

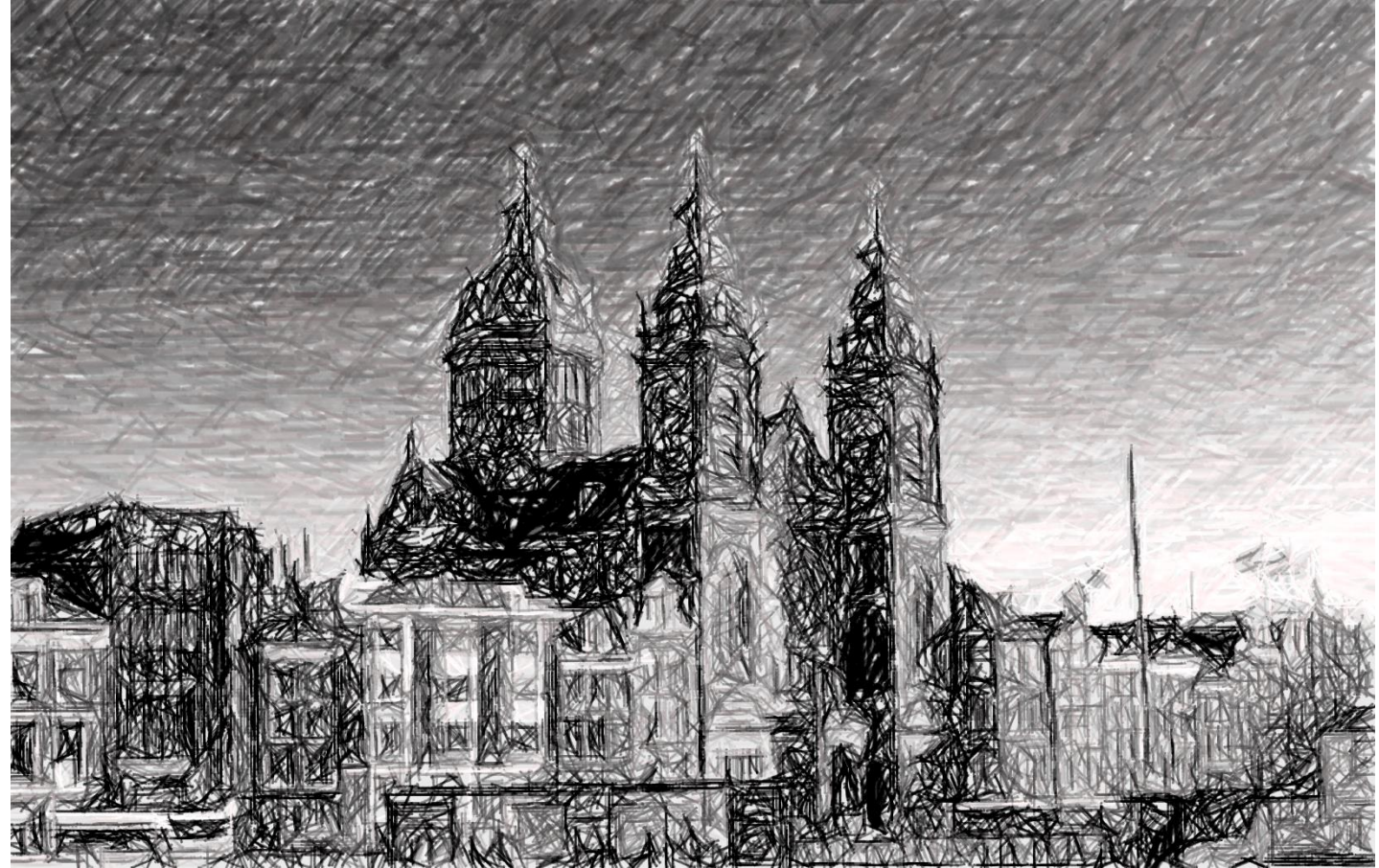
Macroscopic description of the hot and dense QCD matter

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Technical University of Munich
“**Topical Lectures @ Nikhef**”, 07/06/2023



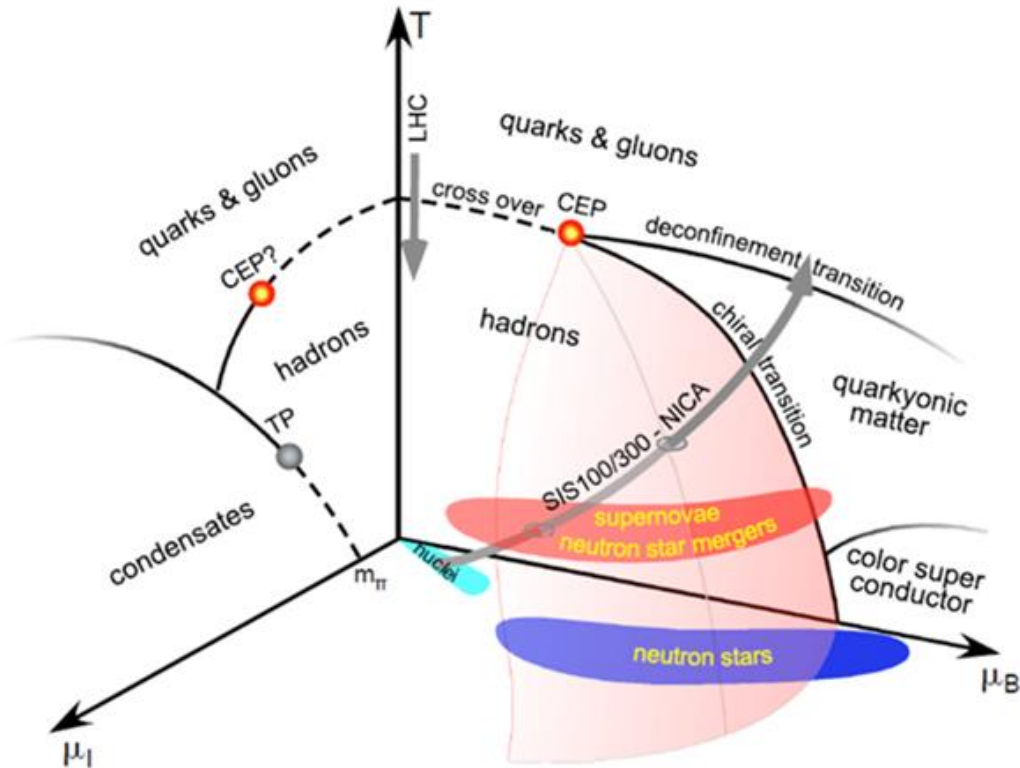
Outline

- QCD phase diagram and QGP
- Heavy-ion collisions
 - Collective phenomena
 - EbyE physics and criticality
 - Femtoscopy
- Macroscopic description

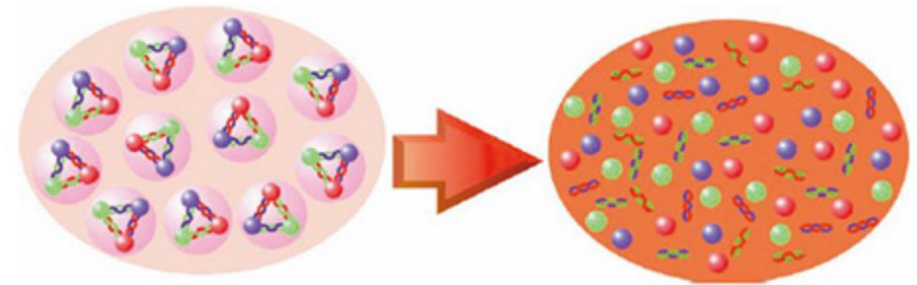


QCD phase diagram and Quark-Gluon Plasma

- Phase diagram of Quantum Chromodynamics (QCD)

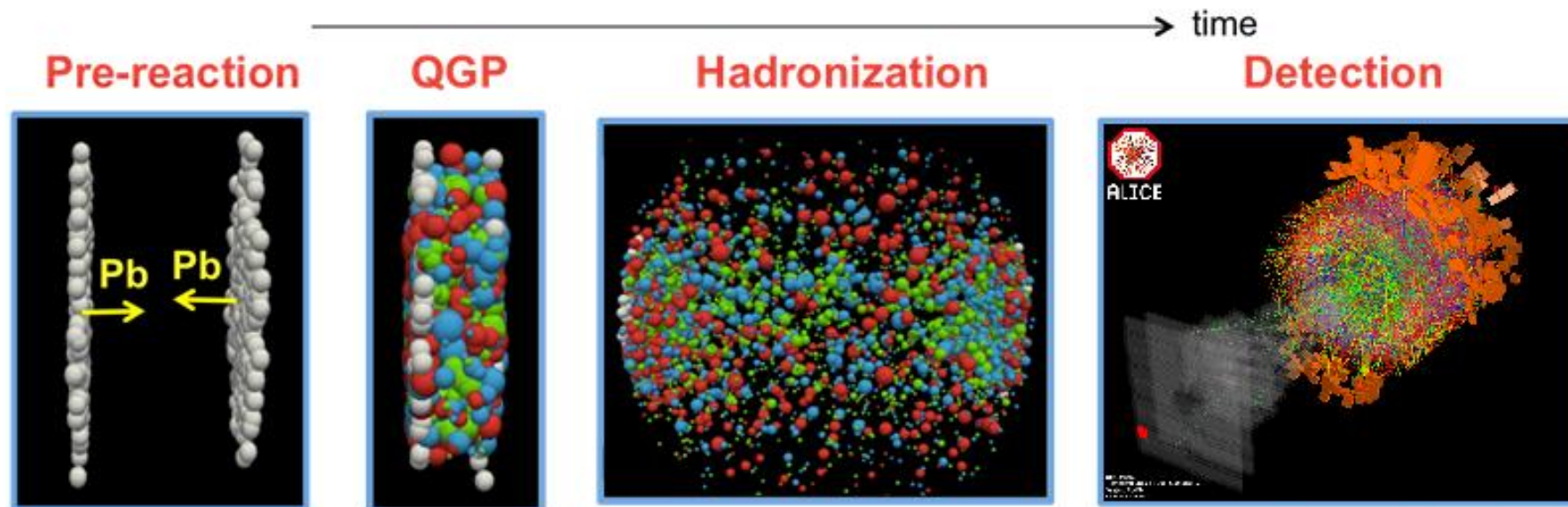


- Quark-Gluon Plasma (QGP): Extreme state of matter in which quarks and gluons can move freely over distances comparable to the size of hadrons



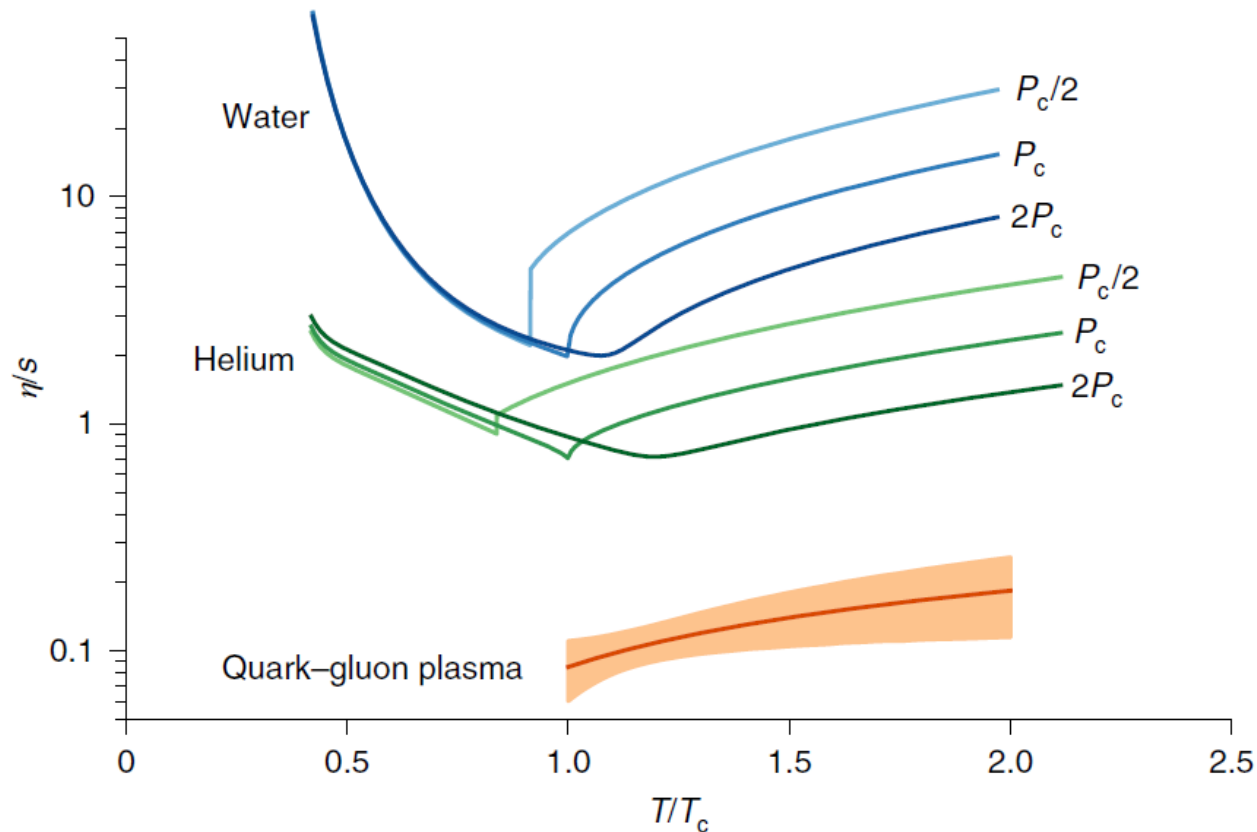
Our mission

- To study the properties of nuclear matter under extreme conditions
 - Quantify the properties of the Quark-Gluon Plasma
 - Demystify the nature of the strong nuclear force
 - Shed light on the evolution of the early Universe
 - Explore the QCD phase diagram



