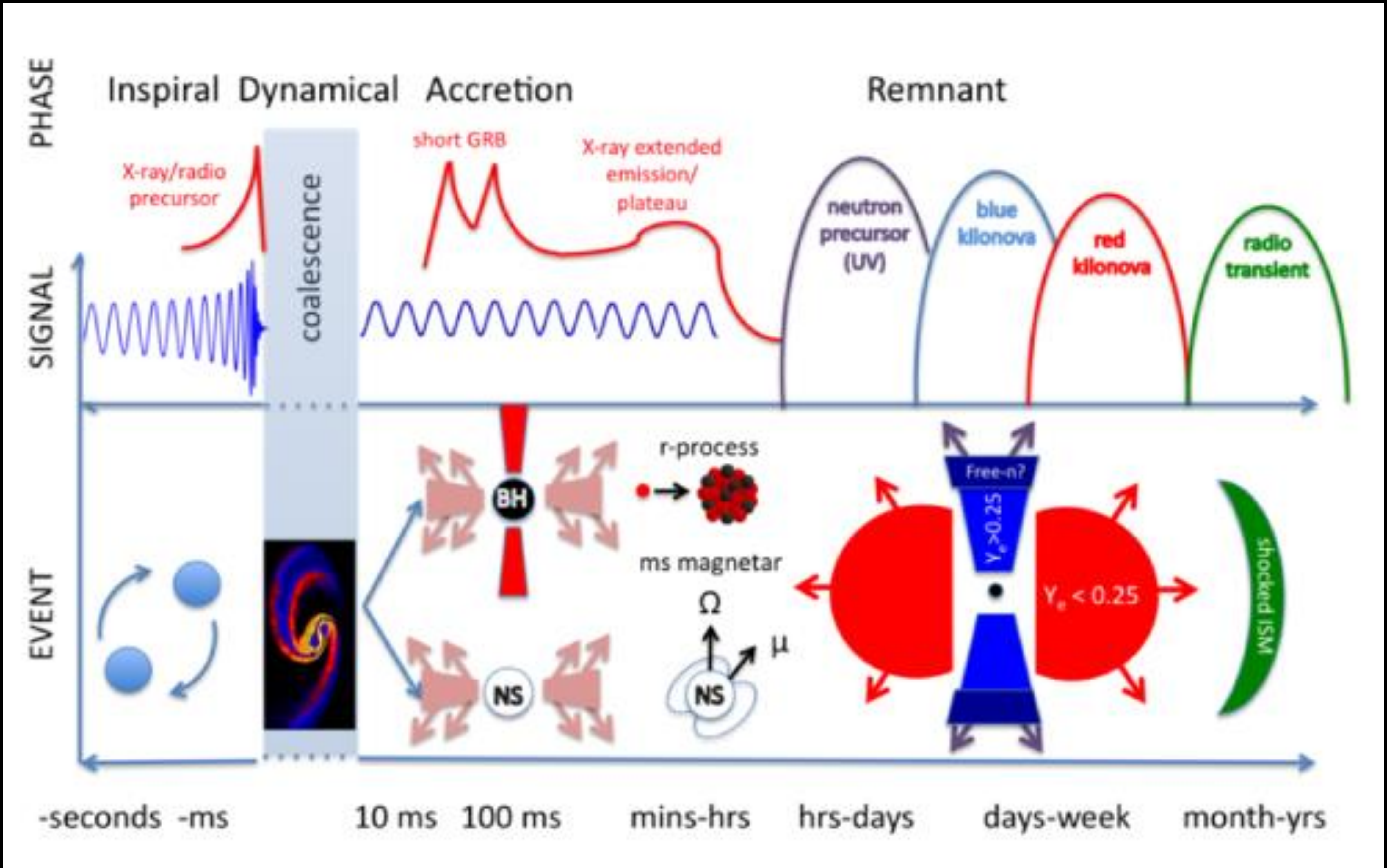


Image: Fernandez & Metzger 2016

Nuclear Physics and Neutron Stars

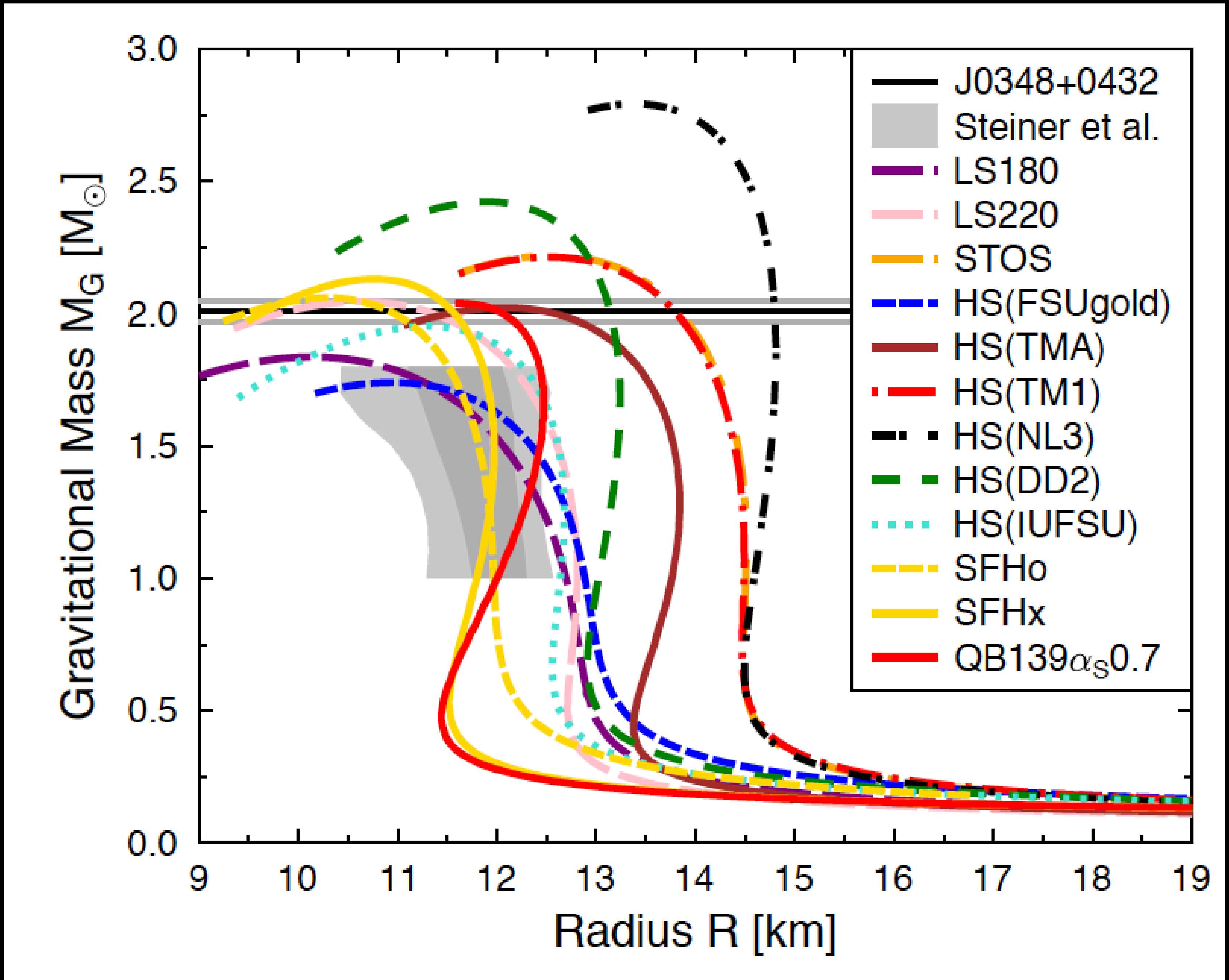


Image: Fischer et al 2014

NS radii <-> Nuclear Interactions

Neutron Star Binaries: r-process

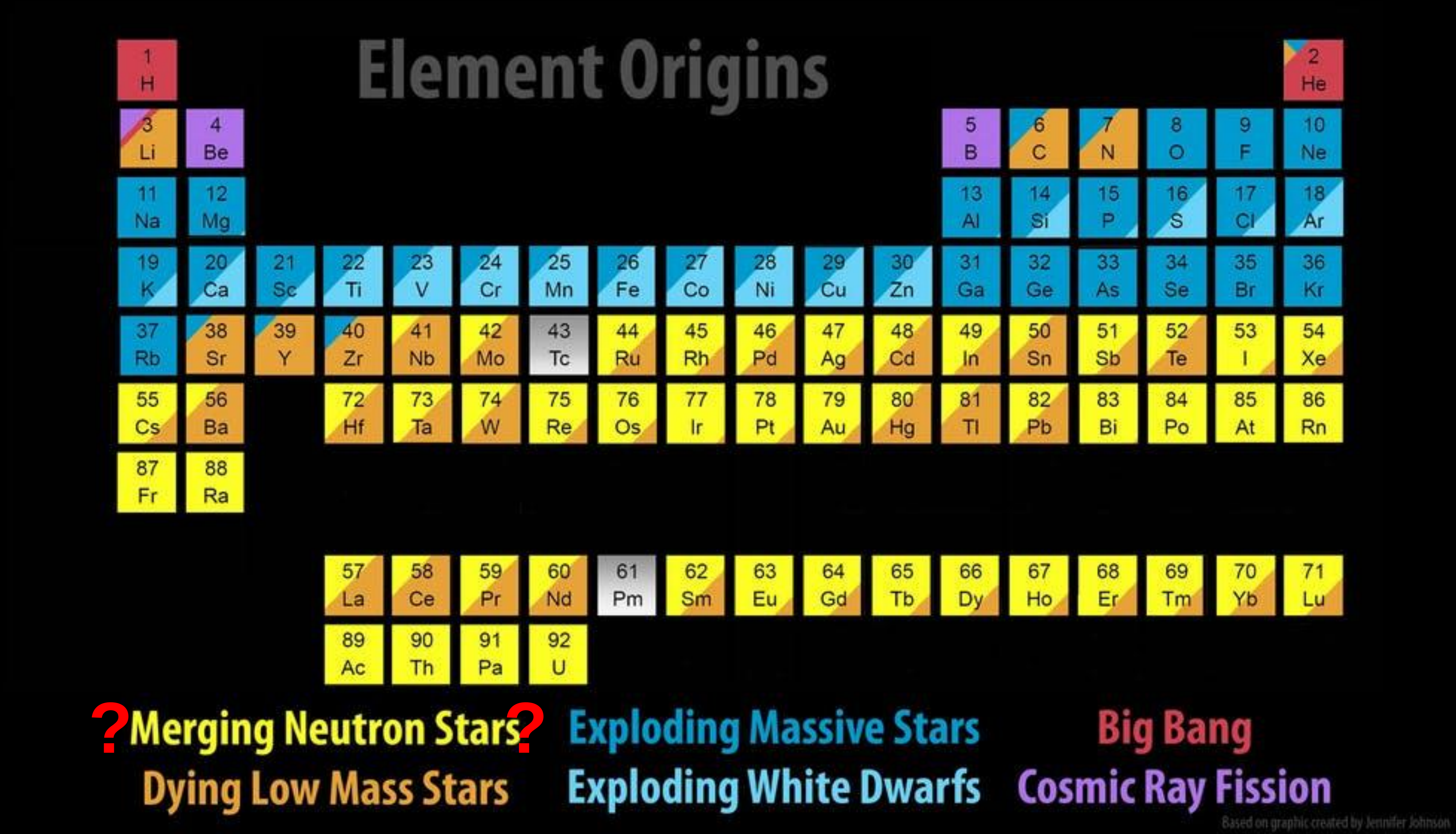
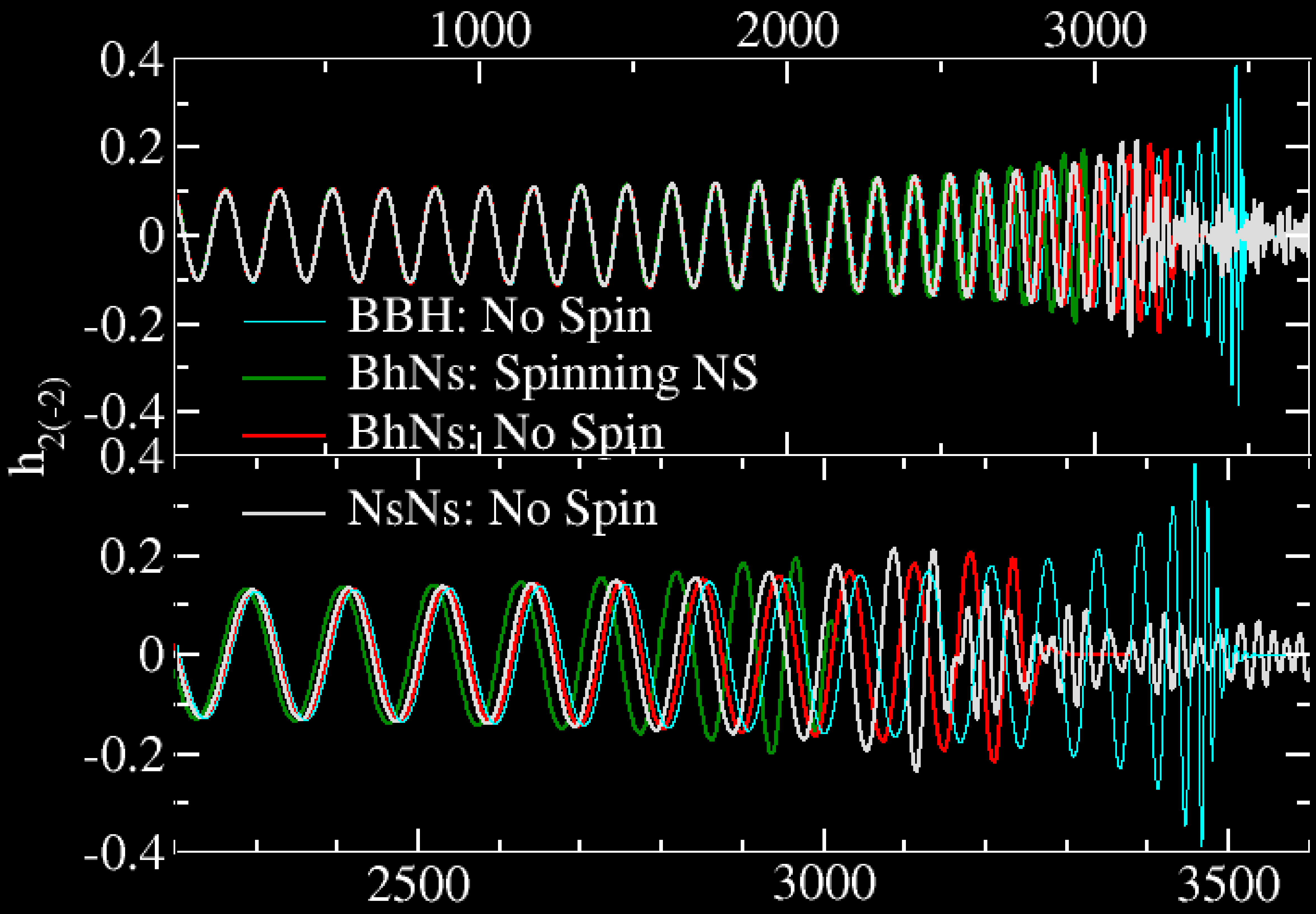


Image: J. Johnson

6. Neutron star binaries: Inspiral signal

Gravitational wave signal : Dephasing

Equal mass systems, NS radius = 14.4km



Inspiral: Tides

- Additional GW emission due to static tidal deformation [proportional to tidal deformability $\Lambda \sim R^5$, tidal correction is a 5PN term - $O(v^{10}/c^{10})$]
    - Marginally detectable by LIGO - requires loud events, or multiple events
- Dynamical tides
    - Resonance with single excitation mode of NS just before merger (Hinderer et al. 2017) : enhanced static tidal effect
    - Many resonance between multiple (4?) modes of neutron stars (N. Weinberg et al.) : effect of unknown strength, long before merger
- Current LIGO templates are calibrated to NS merger simulations - should capture the first two effects (though maybe not well enough for future observations…)

