Quantum Fields in Compact Stars

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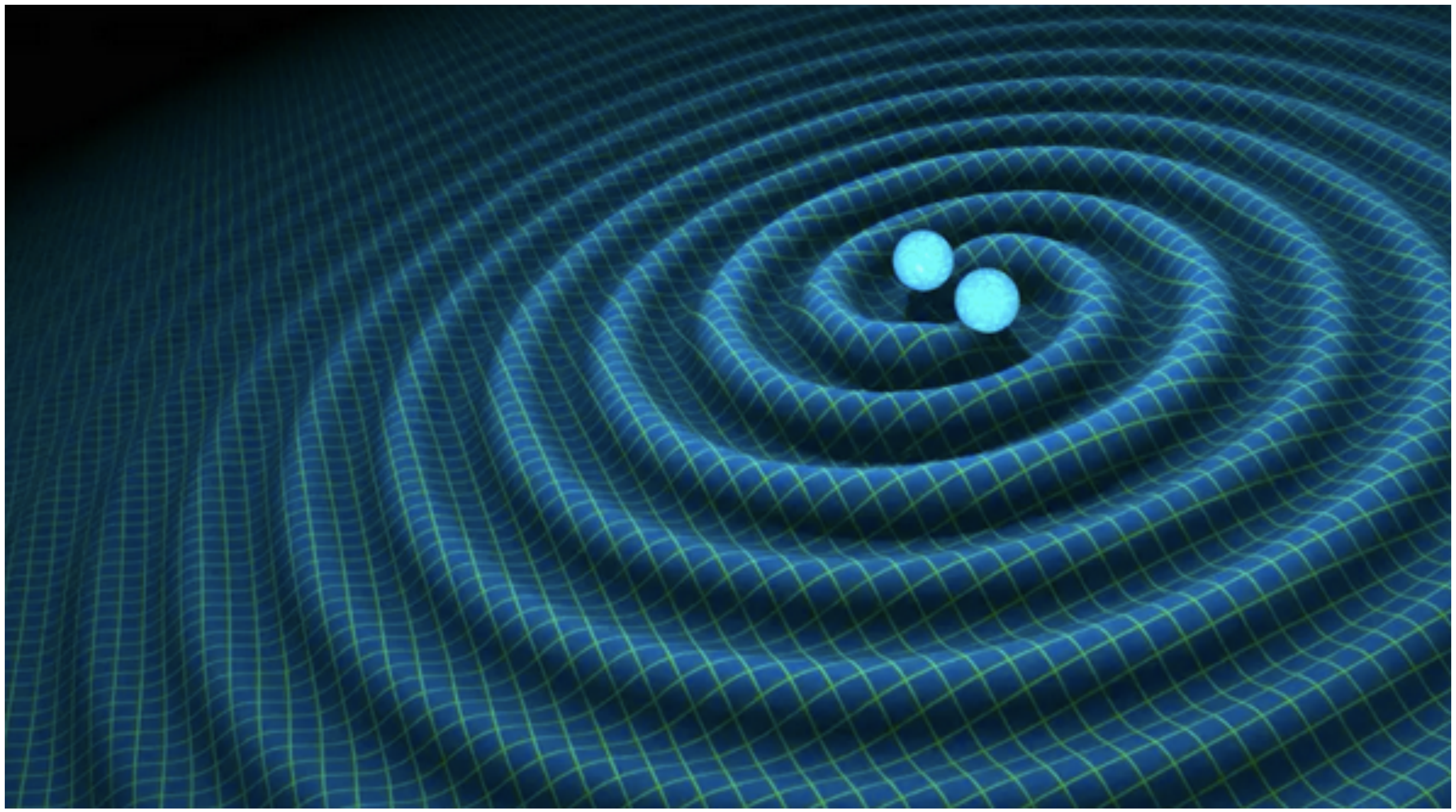
We detect a GW signal… with no electromagnetic counterpart



How do we decide if it's a Neutron star or a Black hole?