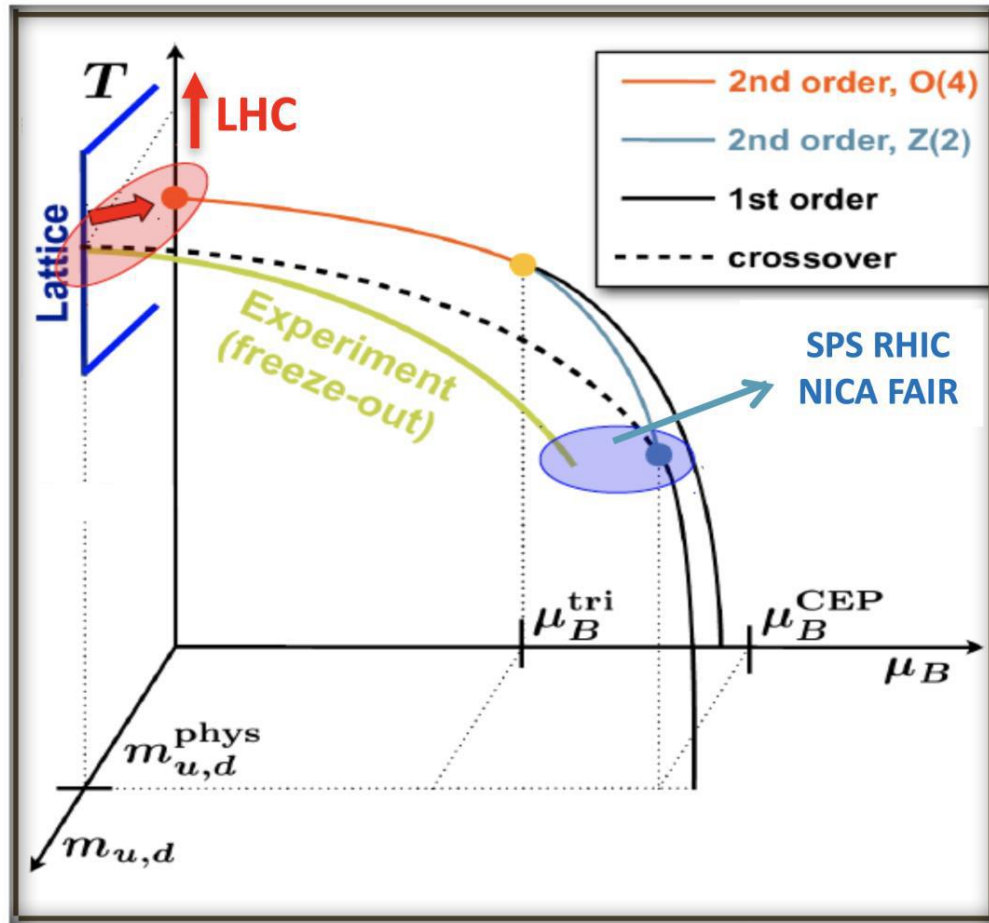
Example #1: Transport properties of QGP

- Temperature dependence of QGP's specific shear viscosity (η/s) is smallest of all known substances



Bernhard, J.E., Moreland, J.S. & Bass, S.A. *Nat. Phys.* **15**, 1113–1117 (2019),

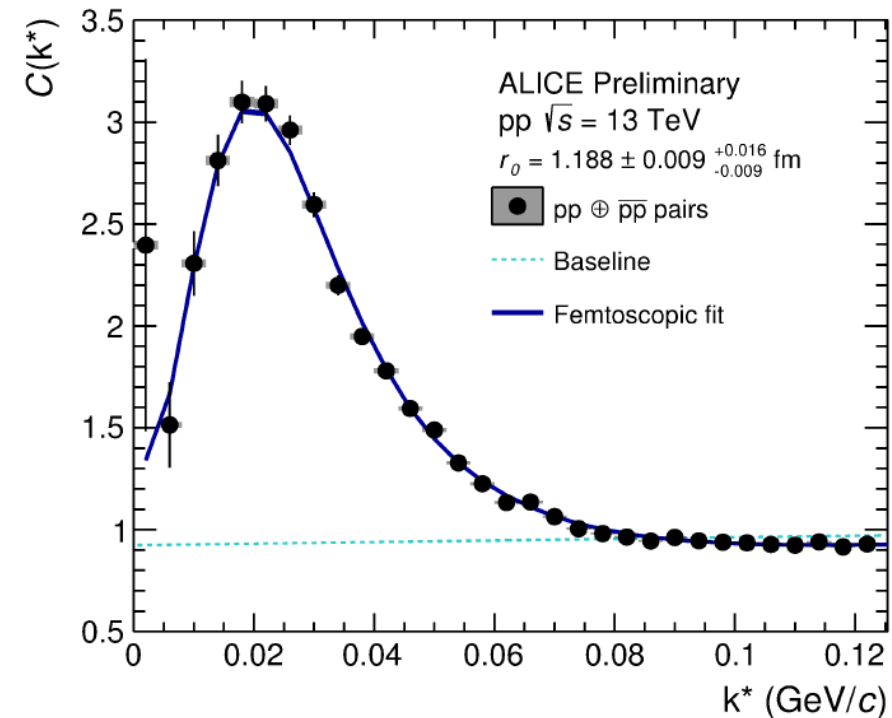
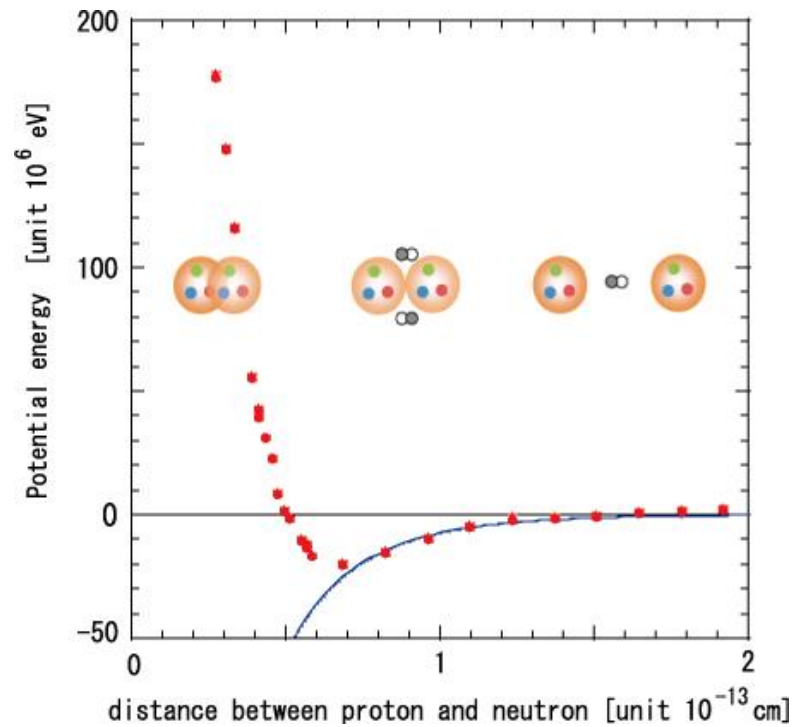
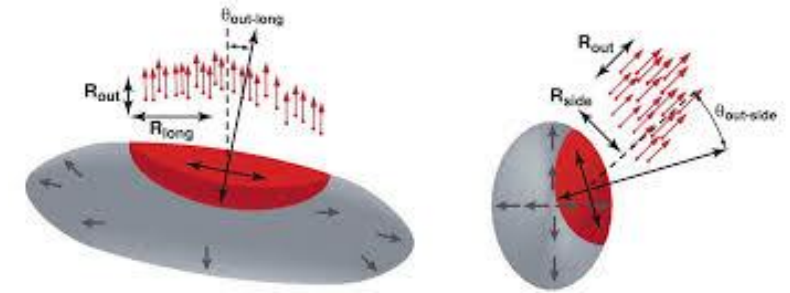
Example #2: Nature of QCD phase transitions



- QCD phase diagram of strongly interacting nuclear matter can be explored in ultrarelativistic heavy-ion collisions
- QGP phase is probed as a function of temperature and baryon chemical potential
- What is the nature of phase transitions in QCD phase diagram (smooth cross over, 1st or 2nd order phase transition, etc.)?
- Existence of critical point?

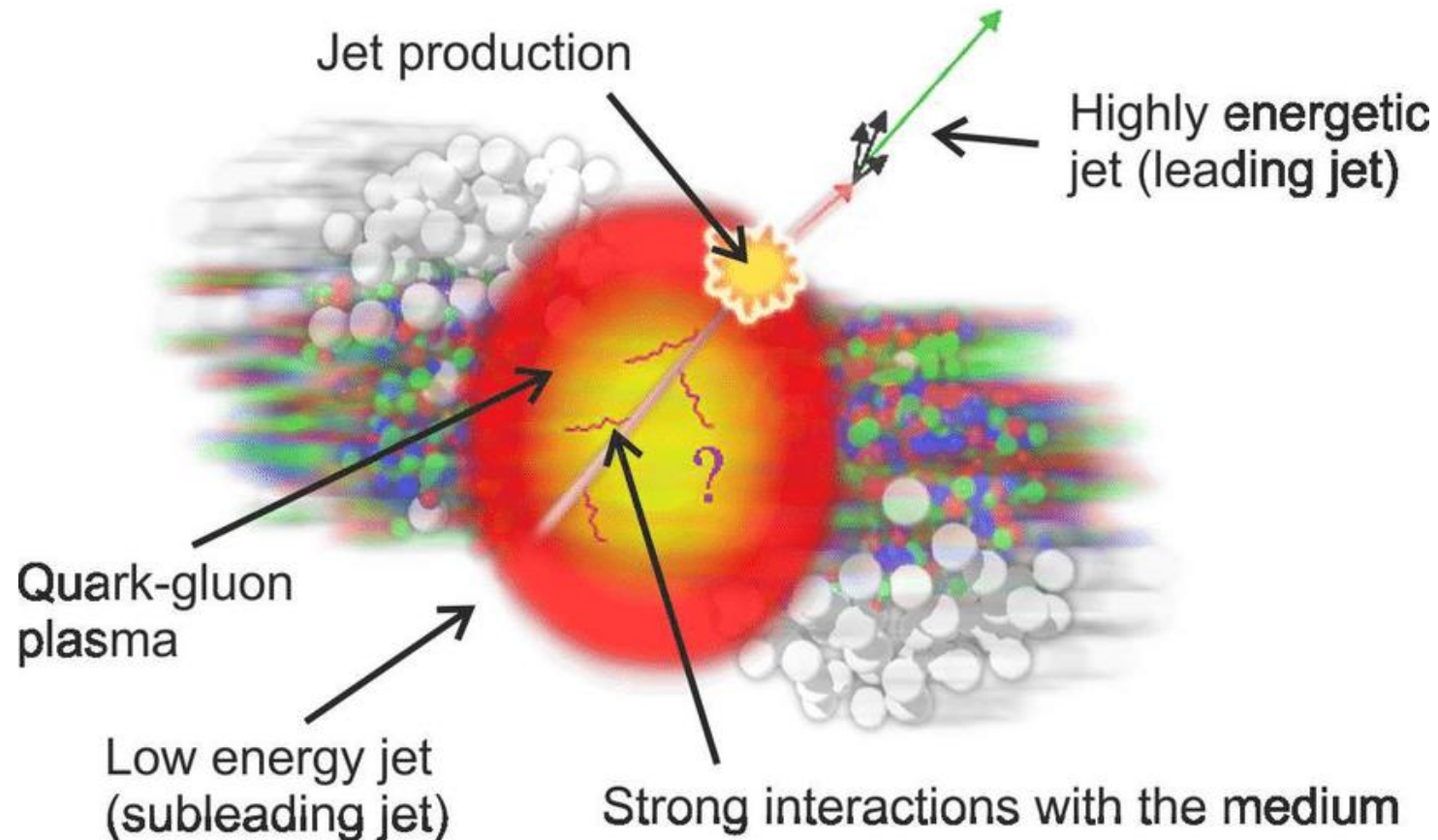
Example #3: Properties of strong nuclear force

- **Femtoscscopy:** How to quantify the details of:
 - Particle-emitting source in nuclear collisions
 - Interactions among produced particles (strong nuclear force, Bose-Einstein correlations, Coulomb force, ...)



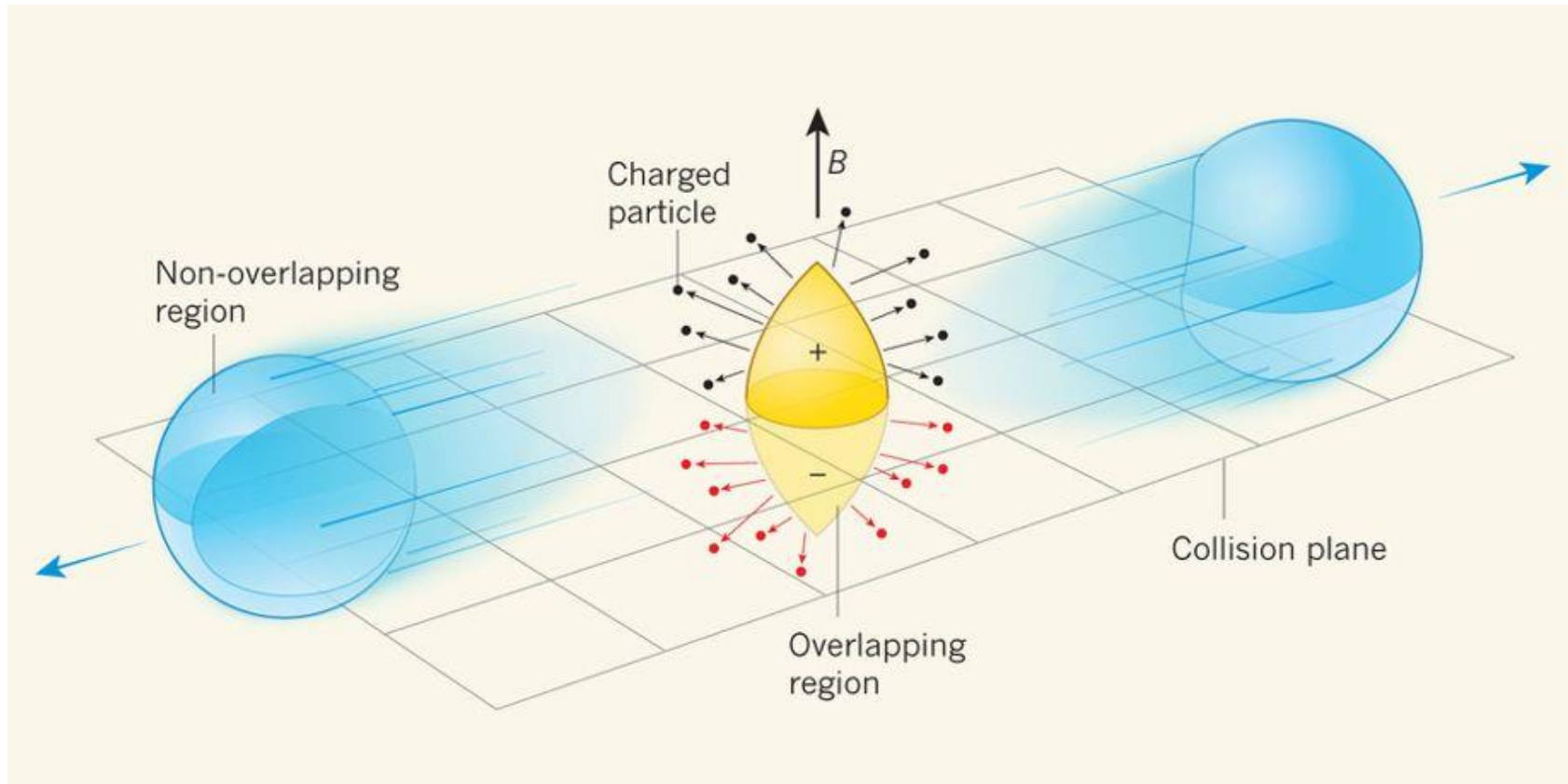
Example #4: Jet quenching

- Reduction of jet energy as it traverses and interacts with medium produced in heavy-ion collisions
 - Jet \sim 'high-energy parton in ultra-relativistic collisions'



Example #5: Chiral Magnetic Effect

- Separation of charge in the direction of the magnetic field perpendicular to the plane of collision

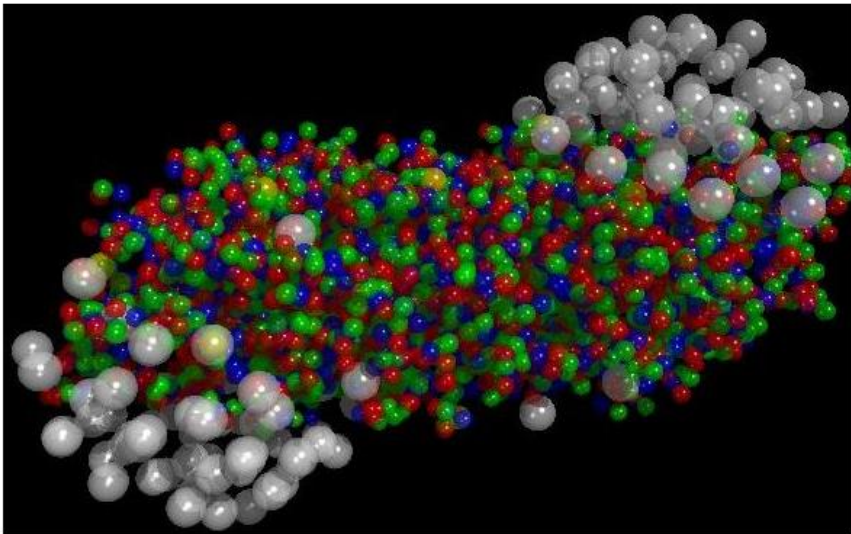


Parity violation in strong interactions?

