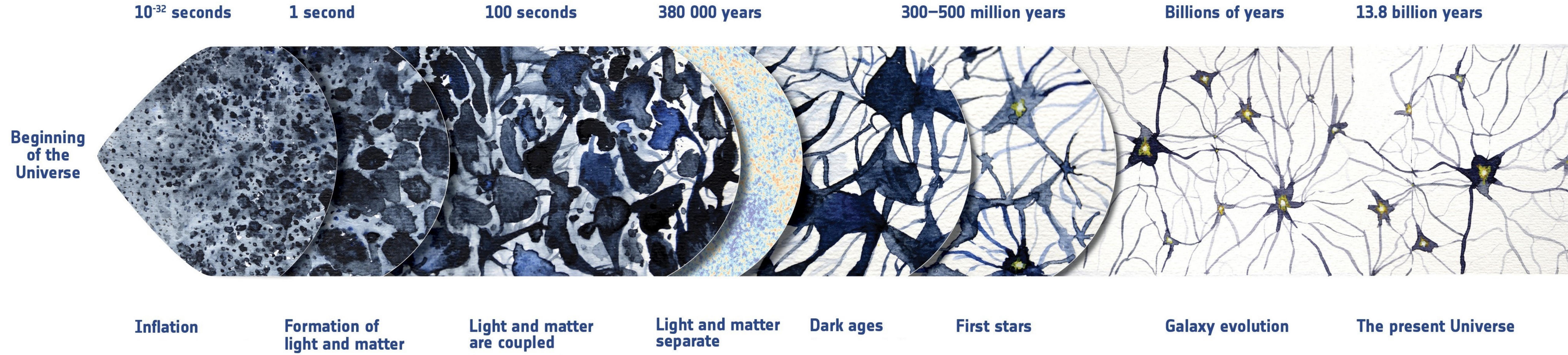
Cosmology

Lecture 1

Pedro Carrilho

The history of the Universe

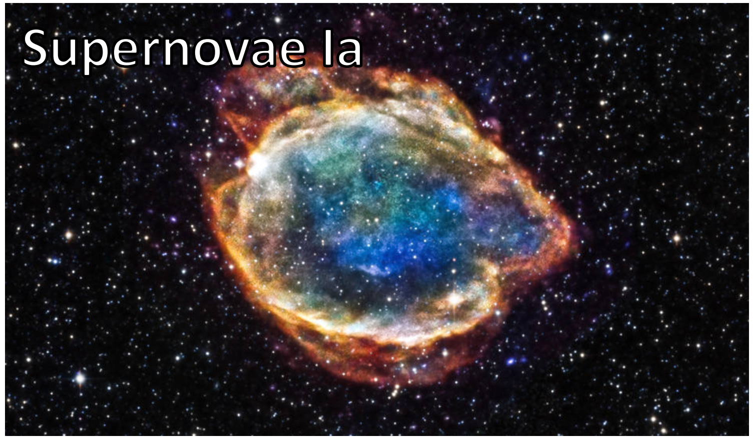


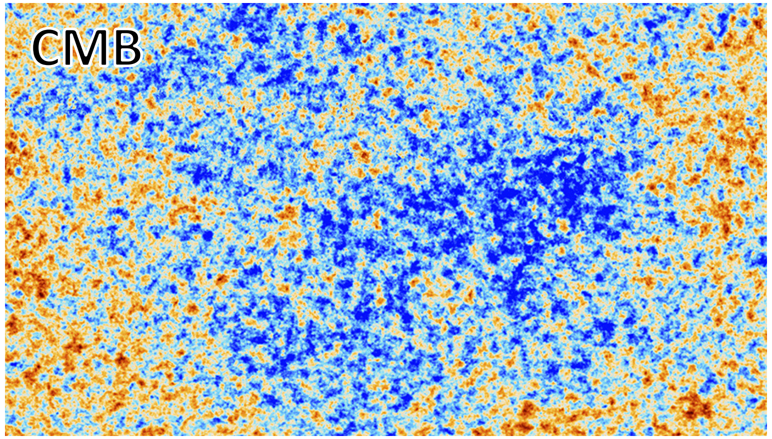
Early	Middle-aged	Late

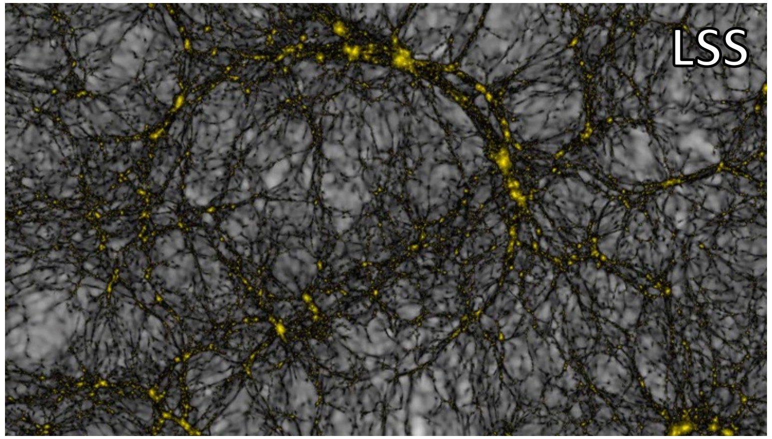
Produce initial fluctuations

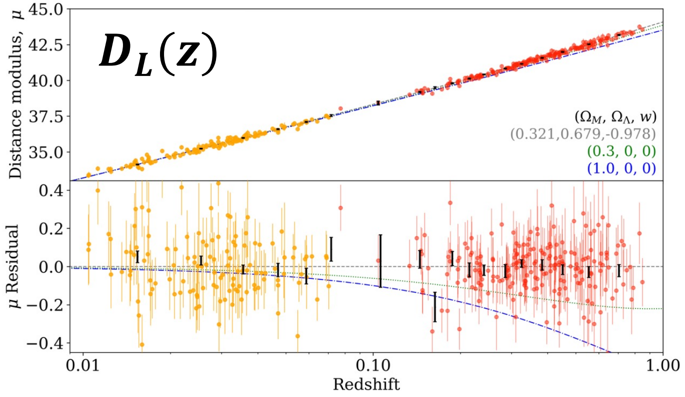
$\delta\rho$ — Processing via acoustic oscillations, etc / Linear evolution → δT_{CMB} — Gravitational collapse, etc / **Non-linear evolution** → δ_{Galaxy}

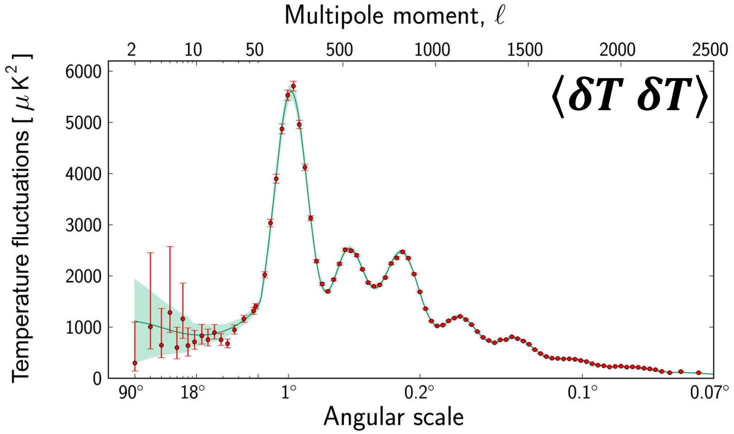
Cosmological probes

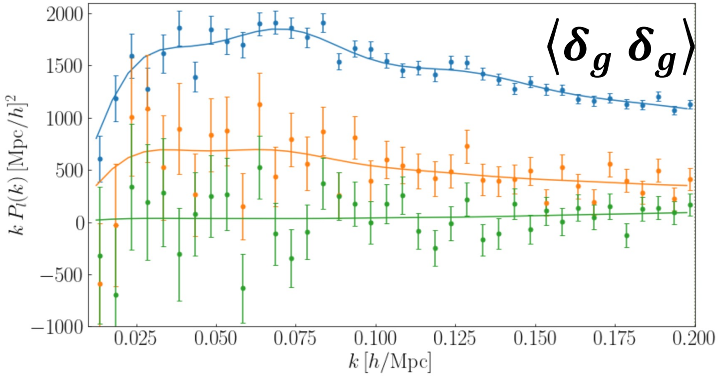












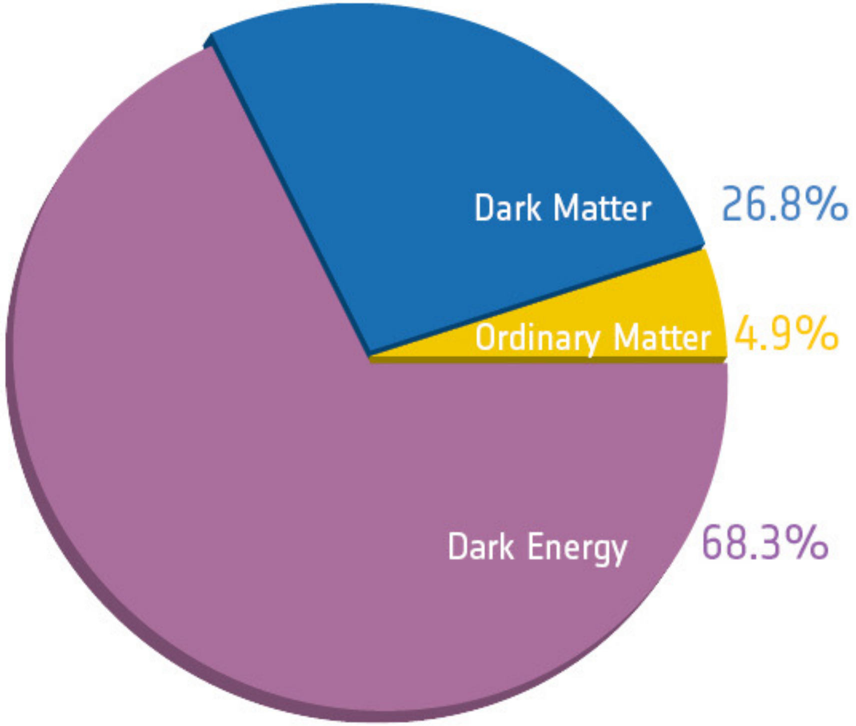
Does a single model fit all observations?

The ΛCDM model

- With 6 parameters you can (almost) describe everything!

