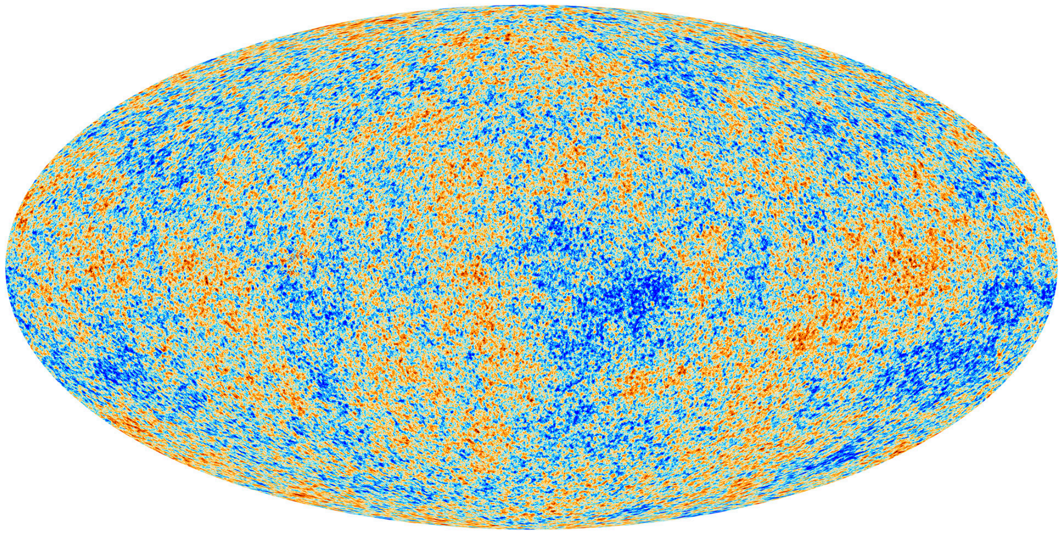
Cosmology

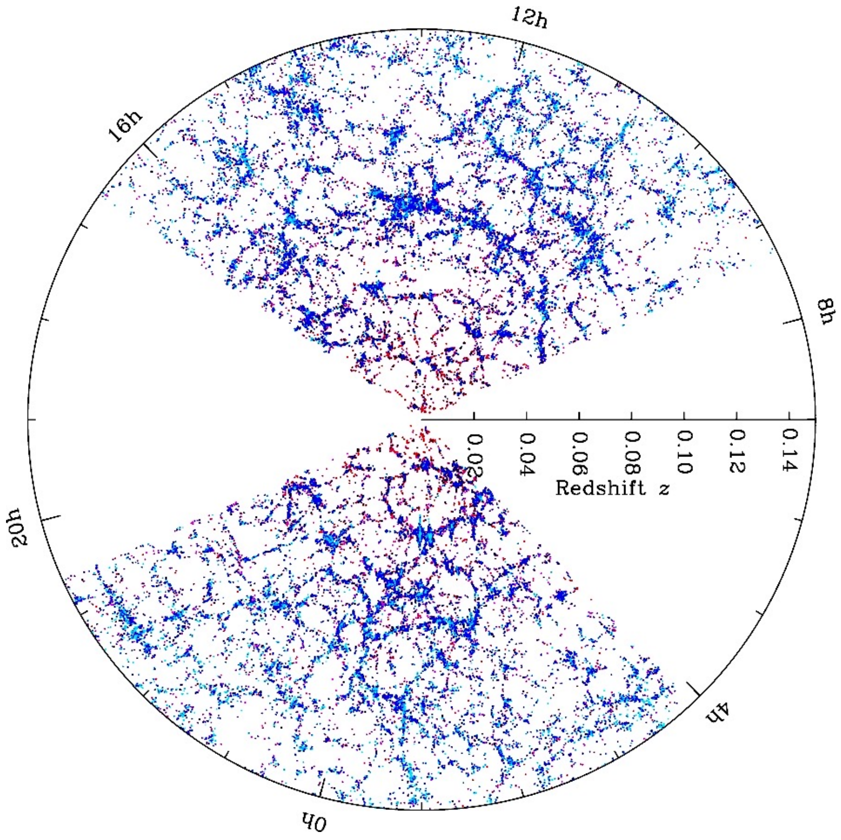
Lecture 2

Pedro Carrilho

The Inhomogeneous Universe

So far, we dealt with a homogeneous and isotropic Universe, but that is only true statistically!





Today, we will explore the **evolution of perturbations** and **how to measure them.**

The Menu

Day 1:

- Expansion of the Universe
- History of expansion in ΛCDM
- Going beyond ΛCDM

Day 2:

- Structure in the Universe
- Evolution of structure in ΛCDM and beyond

Structure in the Universe

Cosmological fluctuations

In the early Universe, perturbations were small

$$\frac{\delta T}{T} \sim 10^{-5}$$

So (until they grow too large), we can use perturbation theory to describe them.

We will be interested in following the evolution of density fluctuations

$$\delta \equiv \frac{\delta\rho(x)}{\bar{\rho}}$$

Which encode information of the growth of structure and the expansion history.

Cosmological fluctuations

We are interested in statistics of perturbations, such as the **correlation function** of a fluctuation field $\boldsymbol{\delta}$

$$\langle \delta(\vec{x})\delta(\vec{x}+\vec{r})\rangle \underset{\text{homogeneity}}{=} \xi(\vec{r}) \overset{\text{Isotropy}}{=} \xi(r)$$

Or its Fourier transform, the power spectrum

$$\langle \delta(\vec{k})\delta(\vec{q})\rangle = (2\pi)^3\delta_D(\vec{k}+\vec{q})P(k)$$

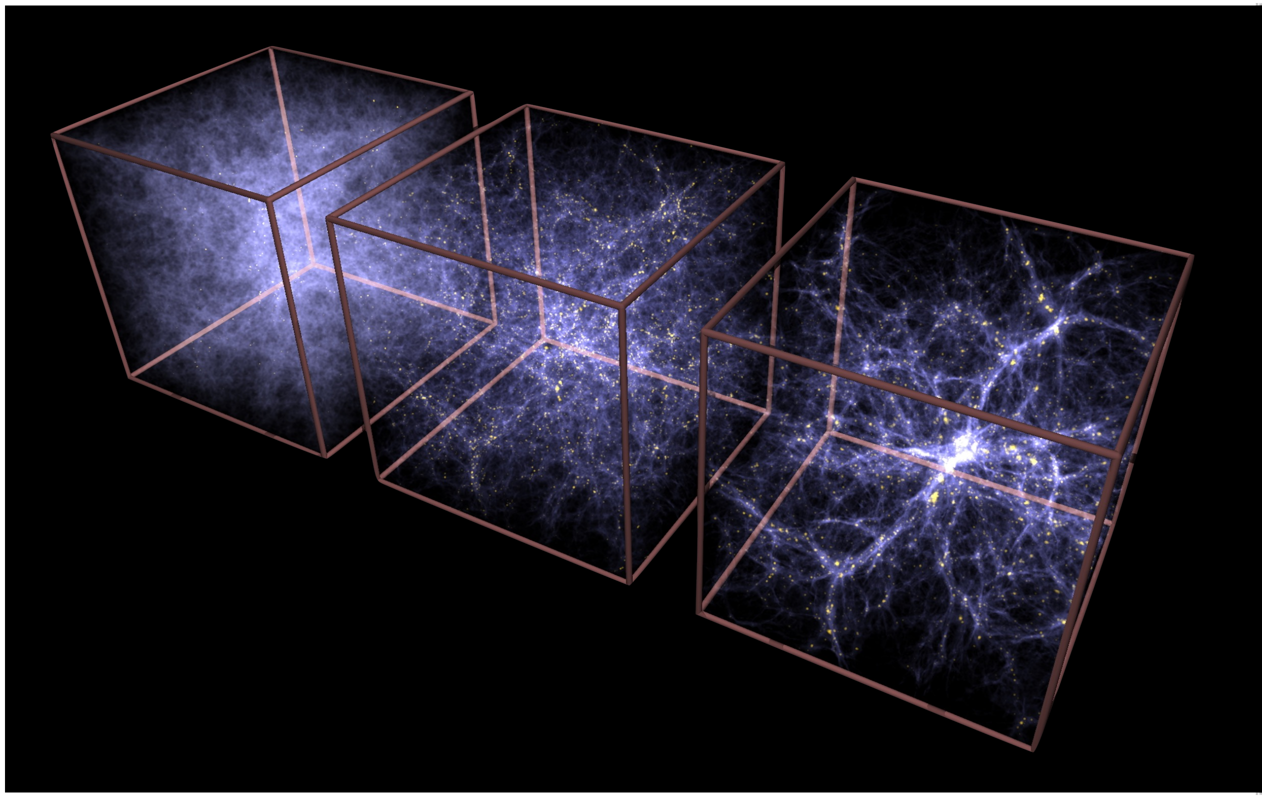
Describes the variance in the field at each scale k.