Equation of state measurements
GW170817

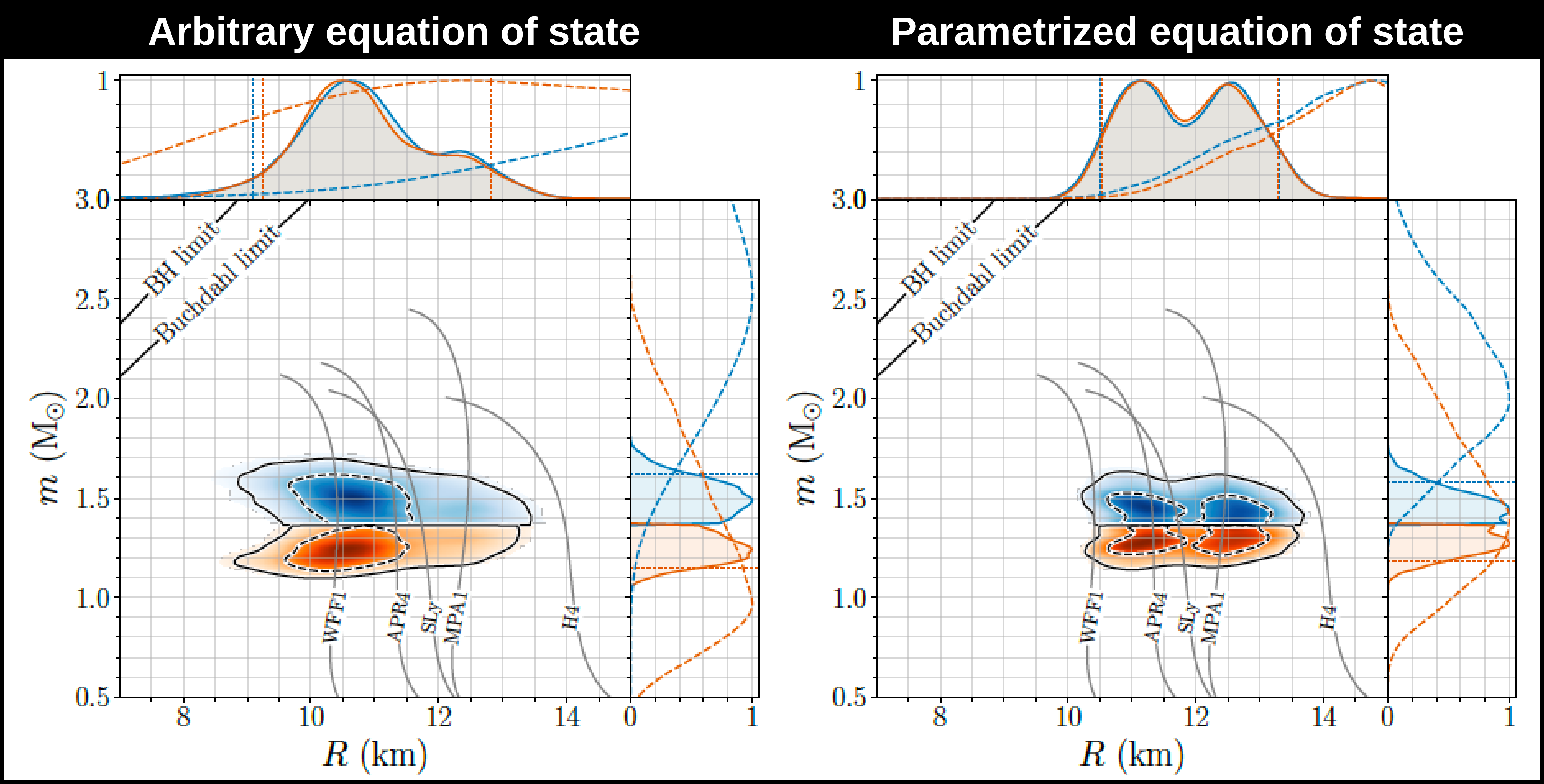


Image: LIGO/Virgo collaboration

Modeling Uncertainties: GW170817

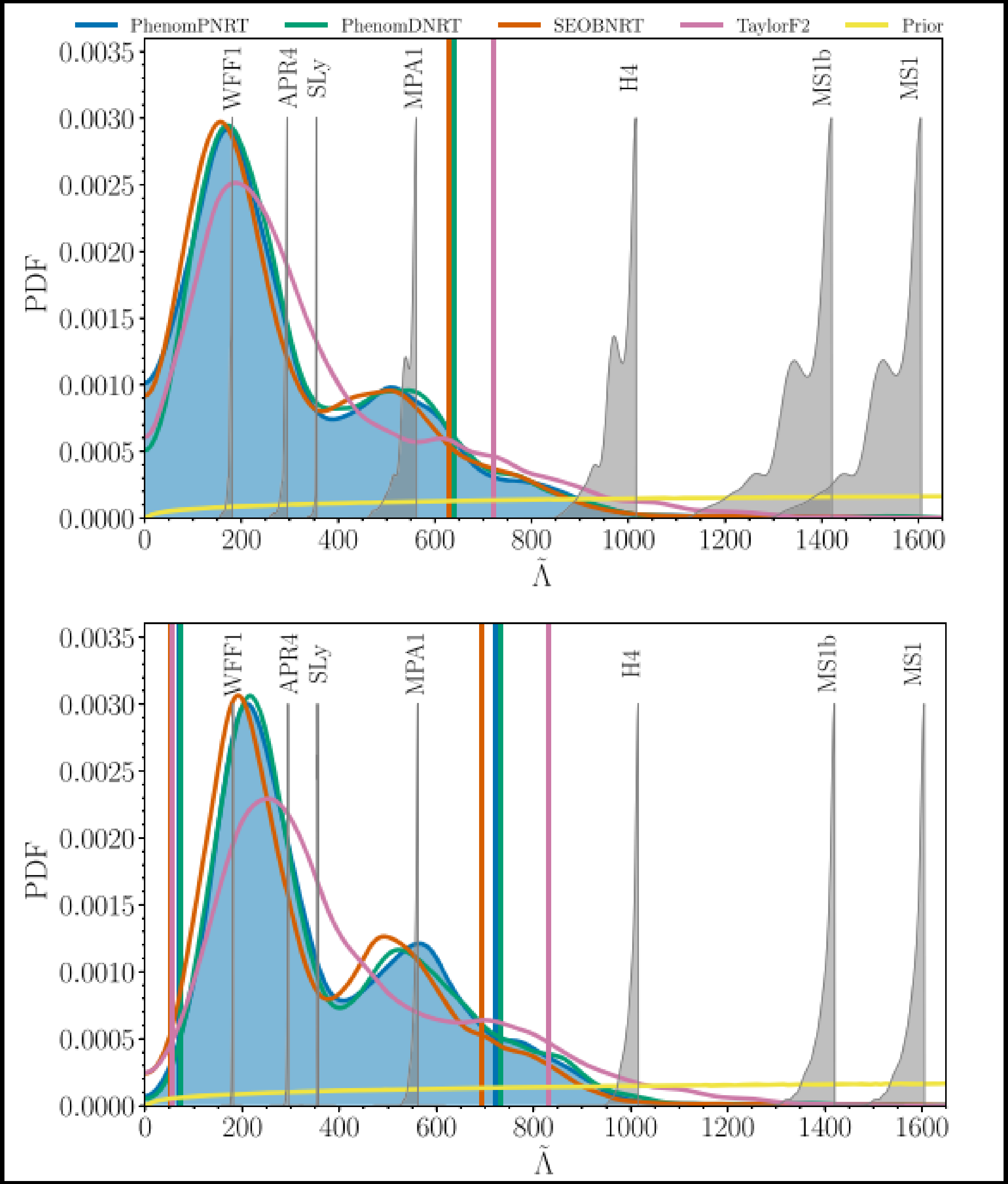
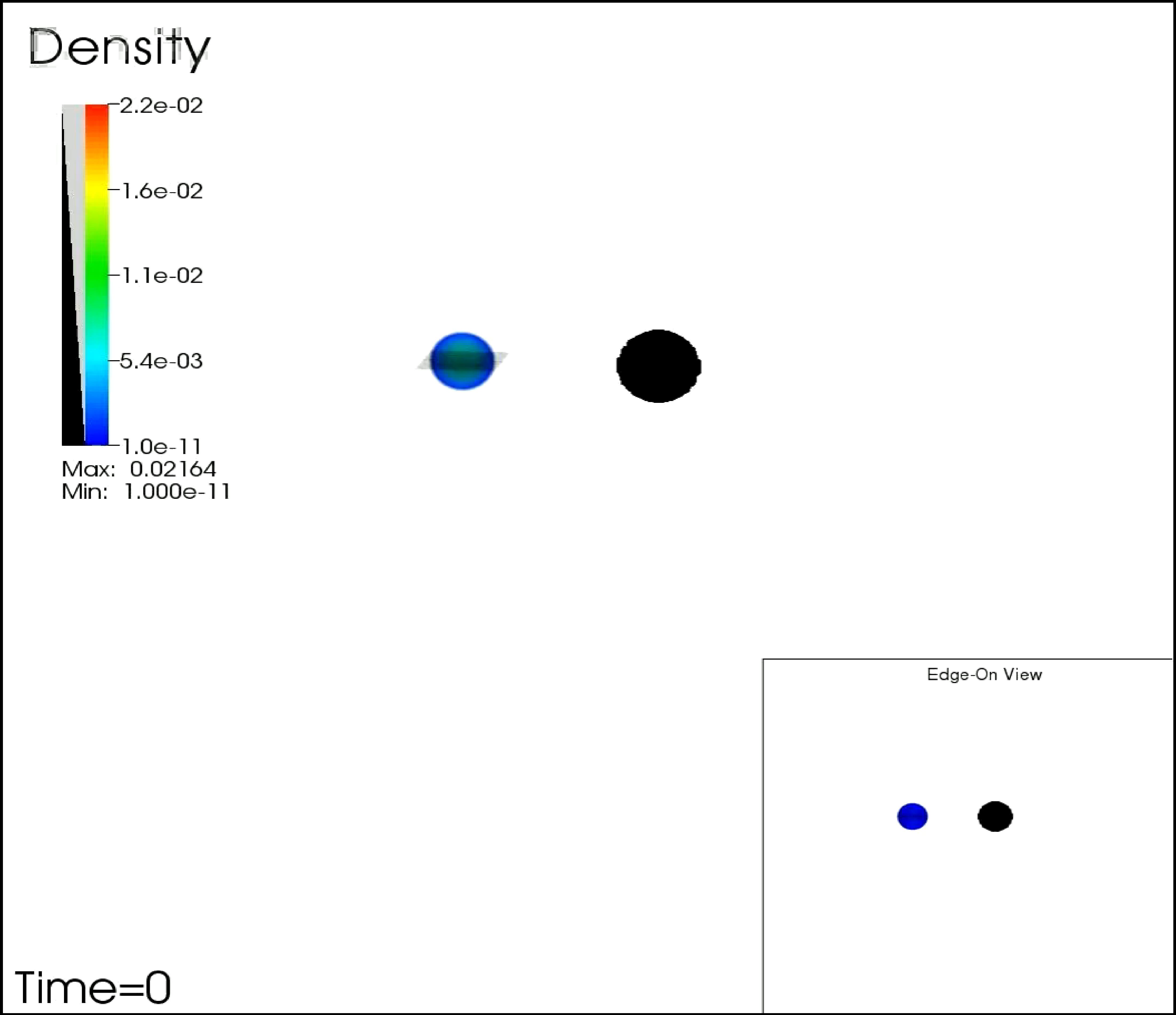


Image: LIGO/Virgo collaboration

7. Merger dynamics : BH-NS


Density
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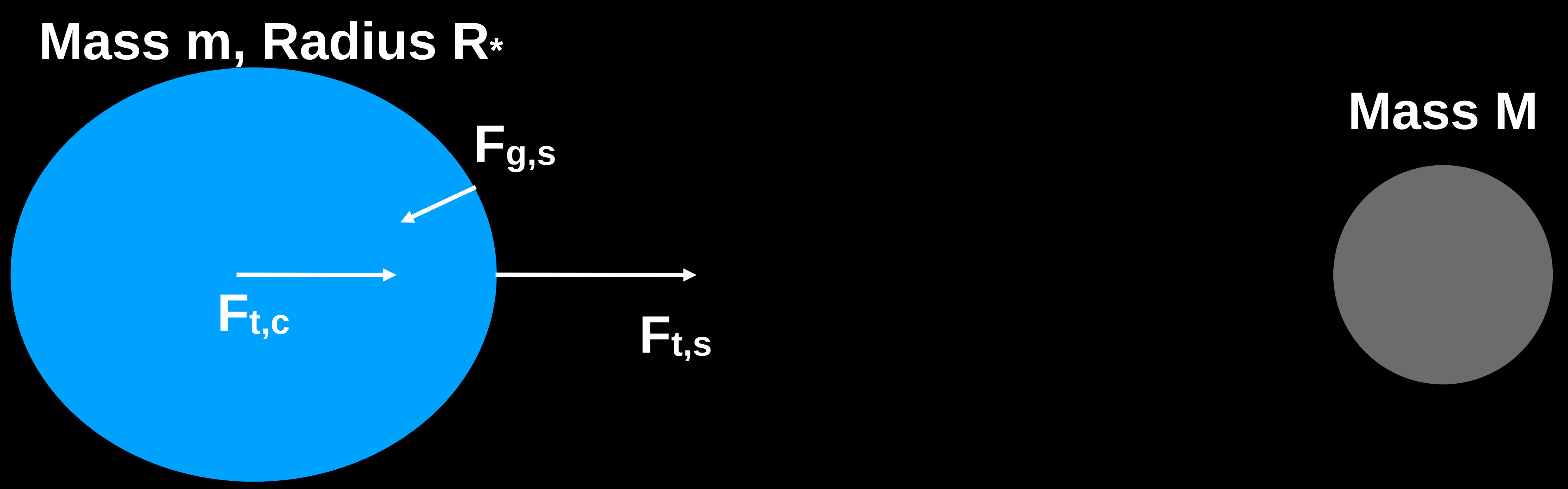
1.6e-02
1.1e-02
5.4e-03
1.0e-11
Max: 0.02164
Min: 1.000e-11
Edge-On View
Time=0

BH-NS merger : Disruption

- Neutron star begins accreting onto BH when it overflows its **Roche lobe**



For m<<M; R*<<(d=Separation)

$$a_{t,s} - a_{t,c} \approx \frac{2GM}{d^3} R_* \qquad a_{g,s} \approx \frac{Gm}{R_*^2}$$

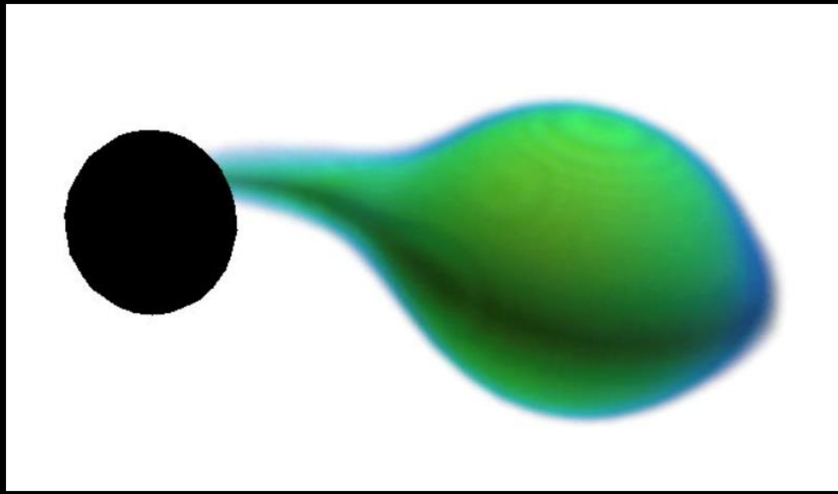
Tidal force = gravitational self-force when

$$d^3 = 2\frac{M}{m} R_*^3$$

BH-NS merger : Disruption

- Neutron star begins accreting onto BH when it overflows its **Roche lobe**

$$d_{\rm dis} \approx \left(\frac{3M_{\rm BH}}{M_{\rm NS}}\right)^{1/3} R_{\rm NS}$$

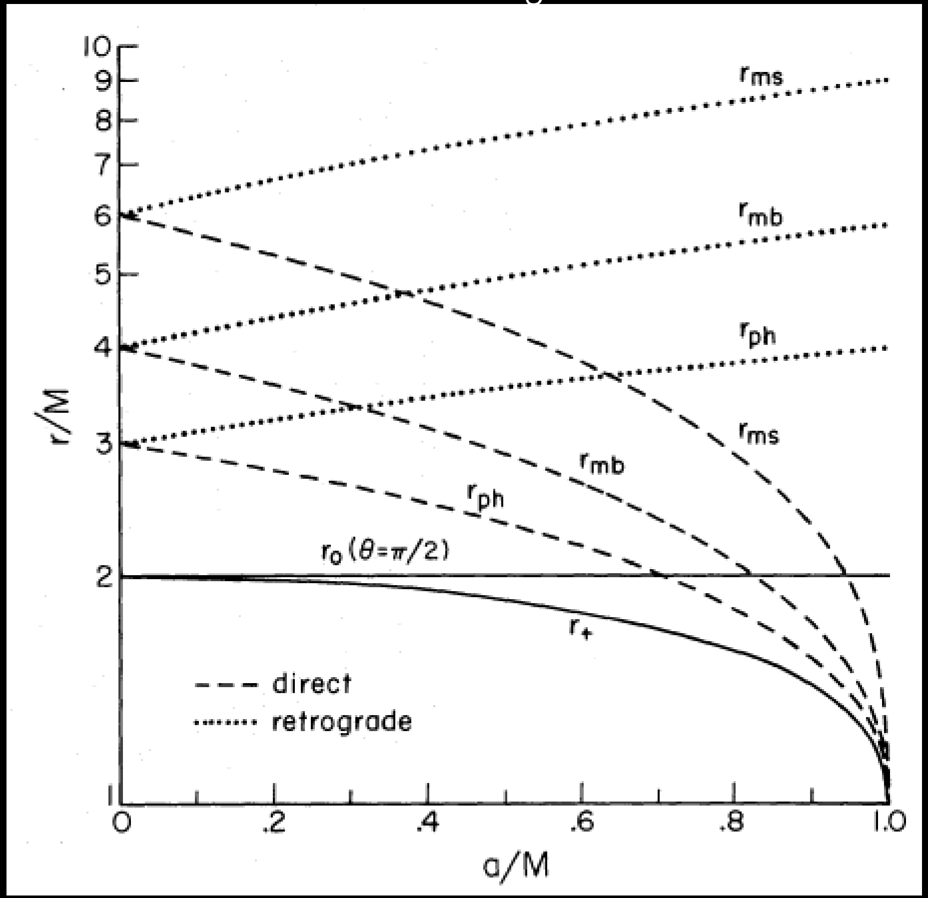


- In GR simulations, accretion is always unstable (for e~0)

Image: Bardeen et al 1972

- For BH spacetime, there is an innermost stable circular orbit (ISCO) at

$$d_{\rm ISCO} = (1-9)\frac{GM_{\rm BH}}{c^2}$$



BH-NS merger : Disruption

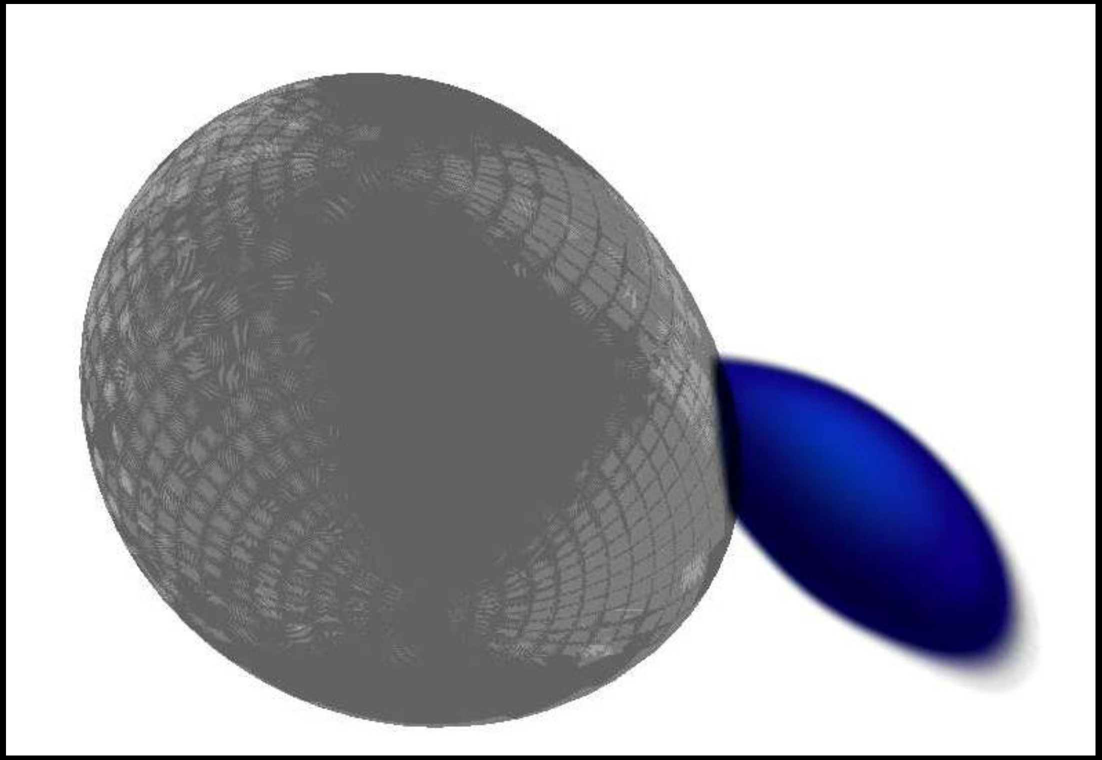
$$d_{\rm dis} = \left(\frac{3M_{\rm BH}}{M_{\rm NS}}\right)^{1/3} R_{\rm NS} \gtrsim (1-9)\frac{GM_{\rm BH}}{c^2} = d_{\rm ISCO}$$

- Consequences:
  - Large NSs are **more likely** to disrupt
  - High mass BHs **prevent** disruption
- Put units back in:
  - M_{BH} ~ (5-30) $M_{\odot}$, M_{NS} ~ (1-2) $M_{\odot}$, R_{NS} ~ 10km -> d_{dis}~(20-30) km
  - GM_{BH}/c^2 ~ 1.5km ($M_{BH}/M_{\odot}$) -> d_{ISCO} ~ (8-400) km

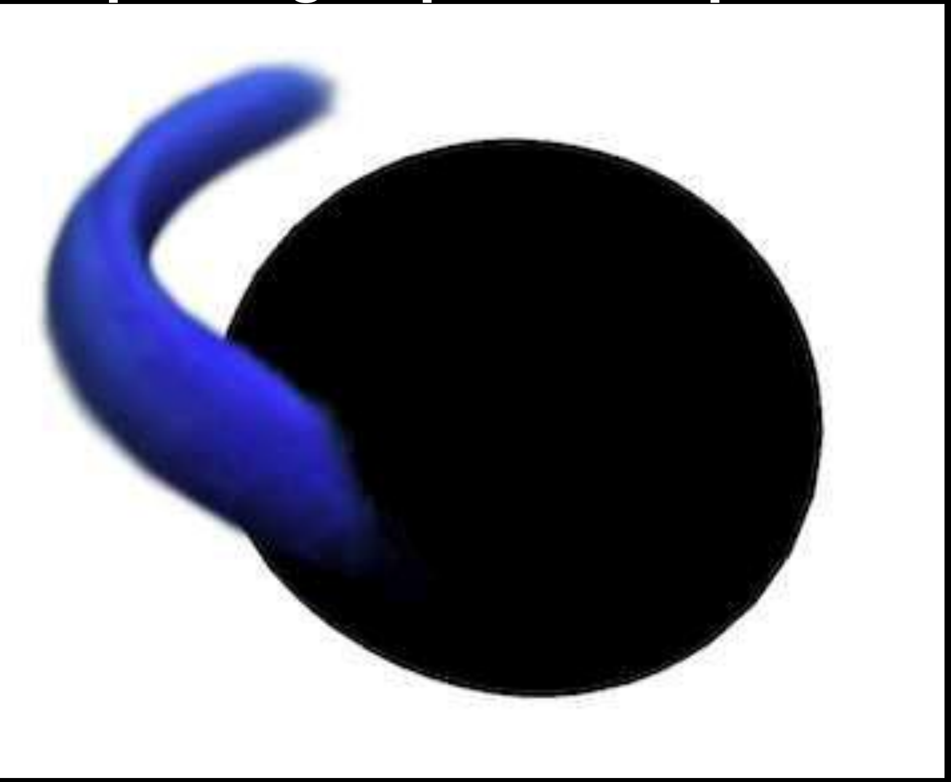
BH-NS merger : Disruption

- ISCO location:
  - Close to BH for maximal spin, prograde orbits
  - Far for maximal spin, retrograde orbits
  - For misaligned spins, computing the ISCO for the component of the spin aligned with the orbital angular momentum is ok
    - Better approximation: Use innermost stable spherical orbit (ISSO, Stone et al. 2012)