$\Omega_b h^2$	0.02237 ± 0.00015
$\Omega_c h^2$	0.1200 ± 0.0012
$100\theta_{MC}$	1.04092 ± 0.00031
τ	0.0544 ± 0.0073
$\ln(10^{10} A_s)$	3.044 ± 0.014
n_s	0.9649 ± 0.0042

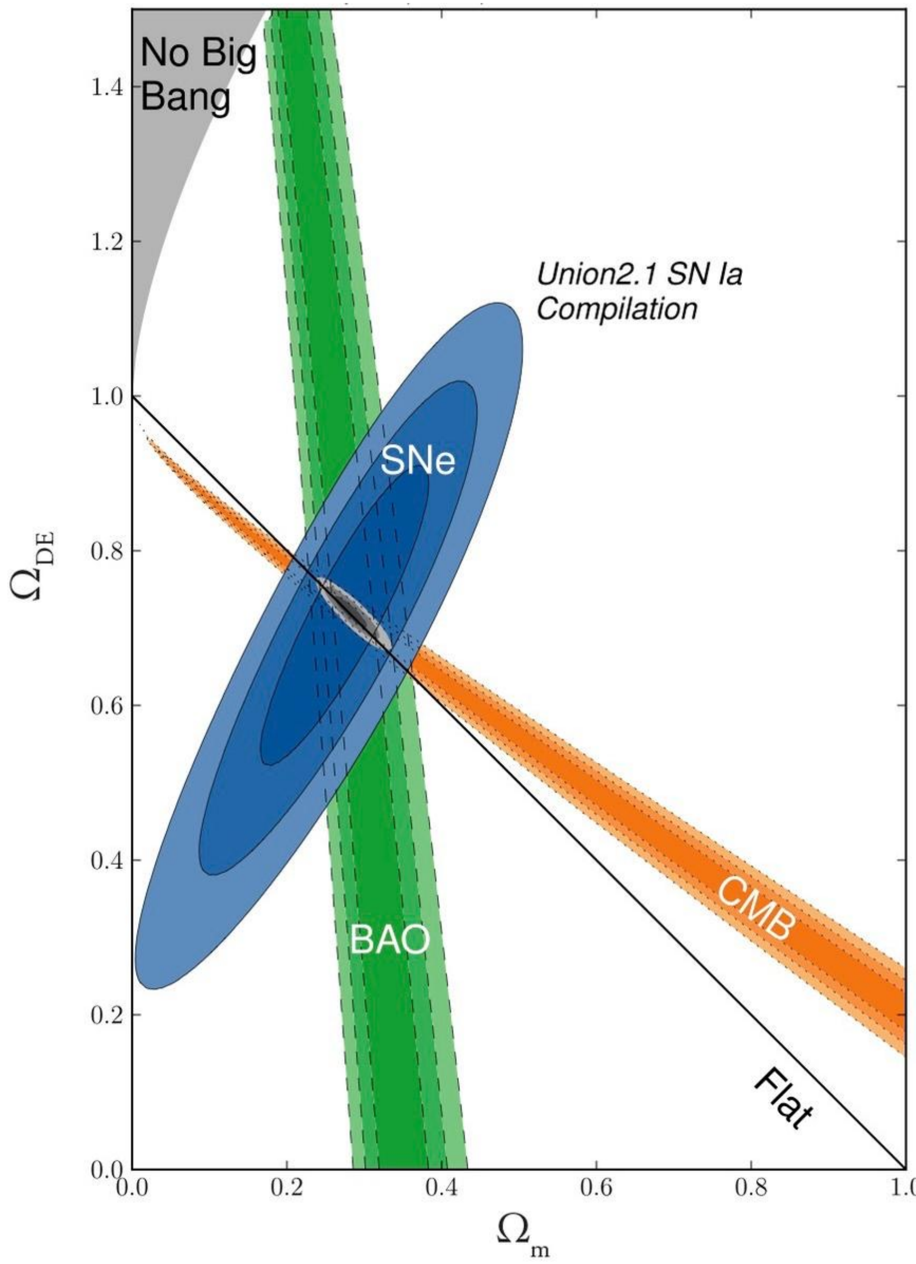
theory



- But you need a lot of dark stuff!

The ΛCDM model

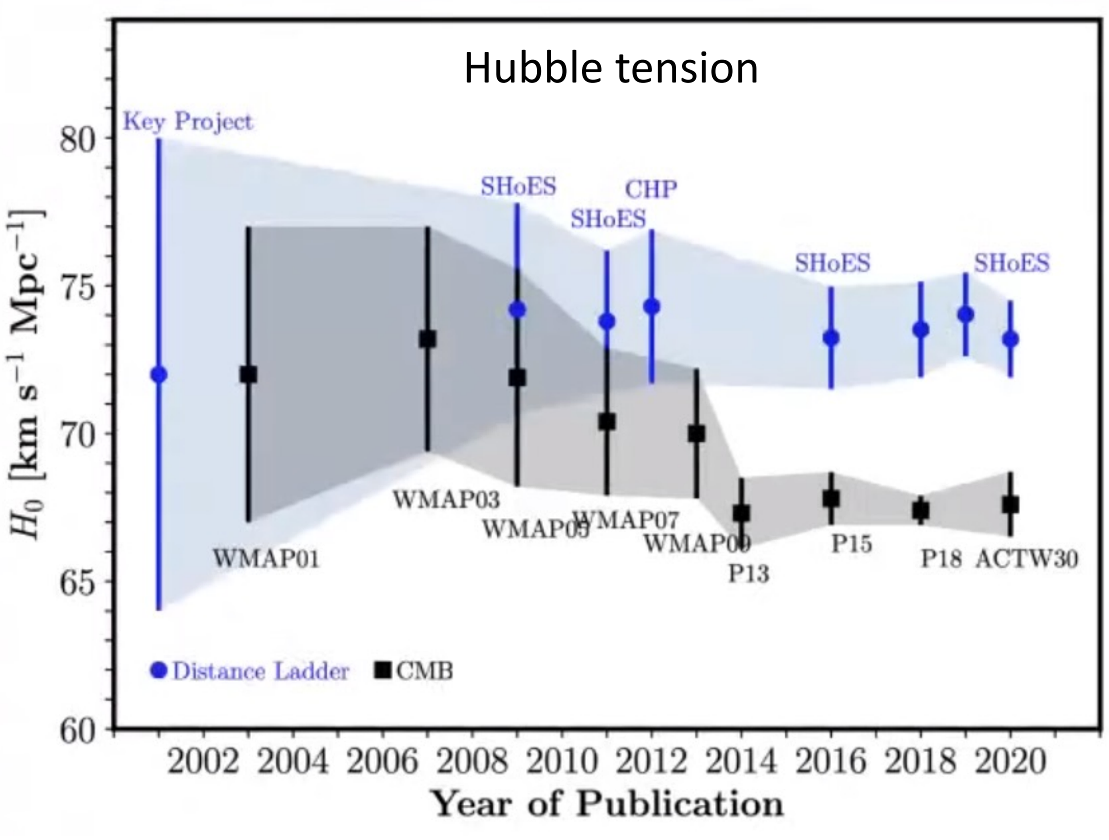
- Ingredients
  - GR: $$G_{\mu\nu} = 8\pi G\, T_{\mu\nu}$$
  - SM: Baryons + Photons + Neutrinos
  - **Cold Dark Matter:** non-interacting cold component
  - **Cosmological constant Λ**: to explain accelerated expansion
  - **Inflation**: to explain initial conditions

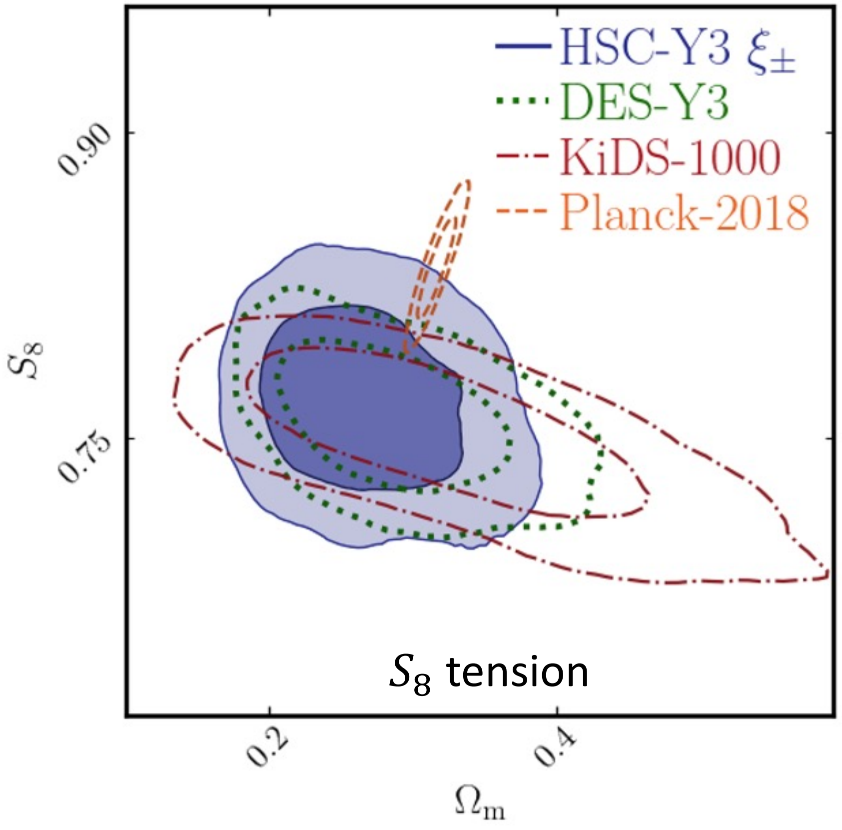


This ΛCDM model has been quite successful, but cracks are appearing!

The troubles of the Universe

Tensions have been found between probes of the early and late Universe in (at least) two cases:





Could these be hints of the nature of the dark sector?

Outstanding questions

What is the nature of dark matter?

What is the explanation for dark energy?