What role does $\hbar$ play in this?

The answer is the Mass

1) White dwarfs: fermions + Newton $\longrightarrow$ Chandrasekhar

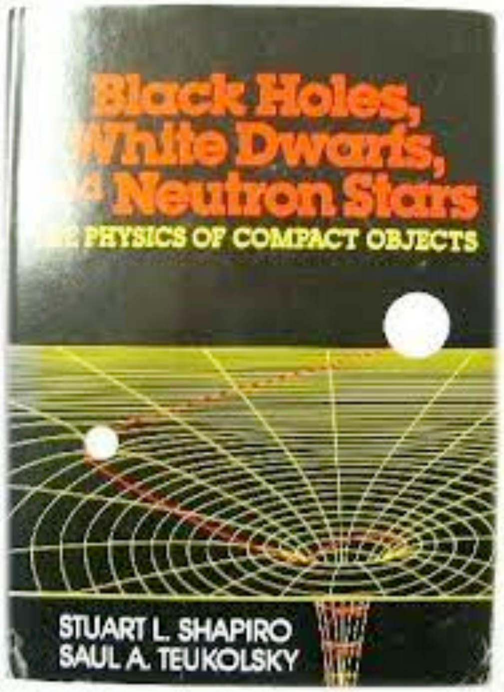
2) Neutron stars: fermions + GR $\longrightarrow$ TOV limit

3) Ultra Compact Objects: $R < 3GM$

We don't know equation of state of QCD

And *classical GR* seems to forbid them:

- Bounds on compactness: Buchdahl's theorem
- Gravitational instability of stable light rings



This talk: Does QFTCS affect 3) ?

Buchdahl's Theorem

[Buchdahl '59]

Static, spherically symmetric isotropic perfect fluid

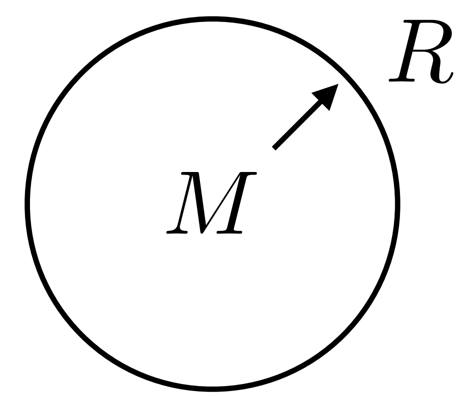
$$T^{\mu}_{\nu} = \mathrm{diag}(-\rho, p, p, p)$$

$$ds^2 = -f(r)dt^2 + h(r)dr^2 + r^2 d\Omega^2$$

Then, assuming nothing about the EOS but only

$$\rho > 0 \quad , \quad \partial_r \rho \leq 0 \quad , \quad G_{\mu\nu} = 8\pi G T_{\mu\nu}$$