QGP discovery: February 2000 @CERN

New State of Matter created at CERN

10 Feb 2000



Geneva, 10 February 2000. At a special seminar on 10 February, spokespersons from the experiments on CERN¹'s Heavy Ion programme presented compelling evidence for the existence of a new state of matter in which quarks, instead of being bound up into more complex particles such as protons and neutrons, are liberated to roam freely.

RHIC Scientists Serve Up 'Perfect' Liquid

New state of matter more remarkable than predicted — raising many new questions

April 18, 2005

TAMPA, FL — The four detector groups conducting research at the Relativistic Heavy Ion Collider (RHIC) — a giant atom "smasher" located at the U.S. Department of Energy's Brookhaven National Laboratory — say they've created a new state of hot, dense matter out of the quarks and gluons that are the basic particles of atomic nuclei, but it is a state quite different and even more remarkable than had been predicted. In peer-reviewed papers summarizing the first three years of RHIC findings, the scientists say that instead of behaving like a gas of free quarks and gluons, as was expected, the matter created in RHIC's heavy ion collisions appears to be more like a *liquid*.

"Once again, the physics research sponsored by the Department of Energy is producing historic results," said Secretary of Energy Samuel Bodman, a trained chemical engineer. "The DOE is the principal federal funder of basic research in the physical sciences, including nuclear and high-energy physics. With today's announcement we see that investment paying off."

"The truly stunning finding at RHIC that the new state of matter created in the collisions of gold ions is more like a liquid than a gas gives us a profound insight into the earliest moments of the universe," said Dr. Raymond L. Orbach, Director of the DOE Office of Science.

Also of great interest to many following progress at RHIC is the emerging connection between the collider's results and calculations using the methods of string theory, an approach that attempts to explain fundamental properties of the universe using 10 dimensions instead of the usual three spatial dimensions plus time.



Secretary of Energy Samuel Bodman

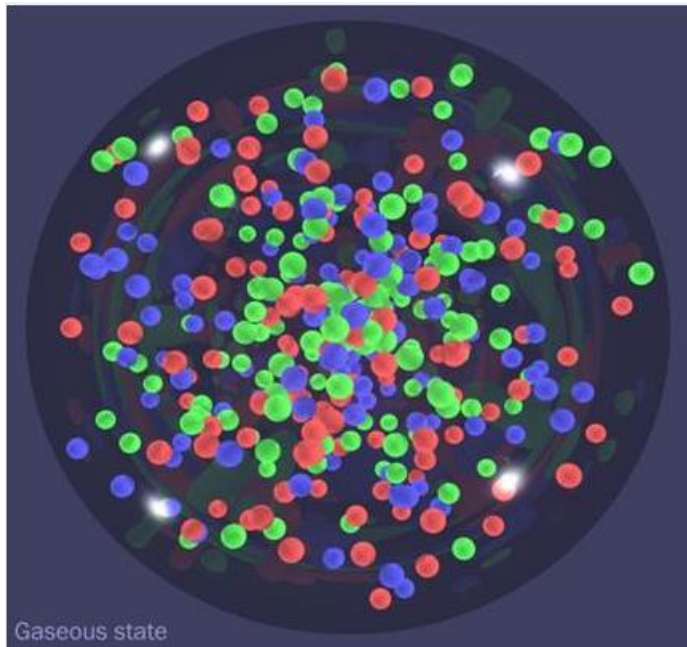
<https://press.cern/press-releases/2000/02/new-state-matter-created-cern>

<https://www.bnl.gov/newsroom/news.php?a=110303>

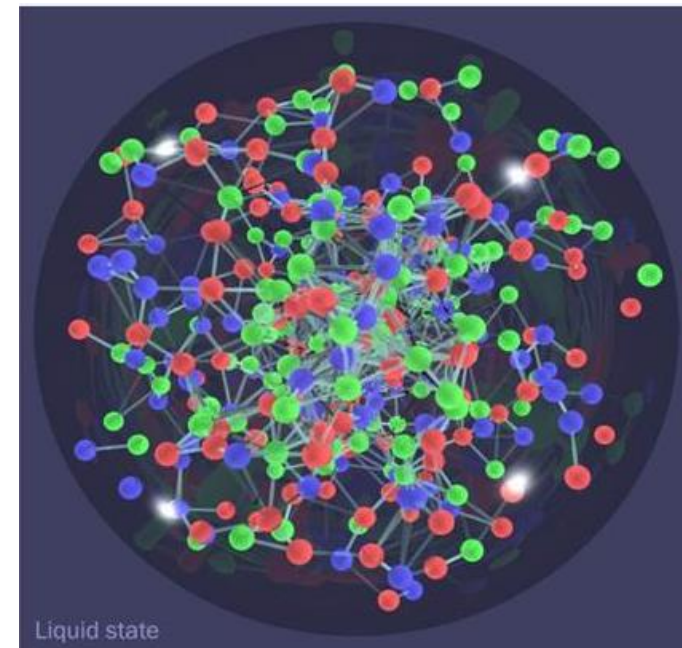
Unexpected QGP properties

- Paradigm shift with results at RHIC energies:

Expected: weakly interacting gas

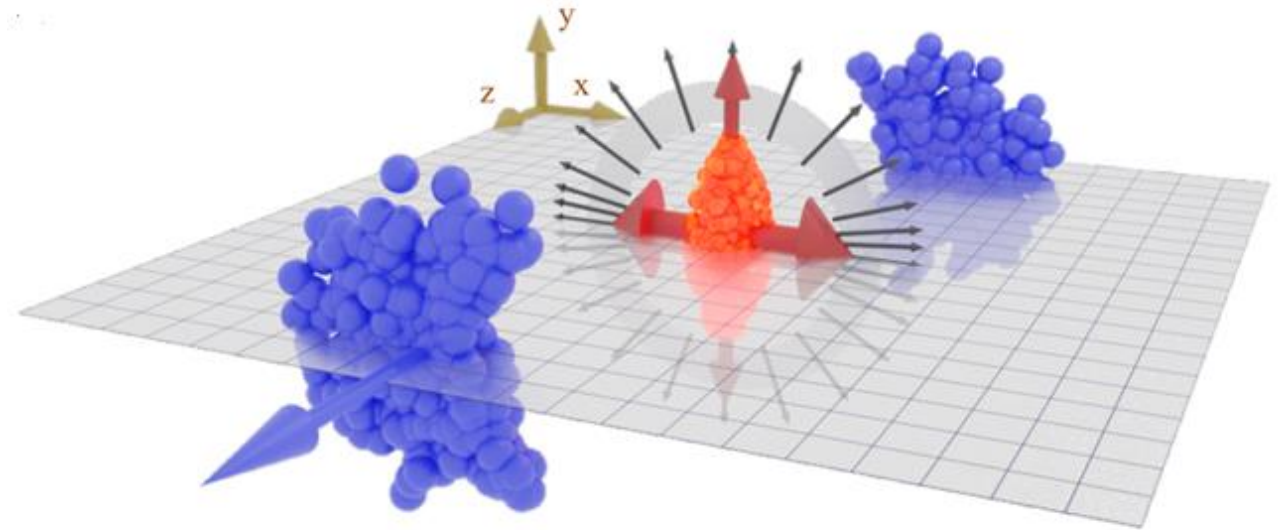


Observed: strongly coupled liquid



Heavy-ion collisions

- **Collective phenomena**
- EbyE physics and criticality
- Femtoscopy



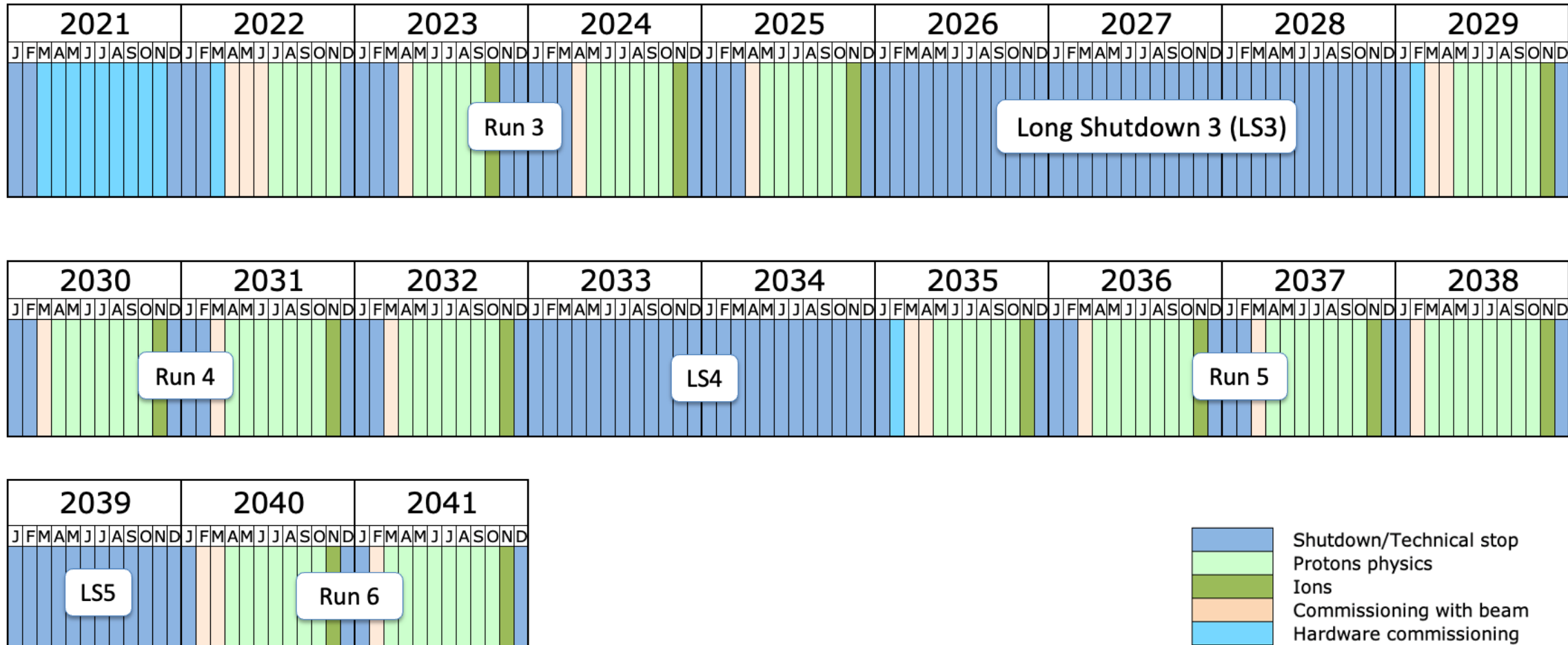
Credits: D.D. Chinellato, ICHEP 2020

Current status at CERN

- In **Run 3** operations at LHC, all four major experiments (ALICE, ATLAS, CMS, LHCb) are taking data
 - p+p collisions at 13.6 TeV (new record for collision energy!)
 - Pb+Pb collisions



Updated LHC timeline

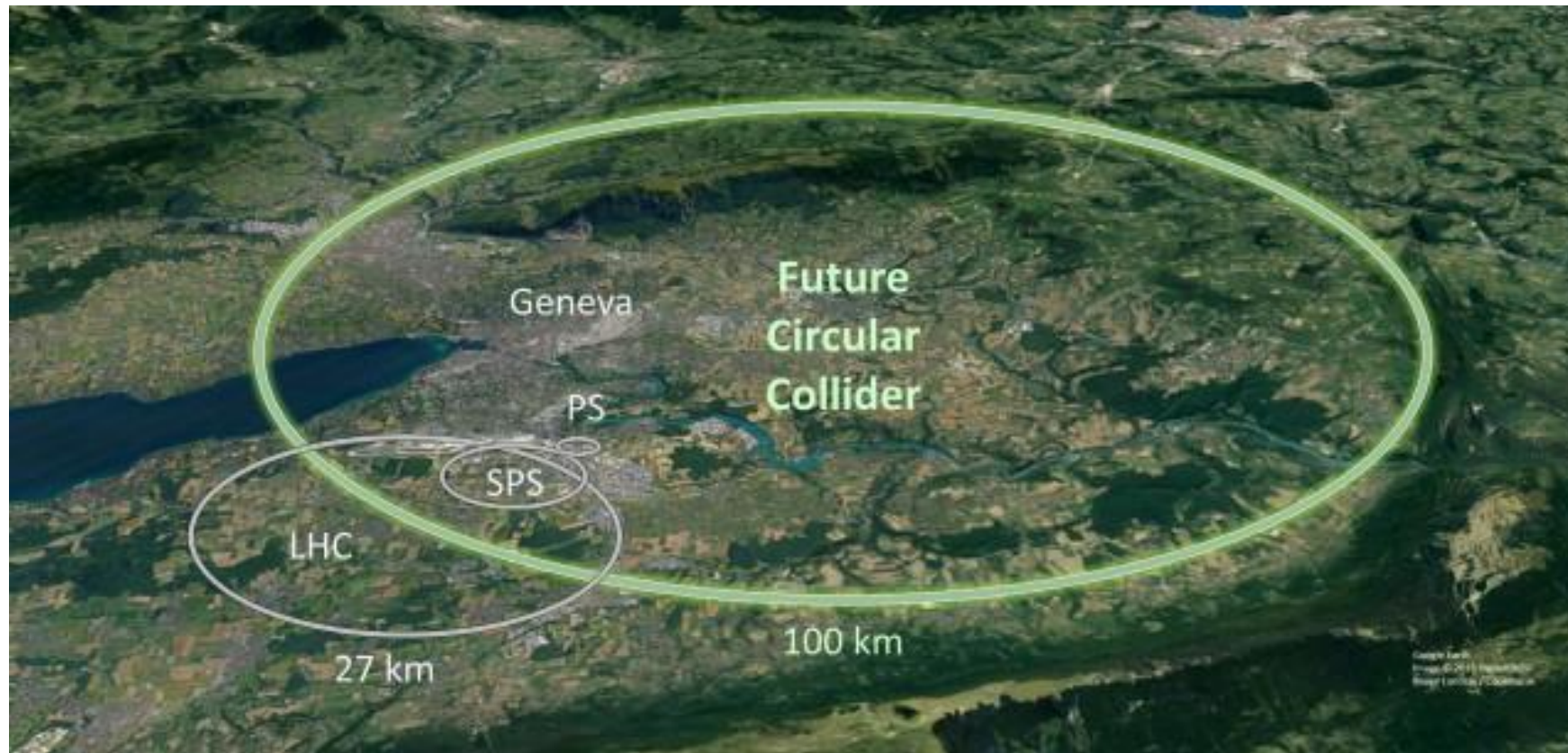


Last update: April 2023

Taken from <https://lhc-commissioning.web.cern.ch/schedule/LHC-long-term.htm>

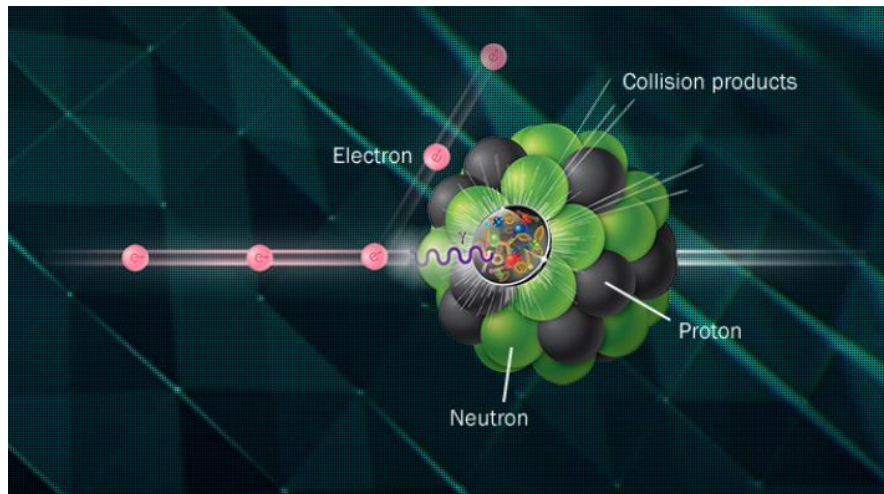
Distant future

- Future Circular Collider (FCC) @ CERN
 - Collisions at 100 TeV – new energy frontier