We often describe its amplitude by the parameter σ_8

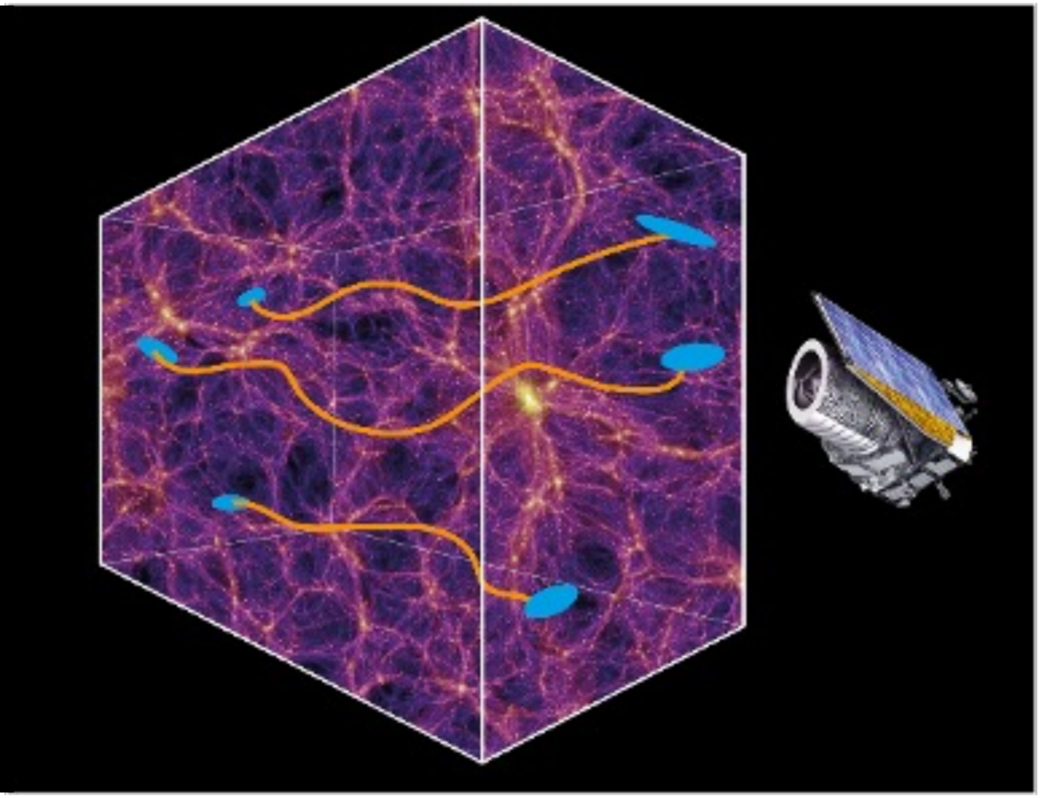
$${\sigma_8}^2 = \frac{1}{2\pi^2}\int \mathrm{d}k\; k^2 P_L(k) W^2_{8\,\mathrm{Mpc/h}}(k)$$

Measuring structure

Two main ways to probe the large-scale structure



Galaxy clustering

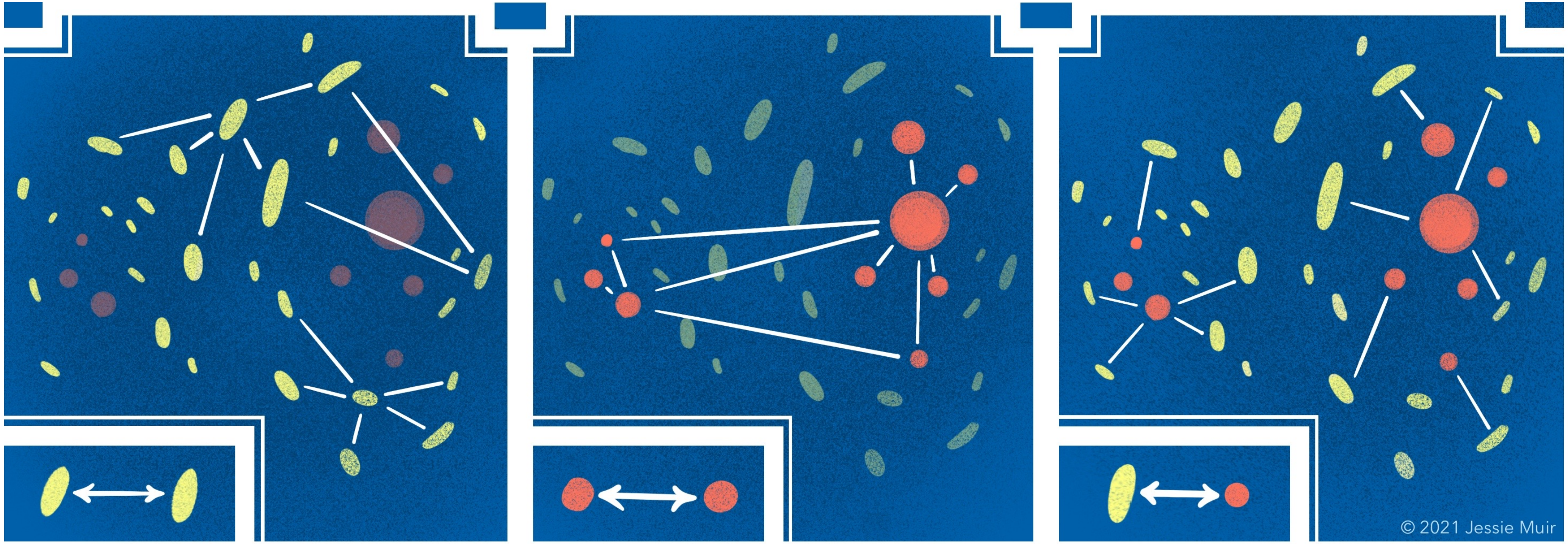


Weak gravitational lensing

Measuring structure

How? We measure correlations

$$\langle\delta\delta\rangle_{obs} \sim \sum_{pairs\ i,j} \delta_i \delta_j$$



WL: Shape – Shape

GC: Position – Position

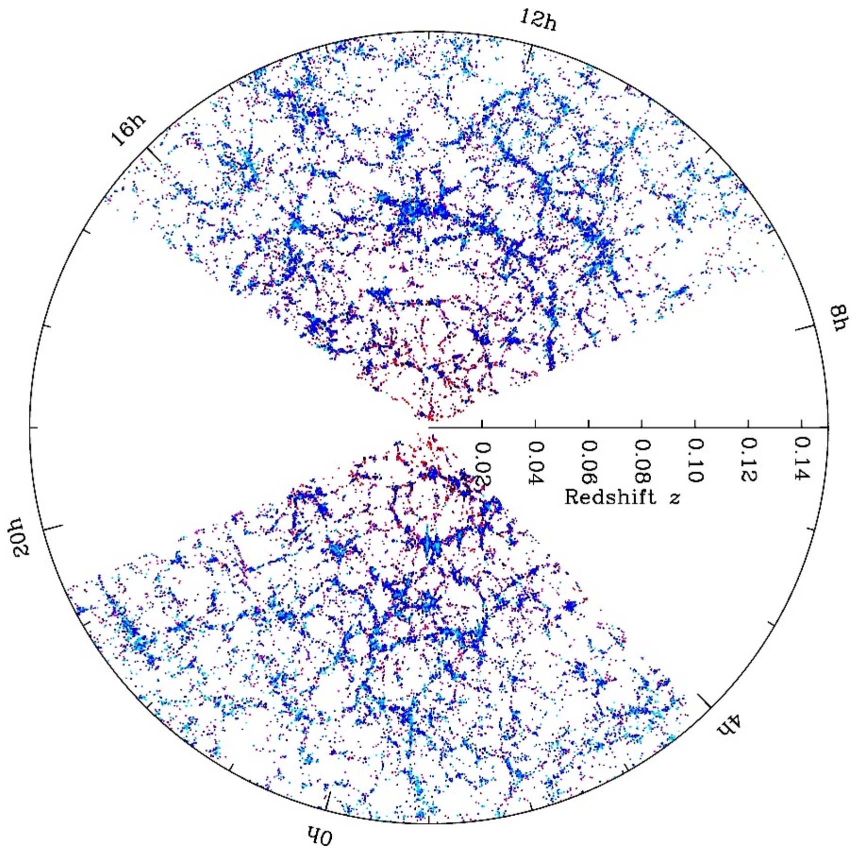
WLxGC: Shape – Position

Measuring structure - GC

Galaxy clustering measures the positions and redshifts of galaxies (also quasars)

Two kinds of clustering

- Spectroscopic galaxy clustering
  - Gives high precision redshifts, so can probe structure in 3D with accuracy
- Photometric galaxy clustering
  - Loses 3D information by not having good redshift determination
  - Can potentially acquire more objects due to faster survey
  - Typically used from weak lensing surveys



Measuring structure - GC

With galaxy clustering, **we do not observe the matter field directly** – we count galaxies!

So what we detect is a biased field

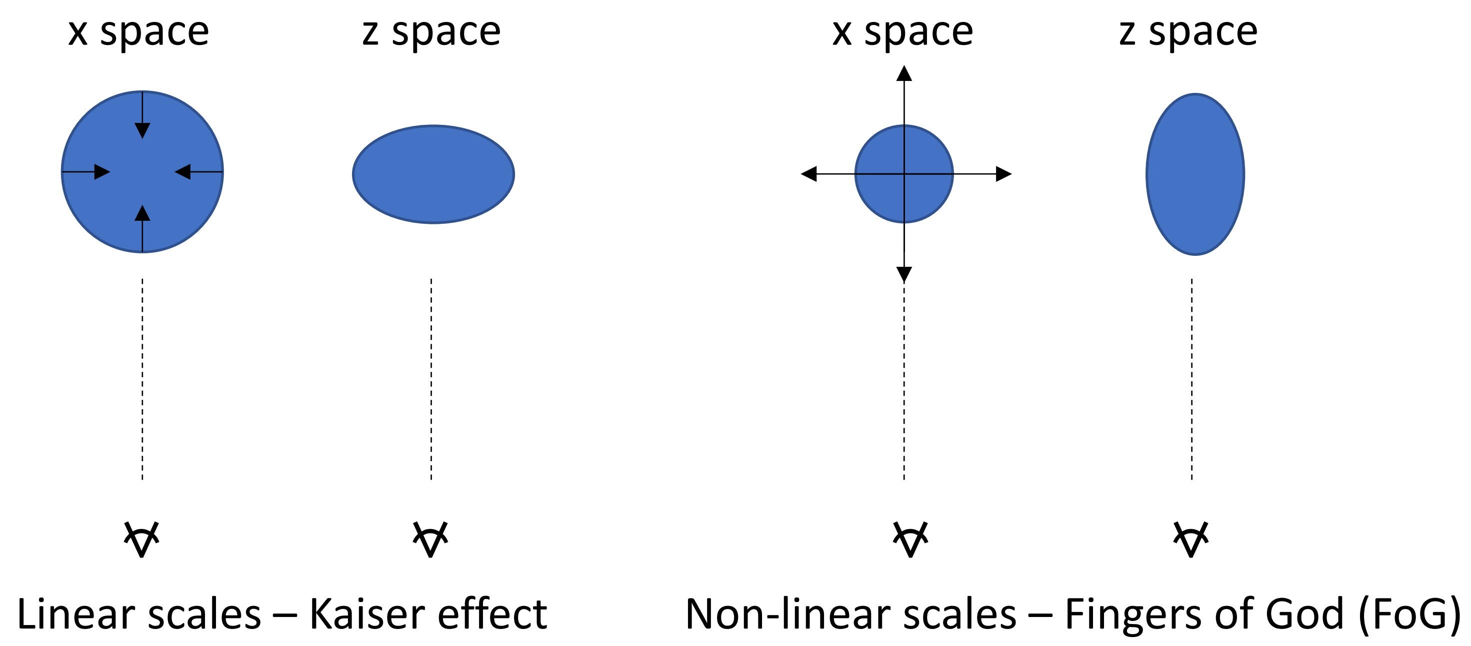
$$\delta_g = \delta_g[\delta] \approx b_1\delta + \frac{b_2}{2}\delta^2 + b_{\mathcal{G}_2}\mathcal{G}_2 + \epsilon + \cdots$$

This is very difficult to predict from first principles, depending on very nonlinear physics, selection, etc

Limits measurement of amplitude of fluctuations, given degeneracy with bias b_1.