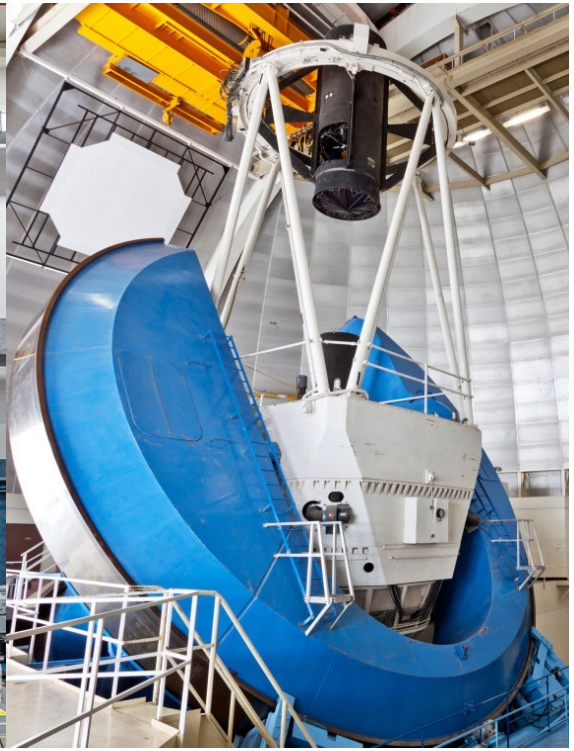
What is the microphysics of inflation?

Is GR the correct theory of gravity on cosmological scales?

The future is bright with new LSS surveys

Euclid

DESI



Their exquisite precision could confirm/resolve **tensions**, **distinguish models** that explain them, **test gravity**, inform **inflationary physics**, and much more…

The (tentative) Menu

Day 1:

- Expansion of the Universe
- History of expansion in ΛCDM
- Going beyond ΛCDM

Day 2:

- Structure in the Universe
- Growth beyond ΛCDM
- LSS surveys: Euclid and DESI

Expansion of the Universe

FLRW spacetime

On sufficiently large scales, we will assume the Universe to be homogeneous and isotropic

This implies that the metric is the **Friedmann-Lemaitre-Robertson-Walker metric**

$$ds^2 = -dt^2 + a^2(t)\left(\frac{dr^2}{1-\kappa r^2} + r^2 d\Omega^2\right)$$

$a(t)$: scale factor, κ: curvature, $a(t = \mathrm{now}) = 1$

$H \equiv \frac{\dot{a}}{a}$: Hubble rate, $H_0 = H(t = \mathrm{now})$: Hubble constant

Can also be written in alternative coordinates

$$ds^2 = a^2(\eta)(-d\eta^2 + d\chi^2 + \chi^2 \mathrm{sinc}^2(\chi\sqrt{\kappa})d\Omega^2)$$

η: conformal time, $\mathcal{H} \equiv \frac{a'}{a}$: Hubble rate

FLRW spacetime

On sufficiently large scales, we will assume the Universe to be homogeneous and isotropic

For the matter stress-energy tensor, this implies it is a perfect fluid

$$T^{\mu\nu} = (\rho + P)u^\mu u^\nu + P g^{\mu\nu}$$

ρ: energy density, P: pressure, u^μ: 4-velocity

Which in FLRW is simply:

$$\left[T^\mu{}_\nu\right] = \mathrm{diag}(-\rho, P, P, P)$$

Cosmic redshift

As the Universe expands, the energy of particles is not conserved.

Let's look at the geodesic equation for a photon

$$p^{\mu}\nabla_{\mu}p^{\nu} = 0$$

In FLRW, the energy measured by a comoving observer is $E = -p^{\mu}u_{\nu}$ and evolves as

$$\frac{dE}{dt} = -H\,E \Rightarrow E \propto a^{-1}$$

So, the redshift of a photon is

$$z \equiv \frac{\nu_e - \nu_o}{\nu_o} = \frac{1}{a_e}$$

Distances

Comoving distance

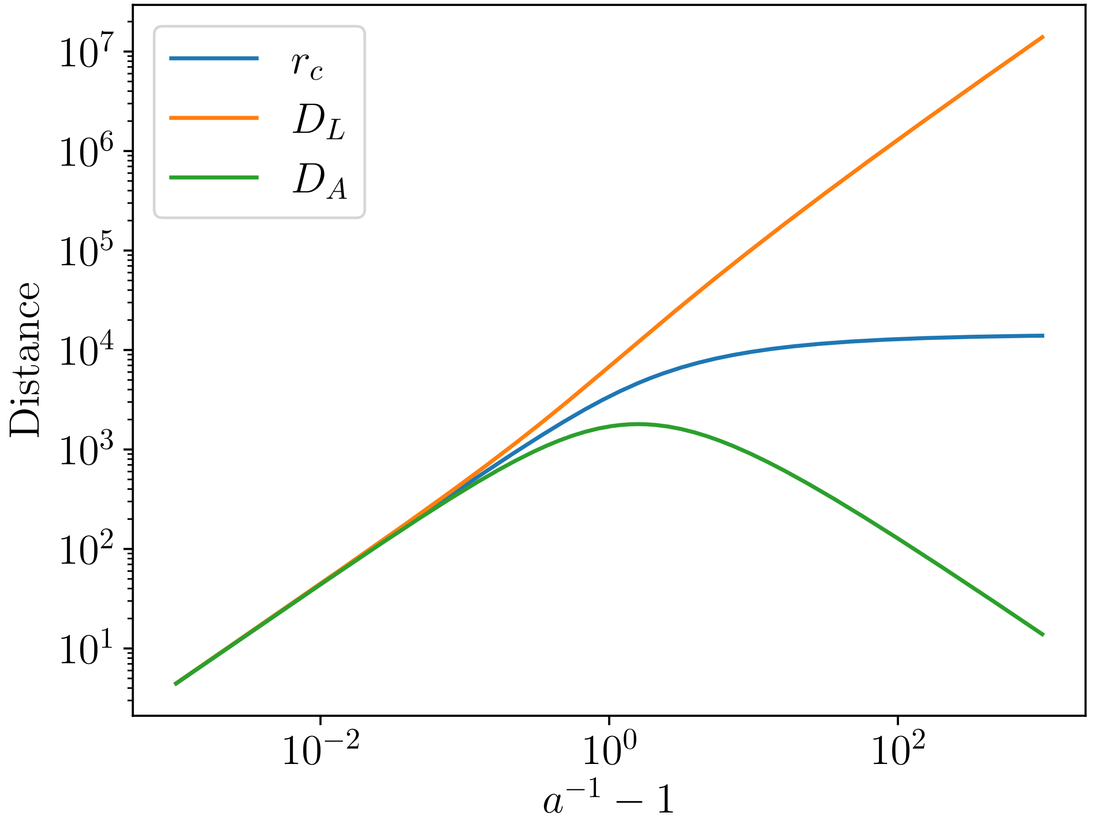
$$r_c = \chi\,\mathrm{sinc}(\chi\sqrt{\kappa}), \qquad \chi = \int_{t_e}^{t} \frac{dt'}{a(t')}$$

Luminosity distance

$$\ell = \frac{L}{4\pi d_L^2}, \qquad D_L = r_c\, a^{-1}$$

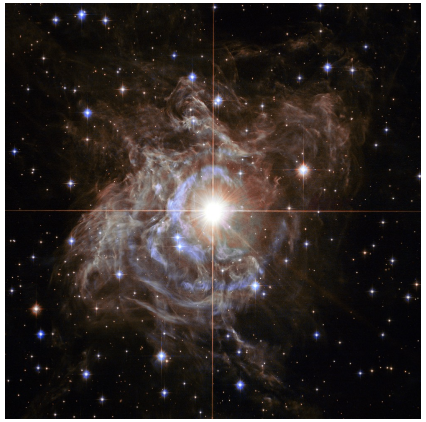
Angular diameter distance

$$\theta = \frac{s}{d_A}, \qquad D_A = r_c\, a$$



How to measure distances?

Close by, one can use **parallaxes**, but otherwise one needs objects with known luminosity or physical size.

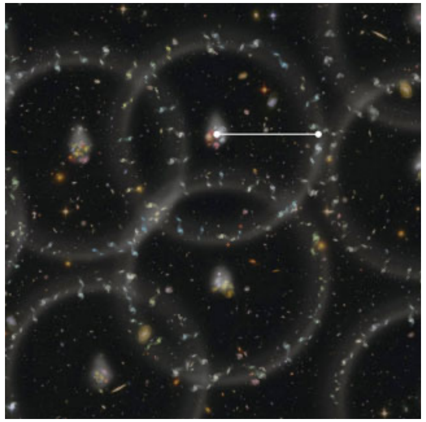


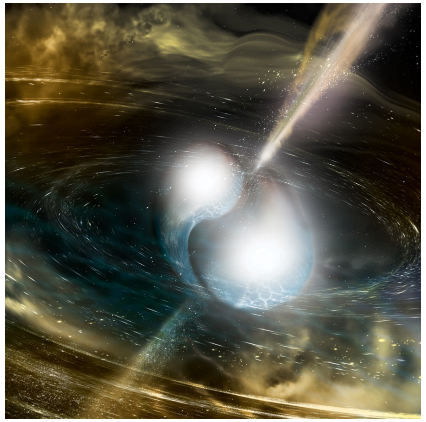
Standard candles: known/calibratable luminosity

- **Cepheids:** known relation between period of pulsation and luminosity
- **Supernovae Ia**: known relation between decay of light curve and luminosity

Standard rulers: Known size

- **BAO**: Baryon acoustic oscillation scale fixed by physics of the photon-baryon plasma





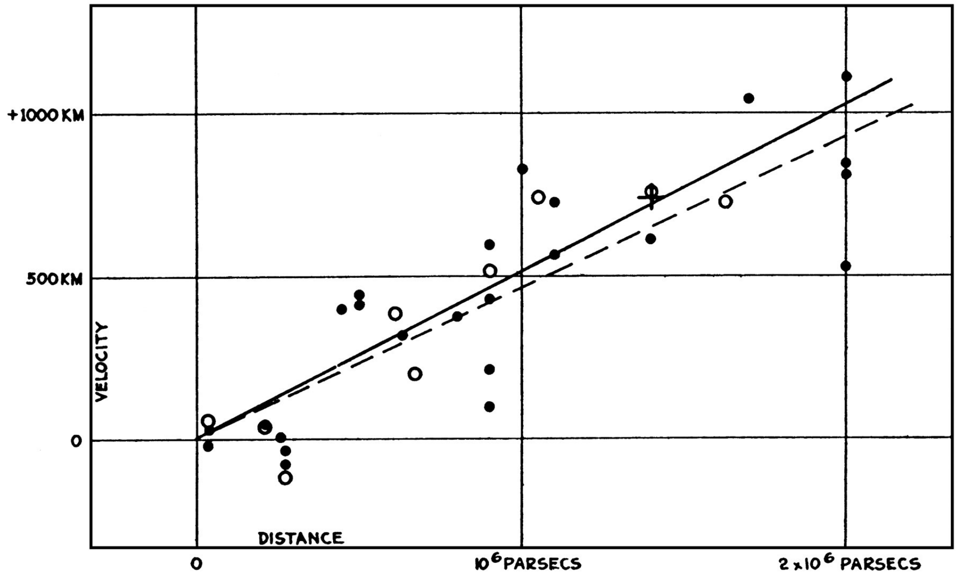
Standard sirens:

- **Gravitational waves**: Distance determined from amplitude of gravitational waves and binary mass.