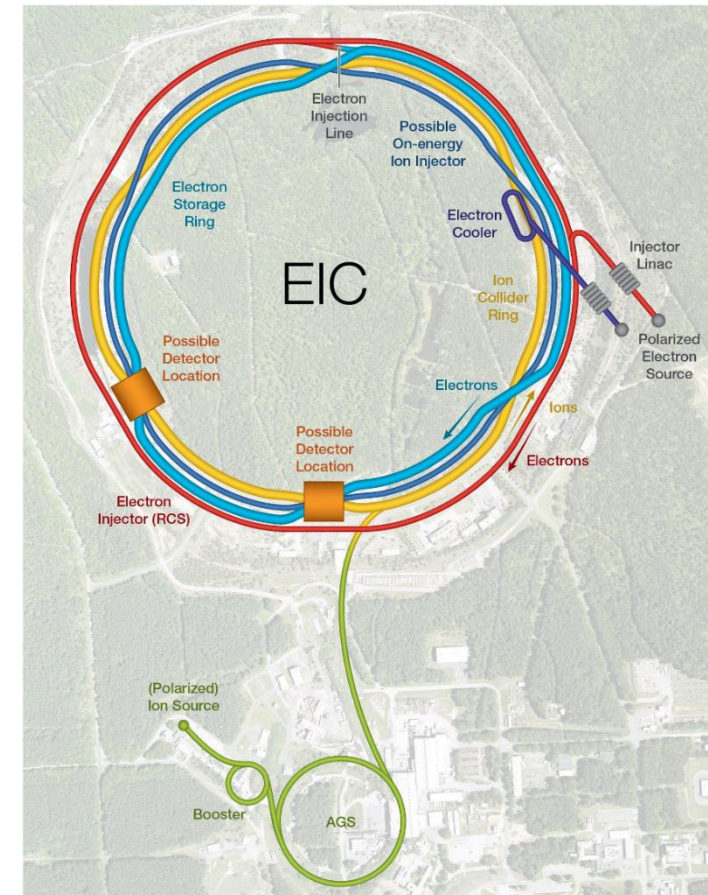


Coming next in US

- The Electron-Ion Collider (EIC) in Brookhaven, United States

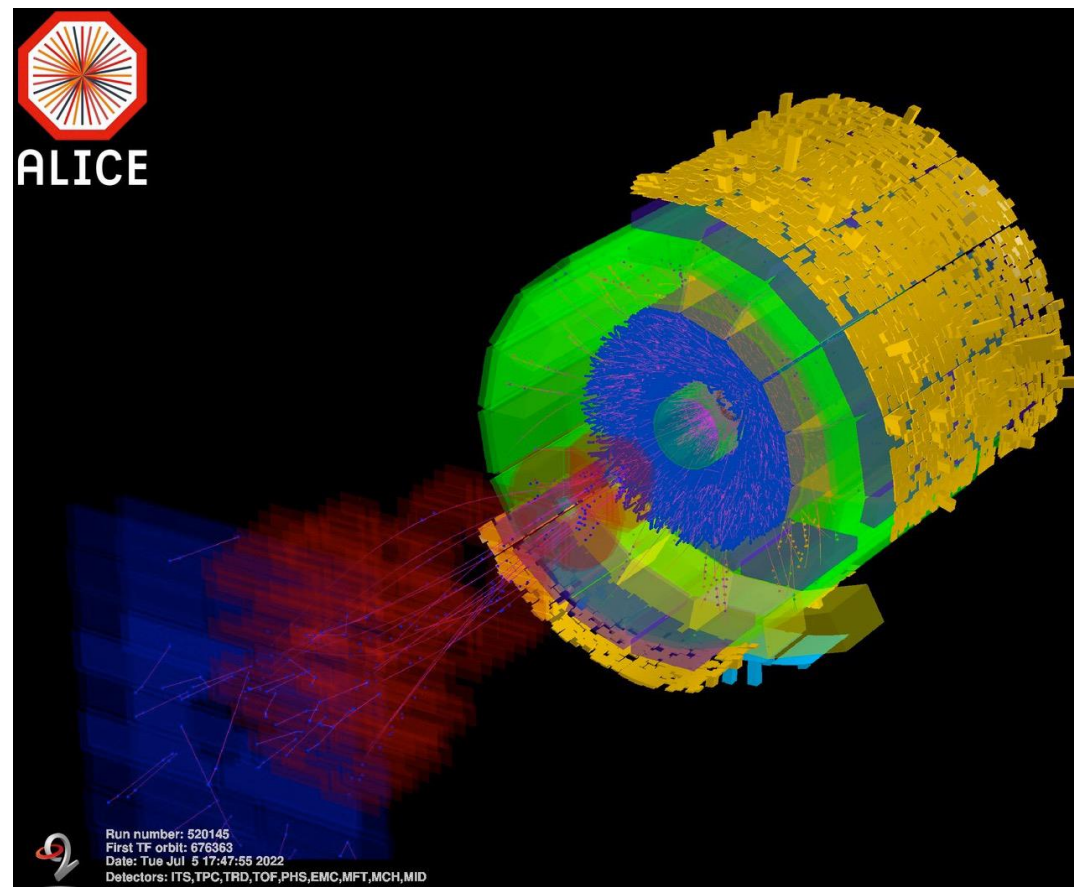
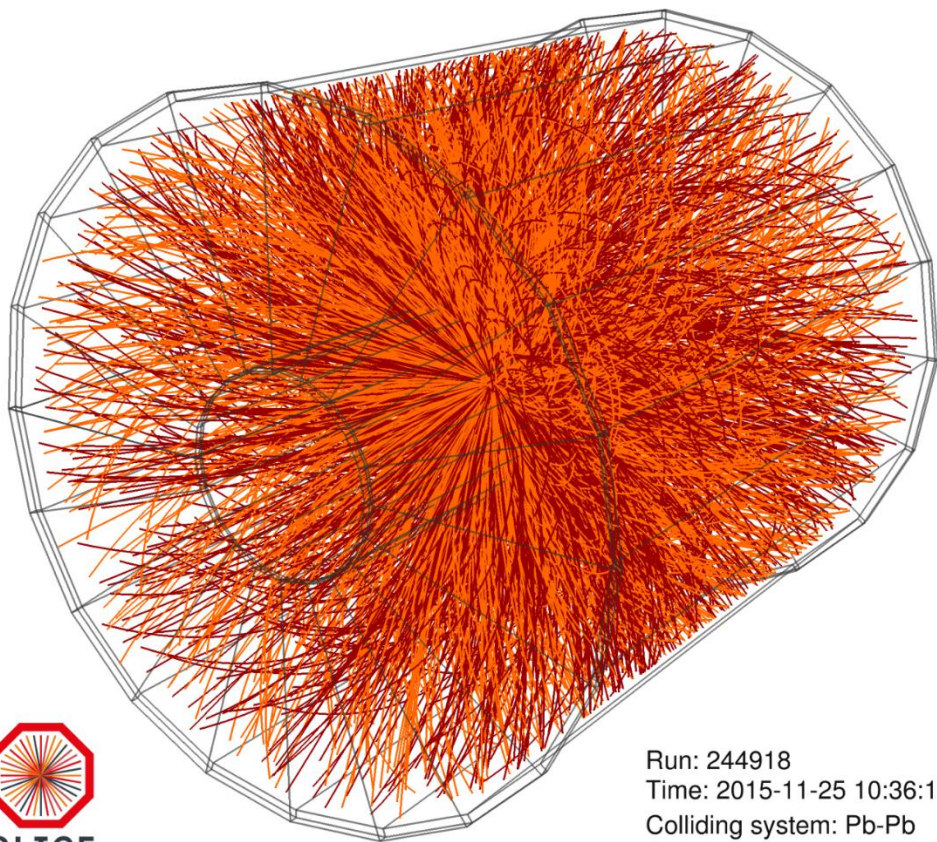


Taken from <https://www.bnl.gov/eic/>



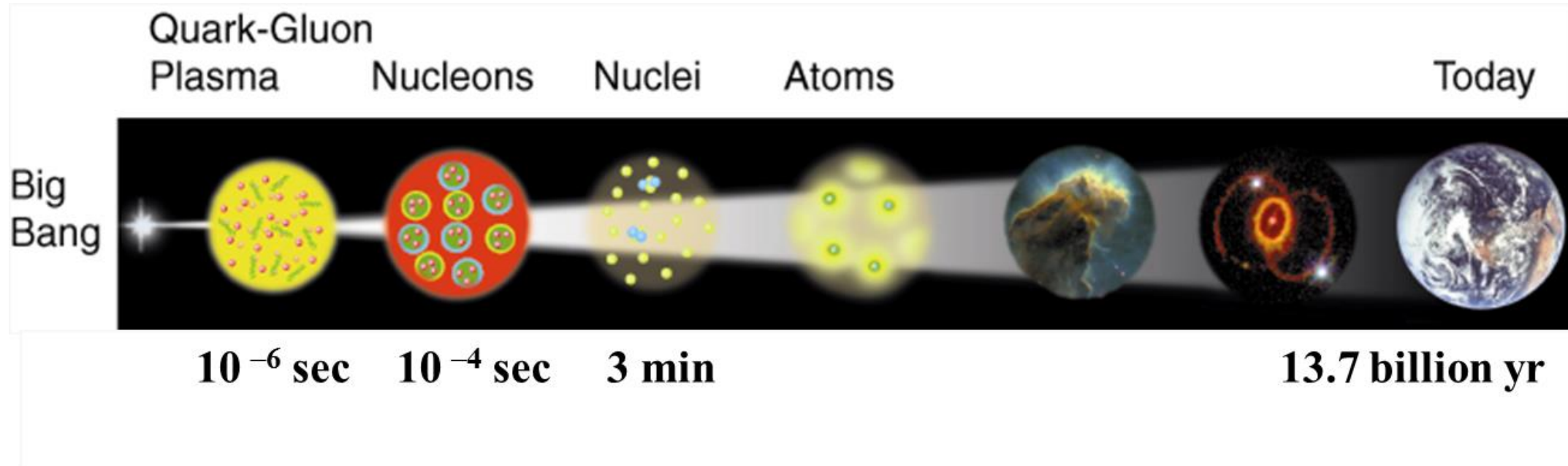
Heavy-ion collisions: Real event display

- How to extract some non-trivial physics out of this mess?



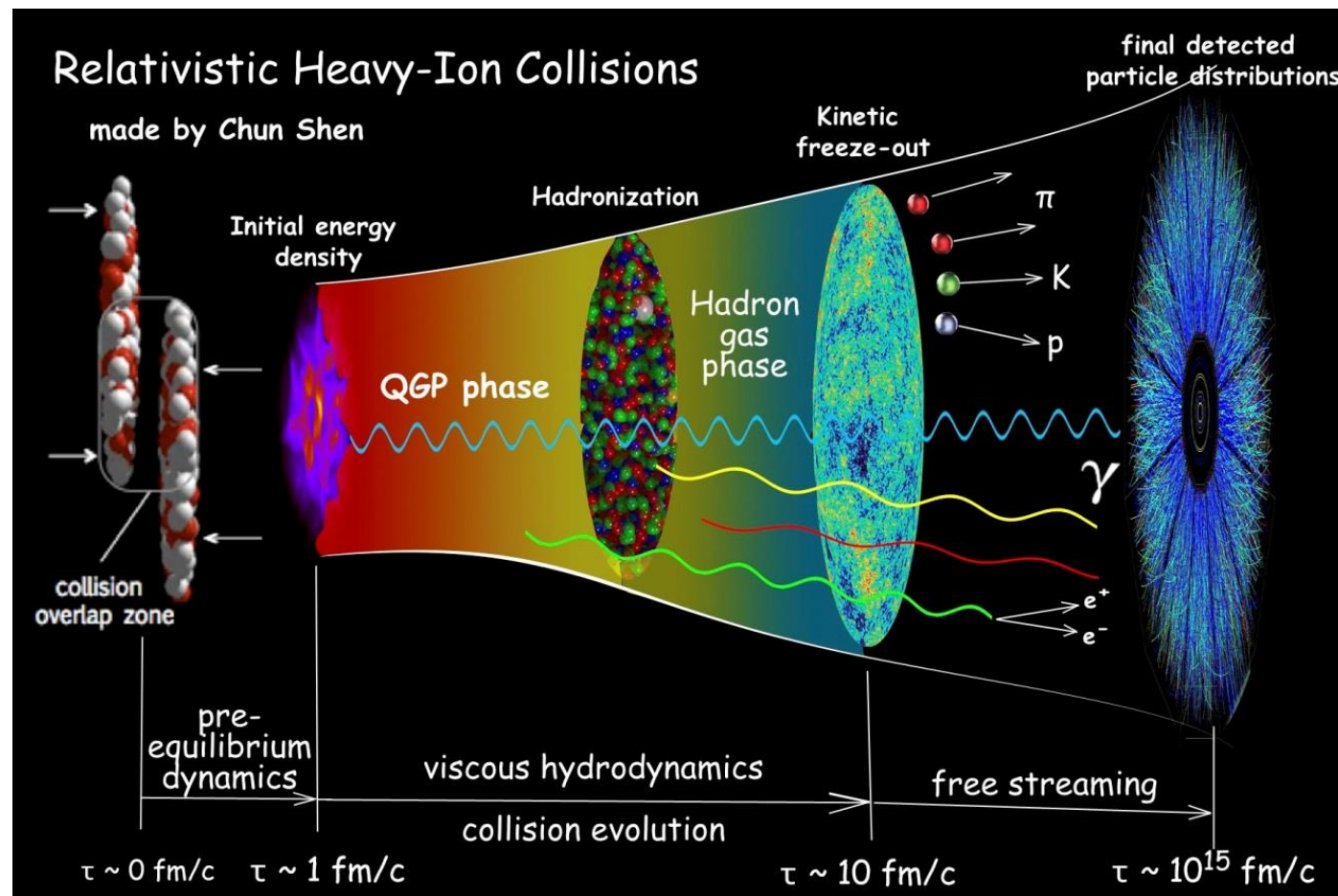
Once upon a time...