Measuring structure - GC

Spectroscopic clustering observes galaxies in redshift space, not their real position



Non-linear effect is hard to model and typically requires adding new bias-like parameters via e.g. EFTofLSS

Measuring structure - GC

The observations then **depend on the velocity** along the line-of-sight in addition to the density!

So, they also probe the **growth rate, $\boldsymbol{f}$**:

$$f = \frac{d \log \delta}{d \log a}$$

We now get an anisotropic power spectrum, depending on $\mu = \hat{k} \cdot \vec{n}$. E.g. for linear scales.

$$P_{\text{Kais}}(k, \mu) = (b_1 + f\mu^2)^2 P_L(k)$$

So, spectroscopic galaxy clustering is most sensitive to

$$\boxed{\boldsymbol{f\sigma_8}}$$

Measuring structure - WL

Weak lensing measures the shear γ observed in galaxy shapes

It is an integrated measure of the density along the line of sight:

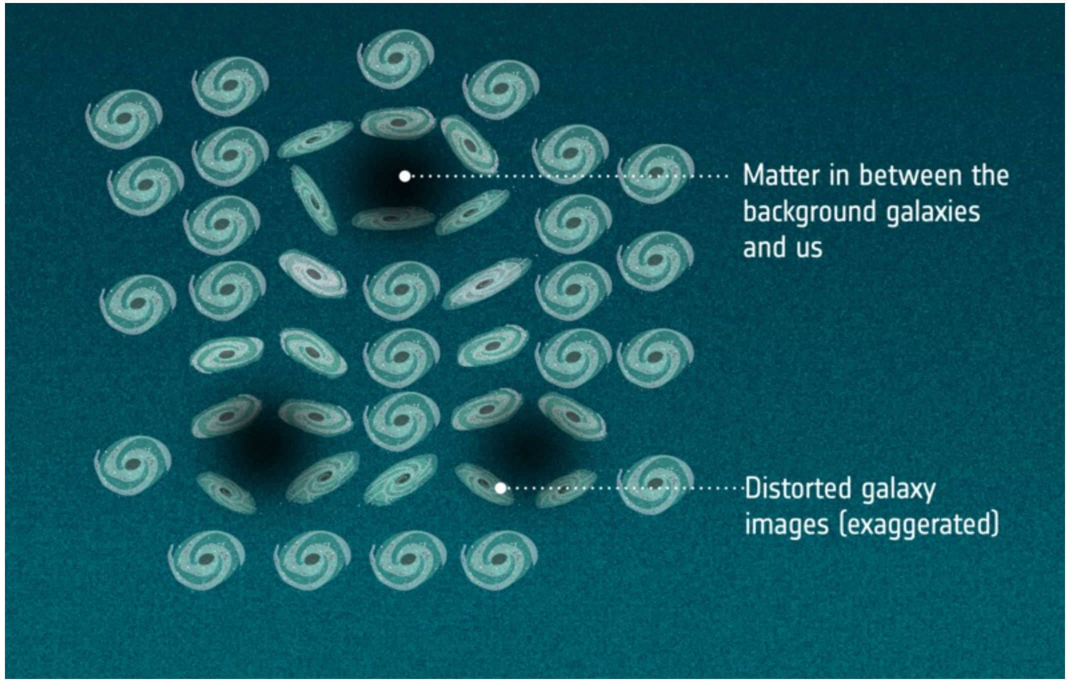
$$C_\ell^{\gamma\gamma} \approx \int_0^{z_{obs}} dz \frac{W^2(z)}{H(z) r_c^2(z)} P_{\delta\delta}\left[\frac{\ell + 1/2}{r(z)}, z\right]$$

with the kernel $W(z)$ being broad in z.

It is an unbiased tracer of the matter!

Probes directly the amplitude σ_8 and is most sensitive to

$$\boxed{\boldsymbol{S_8 = \sigma_8 \sqrt{\Omega_m / 0.3}}}$$



Measuring structure - Nonlinearities

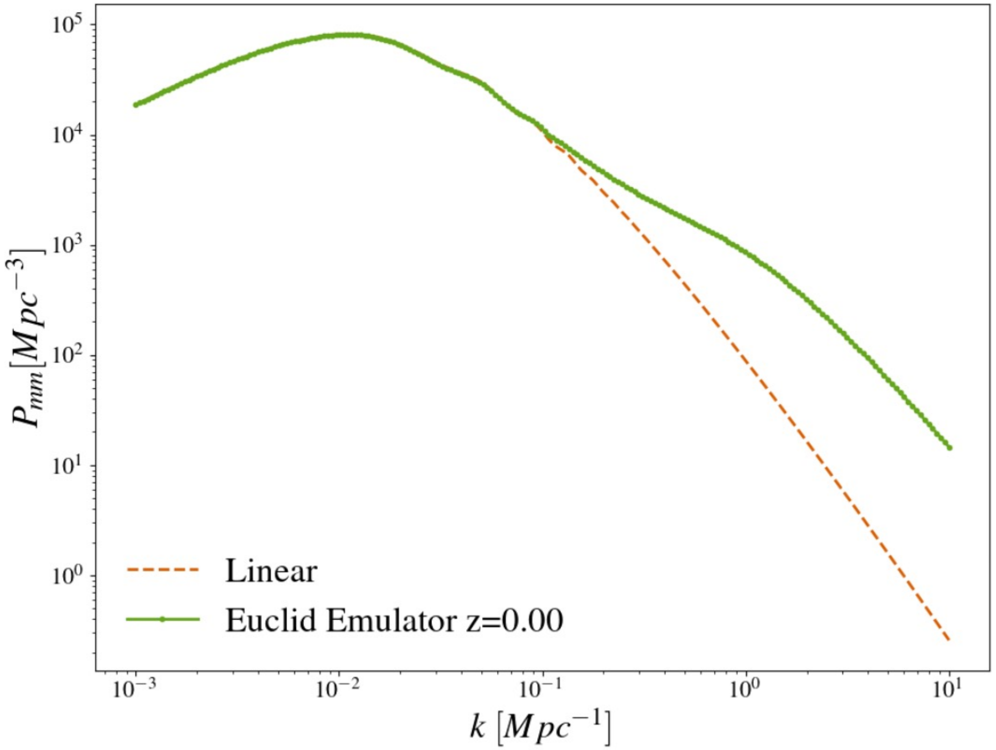
We need to go beyond the linear approximation to predict beyond $k \sim 0.1\ h/\mathrm{Mpc}$

In all cases, we need to predict the **nonlinear power spectrum**

- For dark matter, can be predicted from N-body, but slow
- Can use analytical techniques, but not perfect…

For weak lensing, also need to know baryonic feedback

- Requires huge hydro sims.
- May not represent reality.
- Currently parametrised baryonification models are used.



Euclid

1.2 m space telescope

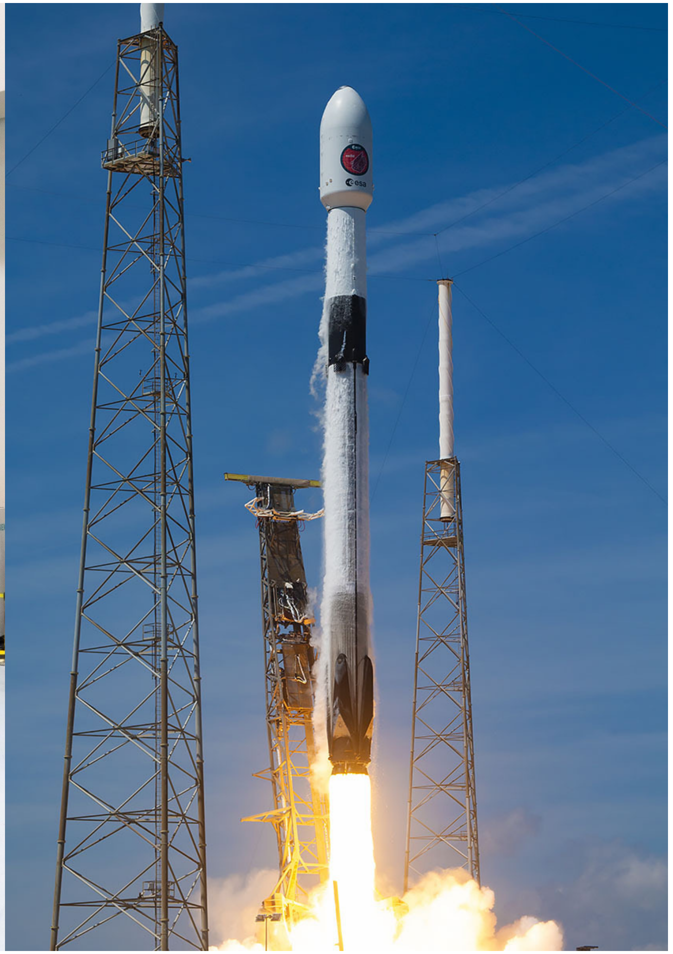
Launched on July 1st 2023

Now living at L2 with JWST and Gaia

First to do weak lensing survey in space!