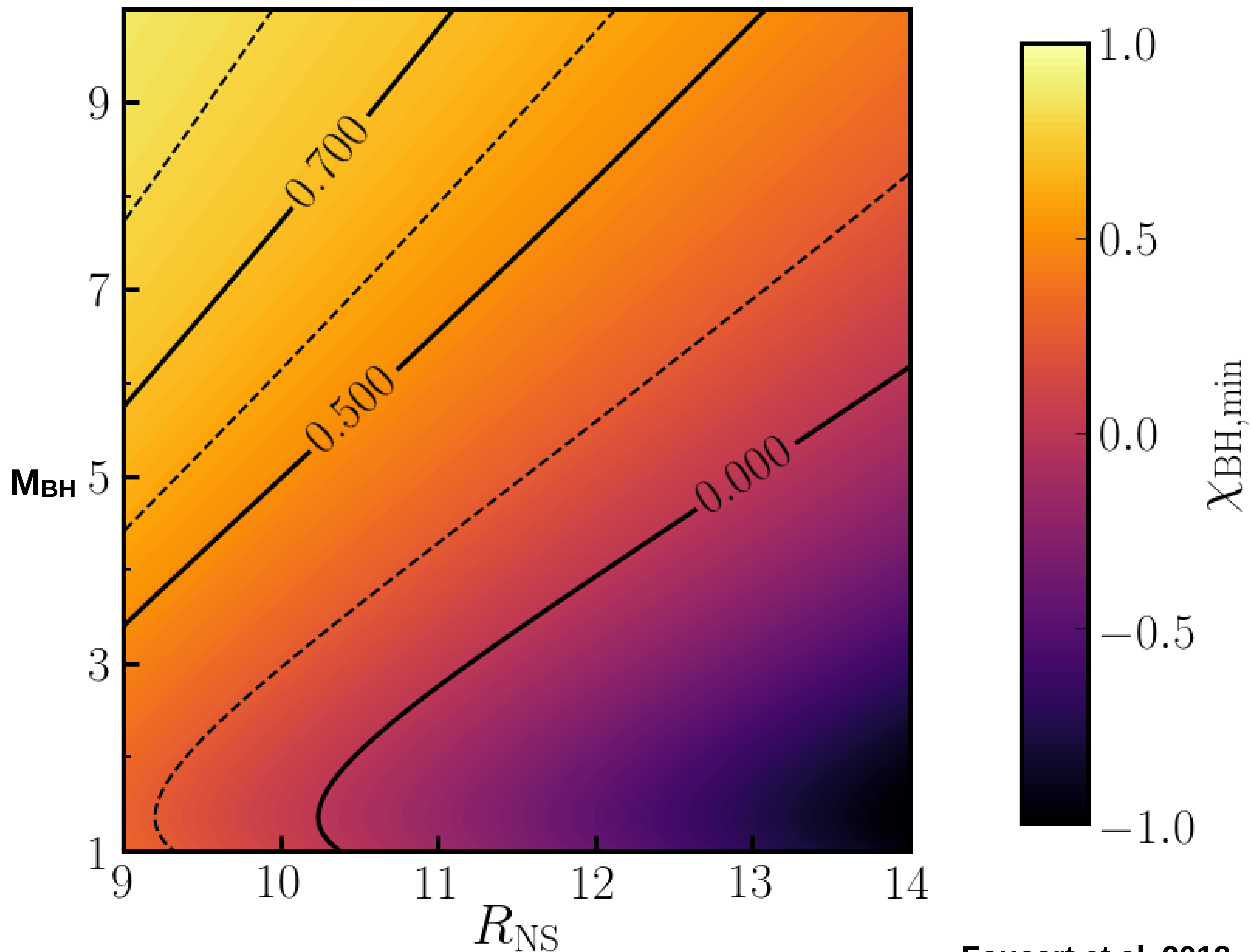
q=6, non spinning, no disruption



q=7, high spin, disruption

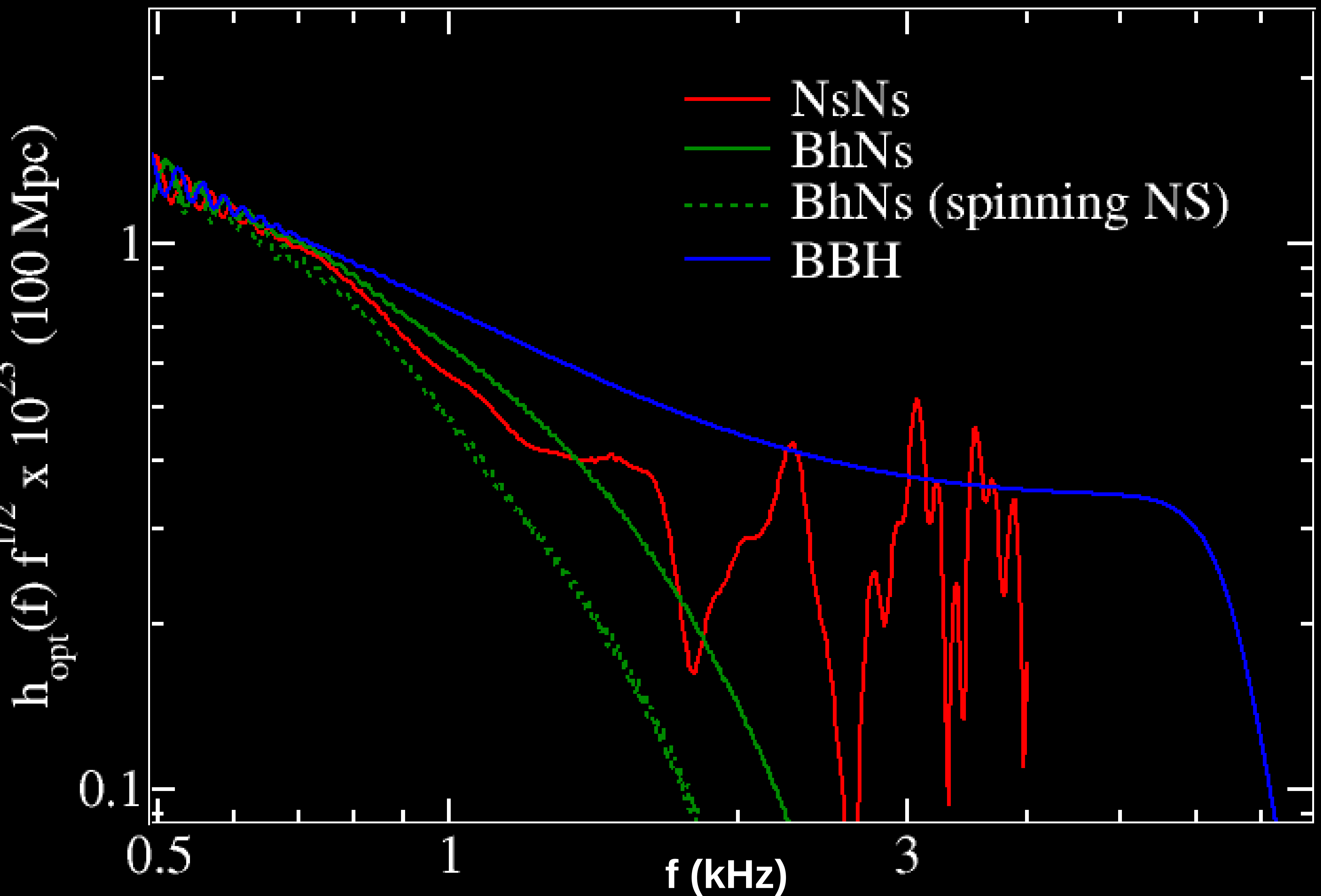


Minimum spin necessary to disrupt a 1.35M☉ neutron star



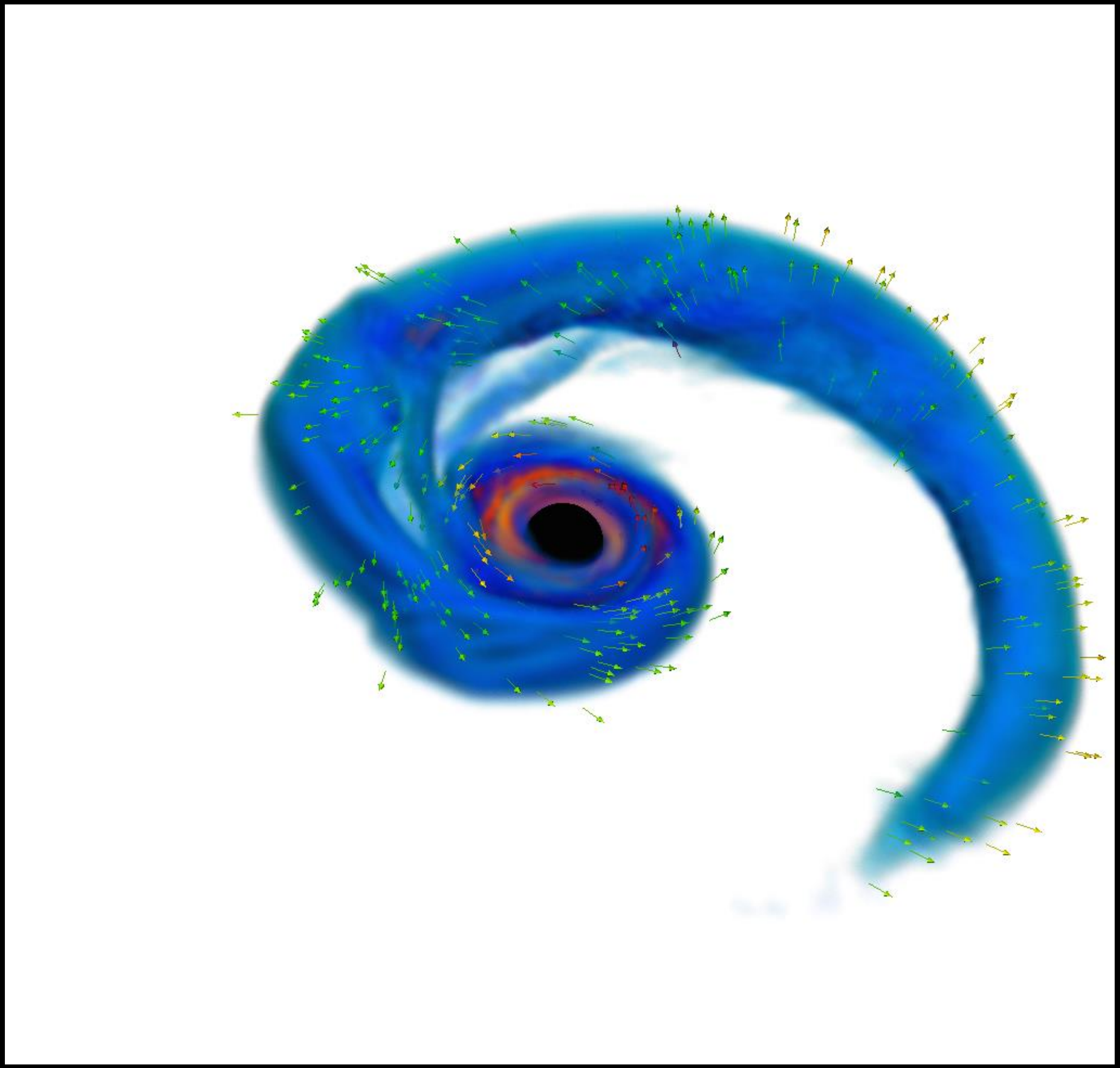
Foucart et al. 2018

BH-NS merger : Disruption in GW



$$f_{\rm dis} \sim 1.6\,{\rm kHz} \left(\frac{M_{\rm NS}}{1.4 M_\odot}\right)^{1/2} \left(\frac{12\,{\rm km}}{R_{\rm NS}}\right)^{3/2}$$

BH-NS merger : Dynamical ejecta



- *Dynamical ejecta:* BH-NS merger unbinds up to ~0.1$M_\odot$
- Ejecta is ~95% neutrons, and cold
- Ejecta average velocity ~(0.1-0.3)c -> $10^{50\text{-}52}$ergs in kinetic energy [-> **radio emission**]

Prediction (fit) for unbound mass: Kawaguchi et al. 2016

Massive ejecta for: large NSs, high mass ratio (if large BH spin)

BH-NS merger : Dynamical ejecta

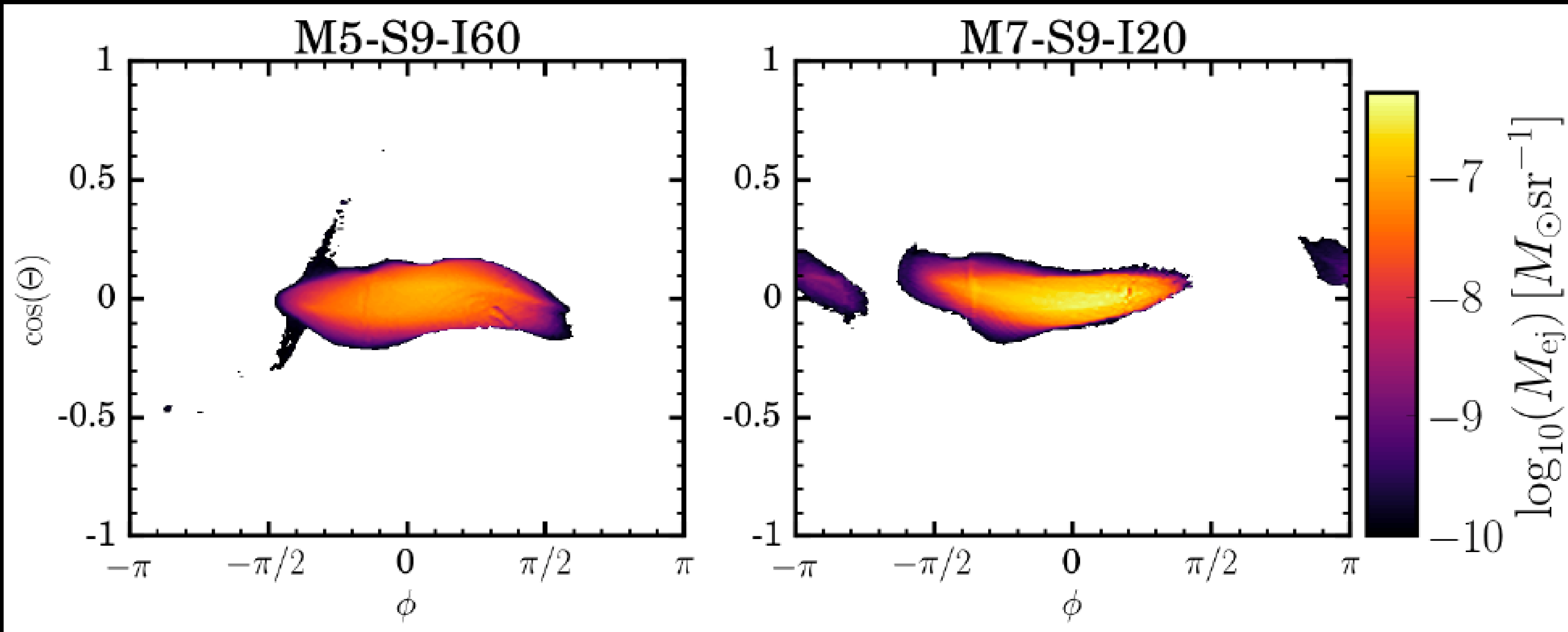
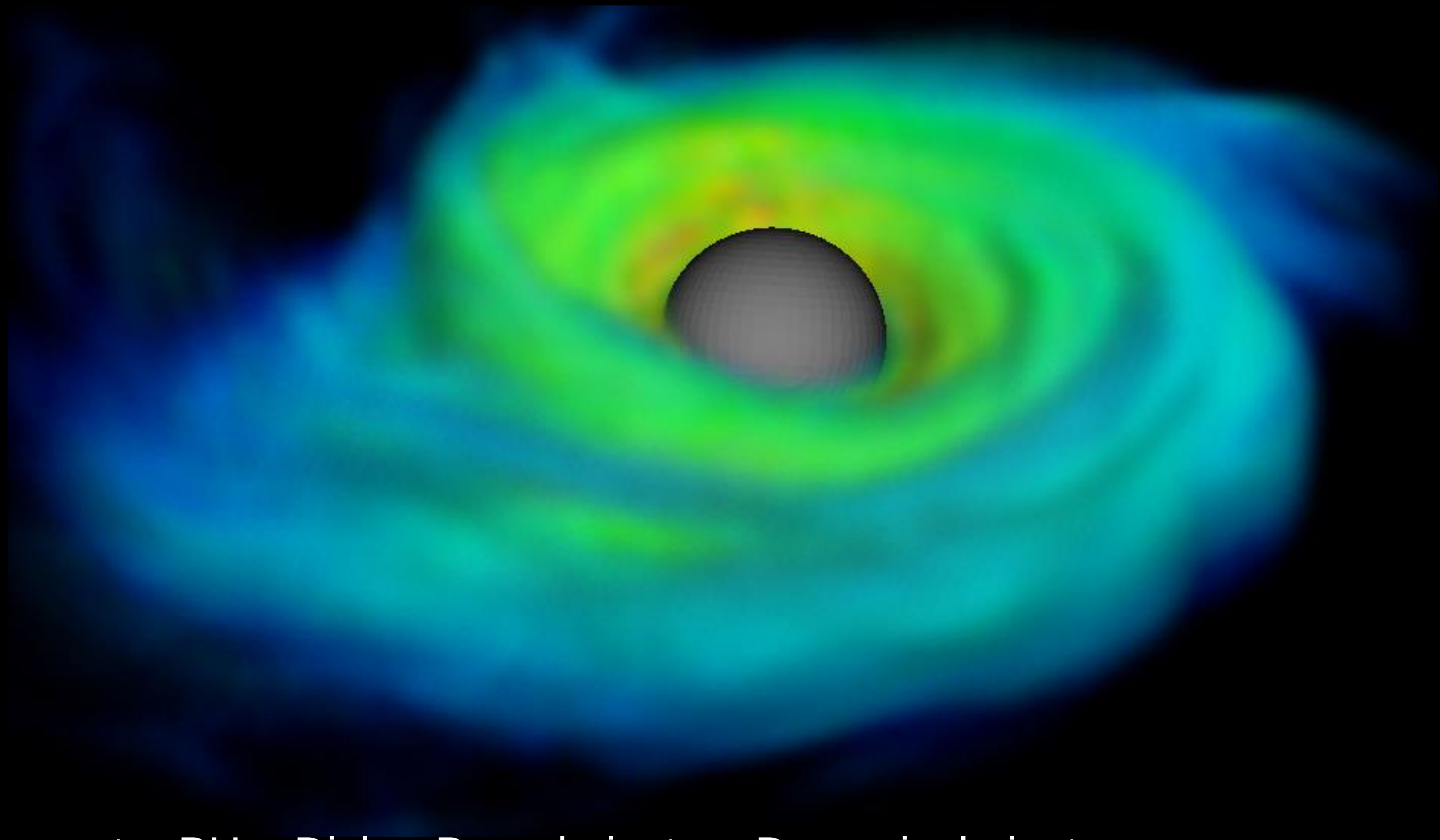


Image: Foucart, Desai et al. 2018

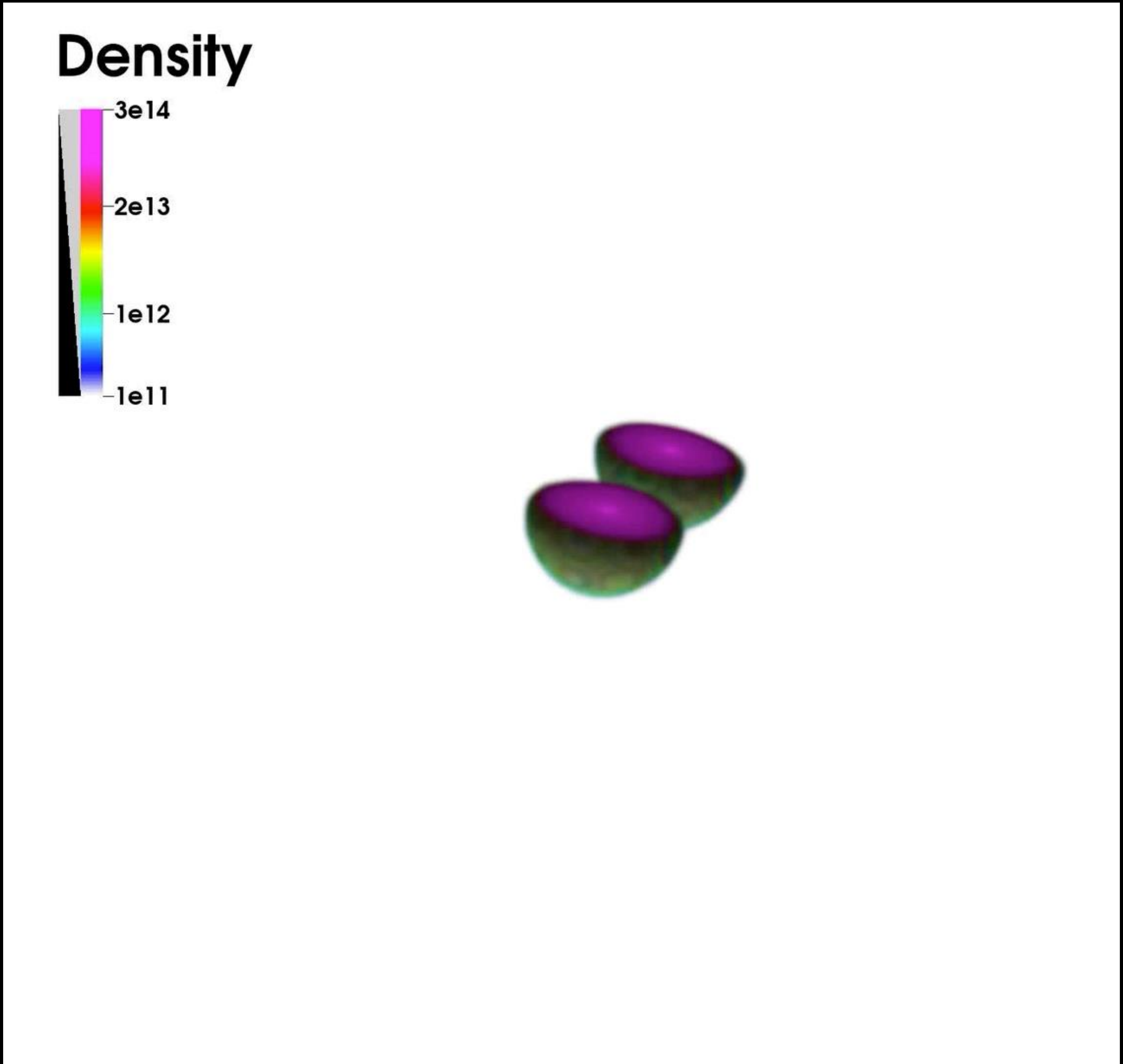
<u>BH-NS dynamical ejecta:</u> Confined within ~20º of the orbital plane