$$\Rightarrow \quad \frac{R}{GM} \geq \frac{9}{4} = 2.25$$

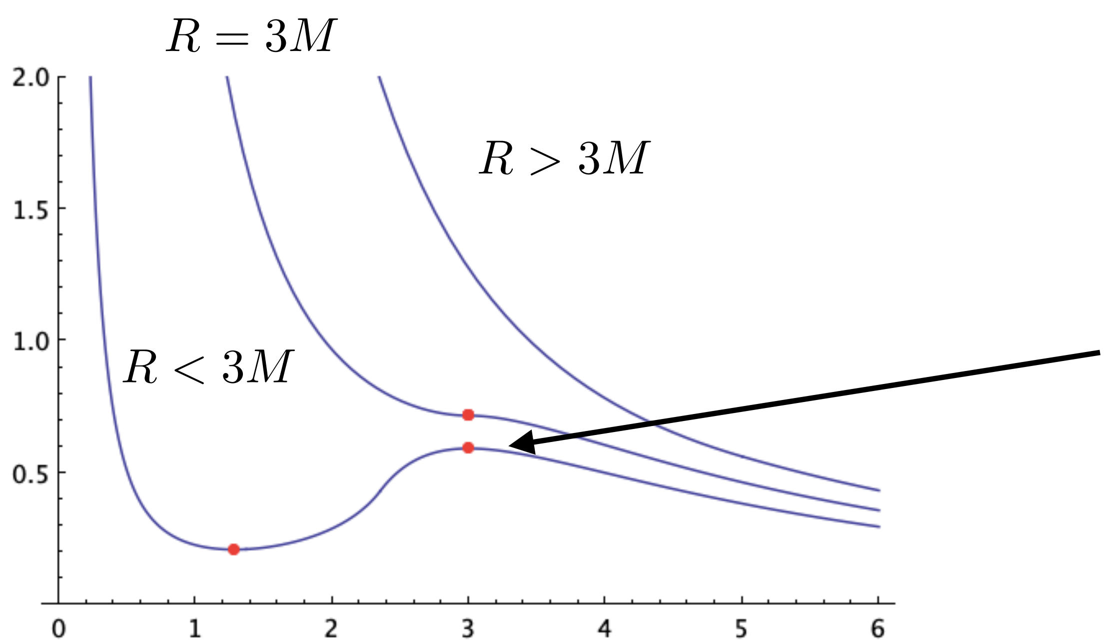


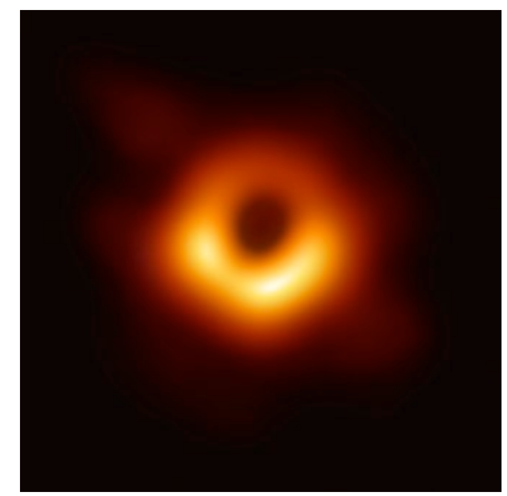
An absolute upper bound on compactness for stars

Light rings

'Photon sphere': light can travel in circles $\longrightarrow$ come in pairs

$$f\,h\,\dot{r}^2 + V(r) = E^2 \quad , \quad V = \frac{L^2}{r^2} f$$





Inner: minimum $\longrightarrow$ gravitational *instabilities* [Keir '14, Cardoso et al '14]

Stars are unstable if $R < 3GM$

‘Schwarzschild star’

Solvable model with both features: *uniform density* star

$$\rho = \text{const} \qquad\qquad ds^2 = -f(r)dt^2 + h(r)dr^2 + r^2 d\Omega^2$$

Unrealistic, but it’s the textbook example of TOV equations

$$f(r) = \left(\frac{3}{2}\sqrt{1-\frac{2GM}{R}} - \frac{1}{2}\sqrt{1-\frac{2GMr^2}{R^3}}\right)^2 ,$$