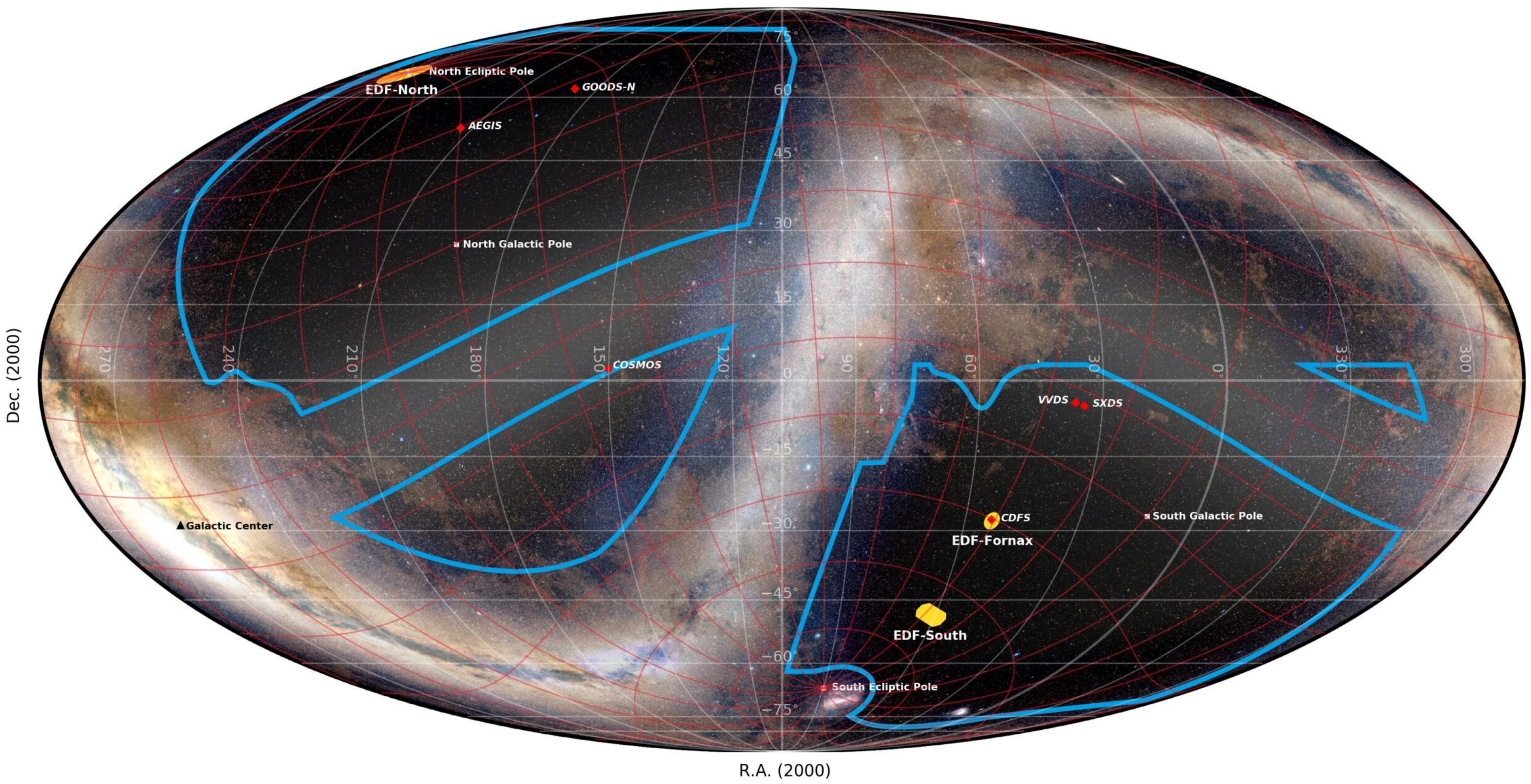
Will measure 1.5 billion shapes of galaxies

Will get 30 million spectroscopic redshifts



Euclid

Will cover at least 15000 sq. degrees of sky (≈1/3 of the sky)



The 15,000 deg.^{2} Euclid Wide Survey, the 53 deg.^{2} Euclid Deep Survey, and the 6 deep auxiliary fields (6.5 deg.^{2}) [Mollweide Celestial]

Euclid Wide Survey region of interest : 16 Kdeg.^{2} compliant with a 15 Kdeg.^{2} survey

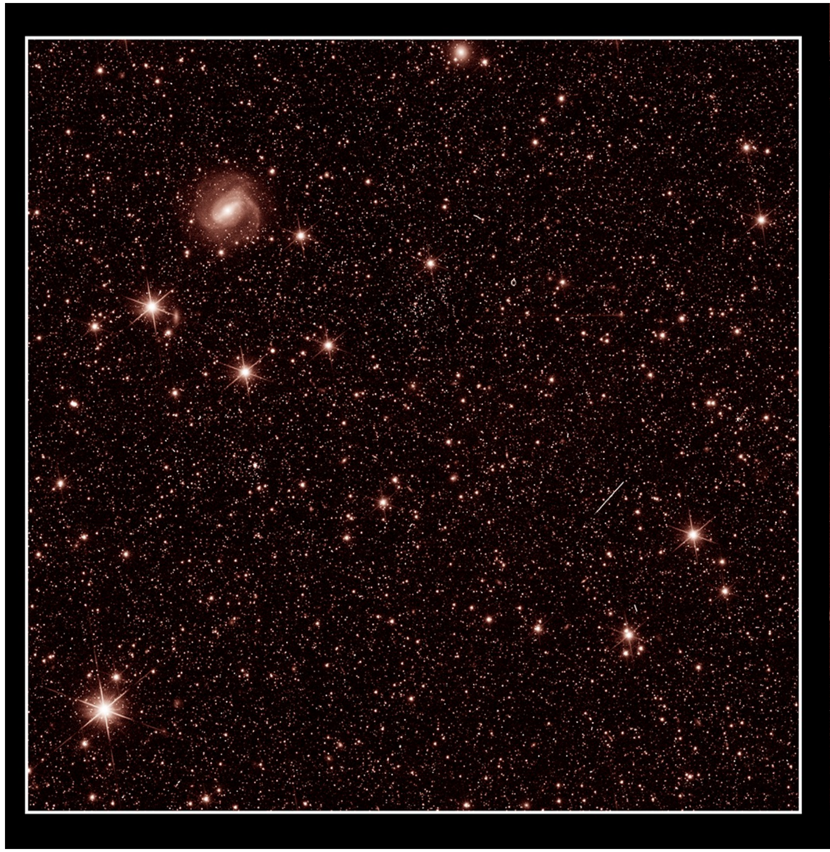
Euclid Deep Fields : North=20 deg.^{2}, Fornax=10 deg.^{2}, South=23 deg.^{2}

Euclid deep auxiliary fields (GOODSN=0.5, AEGIS=1, COSMOS=2, VVDS=0.5, SXDX=2, CDFS=0.5 deg.^{2})