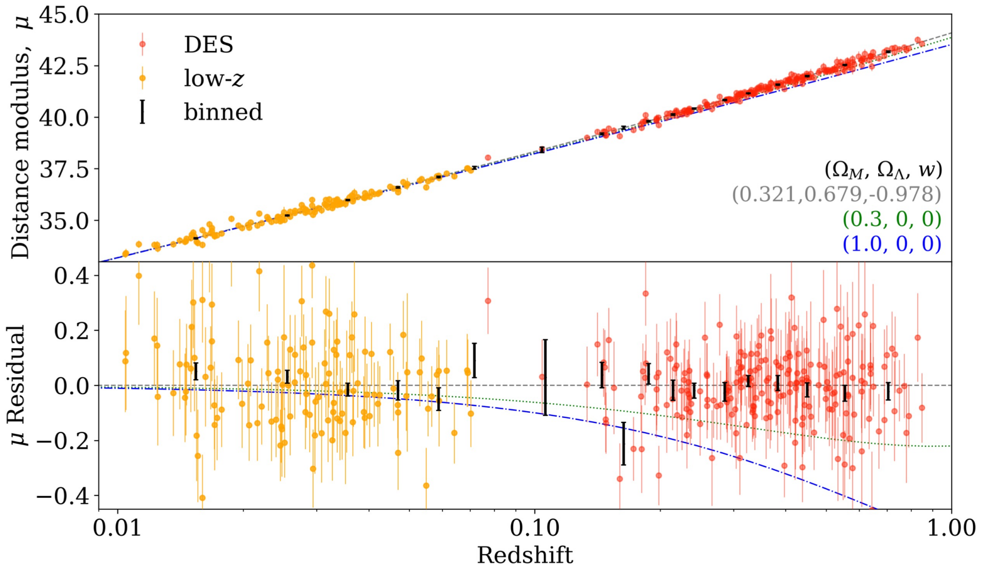
From distances to expansion

Standard candles can measure luminosity distance and constrain the expansion history

$$D_L(z) = H_0^{-1}\left(z + \frac{1}{2}(1 - q_0)z^2 + \cdots\right)$$



Hubble demonstrated Universe expansion, $H_0 > 0$



In 1998, two groups measured acceleration $q_0 < 0$!

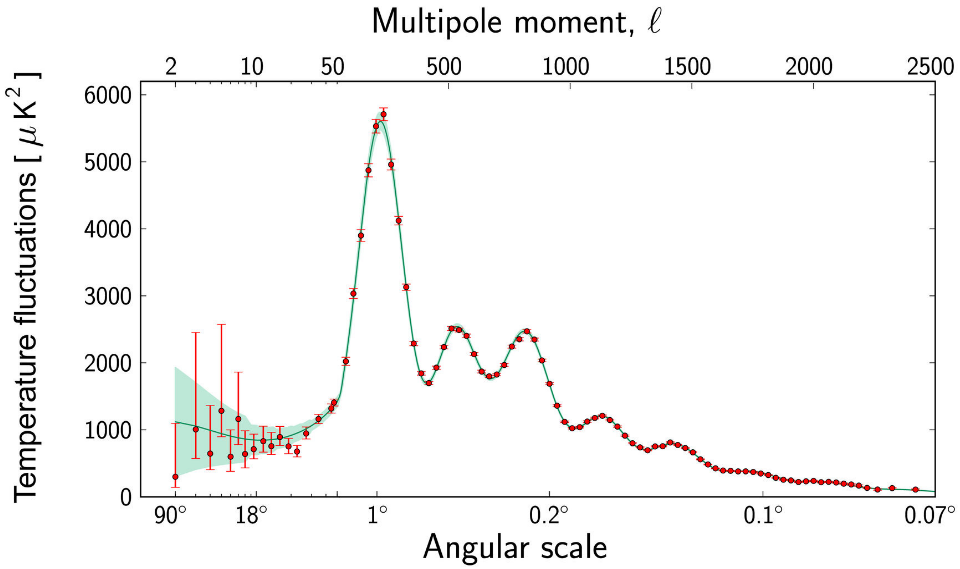
From distances to expansion

Acoustic oscillations can be used to measure angular diameter distance as well as expansion directly.

In the CMB, we see the **acoustic oscillations** in the photon fluid – sound waves frozen at recombination.

The angular scale we see in the peaks is

$$\theta_* = \frac{r_s}{D_M(z_*)}$$



corresponding to the (comoving) sound horizon at recomb.

$$r_s = \int_0^{t_{rec}} \frac{c_s}{a} dt$$

And allowing us to measure the (comoving) angular diameter distance D_M to the CMB.

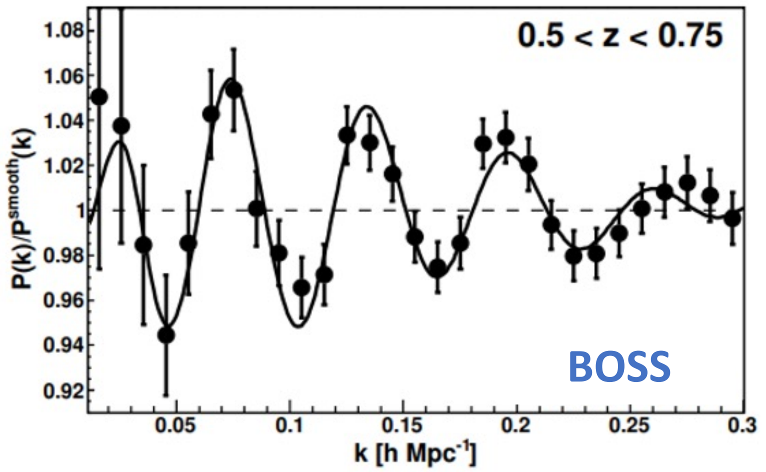
This gives information on the expansion and the curvature of the Universe.

From distances to expansion

In the large-scale structure we see the same acoustic oscillations in the normal matter, but in 3D!

The transverse scale measures the angular diameter distance:

$$\frac{r_d}{D_M(z)} \qquad r_d \gtrsim r_s$$



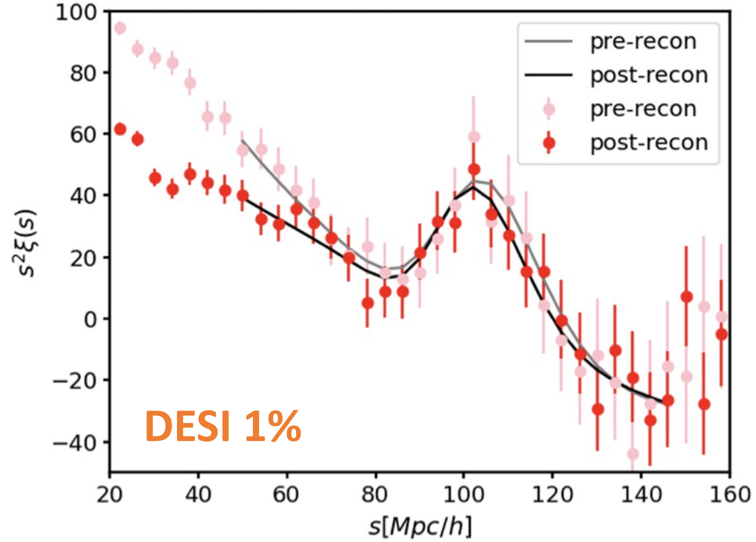
While the radial BAO is sensitive to $H(z)$ directly, probing

$$r_d\, H(z)$$

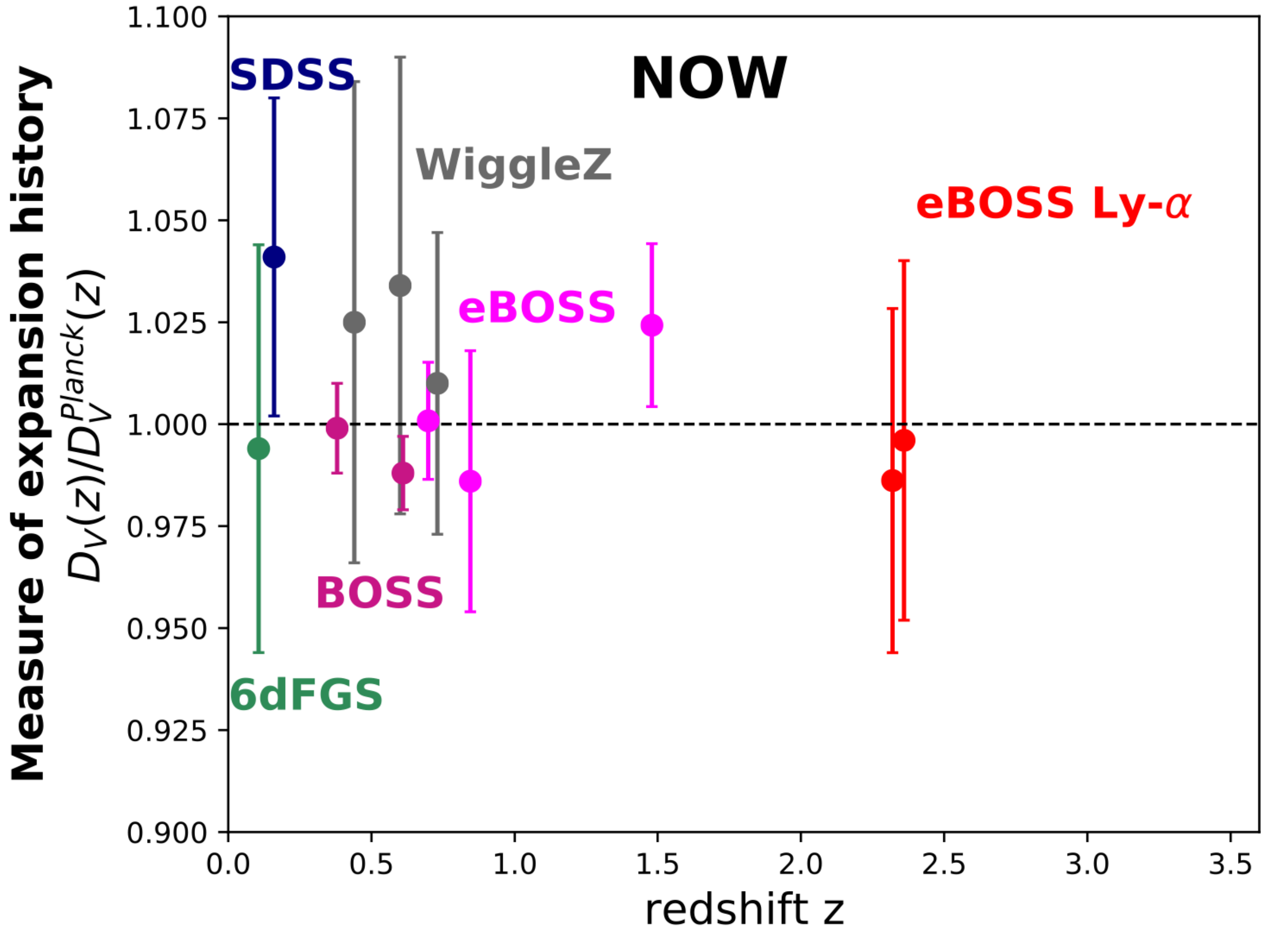
Can also combine them to measure instead