BH-NS merger : Post-Merger remnant



- Remnant = BH + Disk + Bound ejecta + Dynamical ejecta
- Disk is massive ($0.1M_\odot$, except close to disruption limit), compact (<100km), hot (~5MeV)
- Prediction (fit) for mass of matter outside BH: Foucart et al. 2018

8. Merger dynamics : NS-NS


Density
3e14
2e13
1e12
1e11

NS-NS merger : Types of remnants

Outcome of merger <-> [Total mass of binary] / [Maximum mass of NS]

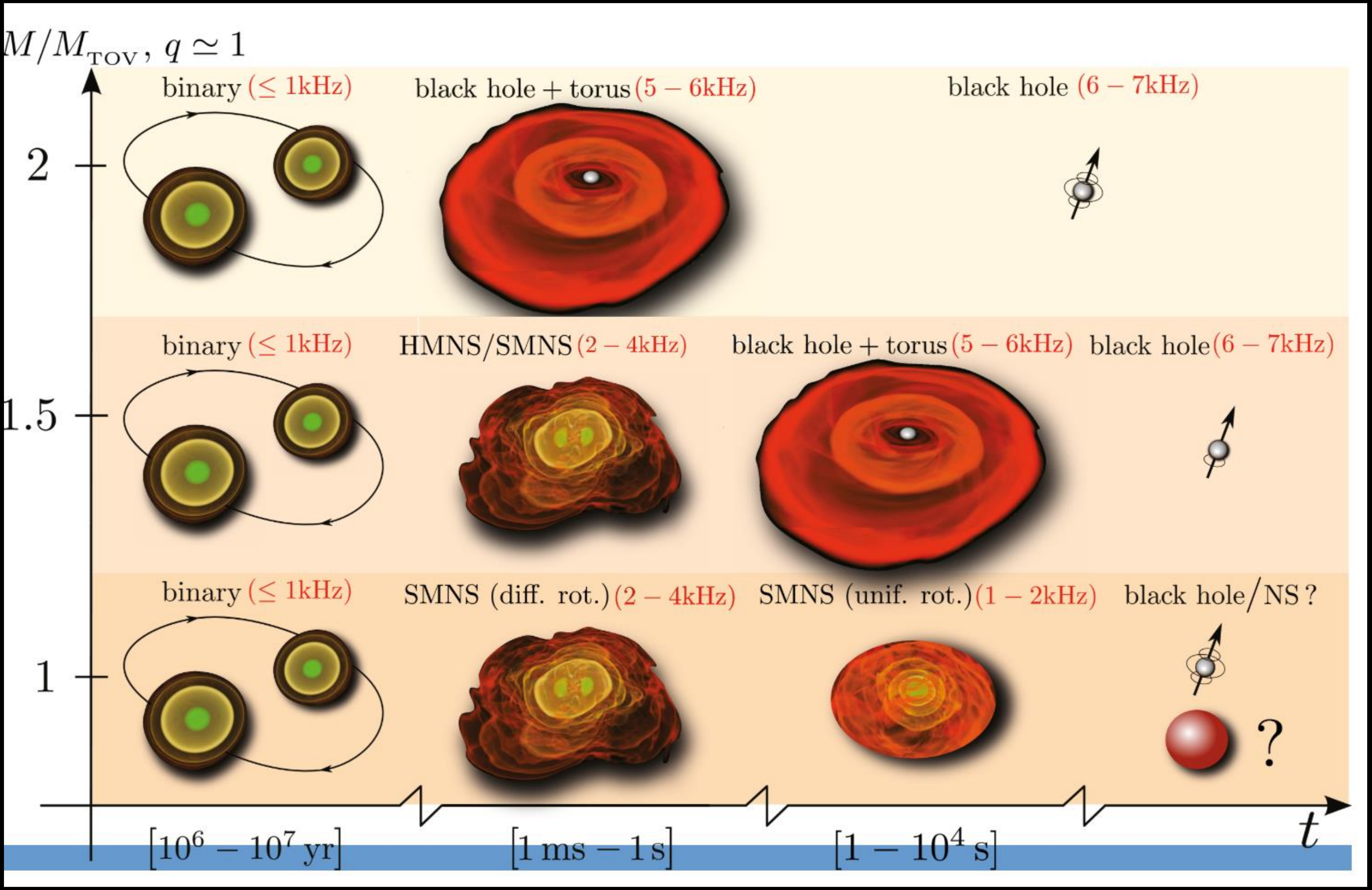
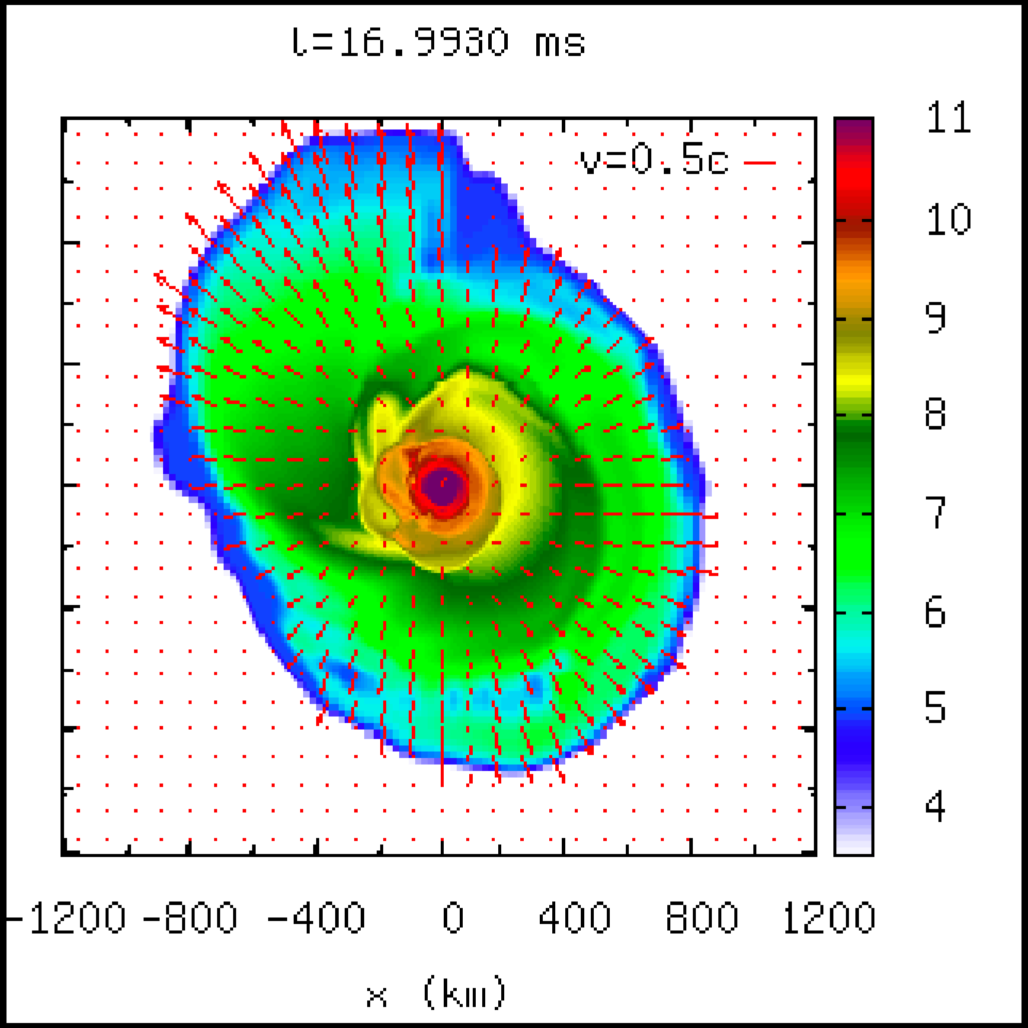


Image: Baiotti et al. 2017

NS-NS merger : Dynamical ejecta



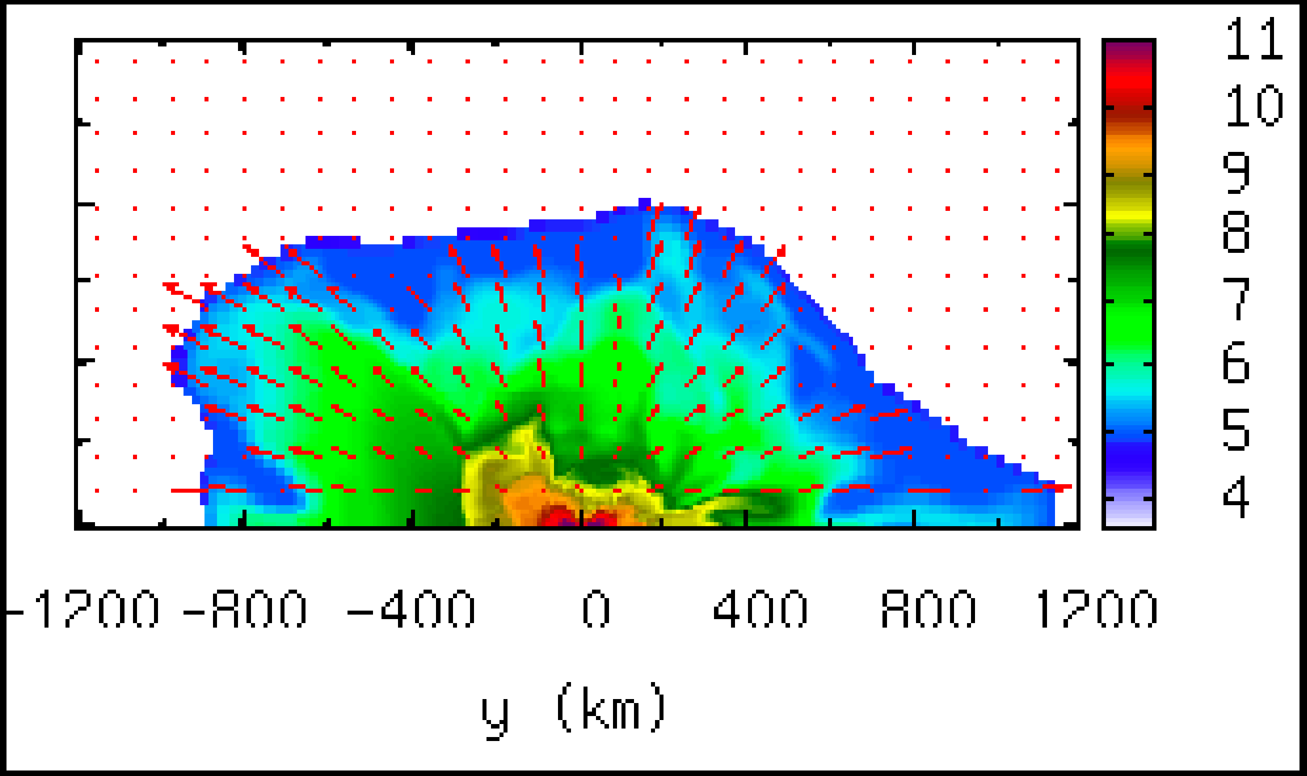


Image: Hotokezaka et al. 2013

<u>NS-NS dynamical ejecta:</u> Mostly isotropic

NS-NS merger : Dynamical ejecta

- Two types of dynamical ejecta in NS-NS mergers
  - Unbound tidal tails
    - Cold, neutron rich, v ~ 0.1c
  - ``Squeezed'' ejecta from contact regions between NSs
    - Hot (shocked), fast (v~0.2c-0.3c)
    - Less neutrons [see later]

<u>Massive dyn. ejecta</u> for: Large stars, asymmetric masses

<u>Massive squeezed. ejecta</u> for: small stars (… but not too small)

Post-merger GW emission (NS-NS mergers)

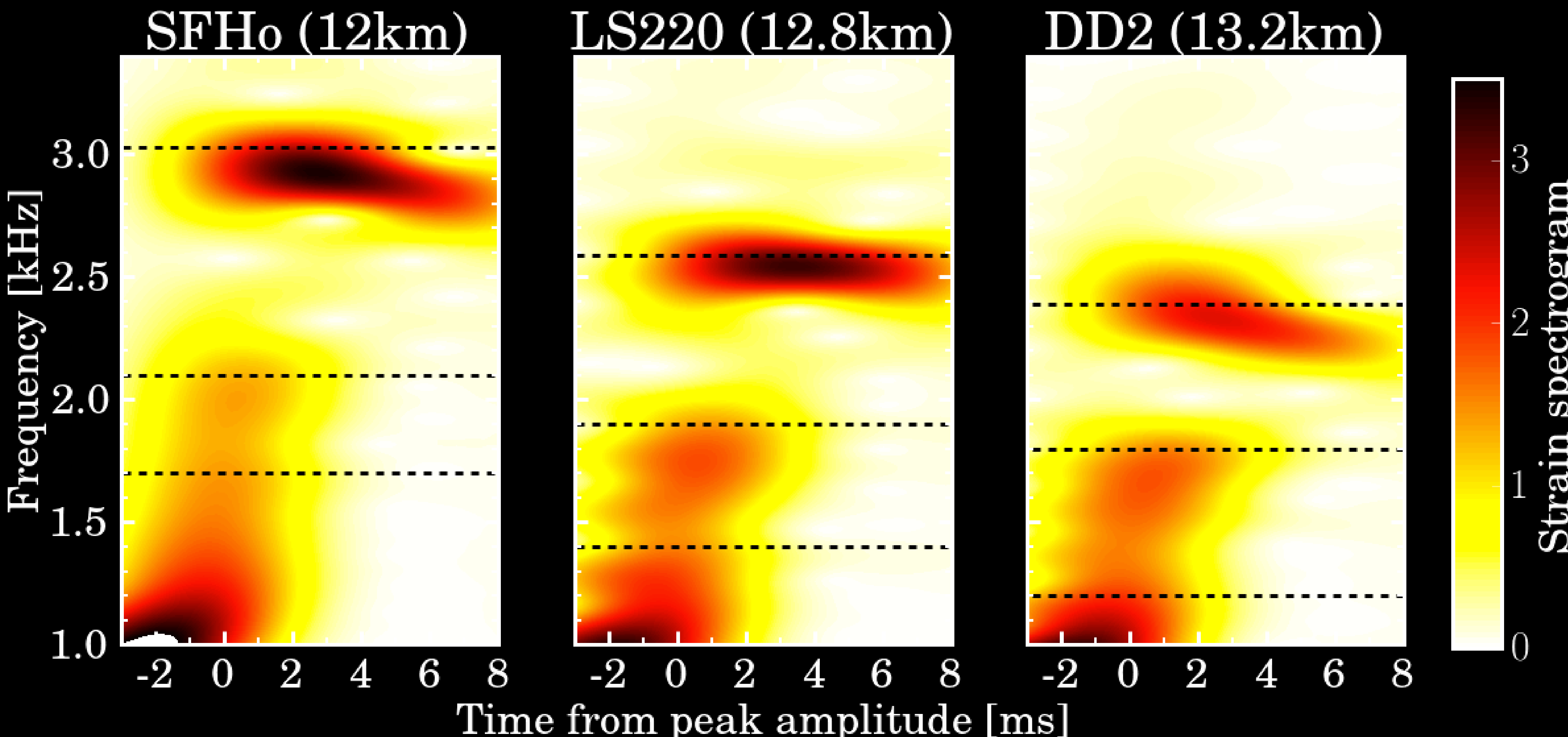
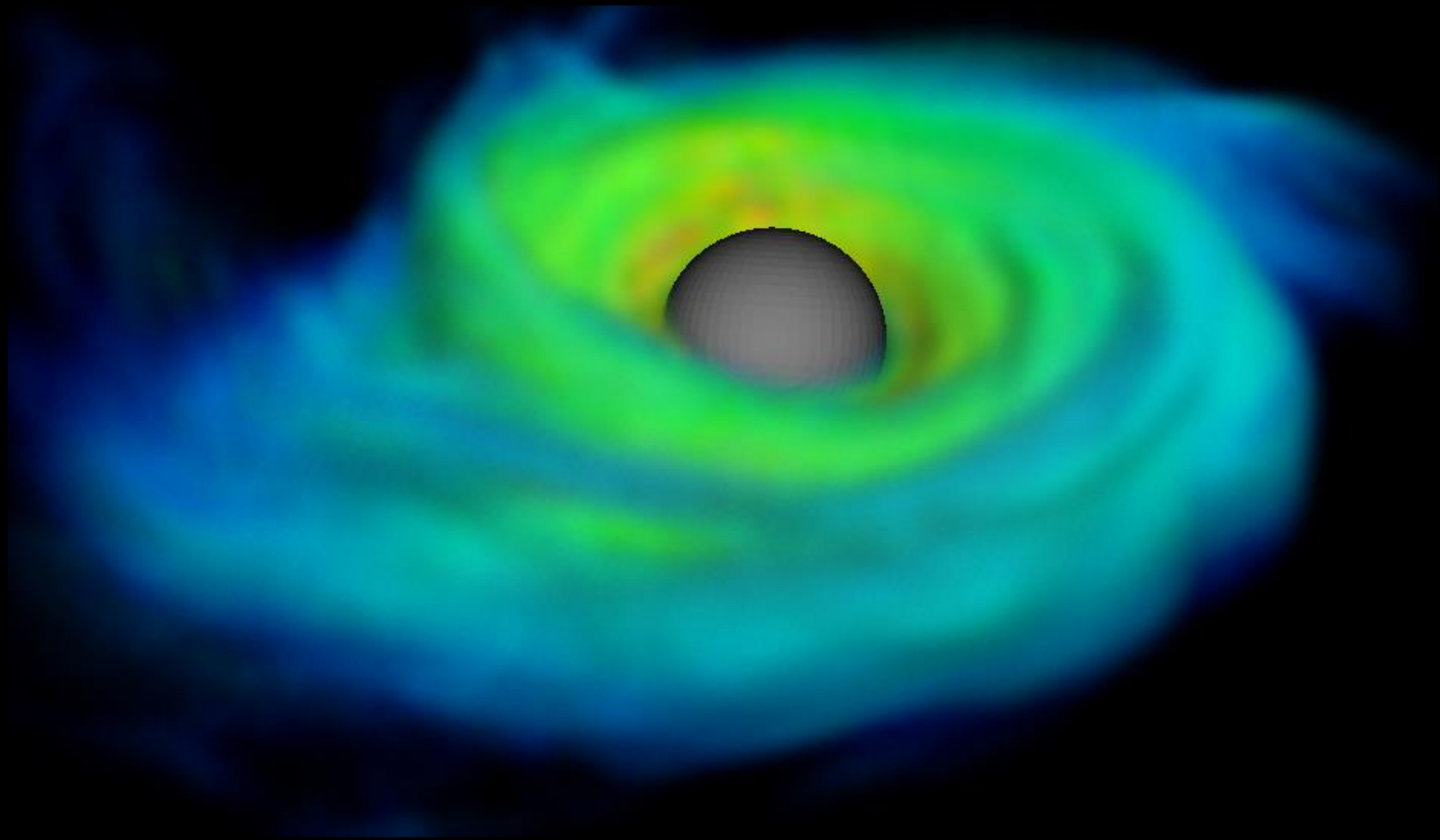


Image: Foucart et al. 2016

- Clear dominant frequency in post-merger signal
- Probe fundamental l=2, m=2 excitation mode of remnant

NS-NS merger : Post-Merger remnant



- Remnant = BH or NS + Disk + Bound ejecta + Dynamical ejecta
- Disk: massive ($0.01M_{\odot}$-$0.1M_{\odot}$, except for rapid collapse), compact (<100km), hot (~5MeV)