Background image: Euclid Consortium / Planck Collaboration / A. Mellinger

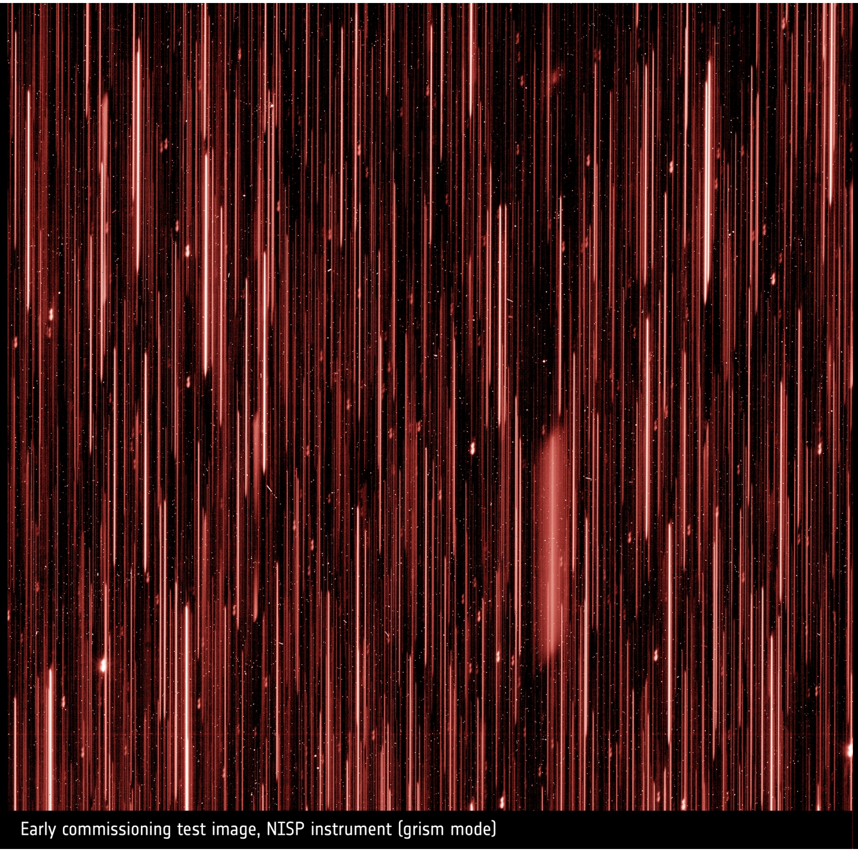
Euclid - 2 Instruments

NISP filters



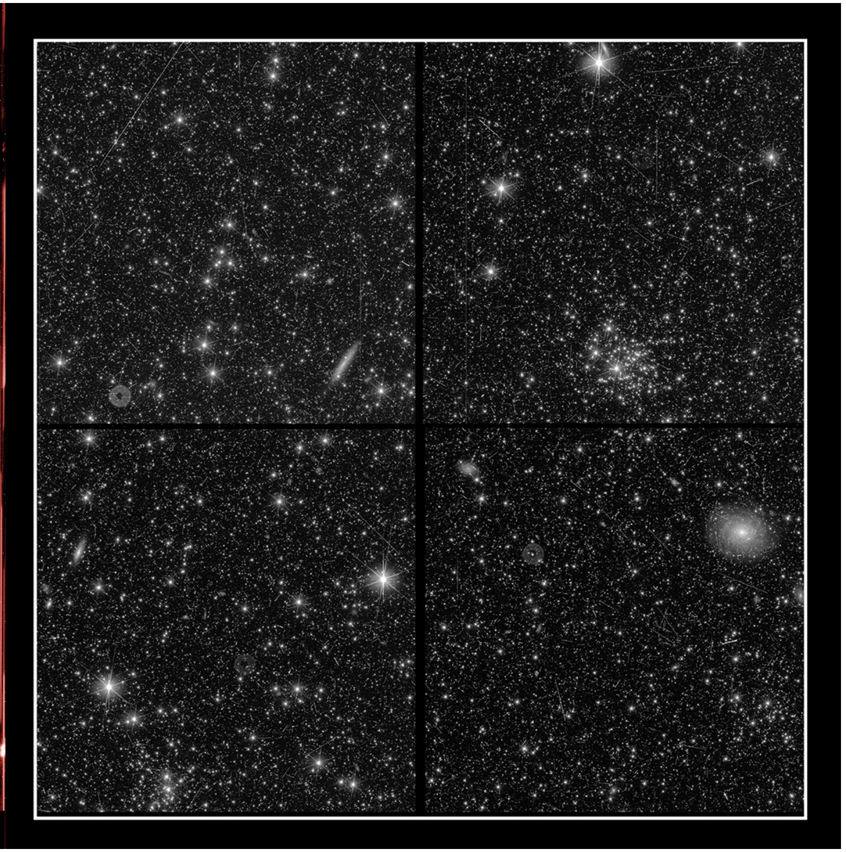
Photometric redshift

NISP grism



Spectroscopic redshift

VIS

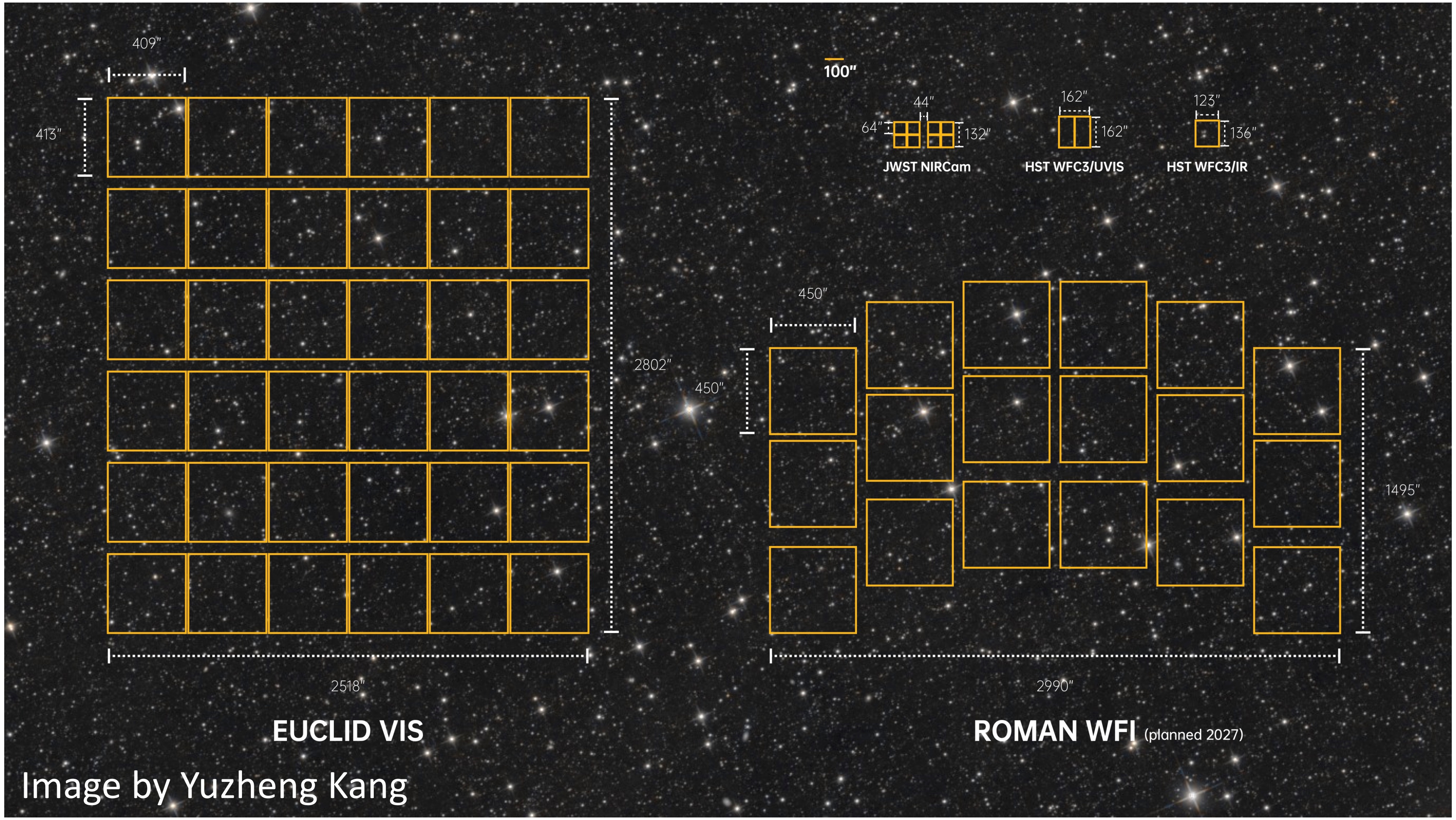


Shape

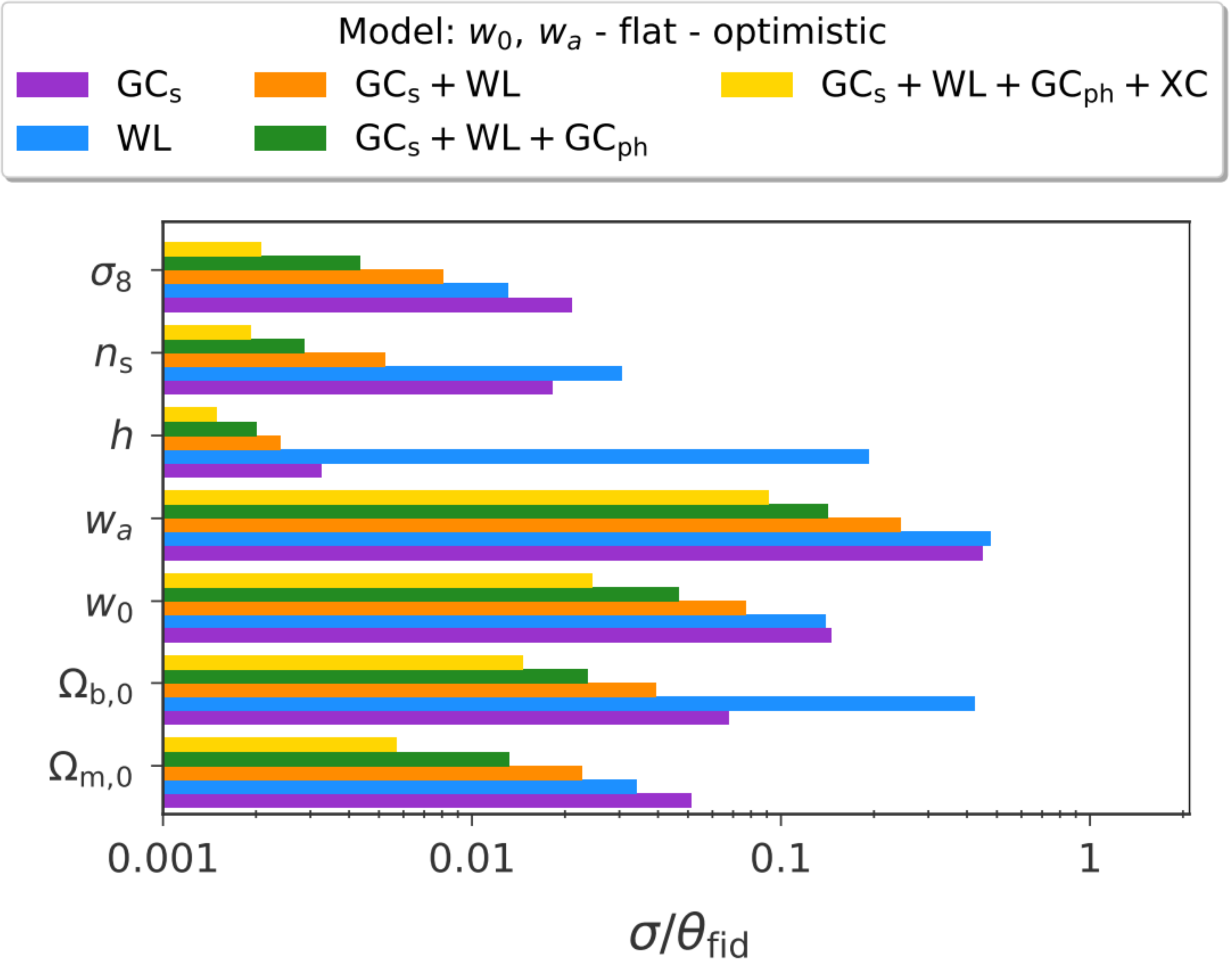
Euclid – Early release observations

Euclid – The power of Euclid

Huge field of view with Hubble-like quality

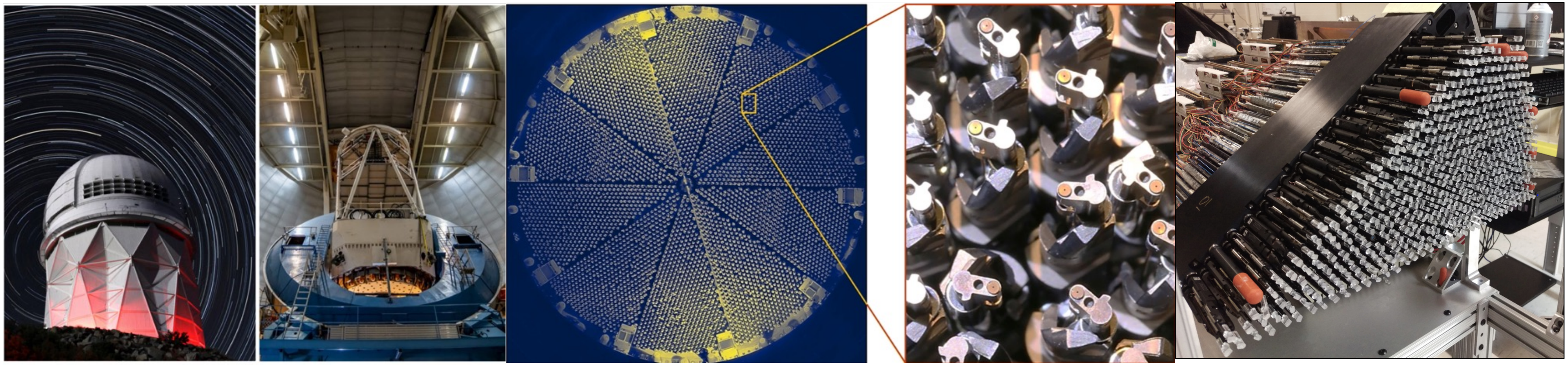


Euclid – forecasts



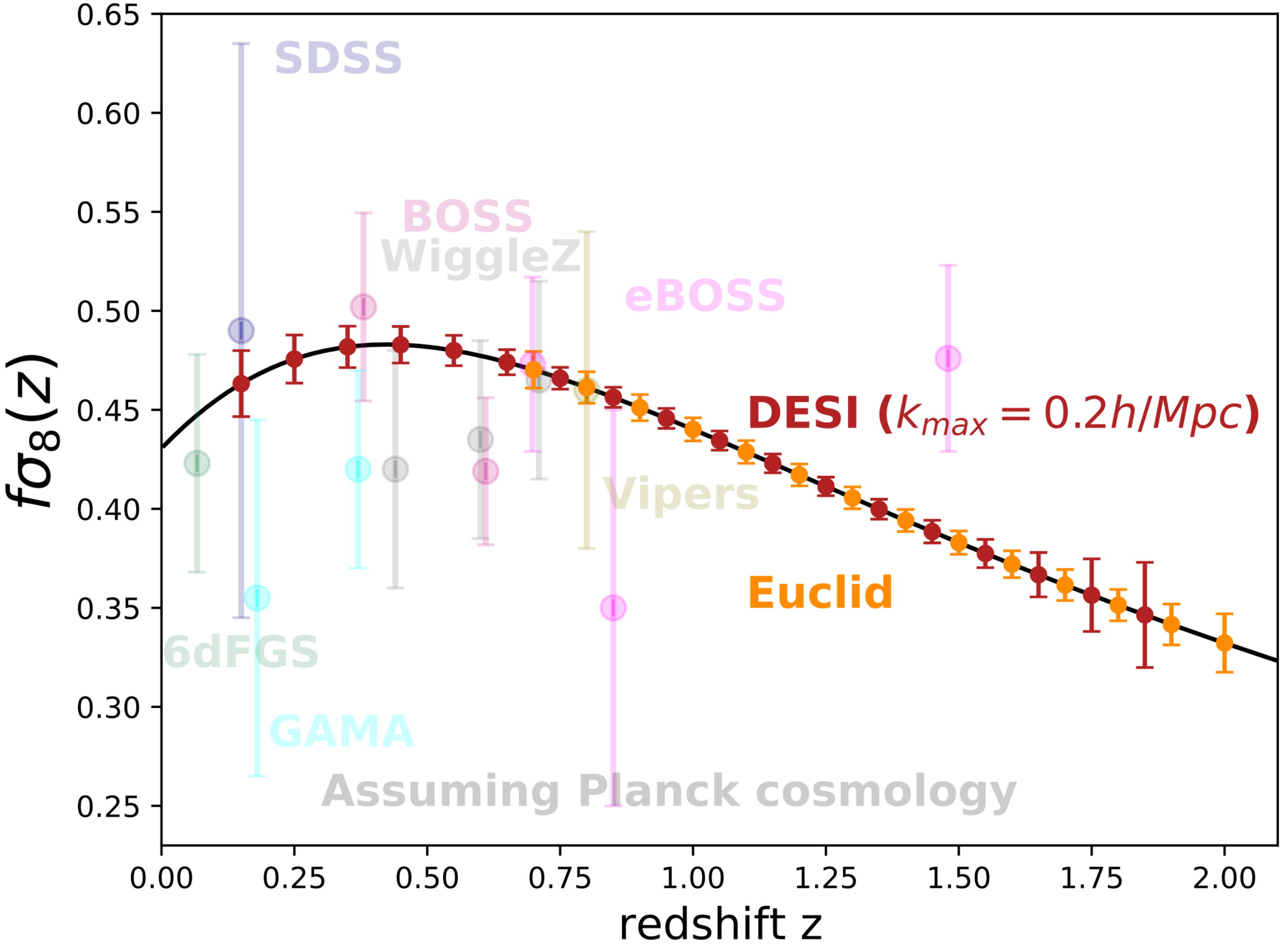
Dark Energy Spectroscopic Instrument (DESI)

- Installed at the Mayall 4m telescope at Kitt Peak, Arizona
- Measures up to 5000 redshifts per pointing over 14000 sq. deg.
- 13.6 million flux-limited sample of galaxies at $z < 0.4$ (BGS)
- 23.7 million color-selected galaxies at $0.4 < z < 1.5$ (LRGs & ELGs)
- 2.8 million Quasars at $z > 0.8$ Ly-α forest at $2 < z < 3.5$



4m Mayall at Kitt Peak, Arizona. Twin to the Blanco, CTIO

DESI – forecasts



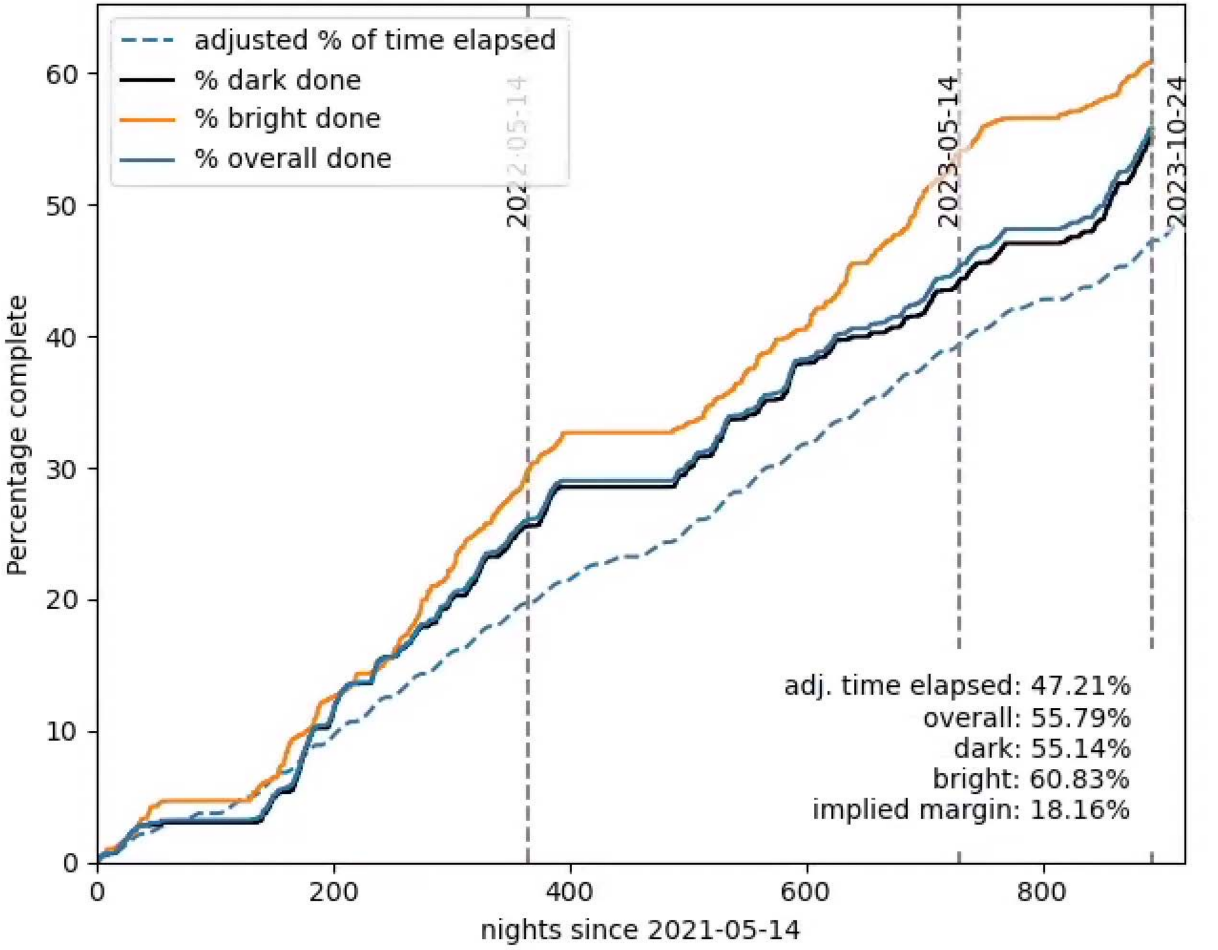
DESI – status

Observations are ahead of time, despite covid and fire.

- BGS (low z) ahead by almost 1 year
- Overall, ahead by around 4 months.

Unblinding of the Year 1 data will happen soon

First papers ~ March/April. **Stay tuned!**



Evolution of structure in ΛCDM and beyond

Perturbations in Cosmology

The theory of cosmological perturbations

The perturbed flat FLRW metric (in Newtonian gauge) is given by

$$ds^2 = a^2(\eta)\left(-\left(1 + 2\Phi(x,\eta)\right)d\eta^2 + \left(1 - 2\Psi(x,\eta)\right)\, d\vec{x}^2\right)$$