$$h(r) = \left(1-\frac{2GMr^2}{R^3}\right)^{-1}$$

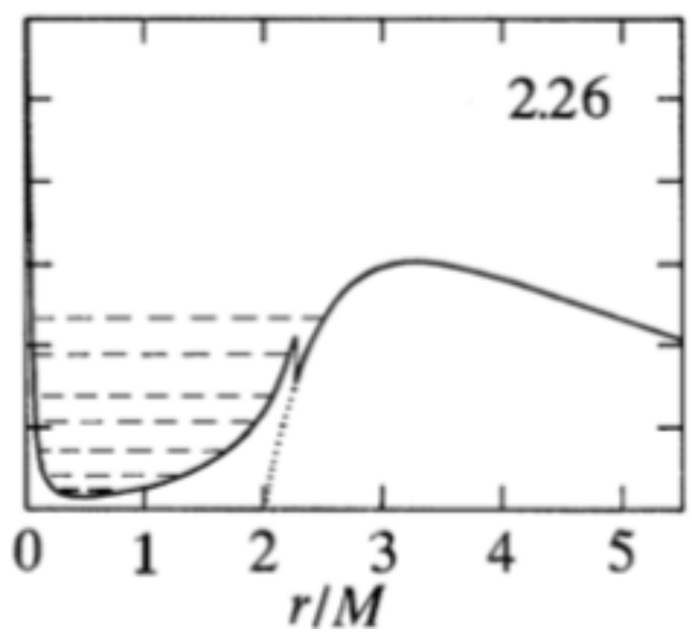
Saturates Buchdahl limit and has two Light rings

Background geometry: Schwarzschild interior / vacuum exterior.

Wave equation

Massless scalar field $\Box\Phi = 0$ $\Phi = \frac{u(r)}{r} Y^{\ell}_{m}(\theta,\varphi) e^{-i\omega t}$

$$-\partial^2_{r_*} u + V(r_*) u = \omega^2 u \qquad \frac{dr_*}{dr} = \sqrt{h(r)/f(r)}$$