- Quark-Gluon Plasma has filled the universe few microseconds after the Big Bang!



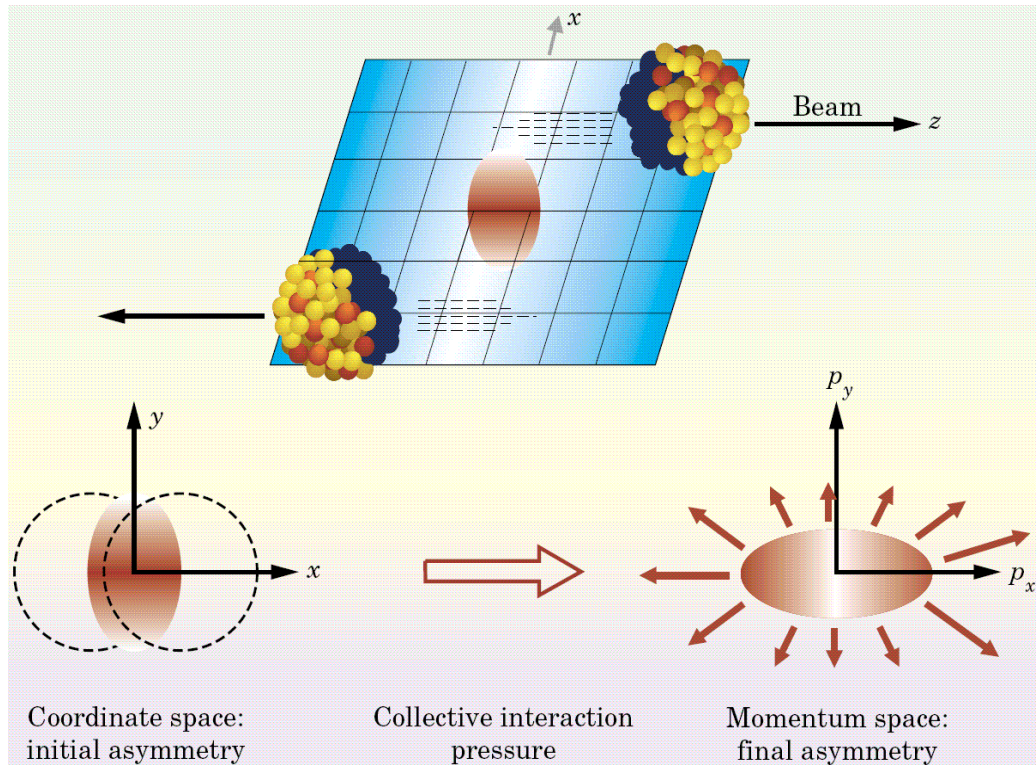
Recreating Universe with heavy-ion collisions?

- Not exactly, but nevertheless there are similarities...



Collective phenomena in heavy-ion collisions

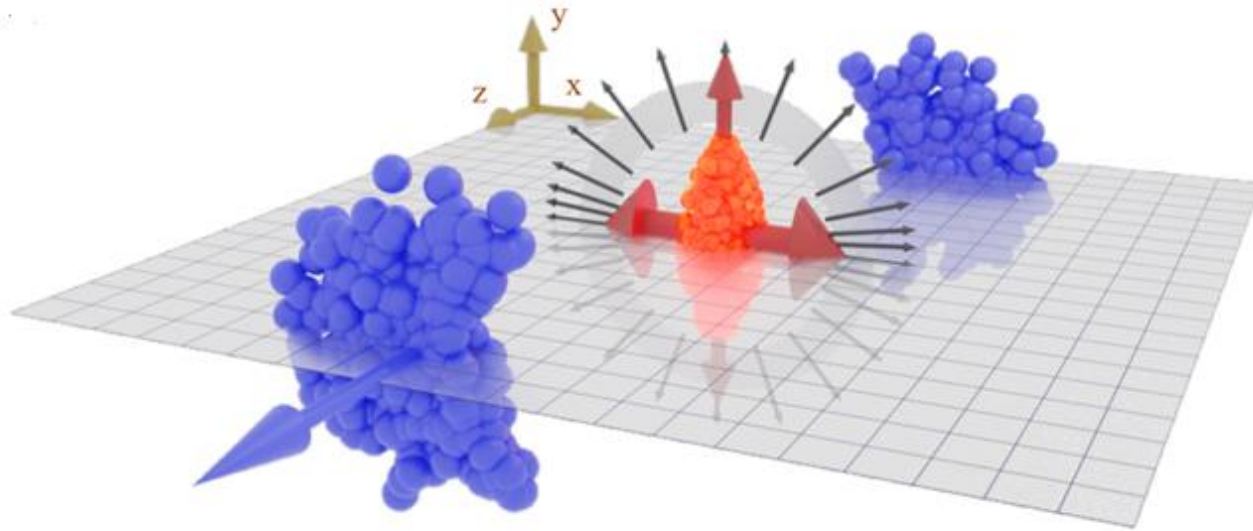
- Transfer of anisotropy from the initial coordinate space into the final momentum space via the thermalized medium:



- Anisotropic flow will develop in heavy-ion collisions only if both of the following two requirements are met:
 - Initial anisotropic volume in coordinate space
 - Thermalized medium

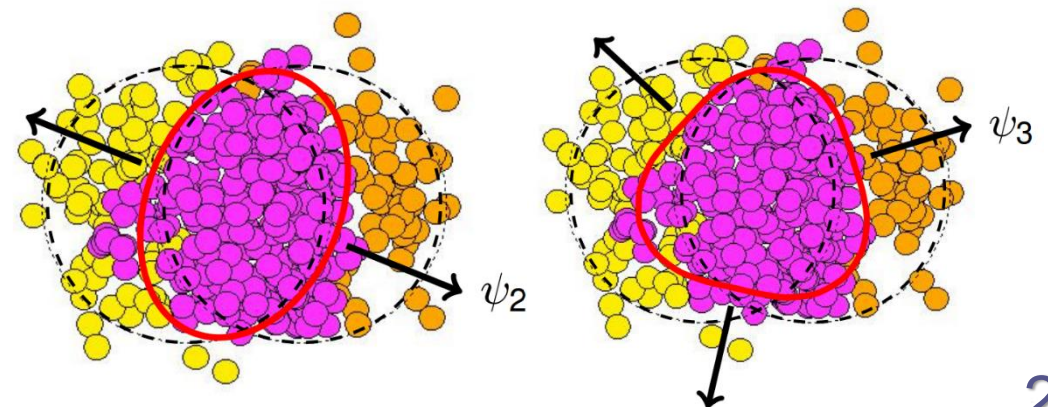
Initial anisotropic volume

- **Non-central** heavy-ion collision is a prime example
 - Trivially (#1): Due to geometry of collision the resulting volume containing interacting matter is anisotropic in coordinate space
 - Trivially (#2): To leading order this anisotropic volume is ellipsoidal



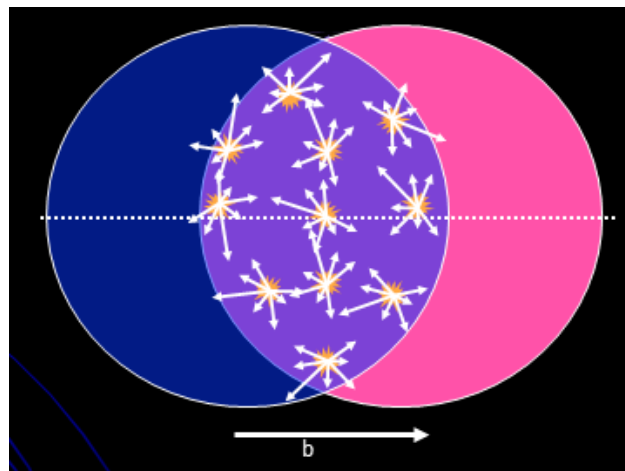
Credits: D.D. Chinellato, ICHEP 2020

- More subtle cases of initial anisotropic volume can occur due to **fluctuations of participating nucleons**

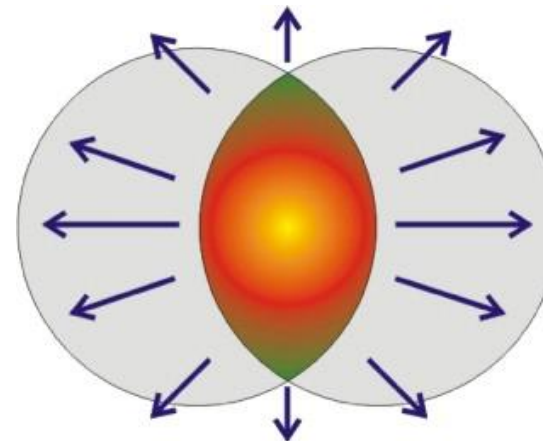
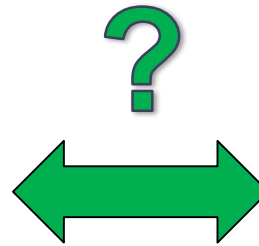


Transfer of anisotropy

- Two conceptually different notions of anisotropy:
 - **Coordinate space anisotropy:** Is the volume containing the interacting particles produced in a heavy-ion collision anisotropic or not?
 - **Momentum space anisotropy:** Is the final-state azimuthal distribution of resulting particles recorded in the detector anisotropic or not?
- A priori these anisotropies unrelated, in practice **correlated**

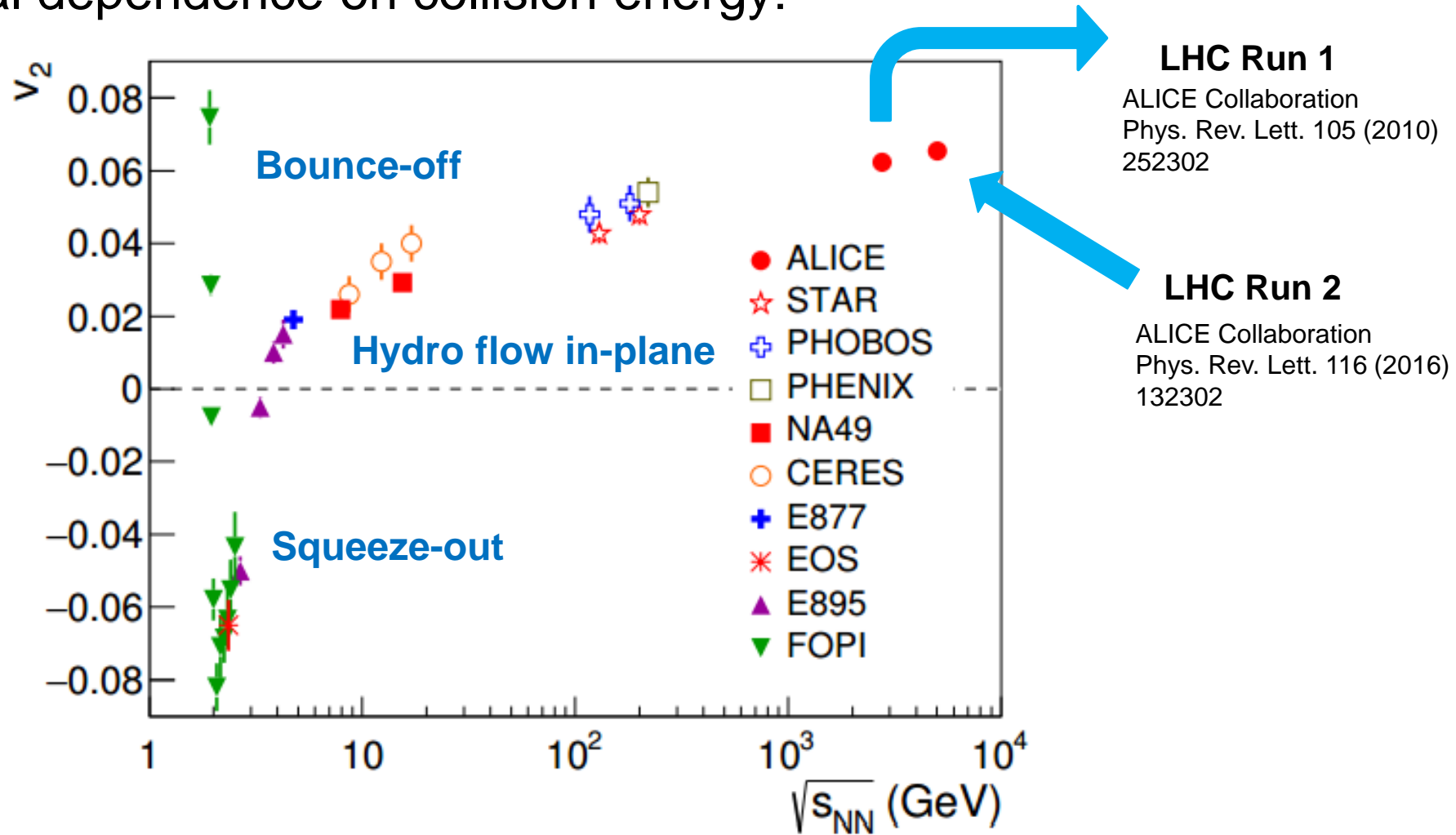


made by Mike Lisa



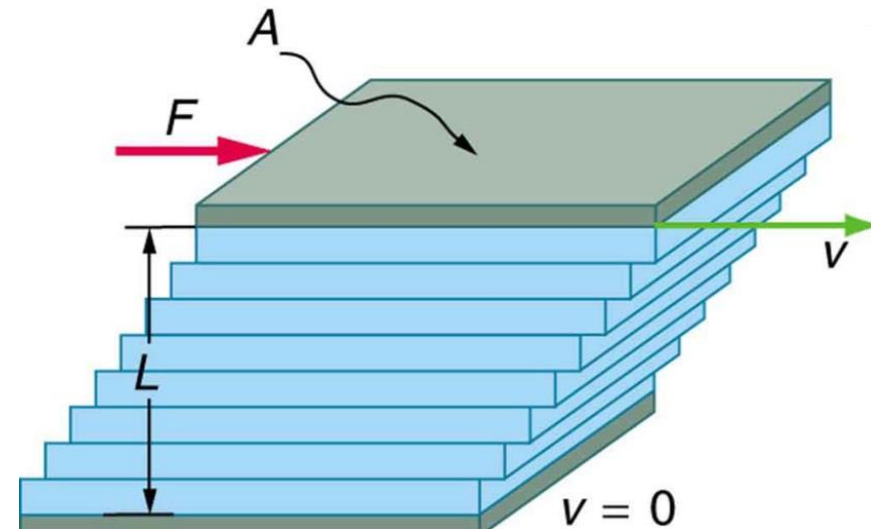
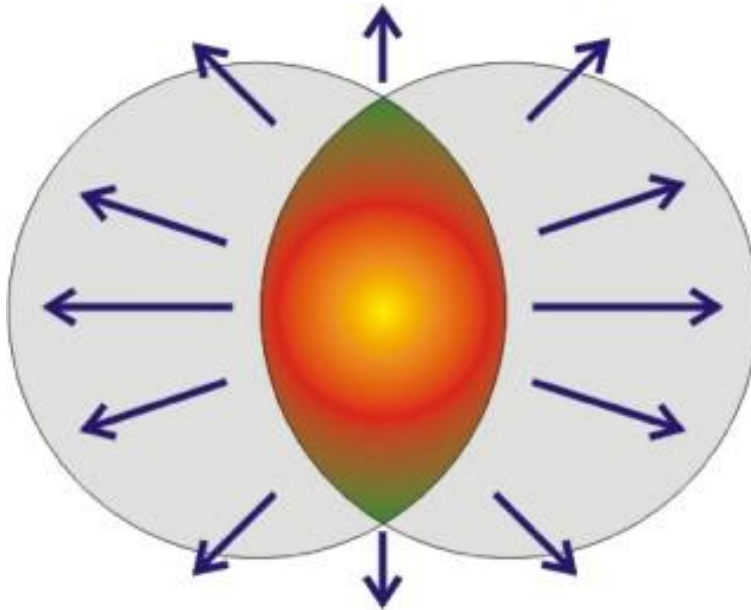
Historical snapshot