We neglect vector (e.g. vorticity) and tensor modes (gravitational waves).

Non-relativistic dynamics like GC care about Φ

Relativistic dynamics like WL are influenced by $\Phi + \Psi$

Perturbations in Cosmology

The stress-energy tensor gets an additional contribution

$$T^{\mu\nu} = (\rho + P)u^{\mu}u^{\nu} + Pg^{\mu\nu} + \pi^{\mu\nu}$$

Anisotropic stress: $\pi_{ij} = a^2 \left(\partial_i \partial_j - \frac{1}{3}\delta_{ij}\nabla^2\right)\Pi$

The remaining quantities are also perturbed

$$\rho = \bar{\rho} + \delta\rho, \qquad P = \bar{P} + \delta P$$

$$u^{\mu} = a^{-1}\left(1 - \phi, \partial^i v\right)$$

Perturbations in Cosmology

Perturbing the Einstein equation to linear order in fluctuations, one gets

00 Eq.: $\nabla^2\Psi - 3\mathcal{H}\Psi' - 3\mathcal{H}^2\Phi = \frac{3}{2}\mathcal{H}^2\delta$

i0 Eq.: $\Psi' + 2\mathcal{H}\Phi - 2(\mathcal{H}^2 - \mathcal{H}')v = 0$

ij Eqs.: $\Psi'' + 2\mathcal{H}\Psi' - \mathcal{H}\Phi' + (\mathcal{H}^2 + 2\mathcal{H}')\Phi = 8\pi G\, a^2\left(\frac{1}{2}\delta P + \frac{1}{3}\nabla^2\Pi\right)$

$\Psi - \Phi = 8\pi G\, a^2\Pi$

Not all are always needed (can be replaced by conservation of stress-energy)

Perturbations in Cosmology

Perturbing the Einstein equation to linear order in fluctuations, one gets

0 Eq.: $$\delta' - (1+w)(3\Psi' - \nabla^2 v) - 3\mathcal{H}(w\rho\delta - \delta P) = 0$$ Continuity equation

i Eq.: $$v' + (1 - 3c_s^2)\mathcal{H}v + \Phi + \frac{1}{\rho(1+w)}\left(\delta P + \frac{2}{3}\nabla^2\Pi\right) = 0$$ Euler equation

We introduced the (adiabatic) sound speed $c_s^2 = P'/\rho'$.