$$D_V(z) = \left[D_M(z)\,\frac{z}{H(z)}\right]^{1/3}$$

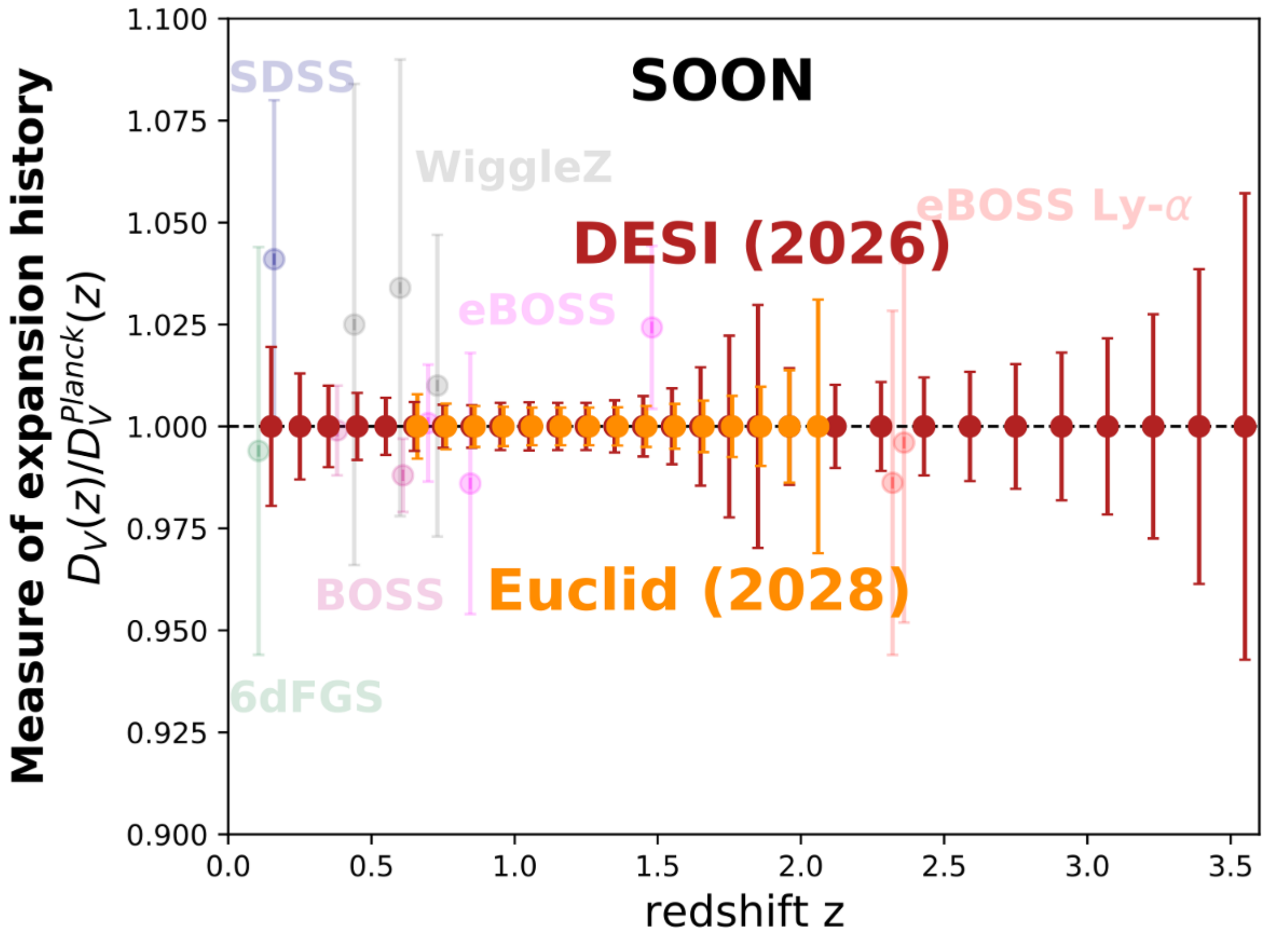
Measuring distances at many redshifts gives the full expansion history!



Status of BAO



Future of BAO



More on Euclid and DESI tomorrow!

Expansion History in ΛCDM

Friedmann equations

General Relativity + FLRW metric gives

$$G_{\mu\nu} = 8\pi G\, T_{\mu\nu} \quad \Rightarrow \quad \begin{aligned} H^2 + \frac{\kappa}{a^2} &= \frac{8\pi G}{3}\rho \\ \frac{\ddot{a}}{a} &= -\frac{4\pi G}{3}(\rho + 3P) \end{aligned}$$

Need $P < -\frac{1}{3}\rho$ to get accelerated expansion!

Conservation of stress-energy gives

$$\nabla_\nu T^{\mu\nu} \Rightarrow \dot{\rho} = -3H(\rho + P)$$

Single fluid solutions

Barotropic fluid

$$P(\rho) = w\rho$$ Equation of state parameter

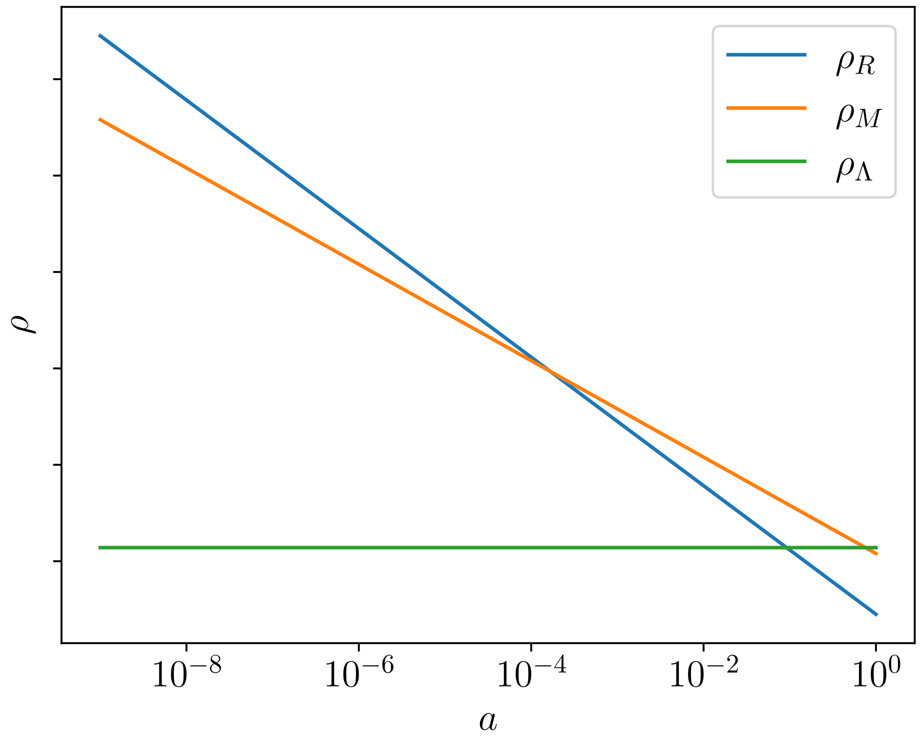
Solutions for different species

$$\dot{\rho} = -3H\rho(1+w)$$

Matter - $w = 0$: $\rho_M = \rho_{M0}a^{-3}$

Radiation - $w = \frac{1}{3}$: $\rho_R = \rho_{R0}a^{-4}$

C.C. Λ - $w = -1$: $\rho_\Lambda = \rho_{\Lambda 0}$



Evolution of fluids in the Universe

Temperature evolution

$$T \propto 1/a$$

Early Universe problems

The flatness and horizon problems

- Why is the Universe so flat?

$$\Omega(a) \equiv \frac{\rho(a)}{\rho_{crit}(a)} = 1 + \frac{\kappa}{(aH)^2}$$

CMB+BAO distances measure this to be

$$\frac{\kappa}{H_0^2} = -0.0001 \pm 0.0018$$

$R_H = (aH)^{-1}$

always grows with standard matter

- Why is the Universe so homogeneous?

$$\chi_{hor} = \int_{-\infty}^{\log a_0} \frac{1}{aH} \, d\log a$$

Too small to explain the large-scale coherence of the CMB

Solution: make R_H decay in the very early Universe!

$$\dot{R_H} < 0 \Rightarrow \ddot{a} > 0$$

Accelerated expansion does exactly this! This is the epoch of **inflation.**

The early Universe and inflation

New problem: Inflation **must end!**

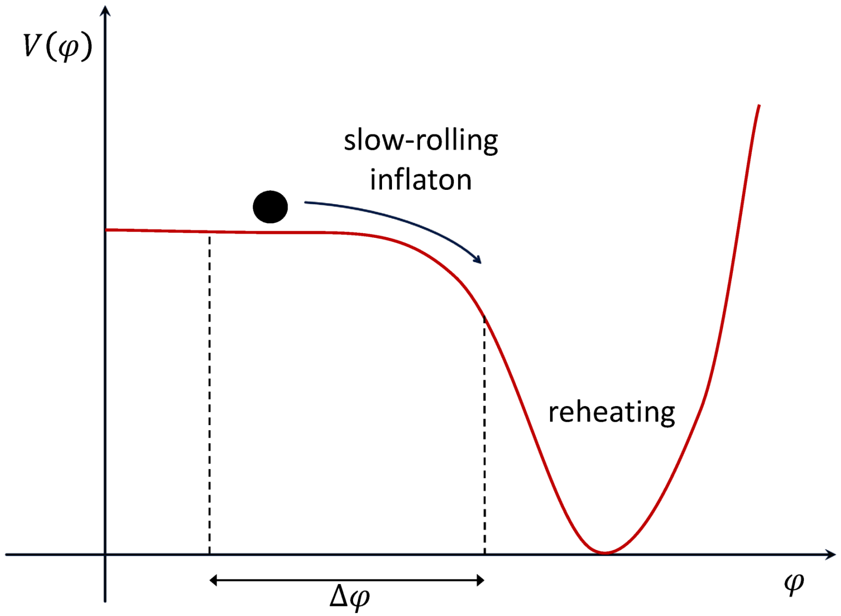
- Cosmological constant can’t work here, need something dynamical

Solution: a new scalar, the **inflaton** $\boldsymbol{\varphi}$

$$S = \int d^4x\sqrt{-g}\left(\frac{R}{2} - \frac{1}{2}\partial\varphi^2 - V(\varphi)\right)$$

$$\Downarrow$$

$$w = \frac{\frac{1}{2}\dot{\varphi}^2 + V(\varphi)}{\frac{1}{2}\dot{\varphi}^2 - V(\varphi)} \xrightarrow[\dot{\varphi}^2 \ll V]{} -1$$



At the end of inflation, **reheating** happens and the inflaton decays to the SM (+ DM?).