- Non-trivial dependence on collision energy:



Hydro flow in-plane

- Non-trivial effect which is sensitive to transport coefficients of QGP (e.g. its shear viscosity)

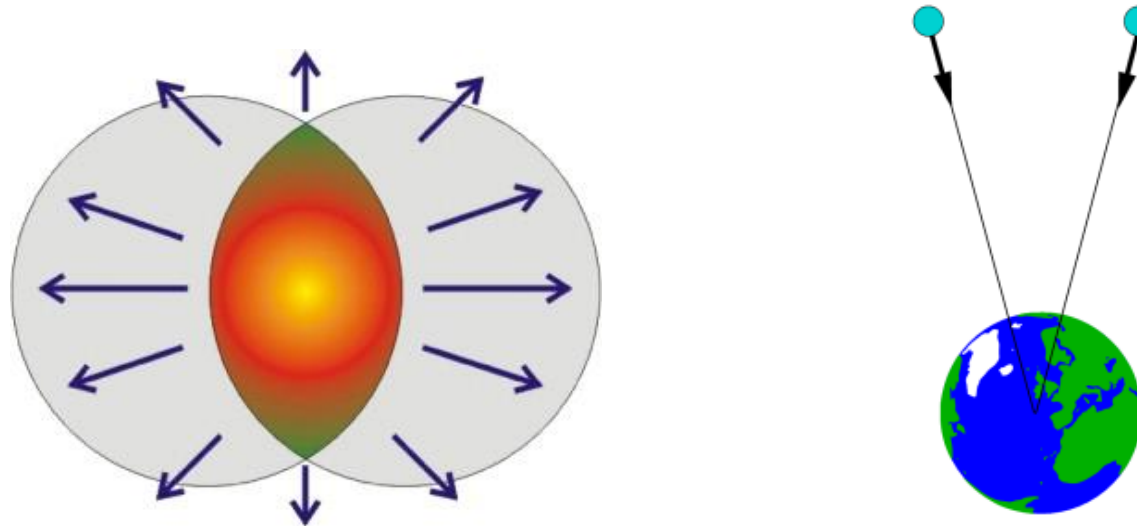


If anisotropic flow has developed, neighboring layers are moving at different relative velocities, parallel displacement is opposed by shear viscosity

large anisotropic flow \Leftrightarrow small shear viscosity

Flow measurements and observables

- The ‘flow principle’: Correlations among all produced particles are induced solely by correlation of each single particle to the collision geometry



- Analogy with the falling bodies in gravitational field (rhs)
- Whether or not particles are emitted simultaneously, or one by one, trajectories are the same

Fourier series

- In flow analysis, anisotropic emission of particles in the transverse plane after heavy-ion collision is described by:

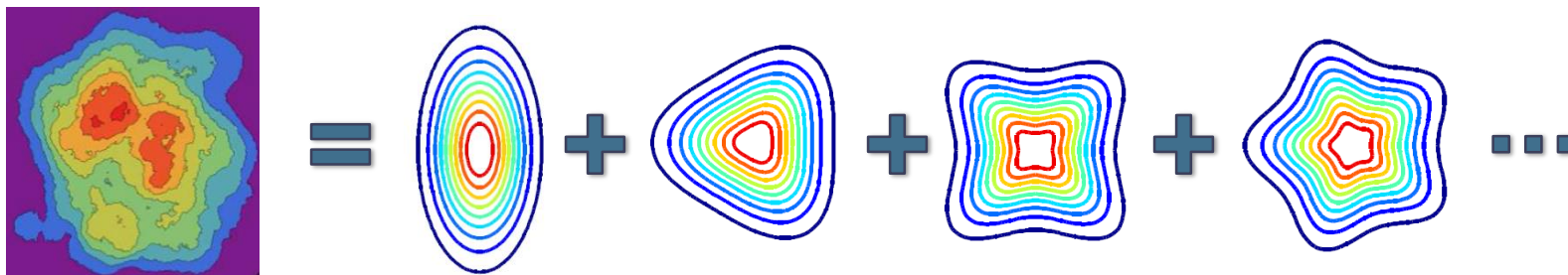
$$f(\varphi) = \frac{1}{2\pi} \left[1 + 2 \sum_{n=1}^{\infty} v_n \cos[n(\varphi - \Psi_n)] \right]$$

- v_n : flow amplitudes
- Ψ_n : symmetry planes
- Anisotropic flow is quantified with v_n and Ψ_n
 - v_1 is directed flow
 - v_2 is elliptic flow
 - v_3 is triangular flow
 - v_4 is quadrangular flow, etc.

S. Voloshin and Y. Zhang, Z.Phys. C70 (1996) 665-672

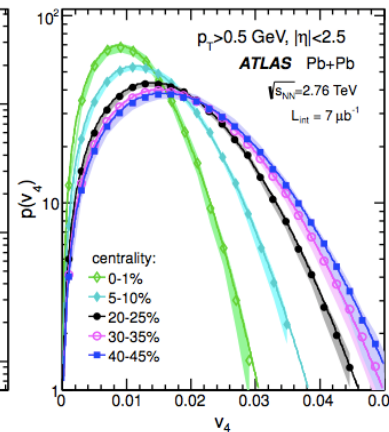
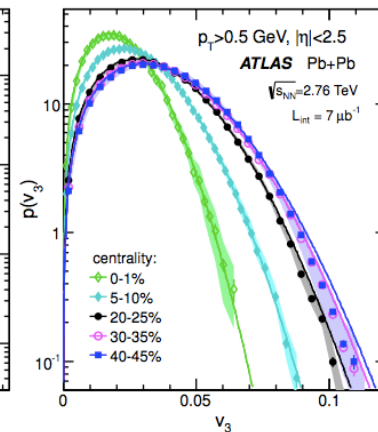
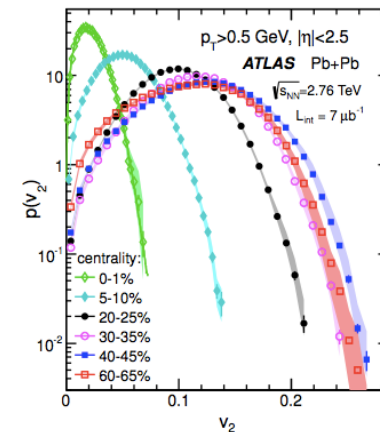
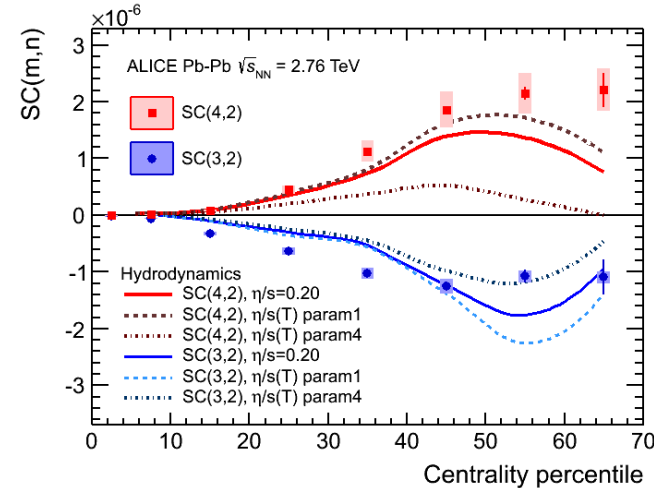
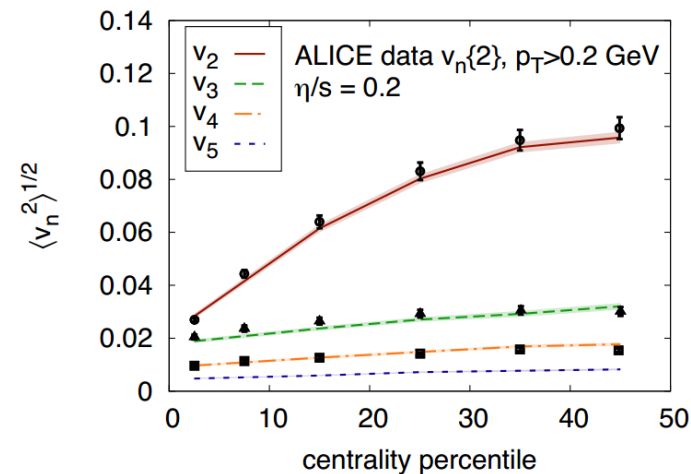
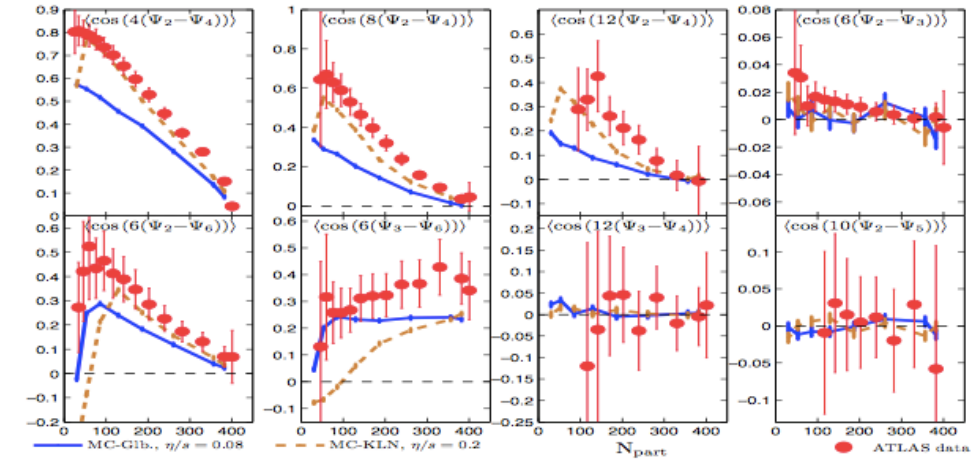
Fourier series

- In **non-central** heavy-ion collisions, due to collision geometry the initial volume is almond shaped (ellipsoidal)
 - Dominant harmonic is v_2 (elliptic flow)
- In **most central (head-on)** collisions, due to fluctuations of participating nucleons any shape can develop, all (lower) order harmonics are equally probable



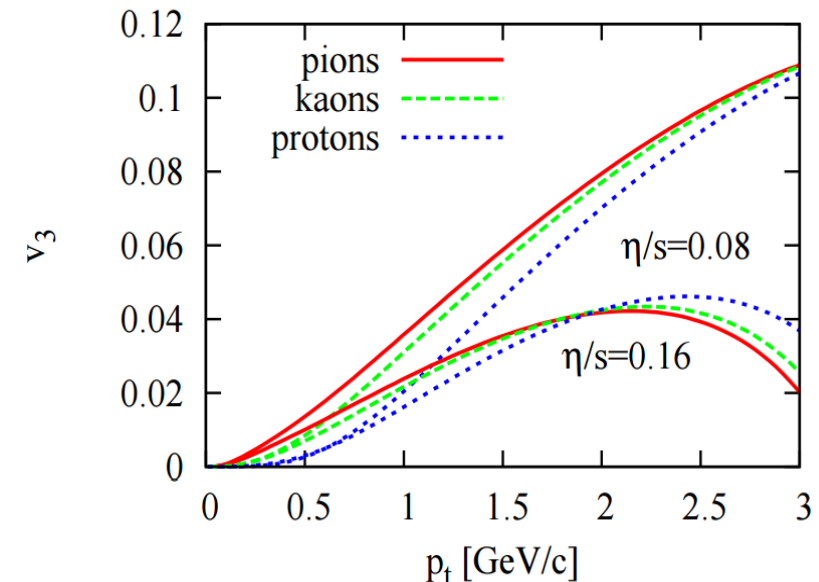
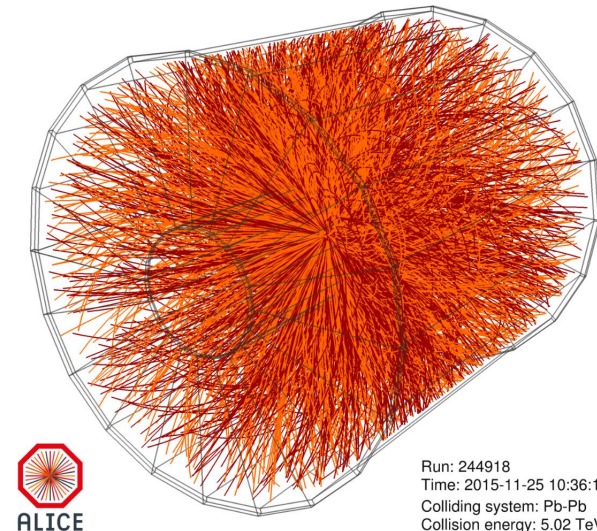
Flow observables

- Individual flow harmonics: $v_1, v_2, v_3, v_4, \dots$
- Correlations between harmonics: $\langle v_m^2 v_n^2 \rangle - \langle v_m^2 \rangle \langle v_n^2 \rangle$
- Symmetry plane correlations: $\langle \cos[mn(\Psi_m - \Psi_n)] \rangle$
- Probability density function: $P(v_n)$
- ...



Why so many flow observables?