9. Kilonovae and r-process nucleosynthesis

EM counterparts to NS mergers

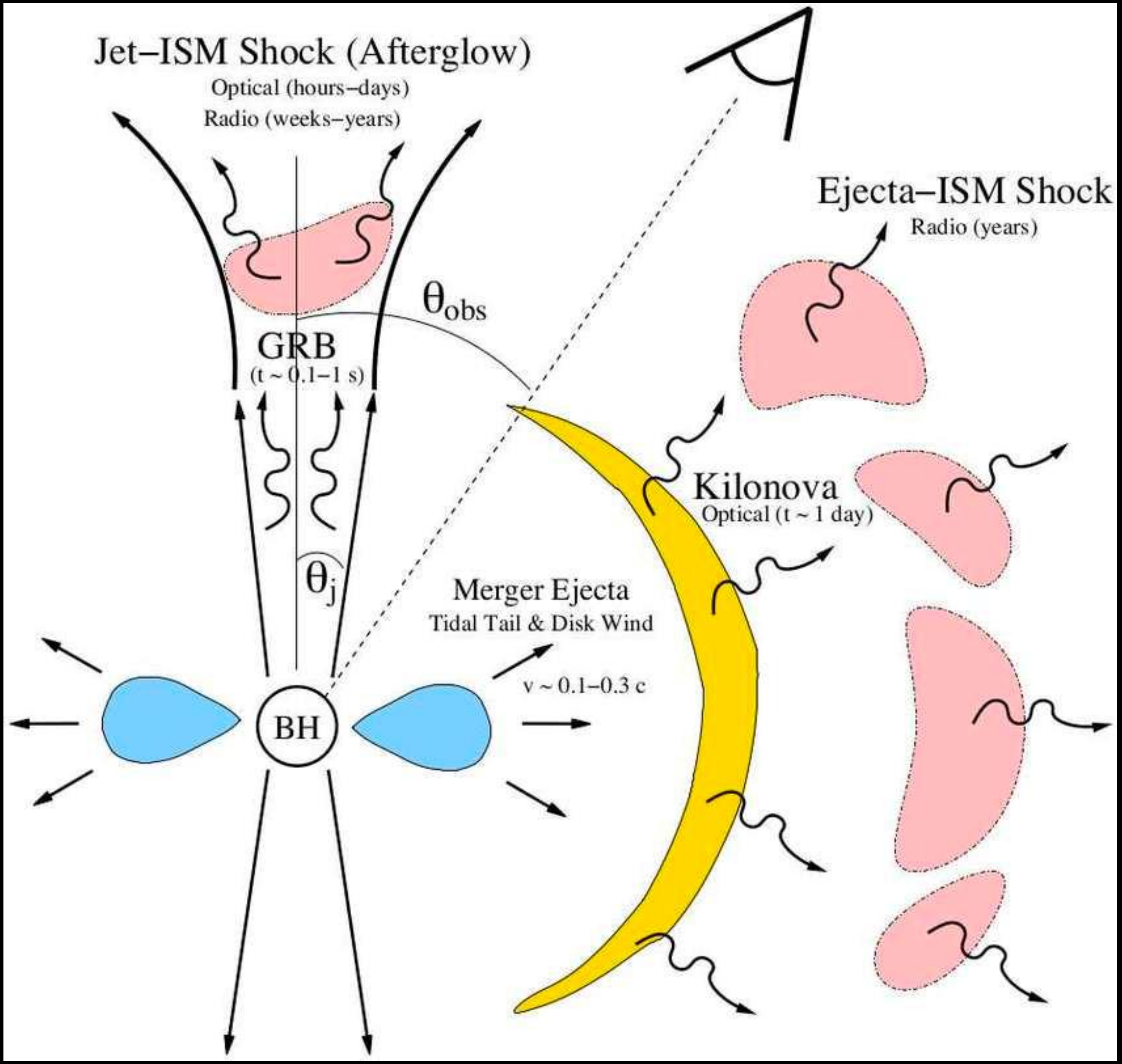
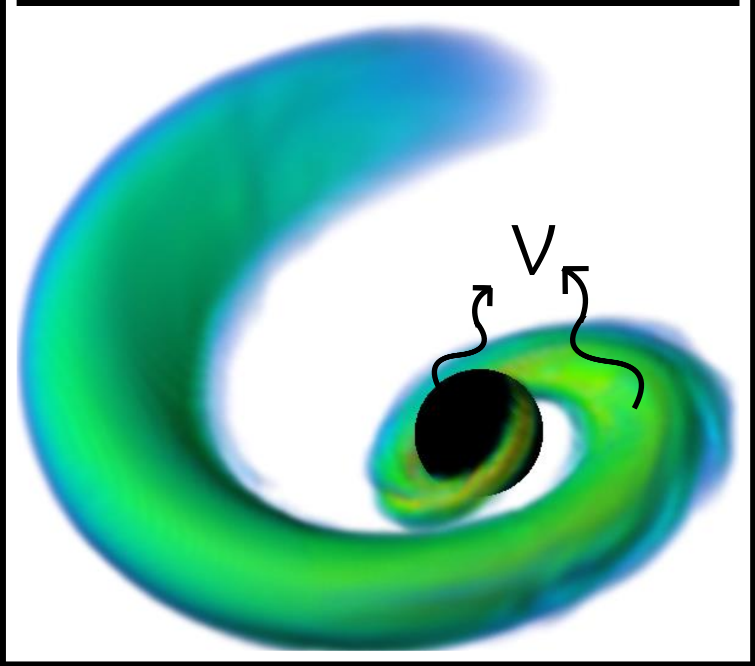


Image: Metzger & Berger 2011

Radioactively powered electromagnetic transients

Merger event produces **unbound outflows**



r-process nucleosynthesis in neutron rich ejecta produces heavy elements

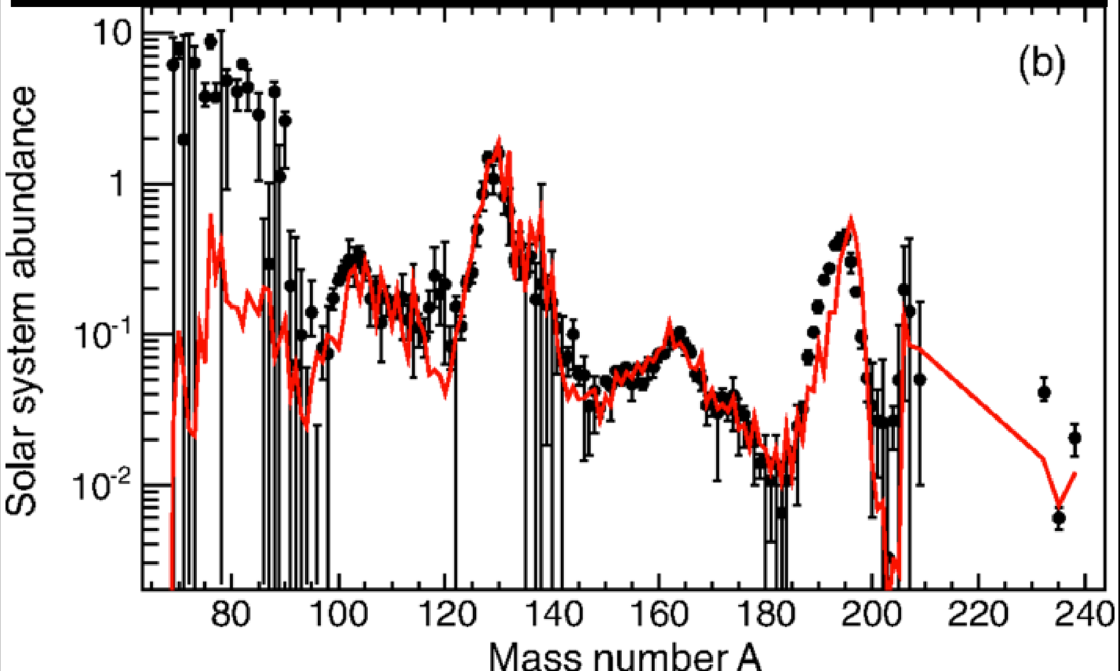
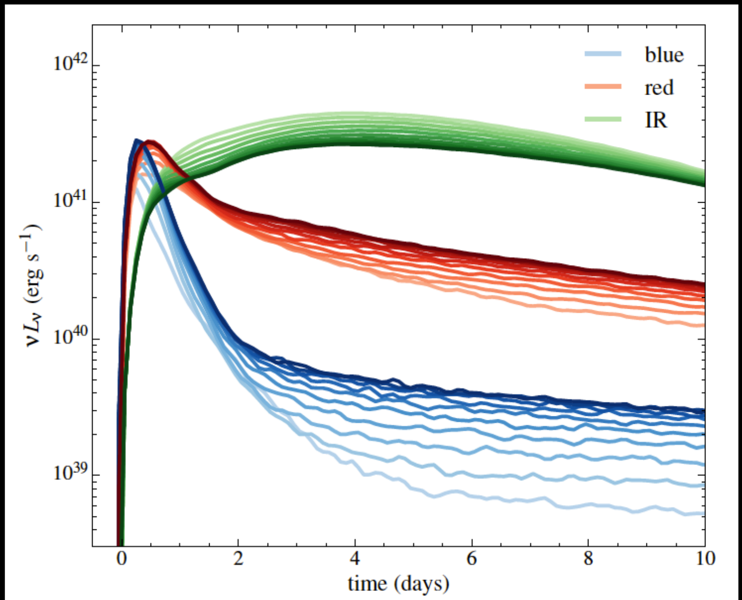


Image: Lorusso et al. 2015

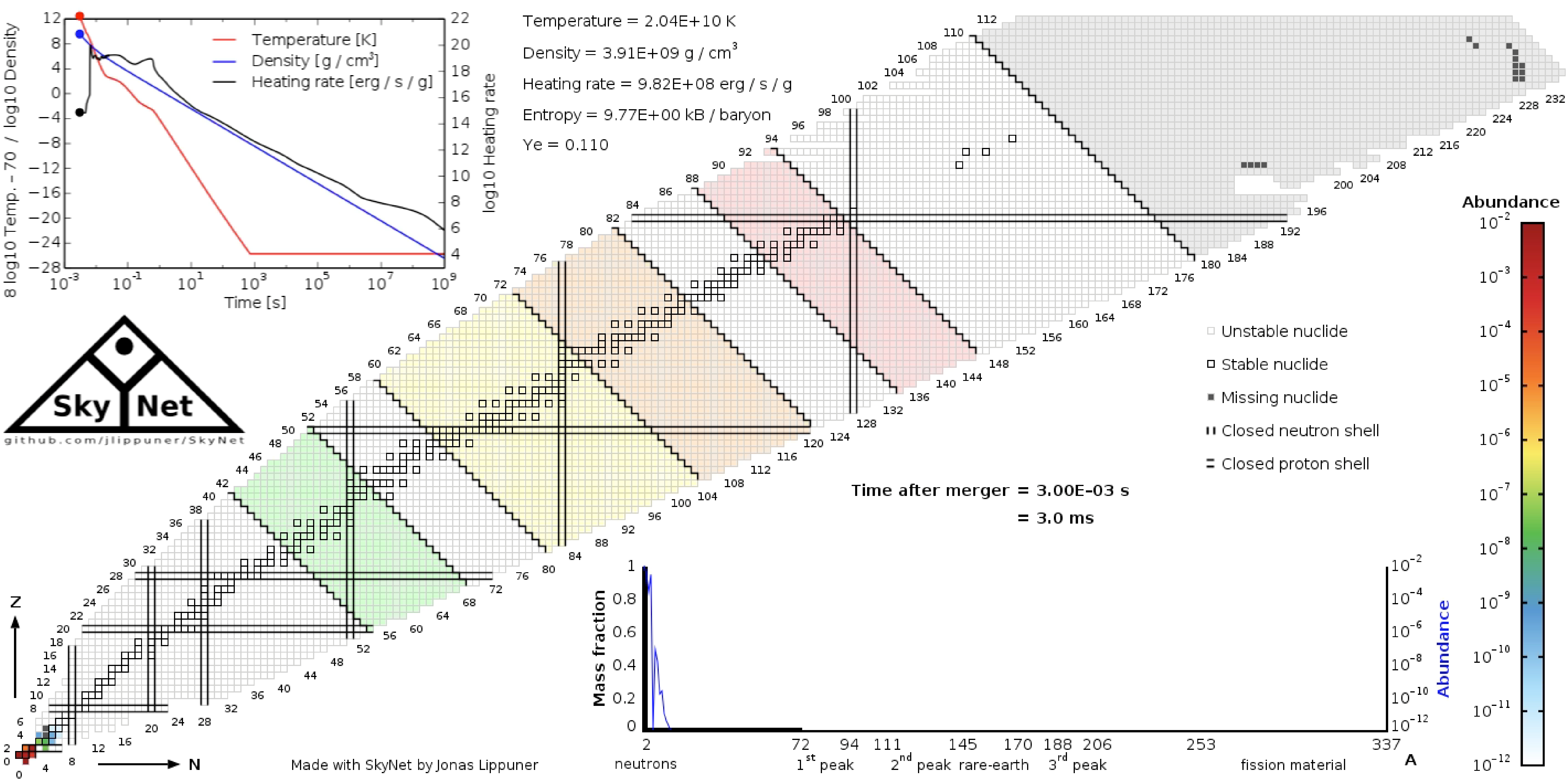
Radioactive decay of heavy elements heats ejecta



Hot ejecta produces **optical/IR transient**

r-process nucleosynthesis and kilonovae

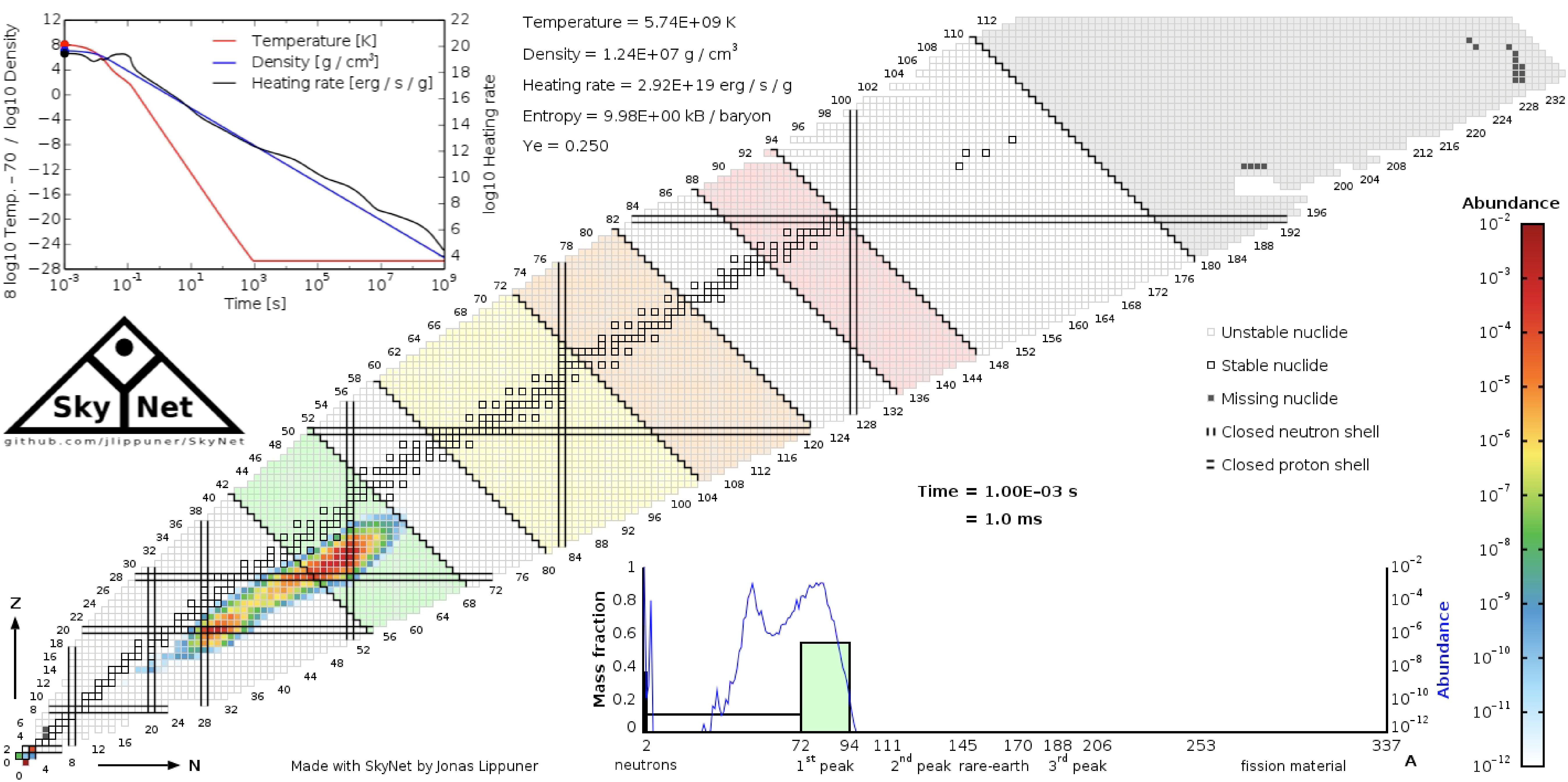
Nucleosynthesis in **neutron rich material** (e.g. tidal ejecta from BH-NS binary)



Visualization: Jonas Lippuner (Caltech), SkyNet code

Nucleosynthesis in neutron poor ejecta

Nucleosynthesis in “**neutron poor” material** (still 75% neutrons…)



Visualization: Jonas Lippuner (Caltech), SkyNet code

Kilonova lightcurves : parameter dependence

- Light curve of transient impacted by the properties of the ejecta:
  - **Opacity** (i.e. composition) : neutron rich -> high opacity
  - **Mass**
  - Expansion velocity
  - Outflow geometry

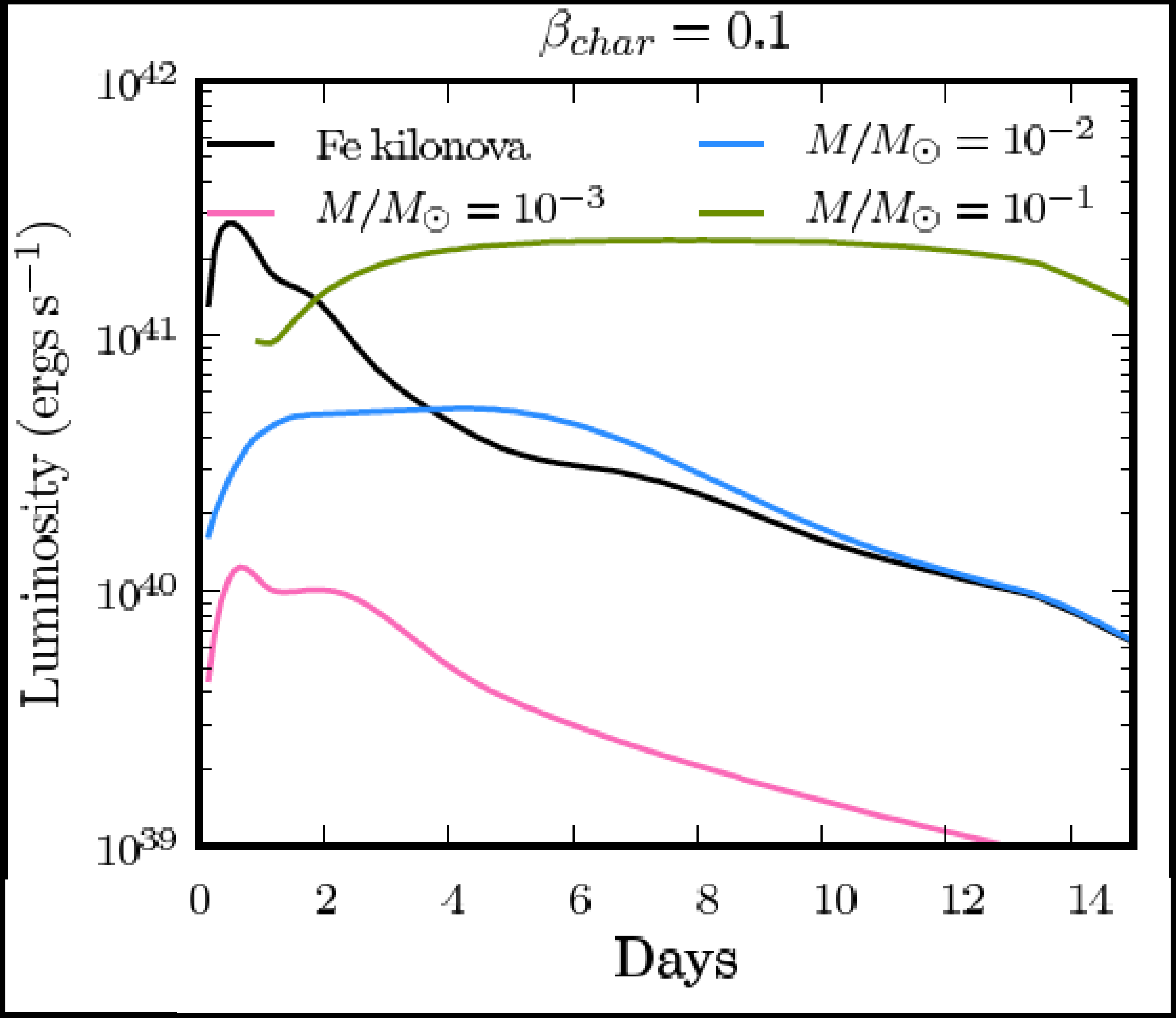


Image: Barnes & Kasen 2013

Kilonova lightcurves : parameter dependence

- Light curve of transient impacted by the properties of the ejecta:
  - Opacity (i.e. composition) : neutron rich -> high opacity
  - Mass
  - **Expansion velocity**
  - Outflow geometry