The middle-aged Universe

After reheating, the Universe is very hot and is **radiation dominated**

$$\rho_{tot} \approx \rho_R \propto a^{-4} \Rightarrow H(a) \propto a^{-2} \Rightarrow a(t) \propto t^{1/2}$$

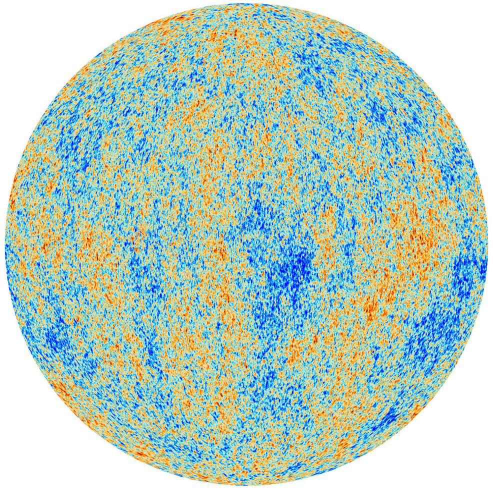
Some key events:

$z = ?$: the abundance of **Dark Matter** is set, either from decoupling (ex WIMPs) or its creation (ex PBHs).

$z \sim 10^9$: **Big Bang Nucleosynthesis** occurs, creating the first composite nuclei.

$z \sim 3000$: **Matter-radiation equality:** we enter the **matter-dominated** epoch

$$\rho_{tot} \approx \rho_M \propto a^{-3} \Rightarrow H(a) \propto a^{-3/2} \Rightarrow a(t) \propto t^{2/3}$$



$z \approx 1100$: **Recombination**: atoms form and the CMB is created.

The late Universe

After the CMB is formed, that is the only light in the Universe and we are in the **Dark Ages**

Structure formation effectively begins during this stage

$z \sim 25$: **First stars** form, **reionization** begins

$z \sim 15$ (but ask JWST): **First galaxies form**

$z \sim 6$: **Reionisation** is complete

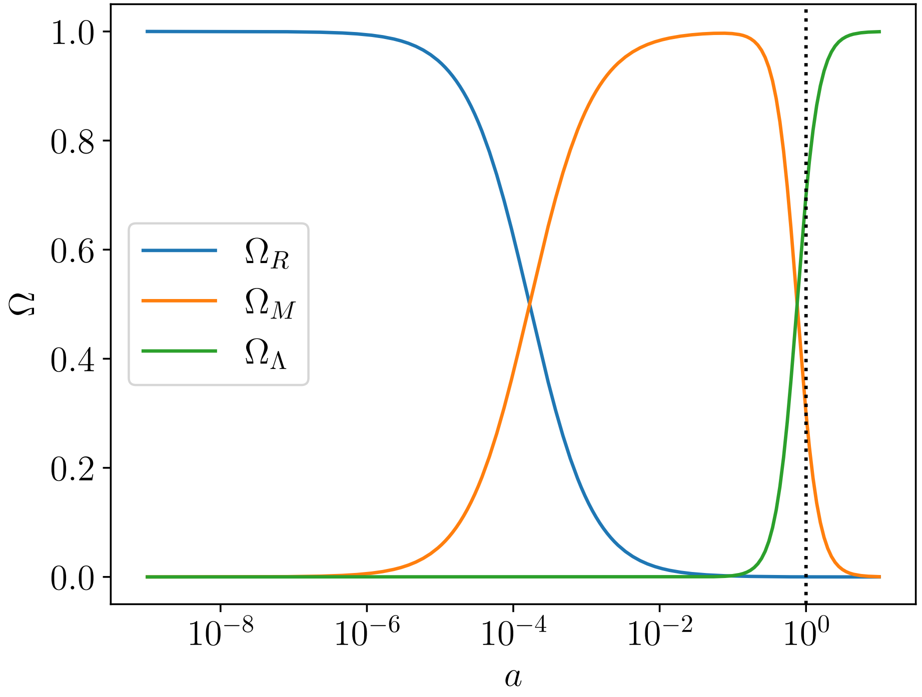
$z \sim 0.7$: **Accelerated expansion begins**

$z \sim 0.3$: **Dark energy domination**

$z = 0$: We observe all of it!

The full story

The history in relative densities, $\Omega_i = \frac{\rho_i}{\rho_{crit}} = \frac{8\pi G\rho_i}{3H^2}$

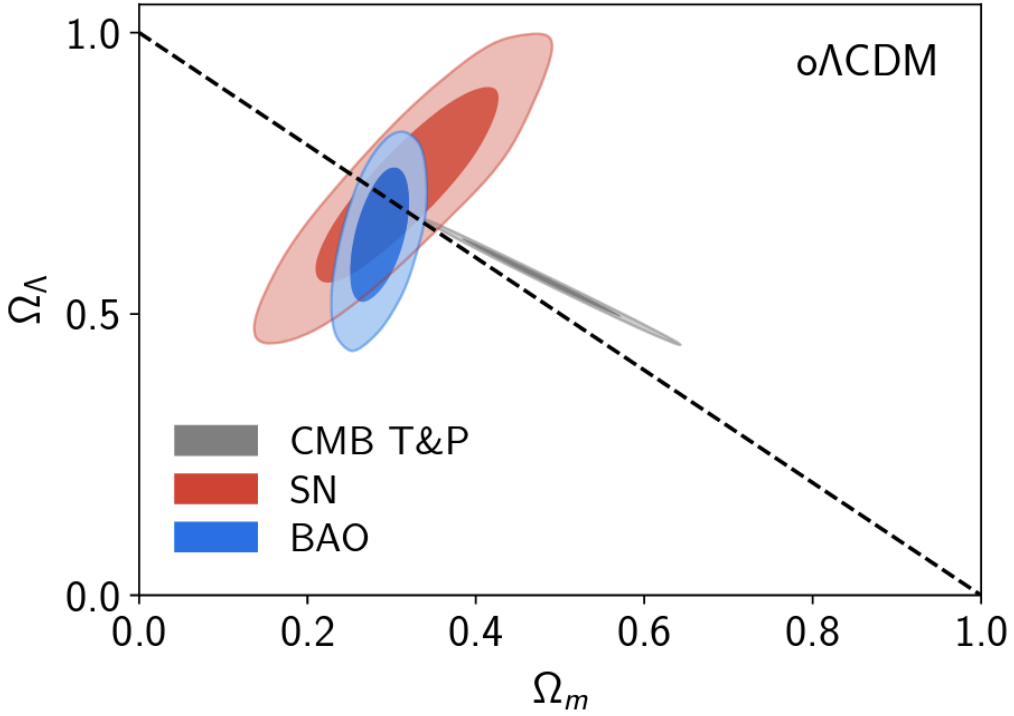


The full evolution of the Hubble rate is

$$H(a) = H_0\sqrt{\Omega_{R0}a^{-4} + \Omega_{M0}a^{-3} + \Omega_{\Lambda 0}}$$

Current concordance

Most observations match the ΛCDM model



But we do not understand the dark sector!

And cosmic tensions are worrying…

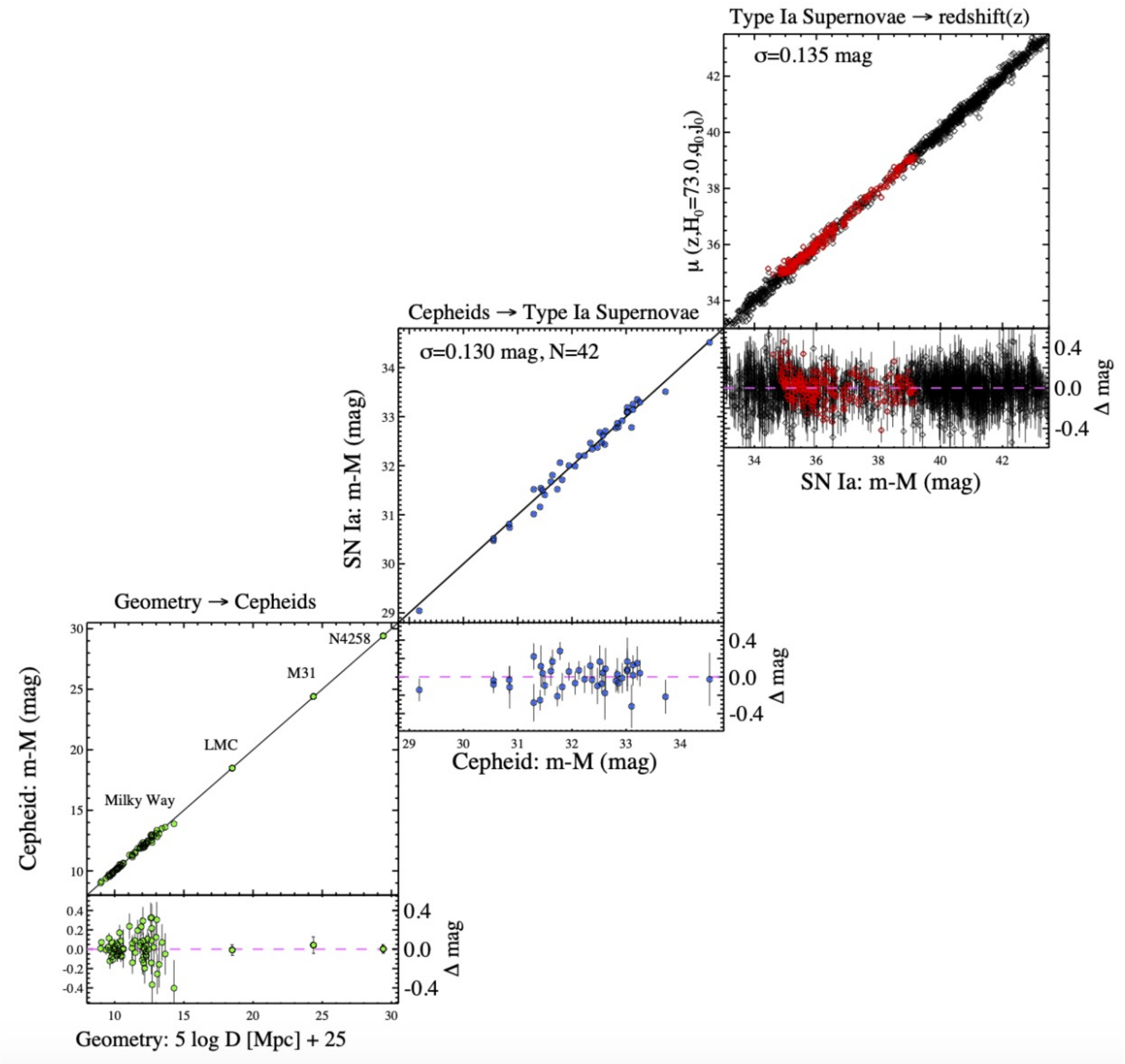
The H_0 tension in detail

What is measured at late times?