To solve for the entire history of the Universe need a set of these per species:

- Baryons, dark matter, photons, neutrinos in ΛCDM
  - For relativistic species, need instead the Boltzmann equation for a full treatment
- Dark energy, scalar fields, etc for beyond ΛCDM.

Evolution of Perturbations

Let's start from the beginning: **primordial fluctuations**

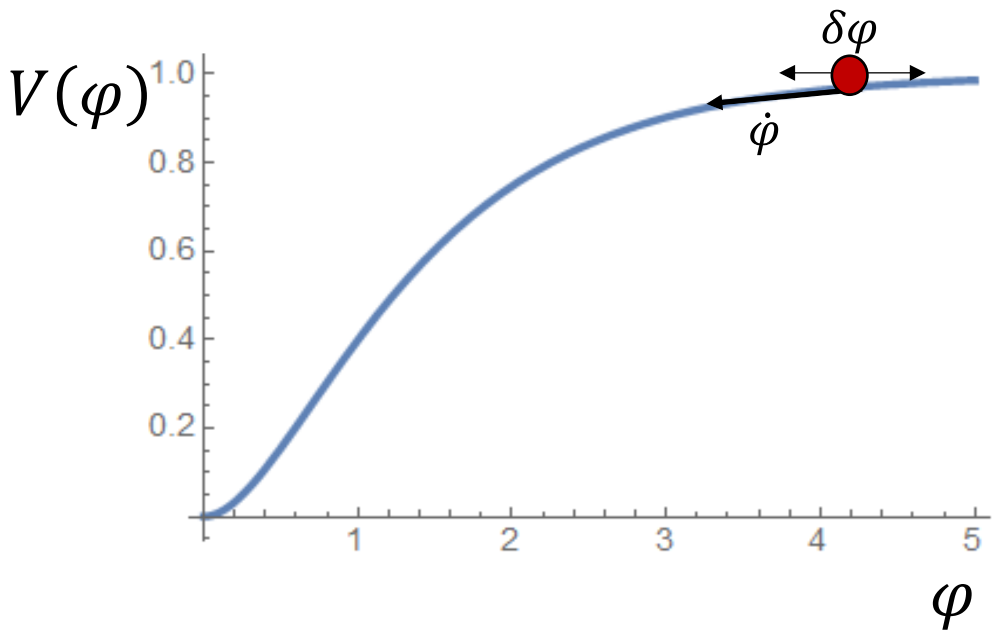
- During inflation, the accelerated expansion enhances quantum fluctuations

- The canonical quantum fluctuations in the field φ are $v = a\delta\varphi$ and obey:

$$v'' + \left(k^2 - \frac{z''}{z}\right)v = 0$$

with $z = a\varphi'/\mathcal{H}$ and $z'' \sim \mathcal{H}^2$ so two regimes:

  - Inside the horizon $k > \mathcal{H}$: small oscillations
  - Outside the horizon $k < \mathcal{H}$: **huge enhancement**



Evolution of Perturbations

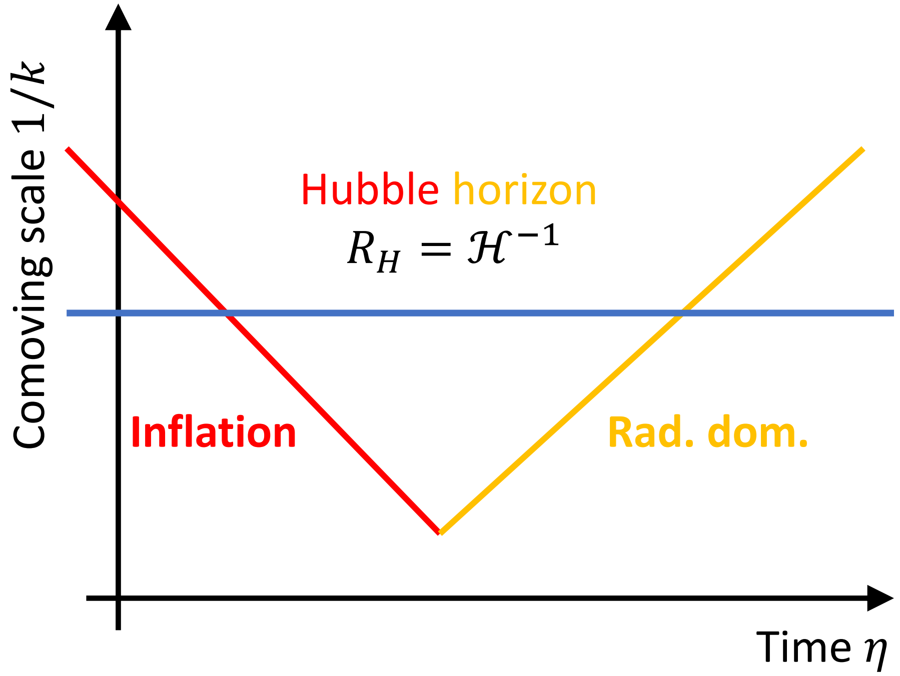
Let's start from the beginning: **primordial fluctuations**

- During inflation, the accelerated expansion enhances quantum fluctuations
- The canonical quantum fluctuations in the field φ are $v = a\delta\varphi$ and obey:

$$v'' + \left(k^2 - \frac{z''}{z}\right) v = 0$$

with $z = a\varphi'/\mathcal{H}$ and $z'' \sim \mathcal{H}^2$ so two regimes:

  - Inside the horizon $k > \mathcal{H}$: small oscillations
  - Outside the horizon $k < \mathcal{H}$: **huge enhancement**



Quantity of interest is $\zeta = v/z$ and it is **conserved outside the horizon**.

Evolution of Perturbations

In more detail, inflation predicts the power spectrum of ζ

$$P_\zeta = A_s\, k^{-3}\left(\frac{k}{k^*}\right)^{n_s-1}$$

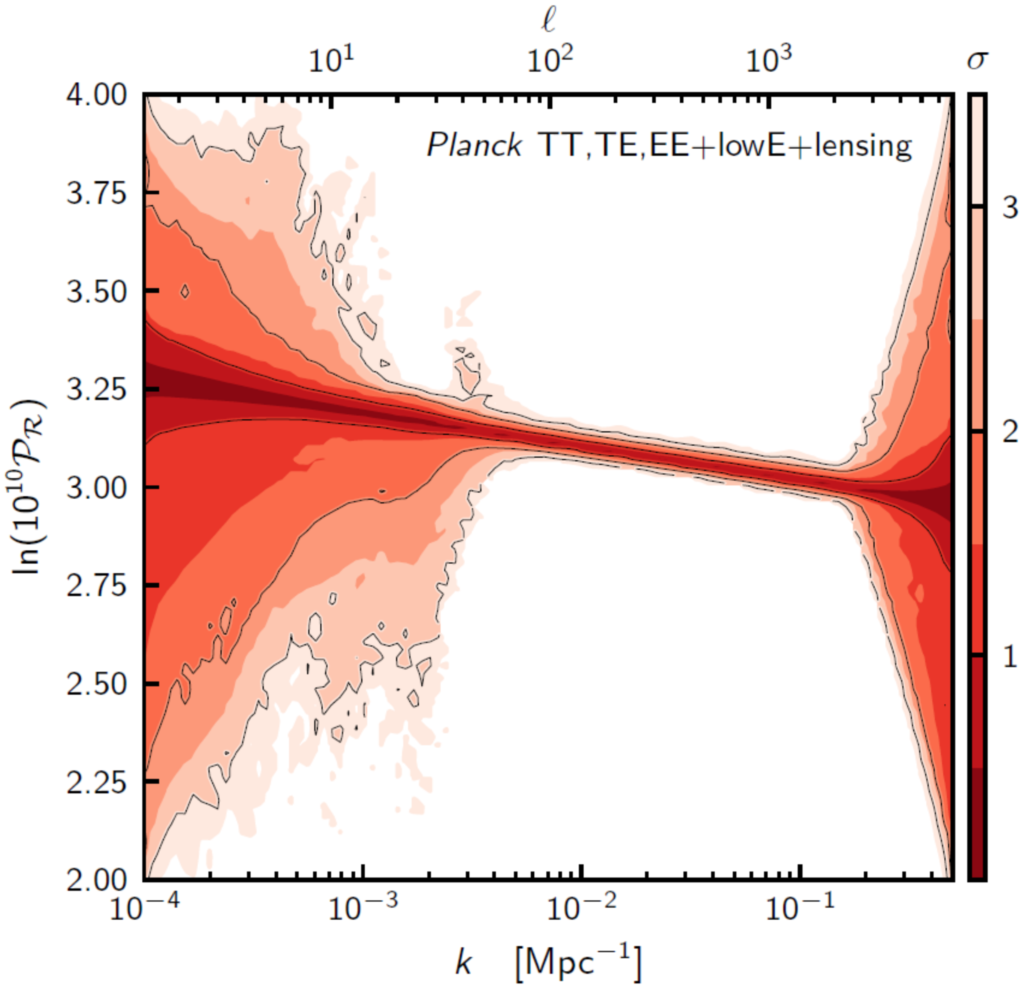
$$A_s = \frac{H_{inf}^2}{4\epsilon M_{pl}^2} \qquad \epsilon \sim \left(\frac{d_\phi V}{V}\right)^2$$

$$n_s = -6\epsilon + 2\eta \qquad \eta \sim \frac{d_\phi^2 V}{V}$$

And the CMB measures

$$\log(10^{10} A_s) = 3.044 \pm 0.014$$

$$n_s = 0.9649 \pm 0.0042$$



Evolution of Perturbations

Right after inflation, most fluctuations we observe now are outside the horizon

$$\delta \sim \zeta = \text{const.}$$

$$P_{\delta,ini}(k) \propto k^{-3} k^{n_s - 1}$$

These are our **initial conditions**

When they re-enter the Hubble horizon, they evolve according to the **sub-horizon** ($k \gg \mathcal{H}$) equations:

$$\nabla^2 \Psi = \frac{3}{2} \mathcal{H}^2 \delta \approx 4\pi G a^2 \rho_m \delta_m$$

$$\Psi = \Phi$$

$$\delta_m' + \theta_m = 0$$

$$\theta_m' + \mathcal{H} \theta_m + \nabla^2 \Phi = 0$$

$$\theta_m = \nabla^2 v$$

Assuming also matter to be all dark and radiation fluctuations to decay very fast.