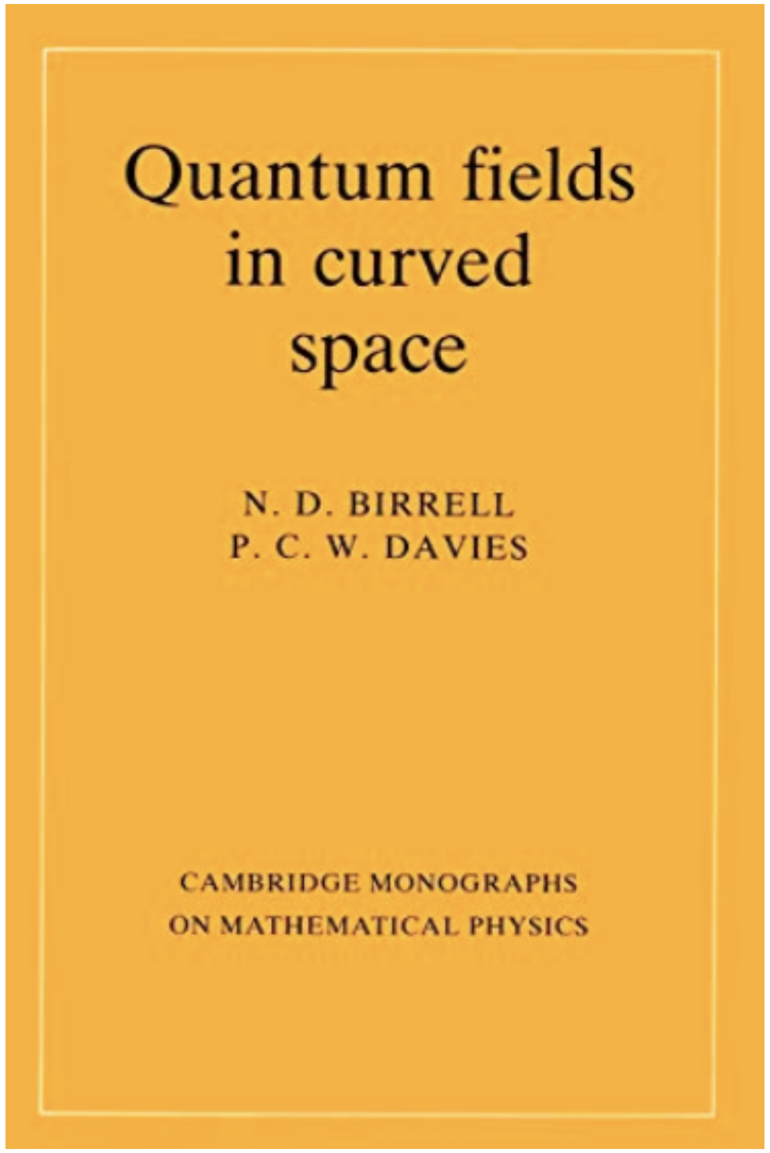


Chandrasekhar-Ferrari ´92

quasi-bound states

If the potential traps classical waves, it should also support vacuum wavefunctions!

QFT in curved spacetime 'Bible'



- Black Holes
- Cosmology
- … no mention of stars!

Let's do this.

QFT in curved spacetime

Semi-classical approximation

$$\mathcal{R}_{\mu\nu} - \frac{1}{2} g_{\mu\nu} \mathcal{R} = 8\pi G \left(T_{\mu\nu} + \langle \hat{T}_{\mu\nu} \rangle \right)$$

Is there some *generic* feature of $\langle \hat{T}_{\mu\nu} \rangle$ for compact objects?

Conformally coupled quantum fields in the *vacuum*

$$\langle \hat{T}^{\mu}{}_{\mu} \rangle = \frac{1}{(4\pi)^2} \left[c\mathcal{F} - a\mathcal{G} - d \, \Box \mathcal{R} \right]$$

$c, a > 0$ fixed by the theory $\quad\quad$ d scheme dependent

But if in addition, the metric is conformally flat

$$\langle \hat{T}^{\mu\nu} \rangle = -\frac{a}{(4\pi)^2}\left[g^{\mu\nu}\left(\frac{\mathcal{R}^2}{2} - \mathcal{R}_{\alpha\beta}\mathcal{R}^{\alpha\beta} \right) + 2\mathcal{R}^{\mu\lambda}\mathcal{R}^{\nu}_{\lambda} - \frac{4}{3}\mathcal{R}\mathcal{R}^{\mu\nu} \right]$$
$$+ \frac{d}{(4\pi)^2}\left[\frac{1}{12} g^{\mu\nu} (\mathcal{R}^2 - 4\mathcal{R}^{;\lambda}_{;\lambda}) - \frac{1}{3}(\mathcal{R}\mathcal{R}^{\mu\nu} - \mathcal{R}^{;\mu;\nu}) \right]$$

The uniform density star conformally flat! (theorem)

Now, although (0.13) is to be found in almost all books on the general theory of relativity and is often discussed in considerable detail, it is remarkable that no mention is made of the fact that the interior metric (0.13) is conformally flat.

[Buchdahl '70]

Conclusion: we know $\langle \hat{T}_{\mu\nu} \rangle$ exactly on this metric.

Classical $T_{\mu\nu}$ in the Buchdahl limit

Consider the classical fluid in the regime

$$R = (9/4 + \epsilon)GM\,, \qquad \epsilon \to 0\,.$$

The inner Light ring moves to the center, and the curvature

$$\mathcal{R}(0) \sim -\frac{1}{r_{\text{int}}^2} \sim -\frac{1}{R^2}\frac{1}{\epsilon}$$

central density is finite but the pressure diverges

$$\rho(0) \sim \frac{1}{GR^2} \quad , \quad p(0) \sim \frac{1}{GR^2}\frac{1}{\epsilon}$$