- Luminosity distance from the cosmic distance ladder at low redshift

Main measurements: SH0ES collaboration

$$H_0 = 73.04 \pm 1.04\ km\ s^{-1}\ \mathrm{Mpc}^{-1}$$



The H_0 tension in detail

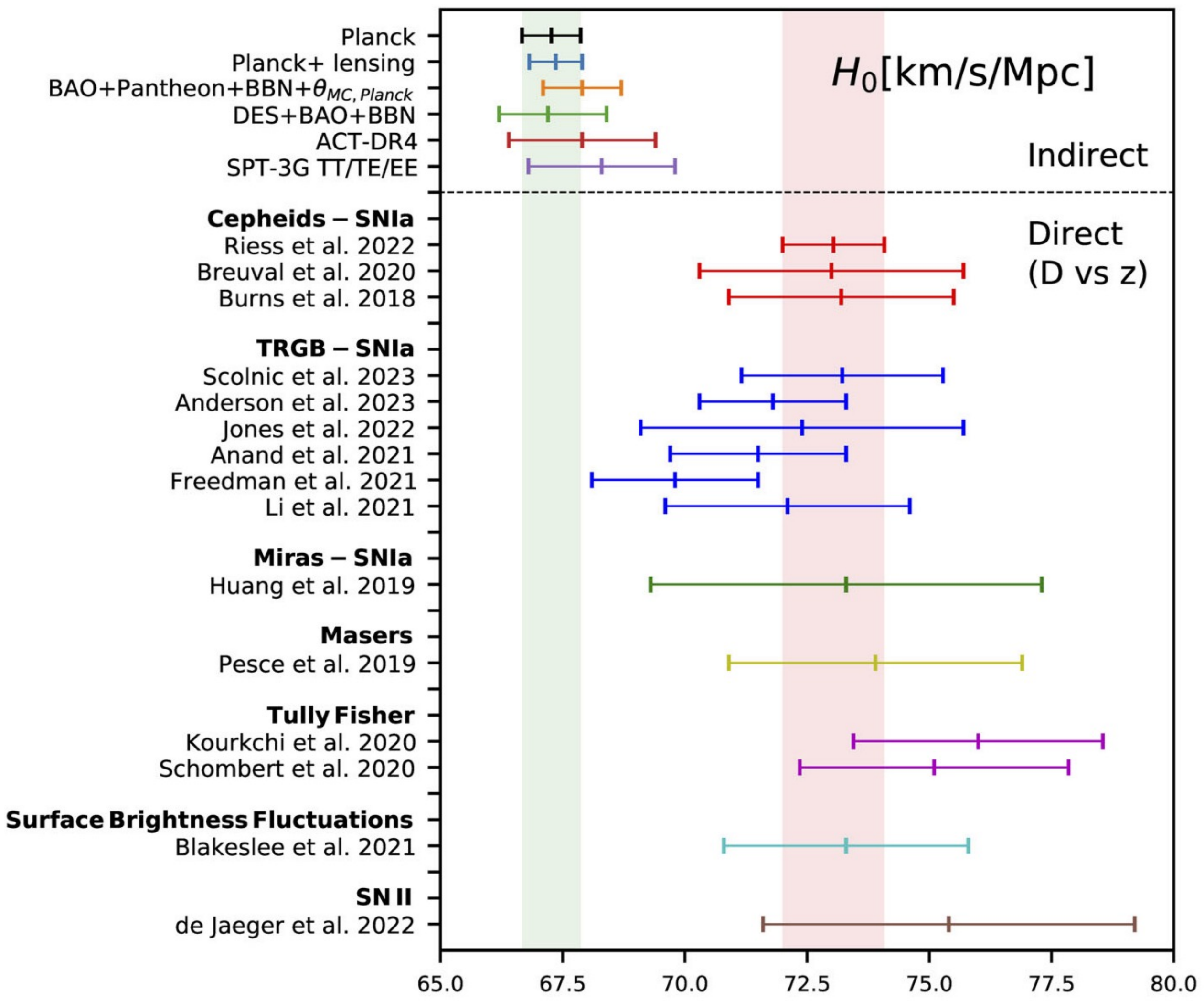
What is measured at late times?

- Luminosity distance from the cosmic distance ladder at low redshift

Main measurements: SH0ES collaboration

$$H_0 = 73.04 \pm 1.04 \; km \; s^{-1} \; \mathrm{Mpc}^{-1}$$

Many other groups get similar result



The H_0 tension in detail

What is measured at early times?

$$\theta_* = \frac{r_s}{D_M(z_*)} = (1.04109 \pm 0.00030)\times 10^{-2}$$

The ΛCDM model predicts

$$r_s = \int_{z_{rec}}^{\infty} \frac{c_s(z)}{H(z)} dz = r_s[\Omega_b h^2, \Omega_M h^2, \Omega_R h^2] \qquad D_M = \int_0^{z_{rec}} \frac{dz}{H(z)} = D_M[\Omega_M h^2, H_0]$$

Can be easily solved for H_0 to infer 67.4 ± 0.5

$$H_0 = 67.4 \pm 0.5 \; km \; s^{-1} \, \mathrm{Mpc}^{-1}$$

Giving a **5σ discrepancy**!

The H_0 tension in detail

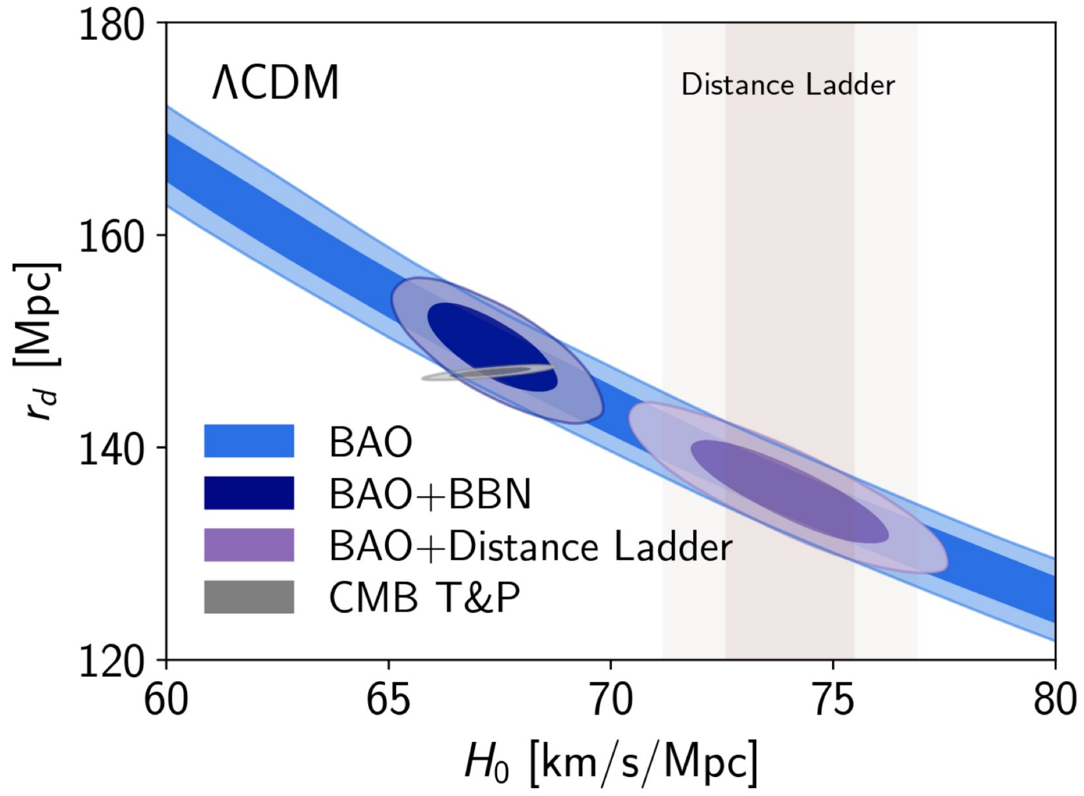
Can BAO help?

- Not by itself, as it measures the combination

$$r_d\, H(z) = r_d\, H_0\sqrt{\Omega_{M0}(1+z)^3+\Omega_{\Lambda 0}}$$

- Unless one adds a measurement of $\Omega_b h^2$ from BBN!

- But still dependent on early Universe probes.



Going beyond ΛCDM

Dark energy problems

The cosmological constant problem – Why is Λ so small?

$$\Lambda_{obs} \sim 10^{-47}\mathrm{GeV}^4$$

$$\Lambda_{vaccum} \sim k_{max}^4 \sim 10^{74}\ \mathrm{GeV}^4$$

$$\Lambda_{obs} = \Lambda_{vaccum} - \Lambda_{counter}$$

Huge fine tuning required!

The coincidence problem – Why now?