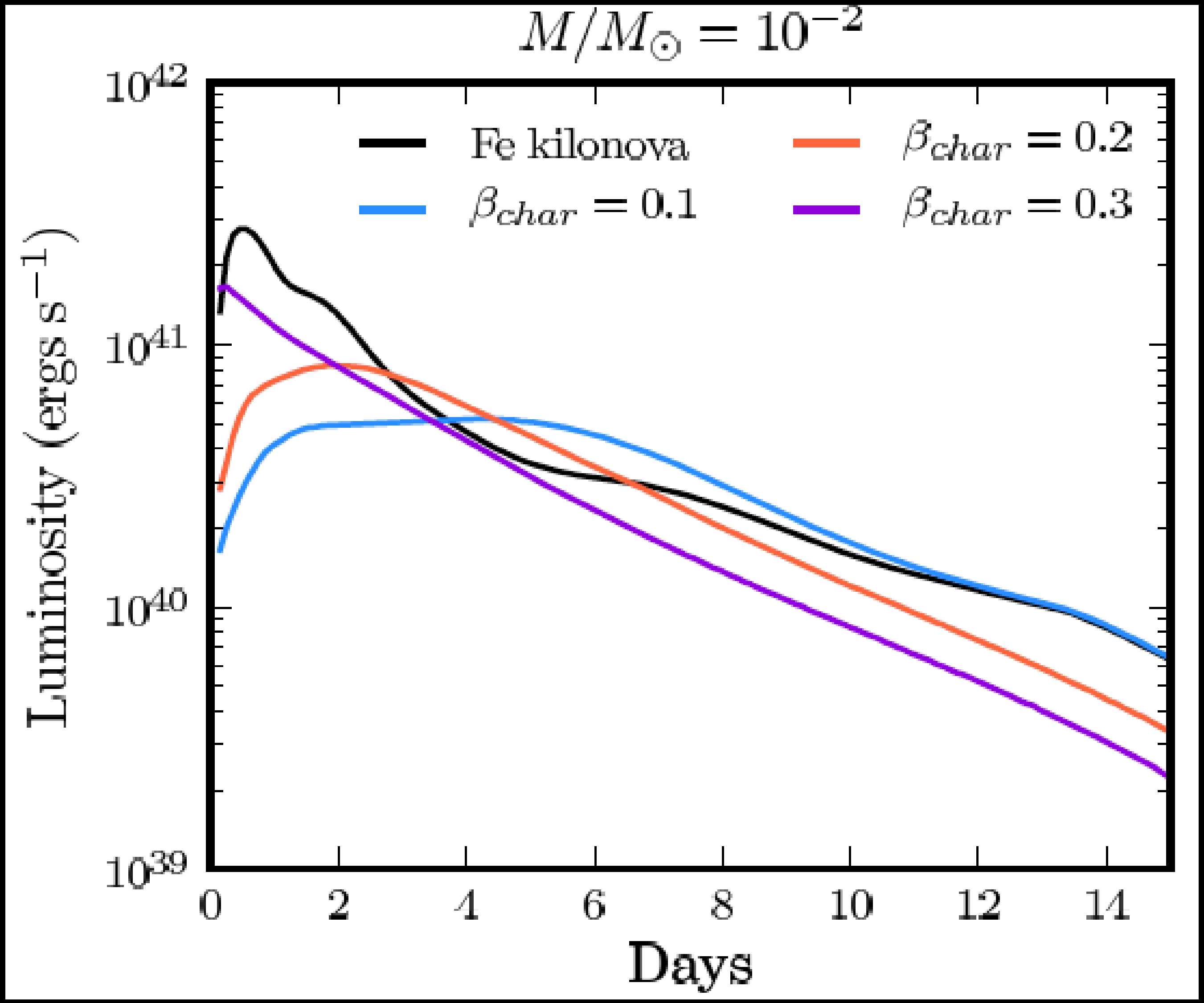


Image: Barnes & Kasen 2013

Outflows in neutron star binaries

NSNS -> NS **NSNS -> BH** **BHNS**

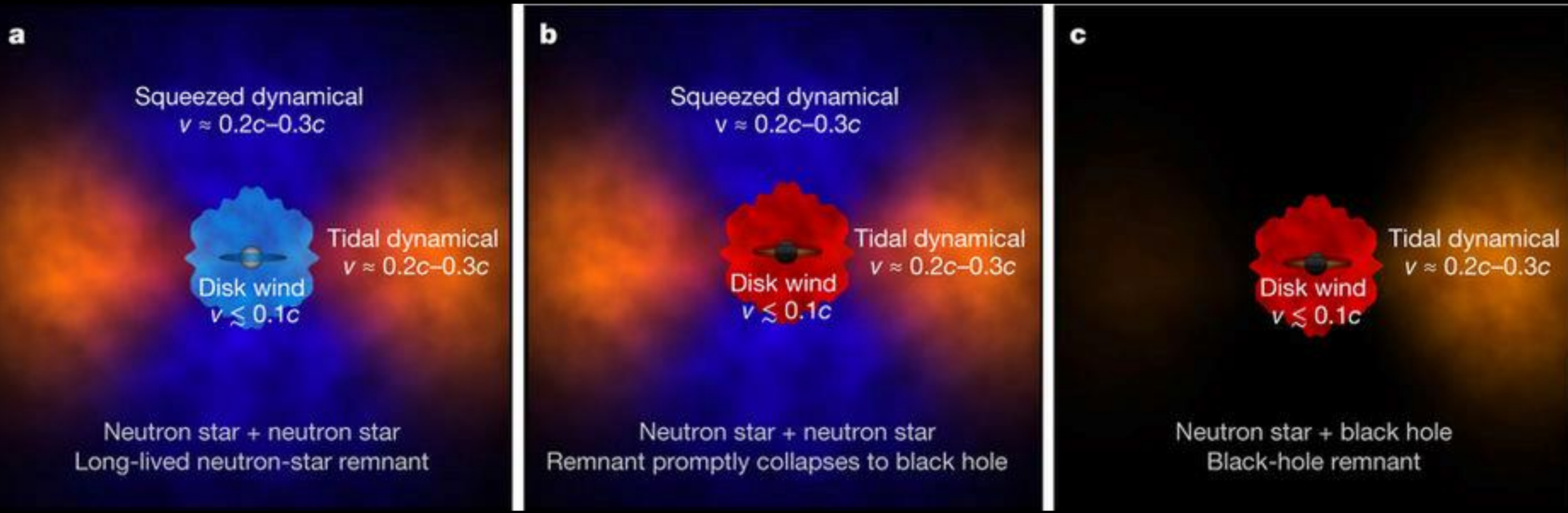
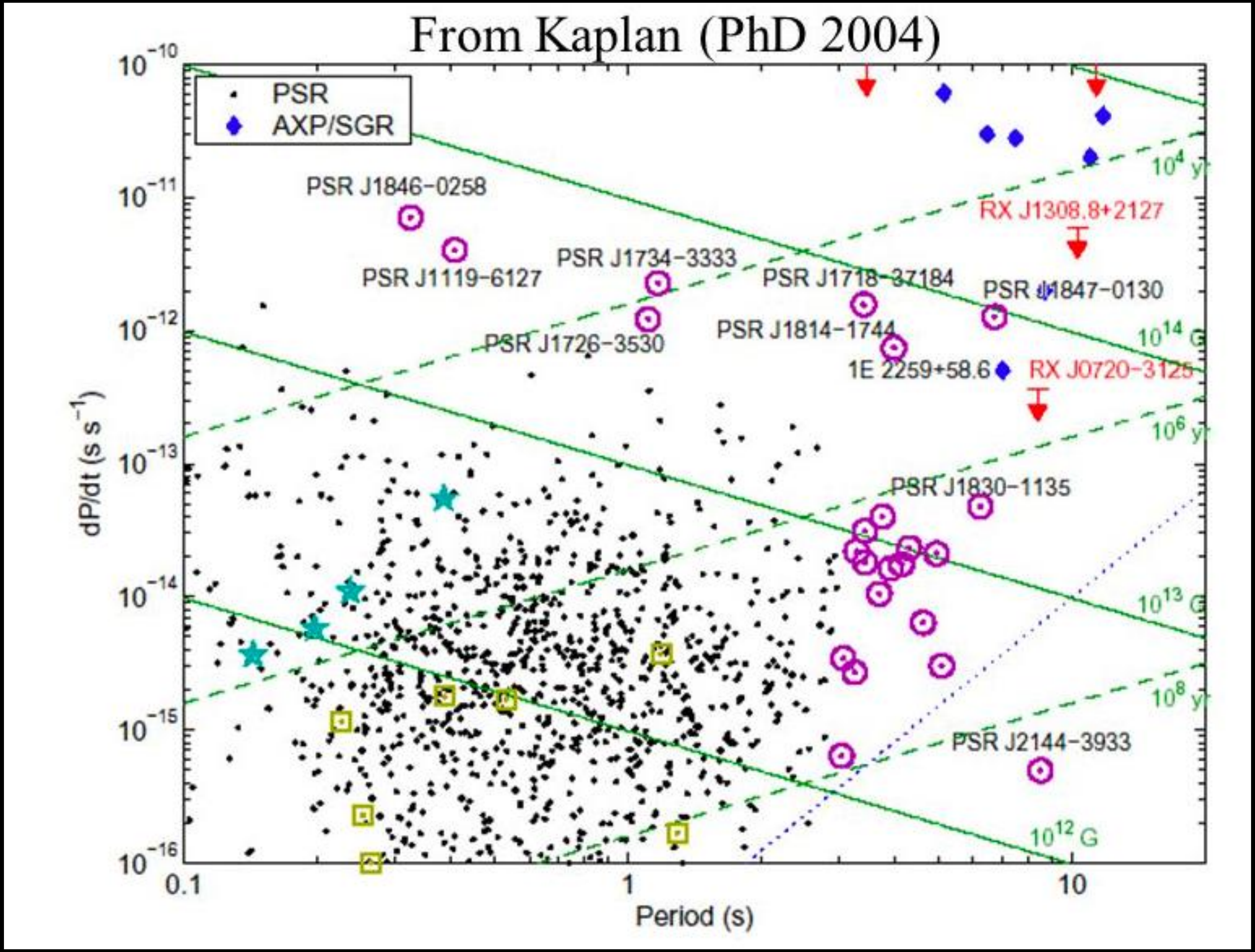


Image: Kasen et al. 2017

Relative contributions of equatorial/polar and dynamical/disk outflows vary with the parameters of the binary

10. Magnetic fields in mergers

Magnetic fields in pulsars



Magnetic fields before merger

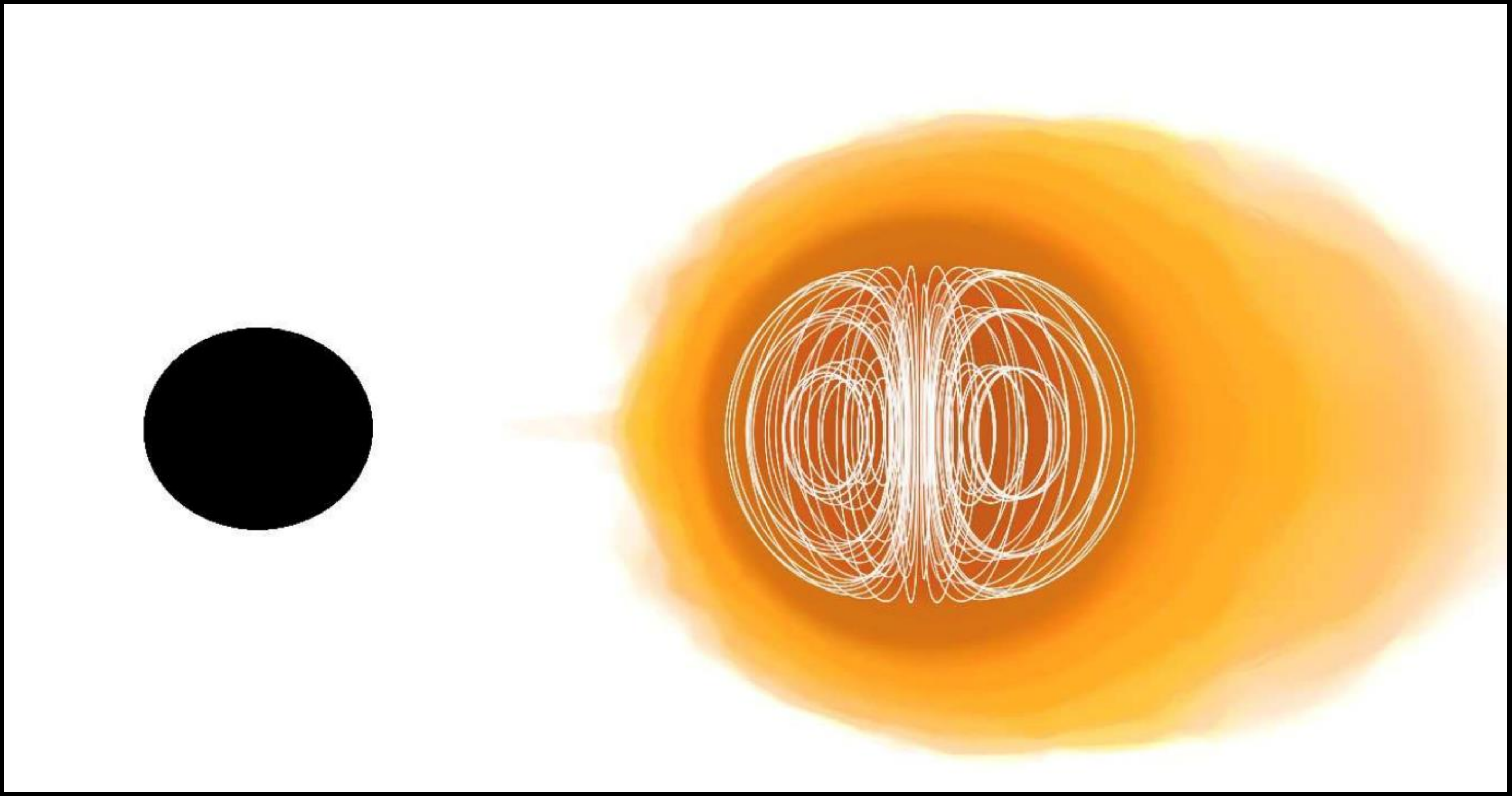
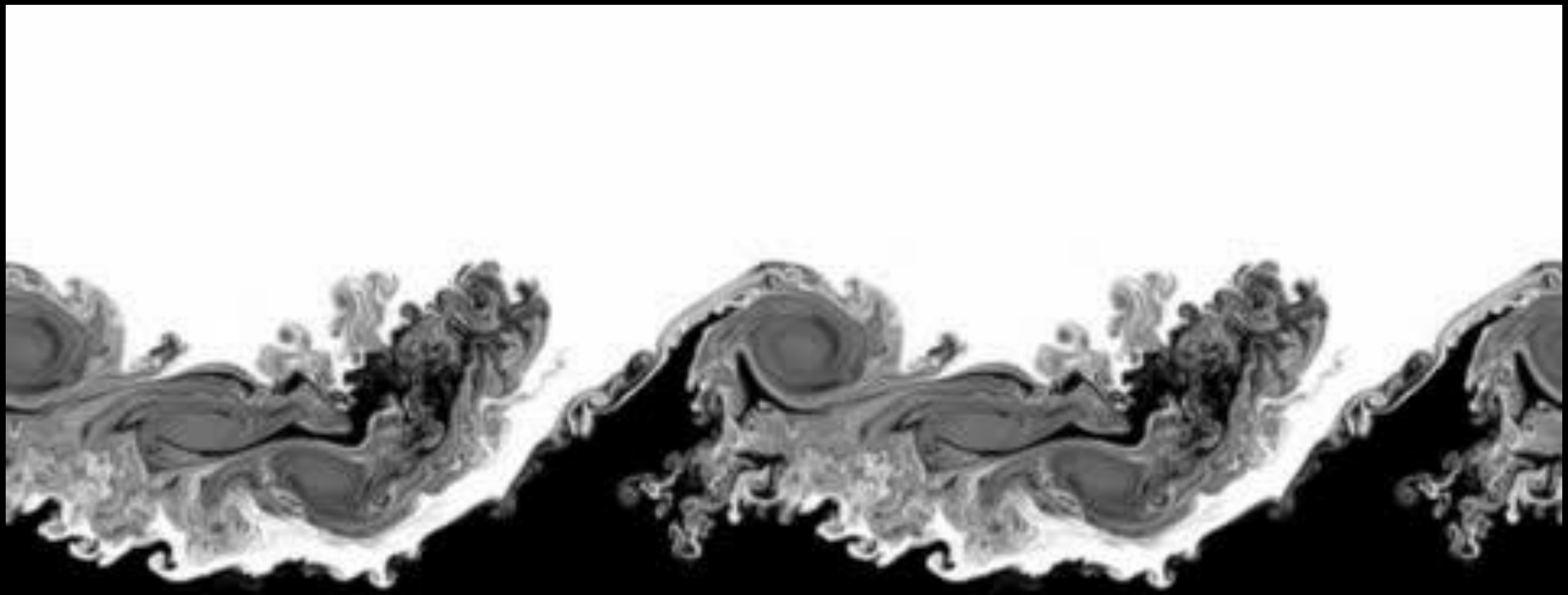


Image: Etienne et al. 2012

Effect of magnetic fields on <u>dynamics</u> of the binary (pre-merger) is <u>negligible</u> unless the magnetic field strength is B~10^{17}G…

<u>Weak EM emission</u> possible due to the interaction of the B-fields from two NSs.

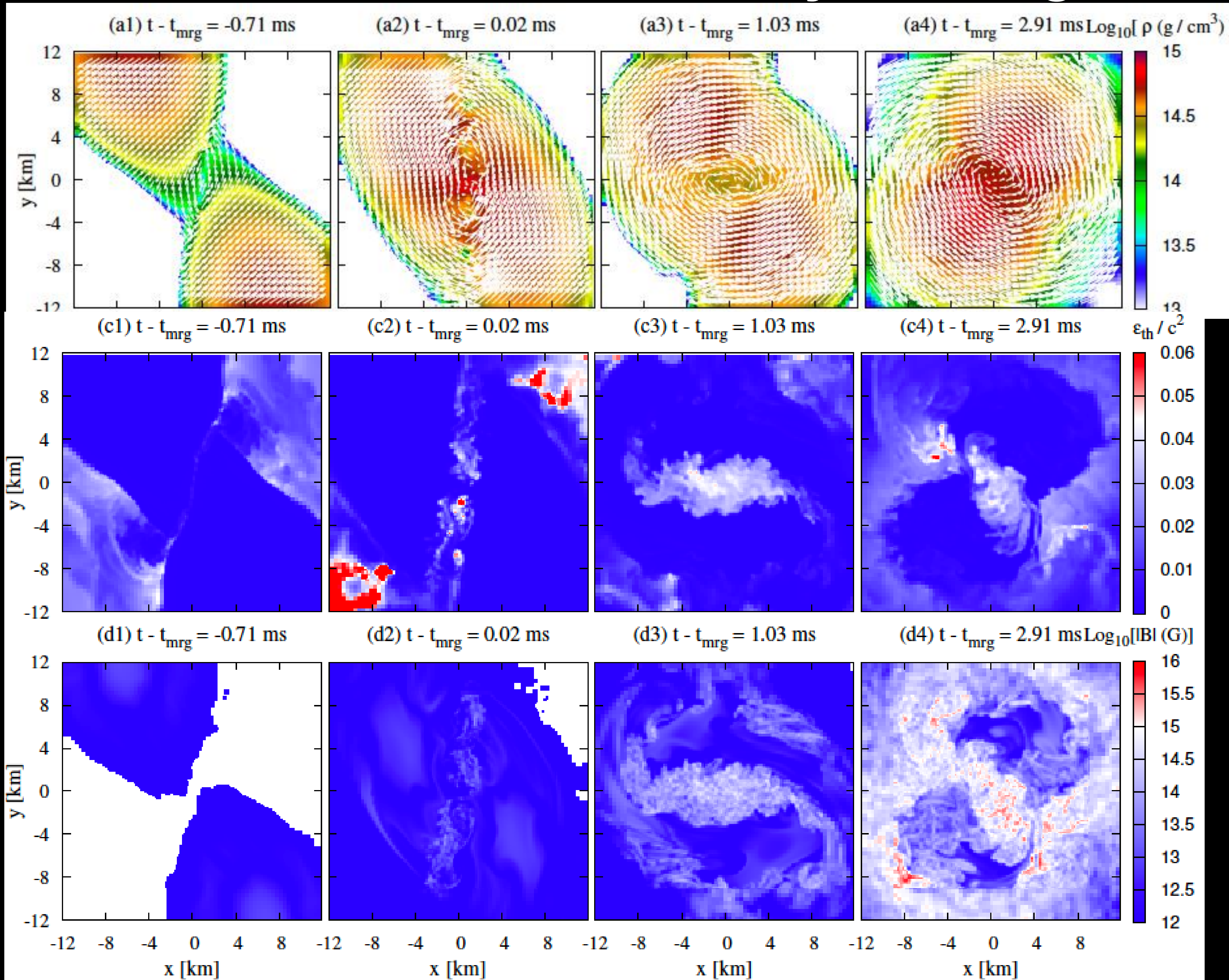
Kelvin-Helmholtz instability



- Fluid instability at the interface between regions moving at different velocities
- Grows on all scales, smallest scales grow fastest!
- Creates <u>strong turbulent magnetic field</u>!

Remant NS could be a magnetar!