- All **independent** flow observables are welcome!
 - Heavy-ion collision is a rather complex system and we cannot describe everything only with few parameters
- Different observables exhibit different sensitivities to QGP properties
 - Example from theory: Transverse momentum dependence of triangular flow is different for different values of QGP's shear viscosity



Independent information

- To constrain the functional form of $f(v_1, \dots, v_n, \Psi_1, \dots, \Psi_n)$, we have to measure all its **valid moments (or cumulants)**
- The most general mathematical result, which relates multiparticle azimuthal correlators and flow degrees of freedom:

$$\left\langle e^{i(n_1\varphi_1 + \dots + n_k\varphi_k)} \right\rangle = v_{n_1} \cdots v_{n_k} e^{i(n_1\Psi_{n_1} + \dots + n_k\Psi_{n_k})}$$

R. S. Bhalerao, M. Luzum and J.-Y. Ollitrault, Phys. Rev. C **84** 034910 (2011)

Open questions:

- What are the smallest collision systems and energies at which QGP can be formed?
- How to extract new and independent constraints from the available heavy-ion data?
- Is the observed universality of flow measurements in vastly different collision systems physical, or just a subtle artifact of using correlation techniques in the randomized data set?

Fluctuations, p.d.f., moments, cumulants

- Properties of random (stochastic) observable v of interest are specified by functional form of probability density function (p.d.f.) $f(v)$
- Different moments carry by definition independent information about the underlying p.d.f. $f(v_n)$

$$\langle v_n^k \rangle \equiv \int v_n^k f(v_n) dv_n$$

- Two completely different p.d.f.'s $f(v_n)$ can have first moment $\langle v_n \rangle$ to be the same, and all higher-order moments different
- Is it mathematically equivalent to specify functional form $f(v_n)$ and all its moments $\langle v_n^k \rangle$?
- A priori it is not guaranteed that a p.d.f. $f(v_n)$ is uniquely determined by its moments $\langle v_n^k \rangle$
 - Necessary and sufficient conditions have been worked out only recently

$$K[f] \equiv \int_0^\infty \frac{-\ln f(x^2)}{1+x^2} dx \quad \Rightarrow \quad K[f] = \infty$$

Krein-Lin conditions (1997)

$$L(x) \equiv -\frac{xf'(x)}{f(x)} \quad \Rightarrow \quad \lim_{x \rightarrow \infty} L(x) = \infty$$

J. Stoyanov, Section 3 in '*Determinacy of distributions by their moments*', Proceedings 2006

2-particle cumulants in general

- Cumulants are alternative to moments to describe stochastic properties of variable
- If 2 p.d.f.'s have same moments, they will also have same cumulants, and vice versa
 - True both for univariate and multivariate case
- X_i denotes the general i -th stochastic variable
- The most general decomposition of 2-particle correlation is:

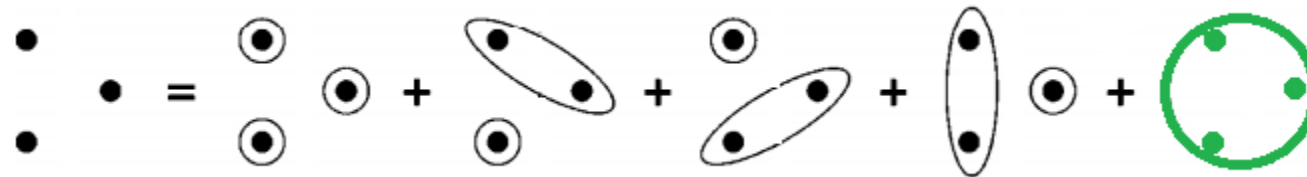
$$\langle X_1 X_2 \rangle = \langle X_1 \rangle \langle X_2 \rangle + \langle X_1 X_2 \rangle_c$$

- By definition, the 2nd term on RHS is 2-particle cumulant
- Cumulants cannot be measured directly, however:

$$\langle X_1 X_2 \rangle_c = \langle X_1 X_2 \rangle - \langle X_1 \rangle \langle X_2 \rangle$$

3-particle cumulants in general

- The most general decomposition of 3-particle correlation is:



- Or written mathematically:

$$\begin{aligned} \langle X_1 X_2 X_3 \rangle &= \langle X_1 \rangle \langle X_2 \rangle \langle X_3 \rangle \\ &+ \langle X_1 X_2 \rangle_c \langle X_3 \rangle + \langle X_1 X_3 \rangle_c \langle X_2 \rangle + \langle X_2 X_3 \rangle_c \langle X_1 \rangle \\ &+ \langle X_1 X_2 X_3 \rangle_c \end{aligned}$$

- The key point: 2-particle cumulants were expressed independently in terms of measured correlations in previous step!

$$\langle X_1 X_2 \rangle_c = \langle X_1 X_2 \rangle - \langle X_1 \rangle \langle X_2 \rangle$$

3-particle cumulants in general

- Working recursively from higher to lower orders, we eventually have 3-particle cumulant expressed in terms of measured 3-, 2-, and 1-particle averages

$$\begin{aligned}\langle X_1 X_2 X_3 \rangle_c &= \langle X_1 X_2 X_3 \rangle \\ &- \langle X_1 X_2 \rangle \langle X_3 \rangle - \langle X_1 X_3 \rangle \langle X_2 \rangle - \langle X_2 X_3 \rangle \langle X_1 \rangle \\ &+ 2 \langle X_1 \rangle \langle X_2 \rangle \langle X_3 \rangle\end{aligned}$$

- In the same way, cumulants can be expressed in terms of measurable averages for any number of particles
 - The number of terms grows rapidly

A bit of math

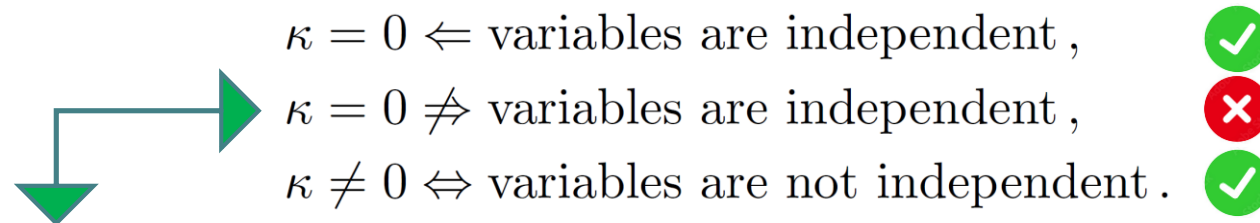
- The mathematical foundation of cumulants is well established!

Theorem: A cumulant $\langle X_i X_j \dots \rangle_c$ is zero if the elements X_i, X_j, \dots are divided in two or more groups which are statistically independent.

Corollary: A cumulant is zero if one of the variables in it is independent of the others. Conversely, a cumulant is not zero if and only if the variables in it are statistically connected.

Kubo, Journal of the Physical Society of Japan, Vol. 17, No. 7, (1962)

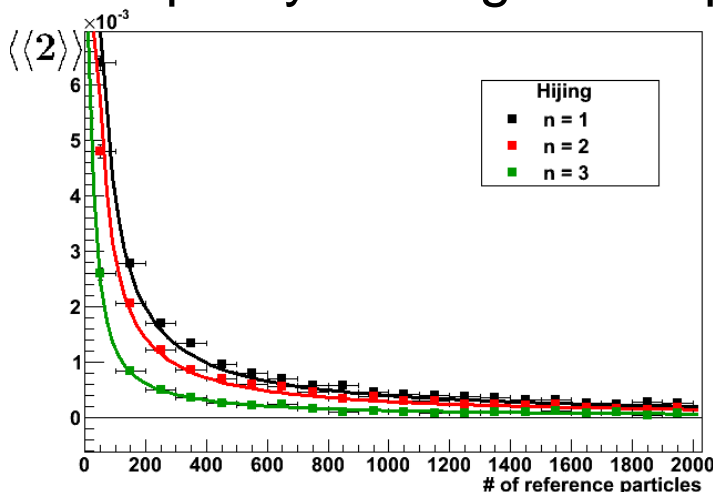
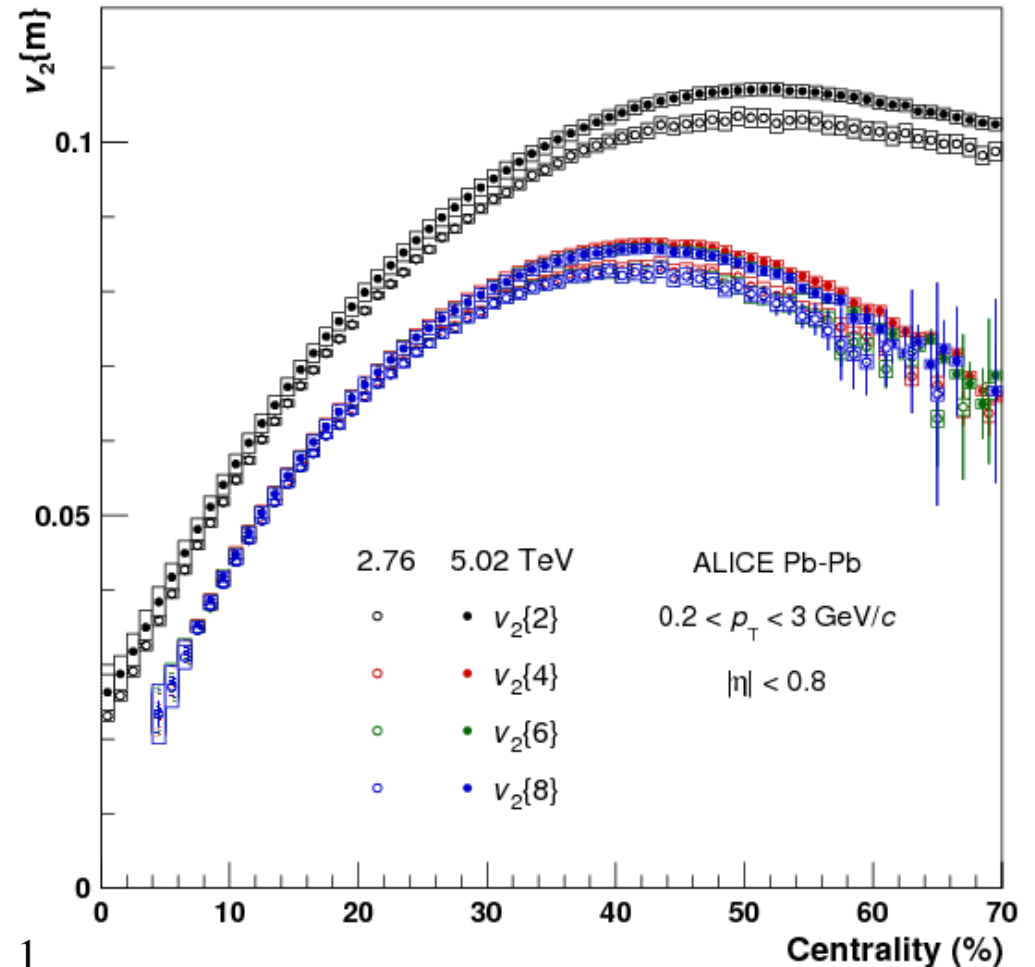
- Careful reading is mandatory, one statement is not covered:

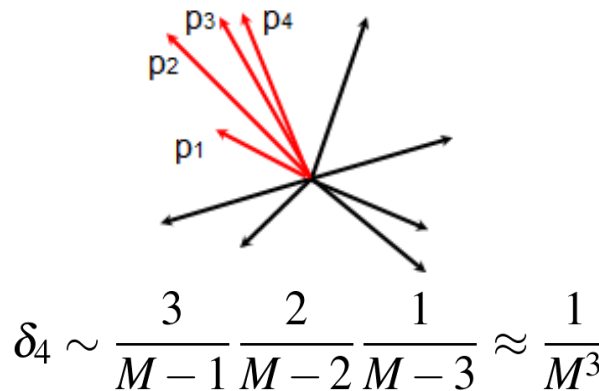


Cumulant can be trivially zero due to underlying symmetries!

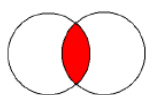
Individual flow harmonics v_n

- Elliptic flow coefficient v_2 of inclusive charged particles as a function of centrality, measured with the two- and multi-particle cumulant methods
- Proof of nontrivial collective effects in heavy-ion collisions
- Multiplicity scaling of few-particle correlations:



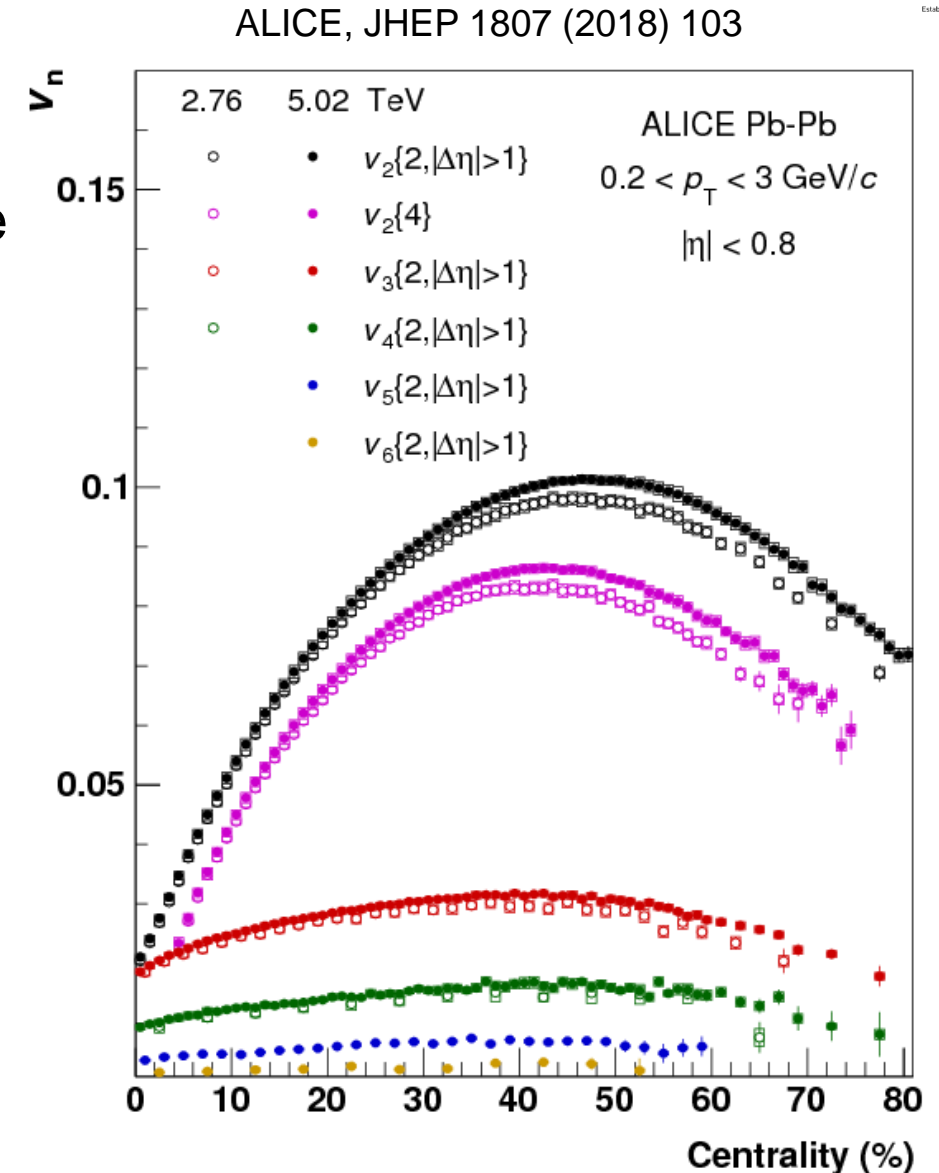
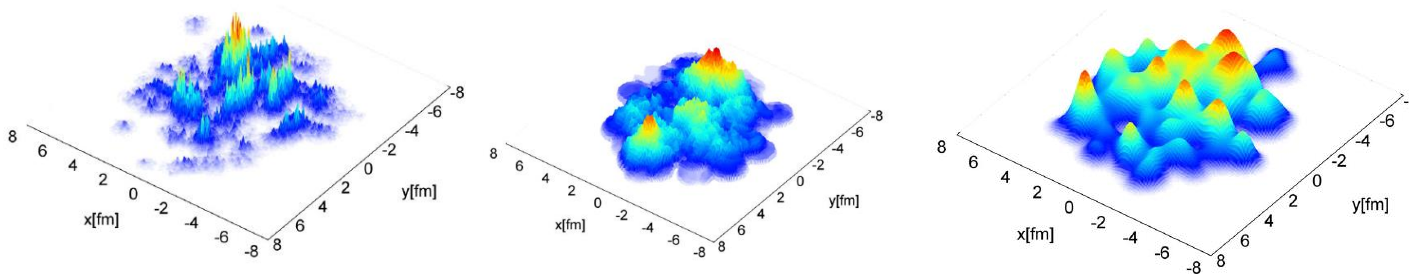


$$\delta_4 \sim \frac{3}{M-1} \frac{2}{M-2} \frac{1}{M-3} \approx \frac{1}{M^3}$$