In the past $\rho_\Lambda \ll \rho_M$

When we observe it $\rho_\Lambda \sim \rho_M$

Dynamical dark energy

Perhaps some symmetry makes $\Lambda = 0$ and we see some other component with negative pressure

We can get inspired by inflation and use a scalar field ϕ

$$S = \int d^4x\sqrt{-g}\left(\frac{R}{2} - \frac{1}{2}\partial\phi^2 - V(\phi)\right)$$

with equation of motion (Klein-Gordon with general potential):

$$\ddot{\phi} + 3H\dot{\phi} + \frac{dV}{d\phi} = 0$$

Leading to a time-dependent equation of state parameter:

$$w(t) = \frac{\frac{1}{2}\dot{\phi}^2 + V(\phi)}{\frac{1}{2}\dot{\phi}^2 - V(\phi)} \xrightarrow[\dot{\phi}^2 \ll V]{} -1$$

Dynamical dark energy

Example potentials

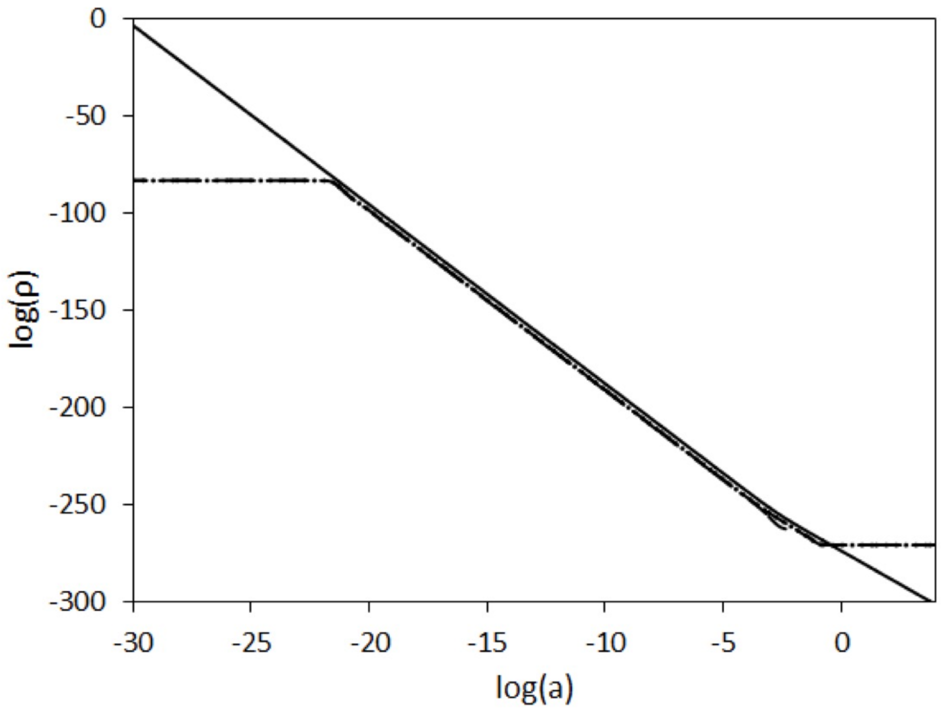
- $V(\phi) = V_0$: Essentially the same as a cosmological constant

- $V(\phi) = V_0\phi^{-n}$: The original model. Includes tracking solutions

$$\frac{\rho_\phi}{\rho_{dom}} \propto t^{\frac{4}{2+n}}$$

- $V(\phi) = V_0 e^{-\lambda\phi}(A + (\phi - \phi_0)^2)$: Includes scaling solutions:

$$\rho_\phi \propto \rho_{dom}$$



Dark energy – parametrised forms

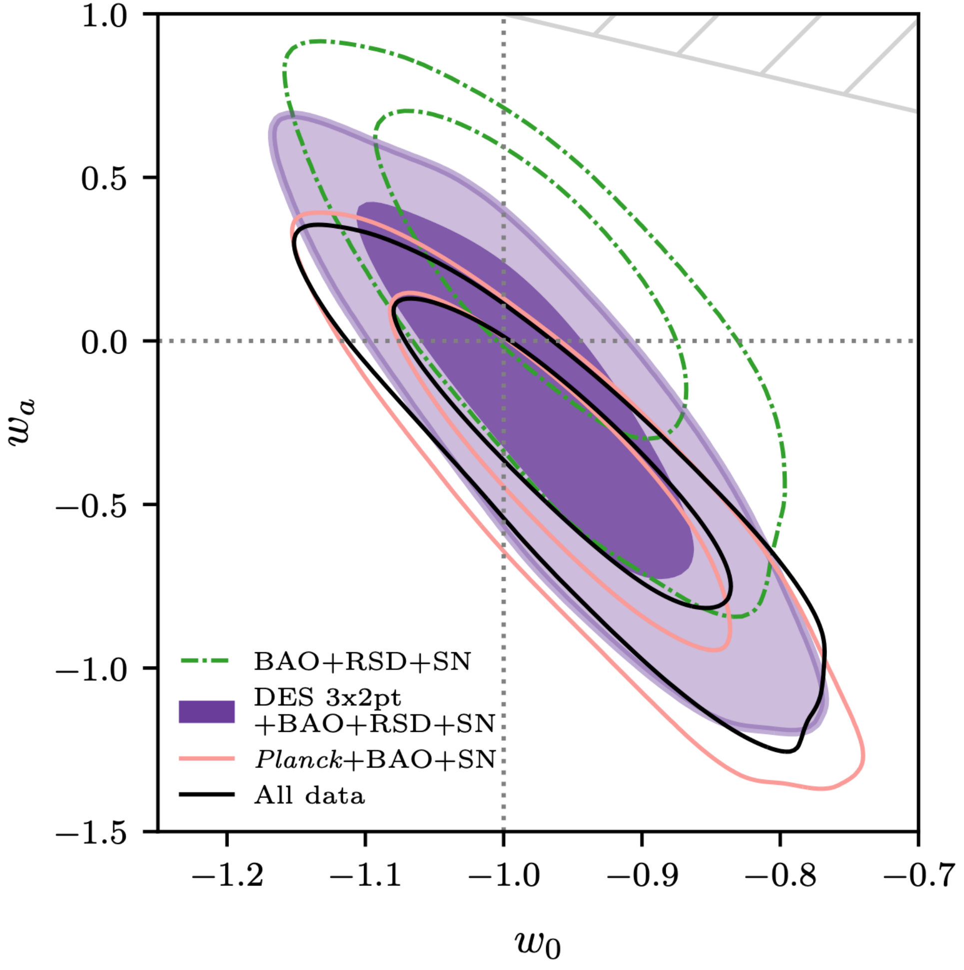
A simple parametrised model of dark energy is called $w_0 w_a$CDM and has

$$w(a) = w_0 + (1 - a)\, w_a$$

Properties

- $w(a = 1) = w_0$
- $w(a \to 0) = w_0 + w_a$
- $\partial_a w = -w_a$
- Captures many different models of dark energy

Dark energy – observational status



Can dark energy solve the H_0 tension?

A very large number of ideas has arisen to resolve the H_0. There are two main groups:

- Late-time solutions
  - Change the distance to last scattering $D_M = \int_0^{z_{rec}} \frac{dz}{H(z)}$
  - Ex: **Dark energy** with $\boldsymbol{w \sim -1.5}$
  - Difficult to reconcile with BAO.
- Early-time solutions
  - Change the sound horizon $r_s = \int_{z_{rec}}^{\infty} \frac{c_s(z)}{H(z)} dz$
  - Ex: **Early dark energy**

$$\boxed{\theta_* = \frac{r_s}{D_M(z_*)}}$$

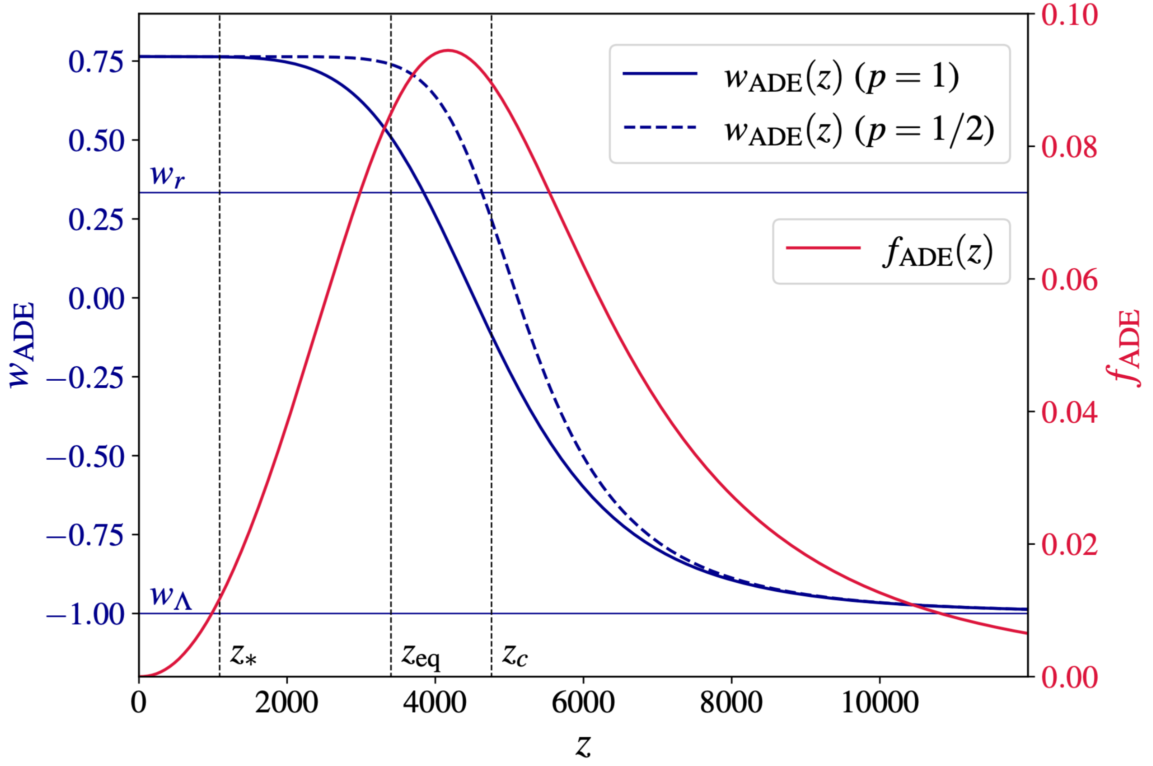
Can dark energy solve the H_0 tension?

Early dark energy

- Increase expansion rate just before recombination with early form of dark energy

$$w(a) = \frac{1+w_f}{\left(1+\left(\frac{a_c}{a}\right)^{3(1+w_f)/p}\right)^p} - 1$$

- Increases $H(z)$ at $z \sim 4000$ and then removes EDE.
- But this scenario is really fine-tuned!
- This type of models only reduces the tension to about 3σ.



New ideas are needed!