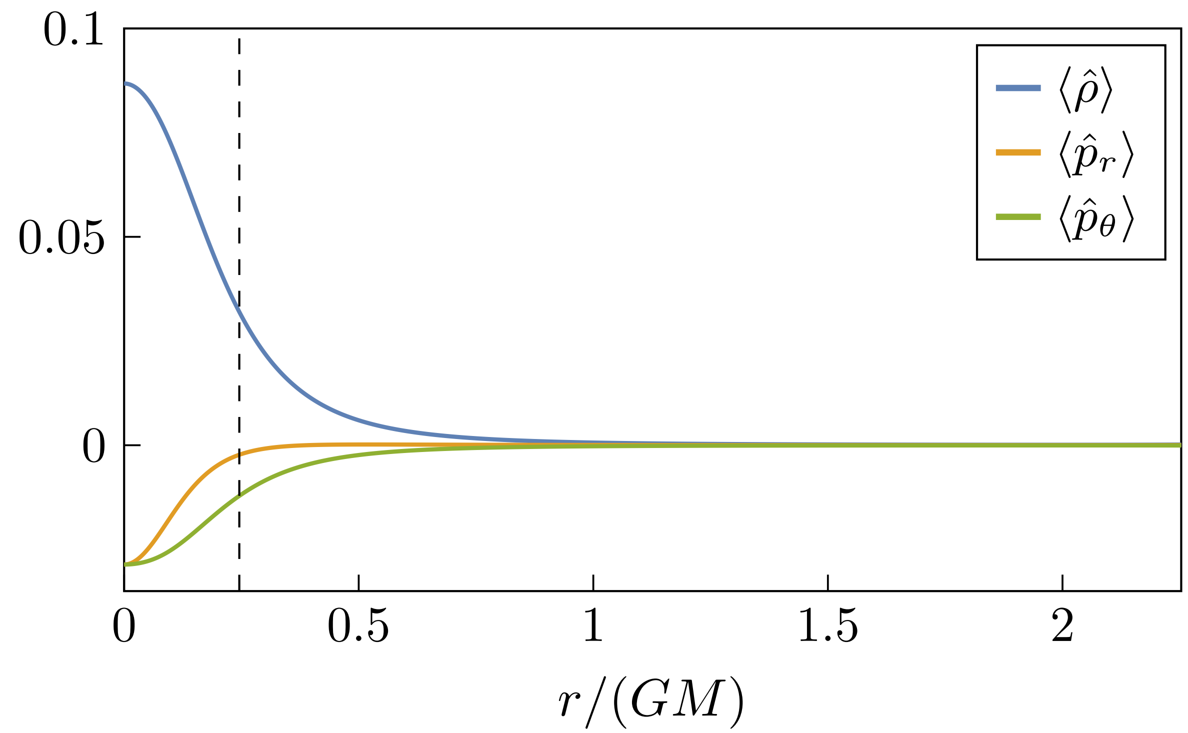
$\langle \hat{T}_{\mu\nu} \rangle$ in the Buchdahl limit

$$\langle \hat{T}^{\mu}_{\nu} \rangle = \mathrm{diag}(-\langle \hat{\rho} \rangle, \langle \hat{p}_r \rangle, \langle \hat{p}_\theta \rangle, \langle \hat{p}_\theta \rangle)$$ not isotropic

Inner light ring fall-off:

$$\frac{\langle \hat{\rho}(r_{\mathrm{in}}) \rangle}{\langle \hat{\rho}(0) \rangle} \sim 10^{-1}$$

quantum trapping.



The quantum terms diverge faster than classical

$$\langle \hat{\rho}(0) \rangle \sim \frac{1}{R^4} \frac{d}{\epsilon^2} \quad , \quad \langle \hat{p}_r(0) \rangle = \langle \hat{p}_\theta(0) \rangle \sim -\frac{1}{R^4} \frac{d}{\epsilon^2}$$

Summary

- Compact objects in GR and some no-go theorems

- Schwarzschild star as background exhibiting these features

- From classical waves to QFT

- Inside inner light ring:

$\langle \hat{T}_{\mu\nu} \rangle$ grows faster than $T_{\mu\nu}$ $\qquad \dfrac{R}{GM} \to \dfrac{9}{4} > 2$

quantum backreaction cannot be neglected *before* a horizon

- What is the backreaction?