Evolution of Perturbations

This results in a single equation for the matter fluctuations during radiation and matter domination

$$\delta_m'' + \mathcal{H}\delta_m' = 4\pi G a^2 \rho_m \delta_m$$

with

$$\mathcal{H}^2 = H_0^2(\Omega_{m0}a^{-1} + \Omega_{r0}a^{-2})$$

Converting the time variable to $y = \rho_m/\rho_r = a/a_{eq}$ we get the Mészáros equation

$$\frac{d^2\delta_m}{dy^2} + \frac{2+3y}{2y(1+y)}\frac{d\delta_m}{dy} - \frac{3}{2y(1-y)}\delta_m = 0$$

with solutions

Rad. dom ($a \ll a_{eq}$): $\delta_m = C_1 + C_2\left(3 + \log\left(\frac{y}{4}\right)\right)$,

Matter dom. ($a \gg a_{eq}$): $\delta_m = C_1(2 + 3y)$

Evolution of Perturbations

But fluctuations at scale k only unfreeze after re-entering the horizon when

$$\frac{k}{\mathcal{H}} \sim k\,\eta_c \sim 1$$

Small scales enter the horizon earlier and start evolving sooner.

Time-dependence is converted to scale-dependence:

Rad. dom ($a \propto \eta$):

$$\delta_m(\eta > \eta_c) \sim C + \log \eta \Rightarrow \delta_m(k) \sim \delta_m^{ini}(k)(C + \log k)$$

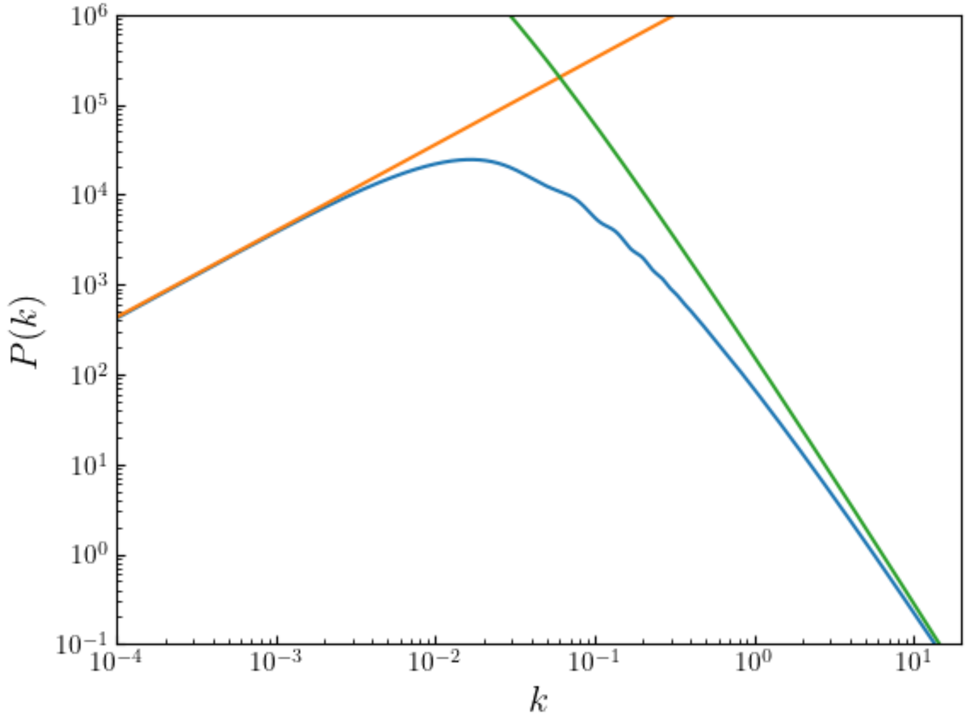
Mat. dom ($a \propto \eta^2$):

$$\delta_m(\eta > \eta_c) \sim \eta^2 \Rightarrow \delta_m(k) \sim \delta_m^{ini}(k)k^2$$

Evolution of Perturbations

Finally, we get the power spectrum

$$P(k) = P_{\delta,ini}(k) \times \begin{cases} (C + \log k)^2 & k > k_{eq} \\ k^4 & k < k_{eq} \end{cases}$$



To do the full calculation requires a Boltzmann solver such as CAMB or CLASS.

Evolution of Perturbations

To get the full growth rate also accounting for full ΛCDM, solve the sub-horizon equations

$$\delta_m' + \theta_m = 0$$
$$\theta_m' + \mathcal{H}\theta_m + \nabla^2\Phi = 0$$
$$\nabla^2\Phi = 4\pi G a^2 \rho_m \delta_m$$

with the correct $\mathcal{H}(\eta)$ including dark energy.

Solving this results in a simple approximation for the growth rate

$$f = \Omega_M(a)^{6/11}$$

Which is valid for alternative expansion histories such as that in $w_0 w_a$CDM!

Measuring $f(a)$ deviating from this implies either modified gravity or more exotic dark energy!

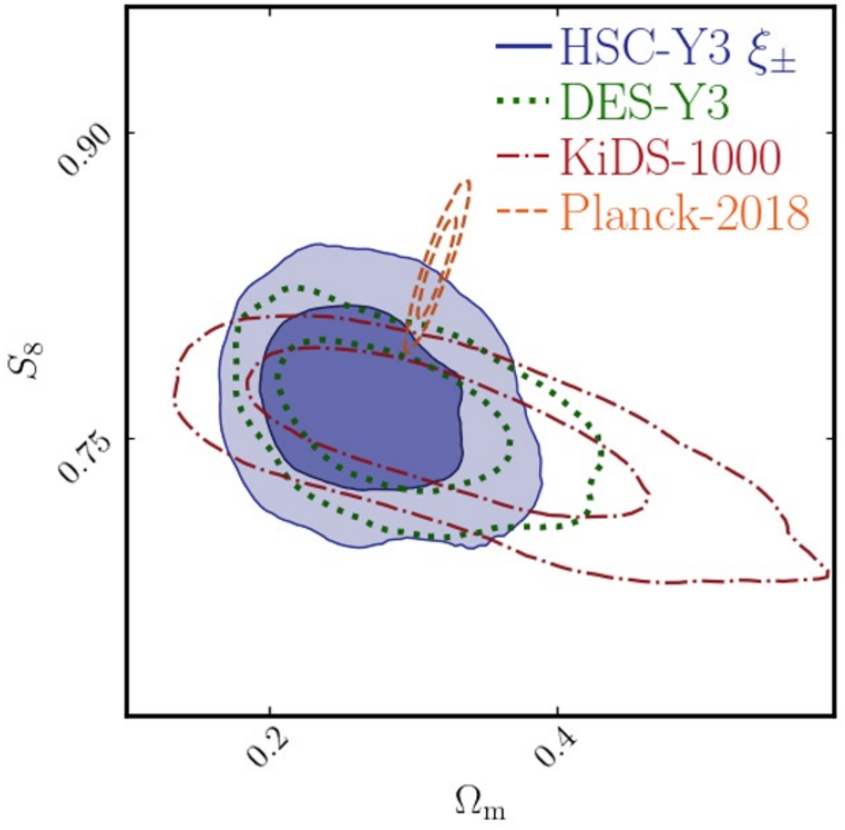
The S_8 tension

What new physics can solve the S_8 tension?

- Anything that suppresses lensing:
  - Modified gravity with Σ:

$$\nabla^2(\Phi + \Psi) = 4\pi G\, \Sigma(a)\, a^2 \rho_m \delta_m$$

- Anything that suppresses growth:
  - Dark friction $\Xi(a)$
  - Weaker gravity at late time $G \to G\, \mu(a)$



General parametrisation beyond ΛCDM

Let us parametrise the sub-Horizon equations for generic deviations

$$a\partial_a\delta_m + \Theta_m = 0$$

$$a\partial_a\Theta_m + \left(2 + \frac{a\partial_a H}{H} + \frac{\Xi(a)}{H}\right)\Theta_m - \left(\frac{k}{aH}\right)^2 \Phi = 0$$

$$\left(\frac{k}{aH}\right)^2 \Phi = \frac{3}{2}\Omega_m \mu(k,a)\delta_m$$

With also a general equation of state $w(a)$.

This system encompasses many modified gravity and dark energy models.

Modified gravity - $f(R)$

$f(R)$ gravity is a general modified gravity with action

$$S = \int d^4x\sqrt{-g}(R + f(R))$$

A very popular model is the Hu-Sawiki model

Model parameter

$$f(R) \propto \frac{R}{AR+1} \approx -16\pi G\rho_\Lambda - f_{R0}\frac{R_0^2}{R}$$

Can be fixed to get ΛCDM expansion

Its perturbations result in a scale-dependent growth of structure through:

$$\mu(k,a) = 1 + \left(\frac{k}{a}\right)^2 \frac{1}{3\Pi(k,a)}$$

$$\Pi(k,a) = \left(\frac{k}{a}\right)^2 + \frac{H_0^2(\Omega_{M0} - 4\Omega_\Lambda a^3)^3}{2|f_{R0}|a^9(3\Omega_{M0} - 4)^2}$$

Interacting dark energy

Dark energy and dark matter could interact with each other

$$\nabla_\mu T^{\mu\nu}_{\mathrm{DM}} = -Q^\nu, \qquad \nabla_\mu T^{\mu\nu}_{\mathrm{DE}} = Q^\nu$$

Dark scattering is a model which exchanges only momentum via elastic scattering.

$$\boldsymbol{Q^i = -(1 + w)\, \sigma_D\, \rho_{DE}\left(v^i_{DM} - v^i_{DE}\right)}$$

This introduces a dark friction term given by

$$\Xi(a) = \left(1 + w(a)\right) \frac{\sigma_D}{m_{DM}} \rho_{DE}(a)$$

Results in a scale-independent modification of growth that can **resolve the $\boldsymbol{S_8}$ tension**!