Kevin-Helmholtz instability in mergers



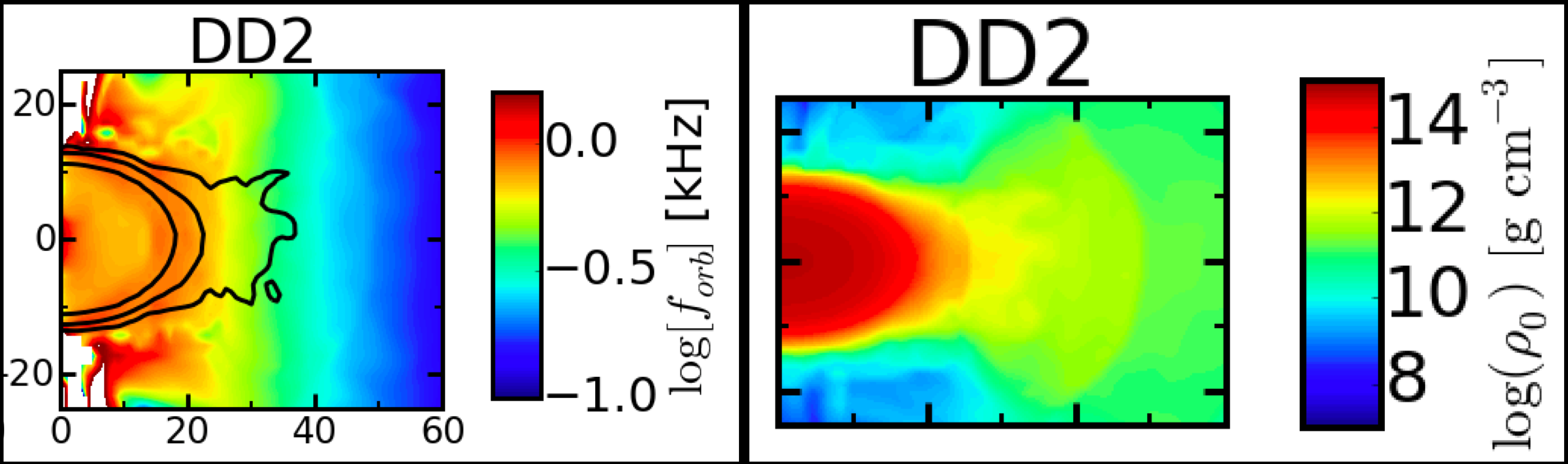
Magneto-rotational instability

- d**Ω**/dr<0 + B-field -> magnetorotational instability [MRI, Balbus & Hawley 1991]
- Creates turbulent magnetic field in post-merger disks with magnetic pressure ~ fluid pressure
- Fastest growing mode: $\lambda = \frac{2\pi c}{\Omega}\sqrt{\frac{b^2}{\rho}}$

$$\lambda \approx 30\,\mathrm{m}\frac{B}{10^{12}\mathrm{G}}\sqrt{\frac{10^{11}\mathrm{g/cm^3}}{\rho}\frac{3M_\odot}{M_{\rm BH}}\left(\frac{R}{50\,\mathrm{km}}\right)^3}$$

Magneto-rotational instability

- Where is the MRI active in a NS-NS merger remnant?



Images: Foucart et al. 2016

- In the accretion disk
- Only in *some* regions of the massive NS remnant

Effects of magnetic fields in merger remnant

- KH instability and MRI grow a turbulent B-field B~10^{16}G in the NS remnant and in the disk
- Causes angular momentum transport, heating
- Consequences:
  - Erases differential rotation in the NS remnant [~10ms]
  - Viscous spreading of the remnant disk [~1s]
  - Causes outflows / winds
    - From rapid redistribution of angular momentum in NS [~10ms]
    - From heating of the disk corona
  - <u>IF a large scale B-field is created</u>, relativistic jets / GRBs

11. Neutrino physics in mergers

Neutrino-Matter interactions

- Accretion disk is optically thick to photons, semi-transparent to neutrinos (optical depth ~ 1-10)
- NS is optically thick to both photons and neutrinos
- **Neutrinos = Main cooling mechanism of the disk**

Important reactions

Charged-current

$$p + e^- \leftrightarrow n + \nu_e$$

$$n + e^+ \leftrightarrow p + \bar{\nu}_e$$

also change composition

$$Y_e = \frac{n_p}{n_p + n_n}$$

Thermal

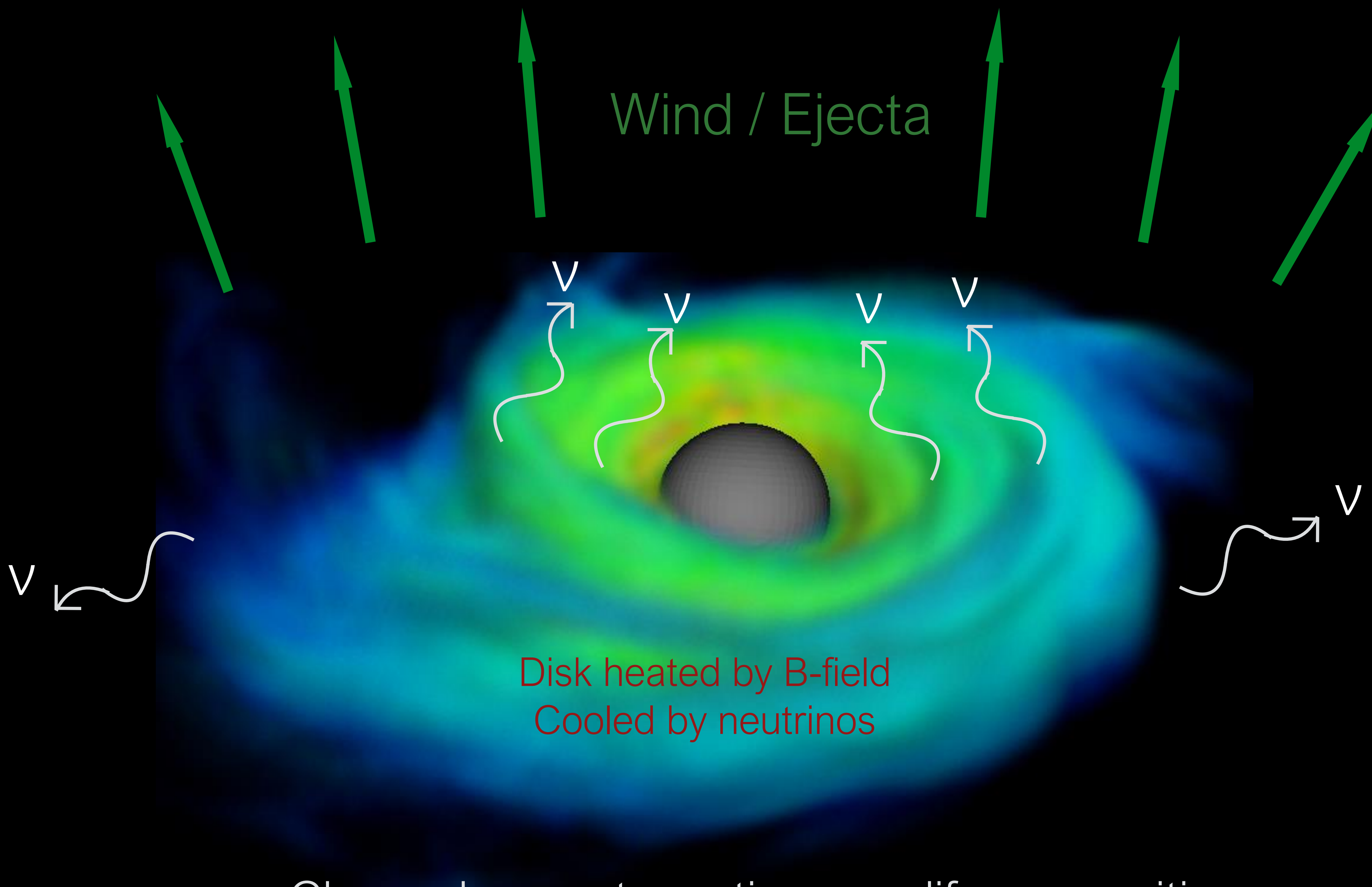
$$e^+ + e^- \leftrightarrow \nu + \bar{\nu}$$

$$N + N \leftrightarrow N + N + \nu + \bar{\nu}$$

+ Scattering on nuclei / nucleons

+ neutrino oscillations

Neutrino-matter interactions



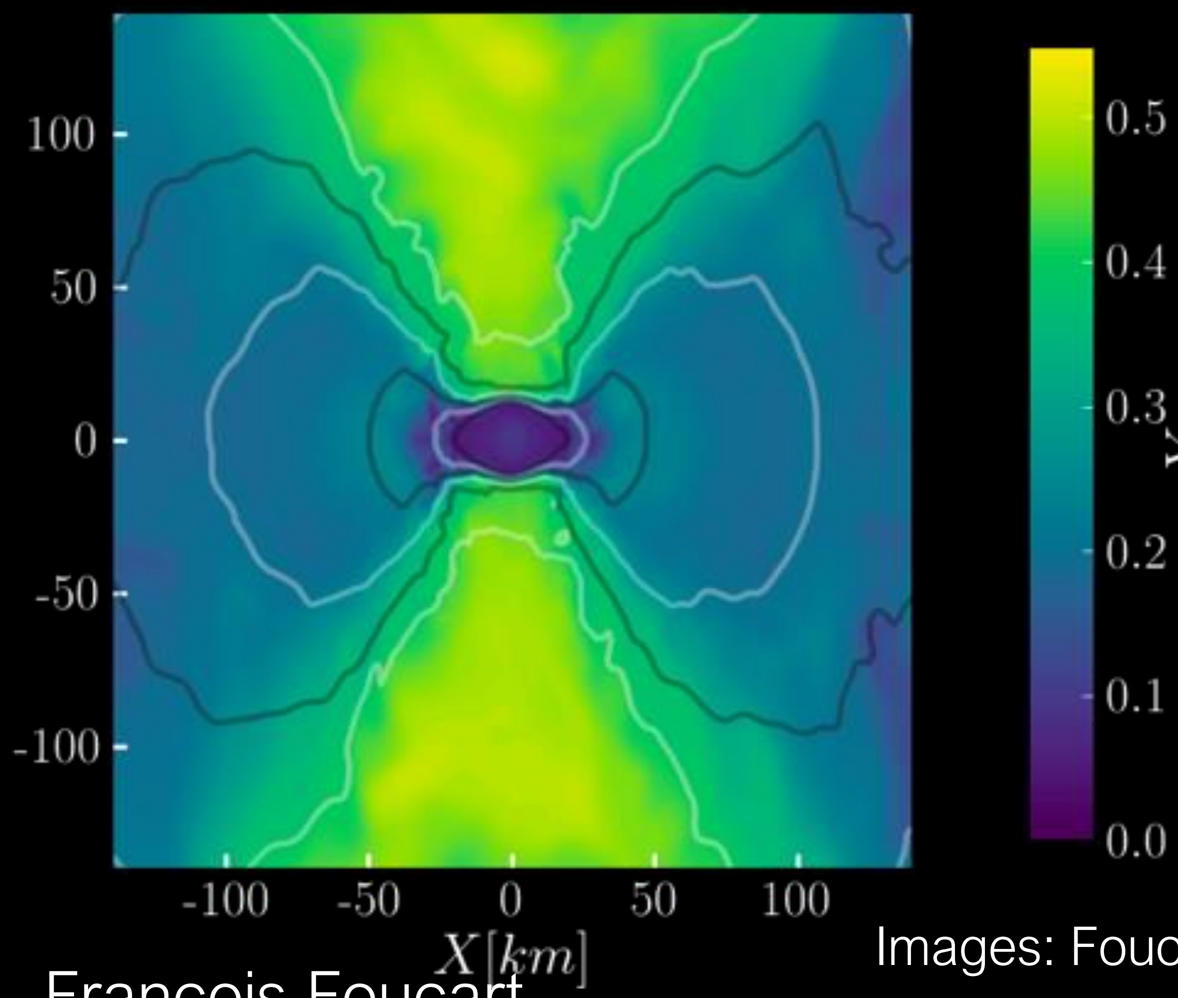
Charged current reactions modify composition:

$$n + e^+ \leftrightarrow p + \bar{\nu}_e \text{ and } p + e^- \leftrightarrow n + \nu_e$$

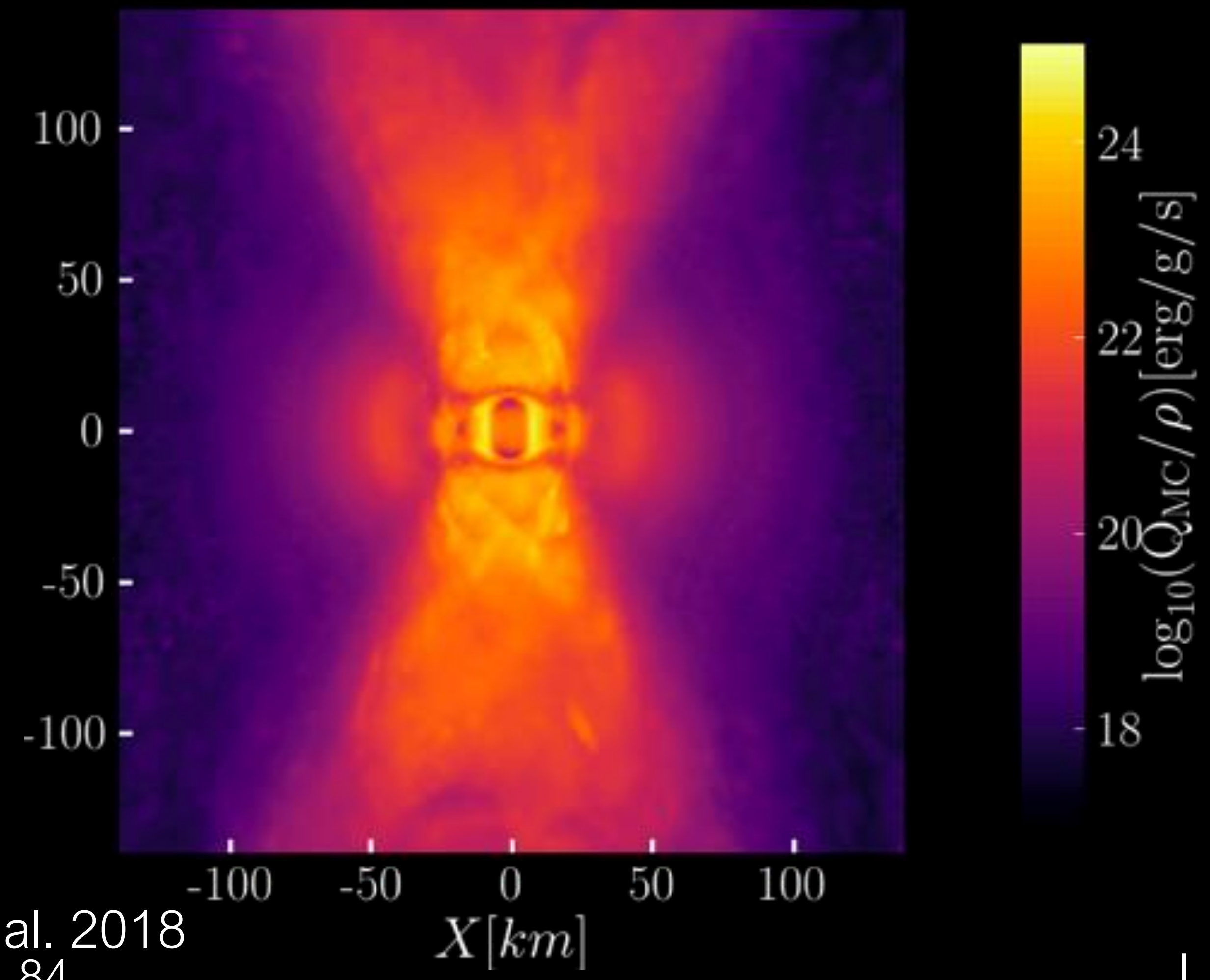
Neutrino-matter interactions

- Cross-sections : strong temperature dependence ($T^{4\text{-}8}$)
- Neutrino **absorption** in low-density regions drives winds
- Neutrino-ejecta interactions change ejecta composition
  - Very important effect for **kilonovae**, **r-process** outcome
- Pair annihilation deposits energy in polar regions (help jets??)

Composition



Energy deposition



Images: Foucart et al. 2018

12. Post-merger evolution

Post-merger evolution

- Unbinds more material
  - Magnetically-driven outflows
  - Neutrino-driven outflows
  - ``Viscous'' outflows
- Powers GRBs?
  - Relativistic jet powered by disk-BH remnant
  - Short GRB may require very low-mass disk (~0.001 solar mass; Gottlieb et al 2023)
  - More massive disks could explain "long-short" GRBs

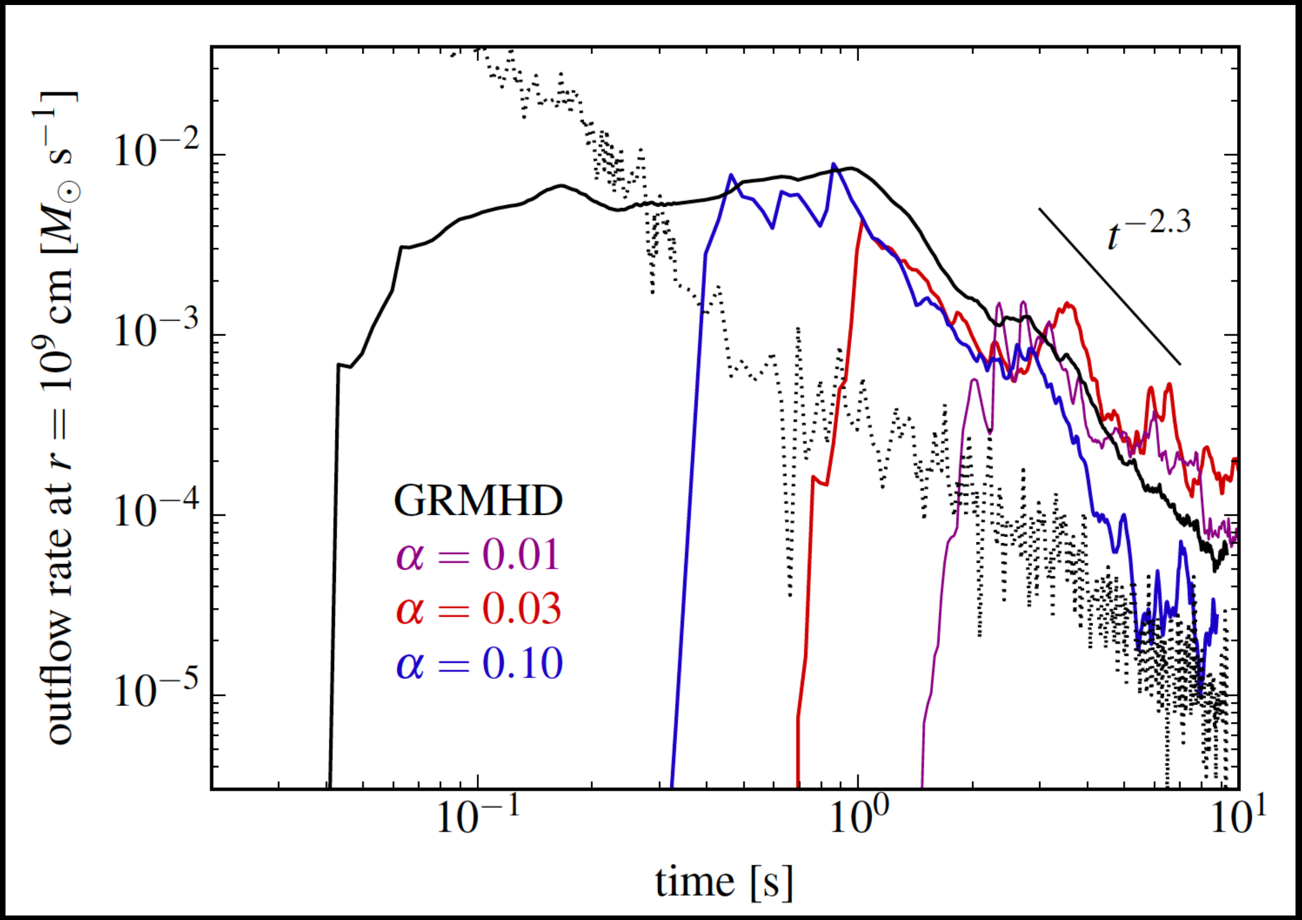


Image: Fernandez et al. 2018

May be the **dominant source of outflows** in NS-NS mergers:

~20% of the mass unbound in B-field driven outflows (Siegel & Metzger, Fernandez et al.)

~20% of the mass unbound in long-term viscous evolution (Fernandez & Metzger, Just et al.)

- NS-NS and BH-NS mergers power powerful GW **and** EM signals
  - Kilonovae
  - GRBs
  - Long-term radio and x-ray emission (disk, ejecta, magnetar)

- Post-merger evolution involves a wide range of physical processes
  - Magnetic fields and MHD instabilities
  - Neutrino physics

- **This makes modeling NS merger a very complex task**

13. Numerical Methods for (magneto)hydrodynamics

MHD Challenges

- High-accuracy shock-capturing method
- Satisfy conservation laws
- Avoid creation of magnetic monopoles
- Resolve relevant MHD instabilities
- Beyond ideal MHD: resistivity, viscosity, heat conduction,…

Conservative shock capturing methods

- Physical (primitive) variables: density, temperature/energy, velocity, electron fraction
- Evolution equations: conservation of baryon number, energy-momentum conservation, conservation of lepton number, Maxwell's equations (with perfect conductivity)

$$\nabla_\mu(nu^\mu) = 0; \quad \nabla_\mu T^{\mu\nu} = 0; \quad \nabla_\mu(nu^\mu Y_e) = 0$$

- All can be written in the form:

$$\partial_t U + \partial_i F^i(U) = S(U)$$

which conserves the spatial integral of U if S(U)=0 and we use an appropriate discretization of the fluxes:

$$\partial_x F^x(U,x) = \frac{F^x\left(U, x+\frac{\Delta x}{2}\right) - F^x\left(U, x-\frac{\Delta x}{2}\right)}{\Delta x}$$

Conservative shock capturing methods

$$\partial_x F^x(U,x) = \frac{F^x\left(U, x+\frac{\Delta x}{2}\right) - F^x\left(U, x-\frac{\Delta x}{2}\right)}{\Delta x}$$

$\boldsymbol{F^x(U, x-\frac{\Delta x}{2})}$

$\boldsymbol{F^x(U, x+\frac{\Delta x}{2})}$

- Ability to capture shock comes from appropriate choice of fluxes on cell faces, generally based on satisfying "jump conditions" between estimates of the fluxes on the left and right side of a face.

$$\boldsymbol{F^x\left(U, x+\frac{\Delta x}{2}\right) = \frac{1}{2}\left(\mathbf{F^x}\left(\mathbf{U}, \mathbf{x}+\frac{\Delta \mathbf{x}^+}{2}\right) + \mathbf{F^x}(\mathbf{U}, \mathbf{x}+\frac{\Delta \mathbf{x}^-}{2})\right) - \mathbf{c}/2\left(\mathbf{U}\left(\mathbf{x}+\frac{\Delta \mathbf{x}^+}{2}\right) - \mathbf{U}(\mathbf{x}+\frac{\Delta \mathbf{x}^-}{2})\right)}$$

Conservative shock capturing methods

- Simplest choice:

$$F^x\left(U, x+\frac{\Delta x}{2}\right) = \frac{1}{2}\left(F^x(U, x+\Delta x) + F^x(U, x)\right) - c\ \left(U(x+\Delta x) - U(x)\right)/2$$

Flux average

Dissipation term

With c = maximum speed at which information moves through the system (sound speed, Alfven speed, light speed… depending on the system of equations).

Addional issues in fluid dynamics

- Complications:
  - Fluxes typically known as function of primitive variables, not U
  - Many advanced methods require interpolation of the primitive variables from cell centers to cell faces
- Consequence:
  - Need to be able to "invert" from U to primitive variables
  - Typically a multi-dimensional root finding problem
- Closing the system of equations requires an equation of state, e.g. $P(\rho, T, Y_e), u(\rho, T, Y_e)$
  - EoS current unknown
  - Discontinuities / phase transitions can impact simulation accuracy

No-monopole condition

- Same issue as for the constraints in Einstein's equations: we need to maintain $\nabla.B = 0$.
  - *Constraint damping* remains an option (= *divergence cleaning*)
  - Better: discretize the equations in such a way that the no-monopole condition is satisfied to roundoff (=*constrained transport)*
    - Hard to do on complex grids (curved meshes, mesh refinement boundary), but generally more robust

Beyond ideal MHD

Non-ideal fluid dynamics often difficult to treat in GR (e.g. simple translation of Newtonian physics are not causal). Many may however be important and are being investigated

- Resistive MHD
- Viscosity due to neutrinos
- Viscosity due to MHD
- Heat transport
- Composition diffusion

Viscosity is also used to approximate the role of MHD turbulence

14. Neutrino transport in merger simulations

Neutrino transport

High cost: (6+1)D problem

$$f_{(\nu)} = f(t, x^i, p^\alpha)$$

and complex collision terms, e.g.

Inelastic scattering

Neutrino-antineutrino annihilation

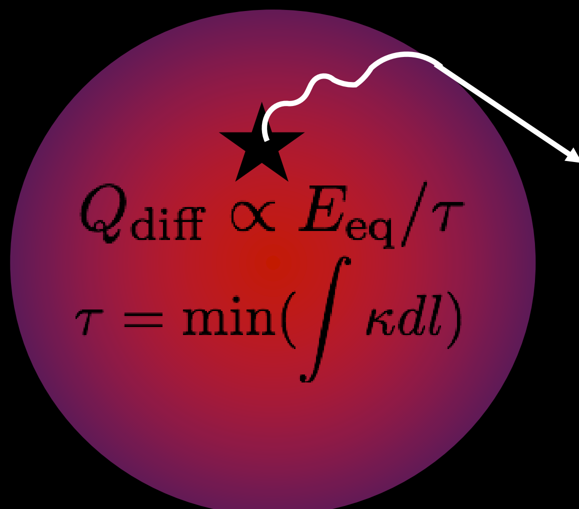
Cross-sections depend strongly on neutrino energy & orientation!

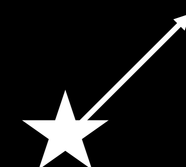
Leakage schemes

Simplest, most common approximation:

Estimate energy and lepton number emission from:

(i) Diffusion limit (ii) Free-streaming





$Q_{\mathrm{free}} = \eta_{\mathrm{free}}$

$$\Rightarrow Q = \left(\frac{1}{Q_{\mathrm{free}}} + \frac{1}{Q_{\mathrm{diff}}}\right)^{-1}$$

Optical depth obtained from approximate solution to

$$|\nabla \tau| = (\kappa_A + \kappa_S)$$

> Leakage schemes are cheap, **but**
> only order-of-magnitude accurate
> No absorption/winds/non-local effects

See Ruffert et al. 1997, Rosswog & Liebendorfer 2003, Sekiguchi et al. 2011, **Deaton et al. 2013**, Neilsen et al. 2014, **Foucart et al. 2014**

Moment formalism (M1)

Relatively cheap, **approximate transport** method.

See Shibata et al. 2011, Foucart et al. 2015

Define moments :
Energy Density E
Flux Density F_i
(optionally) Number Density N

Approximate closure

$$P^{\mu\nu} = \alpha P^{\mu\nu}_{\rm thick} + (1-\alpha) P^{\mu\nu}_{\rm beam}$$

using optically thin/thick limits

Exact evolution equations:

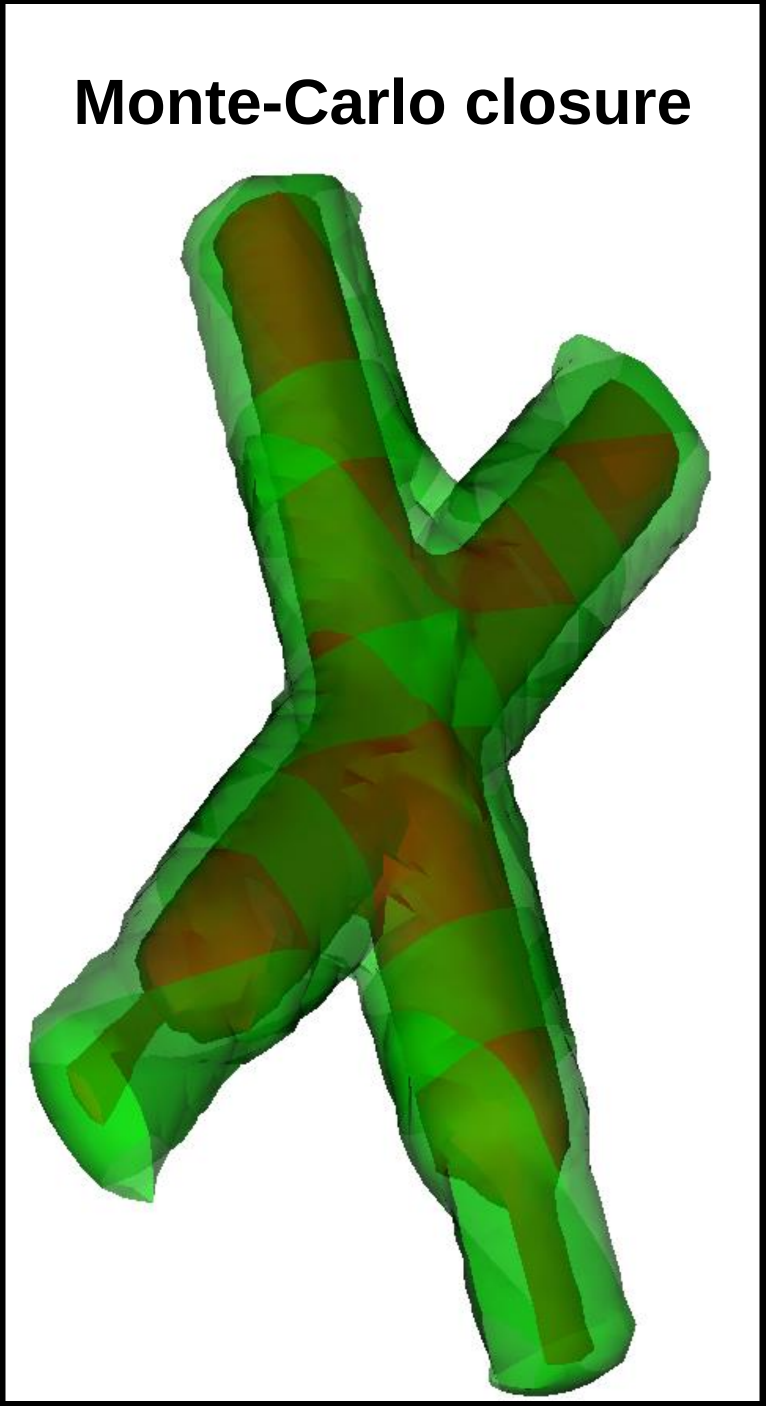
$$\partial_t \tilde{E} + \partial_j \mathcal{F}^j = \text{sources}$$

$$\partial_t \tilde{F}_i + \partial_j \mathcal{P}_i^j = \text{sources}$$

Sources include:
Curvature/redshift terms
Emission/Absorption/Scattering

Impact of current approximations

Crossing beams

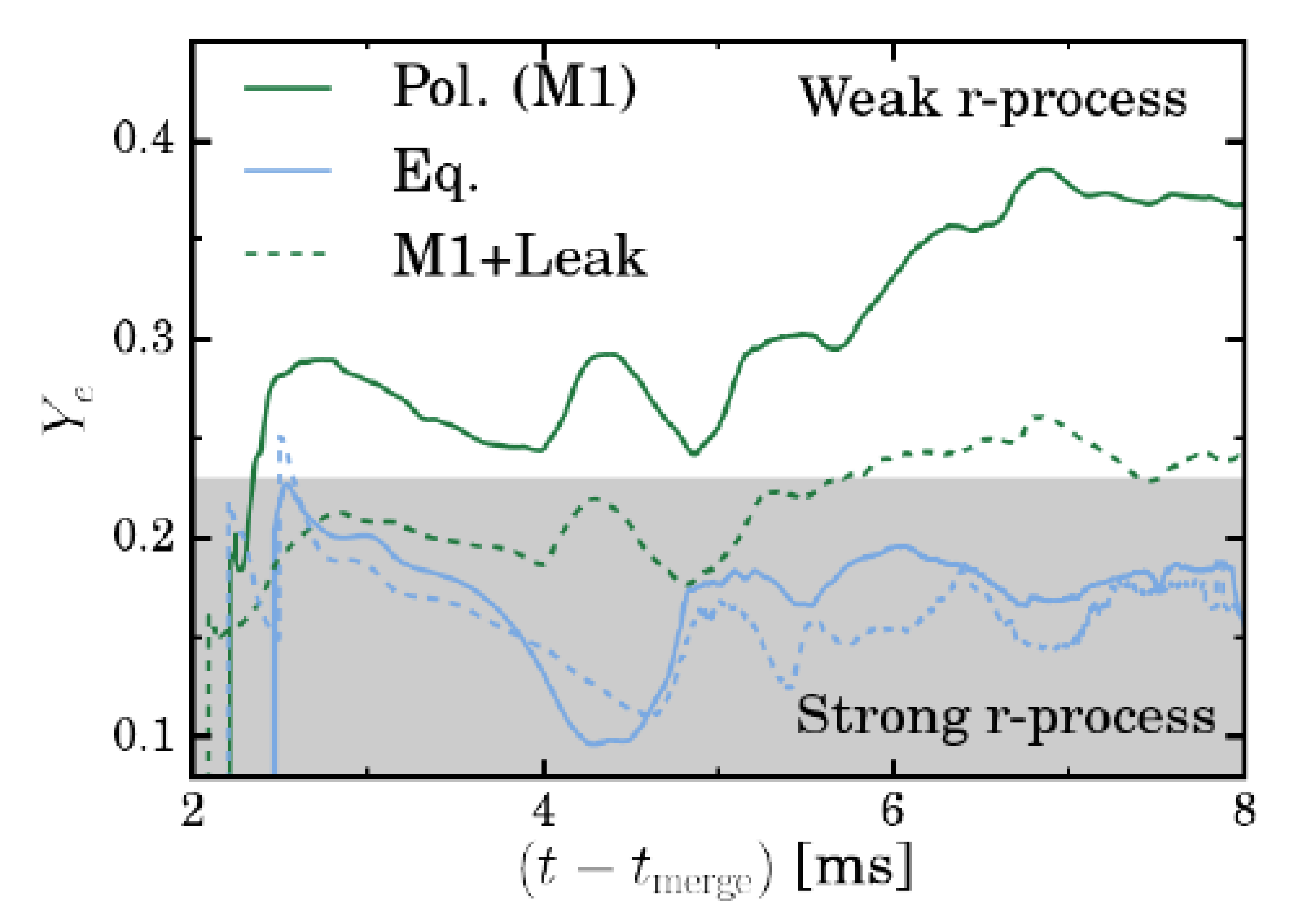


Images: Foucart et al., 2018

Impact of current approximations

<u>Outflow composition (**NSNS**):</u>

Impact of neutrino treatment



Images: Foucart et al., 2017

Monte-Carlo Transport: Formalism

- Sample distribution $f_{(\nu)}$ using superparticles / packets.
- Packets are emitted / transported / scattered / absorbed as individual neutrinos
- Very efficient method *at low-resolution in high-dimensional spaces*

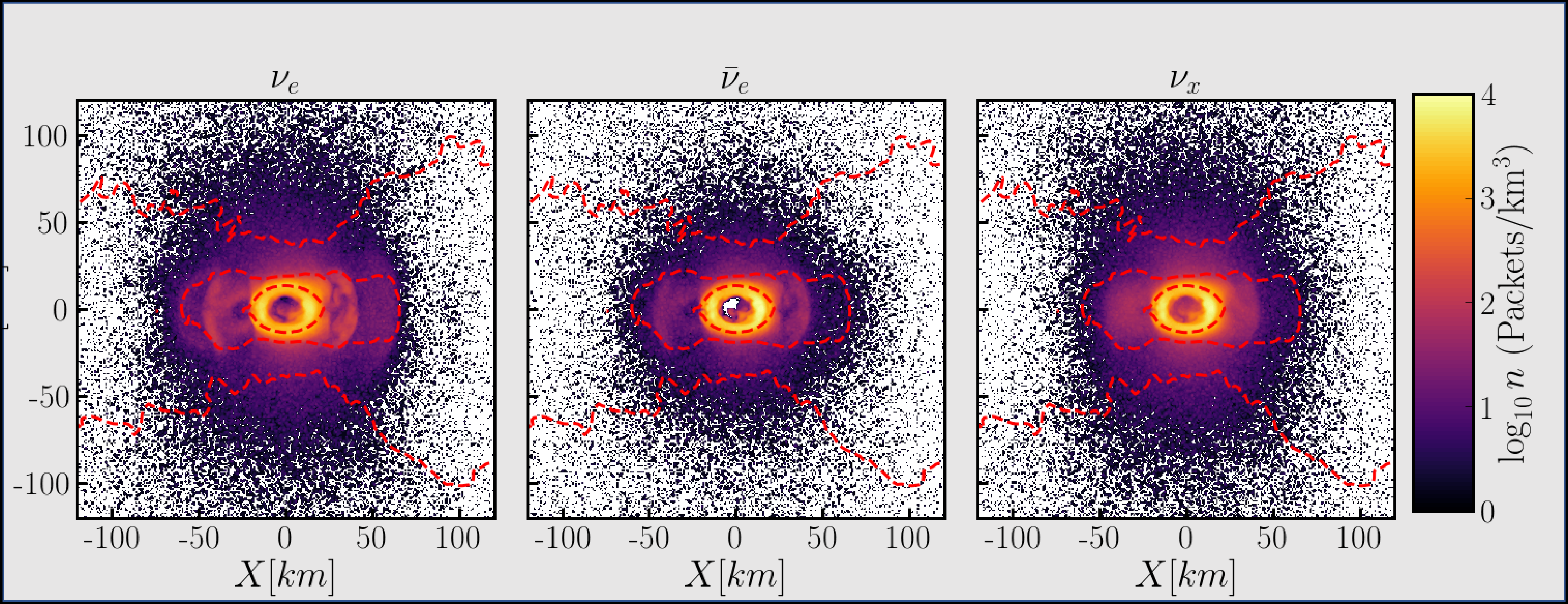


Image: Foucart *et al* (2020)

Monte-Carlo: Implementation Issues

High absorption / scattering opacities expensive to handle without corrections.

SpEC choices:

- High scattering opacity as diffusion instead of many individual scatterings
- High absorption opacity using implicit Monte-Carlo

Stiff coupling to fluid in high opacity regions: also fixed by use of implicit MC

Parallelization difficult due to inhomogeneous distribution of packets

Monte-Carlo Transport: Results

Difference between M1 and MC methods : 10% relative errors (10^8 packets)

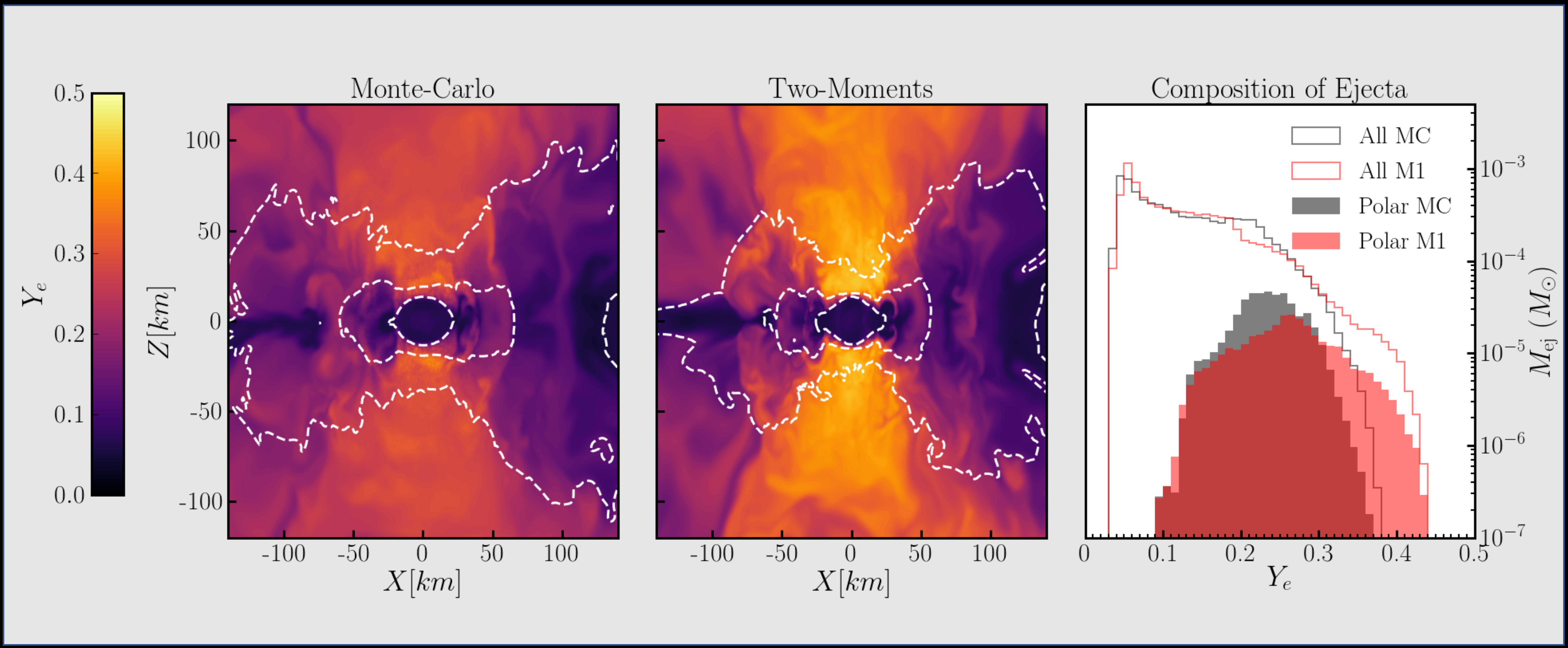


Image: Foucart *et al* (2020)