Individual flow harmonics v_n

- Anisotropic flow coefficients v_n of inclusive charged particles as a function of centrality, for the two-particle and four-particle cumulant methods
- Different centrality dependence of geometry-dominated harmonics (v_2) and fluctuations-dominated harmonics (v_3 , v_4 , v_5 , and v_6)
- Constraints on modeling of initial conditions



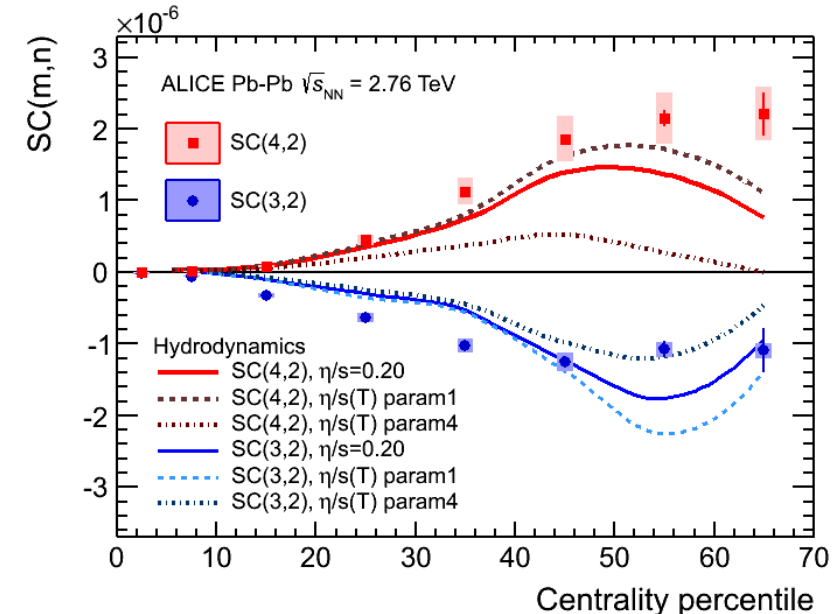
Symmetric Cumulants $SC(m,n)$

- How to quantify experimentally the correlation between two different flow amplitudes?
 - Symmetric Cumulants (Section IVC in Phys. Rev. C **89** (2014) no.6, 064904)

$$\begin{aligned}
\langle\langle \cos(m\varphi_1 + n\varphi_2 - m\varphi_3 - n\varphi_4) \rangle\rangle_c &= \langle\langle \cos(m\varphi_1 + n\varphi_2 - m\varphi_3 - n\varphi_4) \rangle\rangle \\
&\quad - \langle\langle \cos[m(\varphi_1 - \varphi_2)] \rangle\rangle \langle\langle \cos[n(\varphi_1 - \varphi_2)] \rangle\rangle \\
&= \langle v_m^2 v_n^2 \rangle - \langle v_m^2 \rangle \langle v_n^2 \rangle
\end{aligned}$$

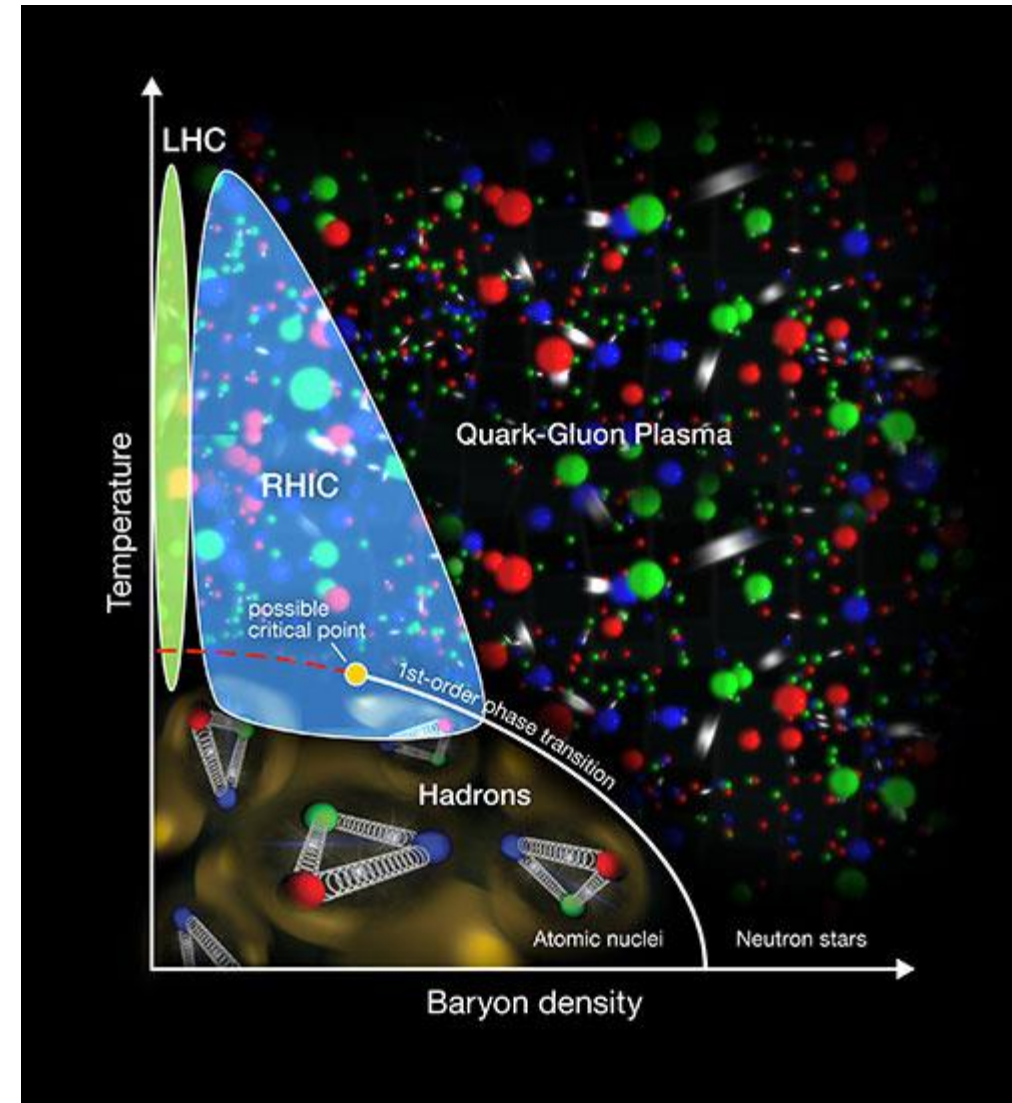
- SC observables are sensitive to differential $\eta/s(T)$ parametrizations
- Individual flow amplitudes are dominated by averages $\langle \eta/s(T) \rangle$
- Independent constraints both on initial conditions and QGP properties

ALICE Collaboration, Phys. Rev. Lett. 117, 182301 (2016)



Heavy-ion collisions

- Collective phenomena
- **EbyE physics and criticality**
- Femtoscopy

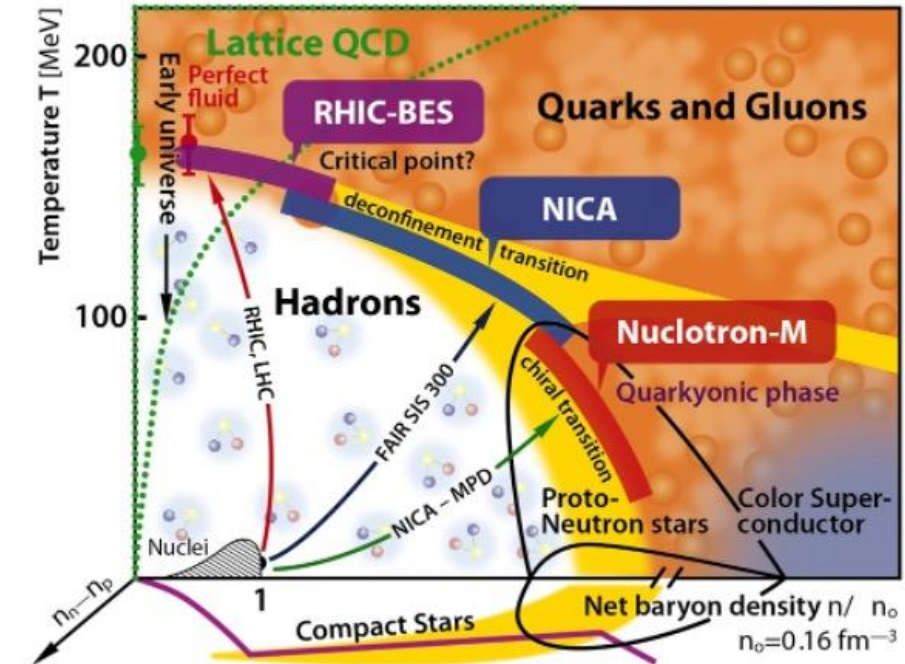


Event-by-event (EbyE) physics

- Physical quantities are expected to display a qualitative different behaviour in case of a phase transition, and can be signalled by anomalous fluctuations and correlations in a number of observables [H. Heiselberg *Phys.Rept.* 351 \(2001\) 161-194](#)
- EbyE fluctuations of multiplicity, net-charge, mean transverse momentum, etc., can be used to probe dynamical fluctuations due to production of quark–gluon plasma (QGP)

Skewness and kurtosis of $\langle p_T \rangle$ fluctuations

- Main idea: Fluctuations in temperature between different phases in QCD phase diagram are inscribed in event-by-event $\langle p_T \rangle$ fluctuations of final-state particles
- Two categories of fluctuations:
 - Statistical – trivial, due to finite multiplicity
 - Dynamical – encode nontrivial physics
- Main challenge: How to disentangle dynamical fluctuations from the ones which are non-thermodynamic in nature (fluctuations of initial positions of participating nucleons, etc.)
- Higher moments of $\langle p_T \rangle$ fluctuations: **skewness** and **kurtosis**



$$\gamma_{\langle p_T \rangle} = \frac{\langle \Delta p_{T,i} \Delta p_{T,j} \Delta p_{T,k} \rangle}{\langle \Delta p_{T,i} \Delta p_{T,j} \rangle^{3/2}}$$

Standardized skewness

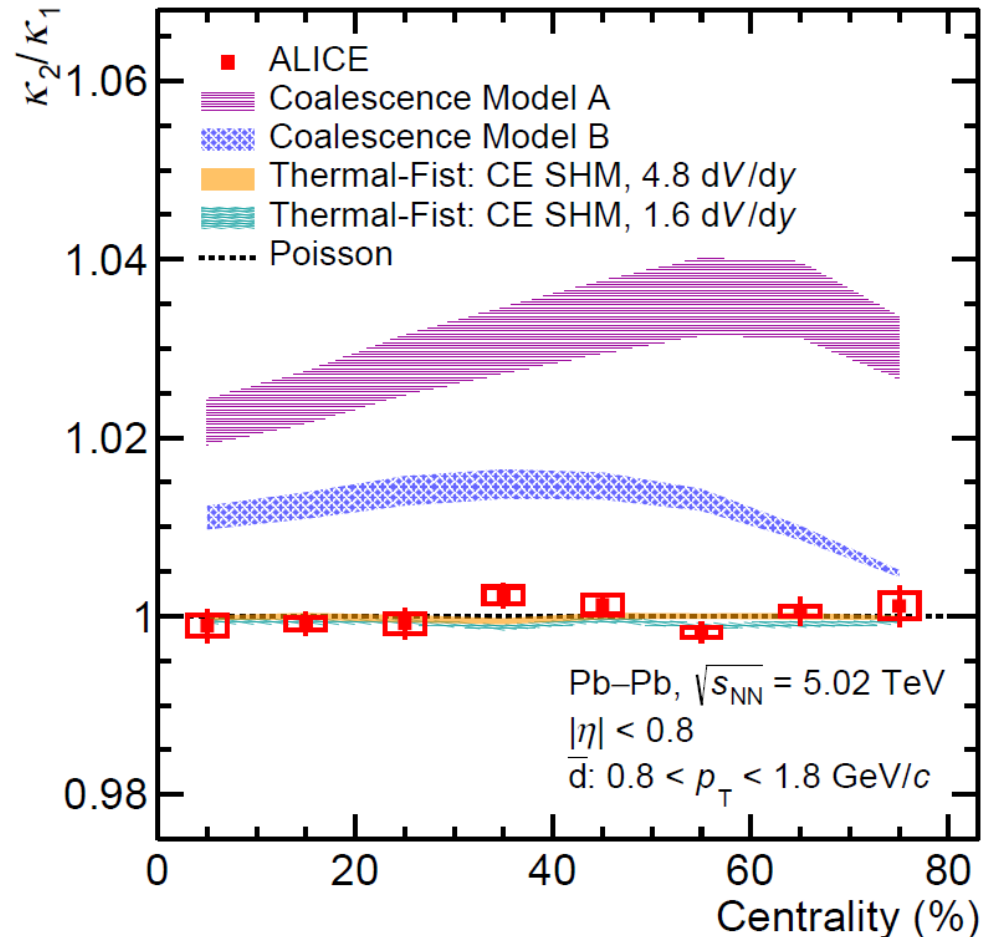
$$\Gamma_{\langle p_T \rangle} = \frac{\langle \Delta p_{T,i} \Delta p_{T,j} \Delta p_{T,k} \rangle \langle \langle p_T \rangle \rangle}{\langle \Delta p_{T,i} \Delta p_{T,j} \rangle^2}$$

Intensive skewness

$$\kappa_{\langle p_T \rangle} = \frac{\langle \Delta p_i \Delta p_j \Delta p_k \Delta p_l \rangle}{\langle \Delta p_i \Delta p_j \rangle^2}$$

Kurtosis

Antideuteron number fluctuations



Higher order cumulants and Pearson correlation coefficient

$$\kappa_1 = \langle n \rangle,$$

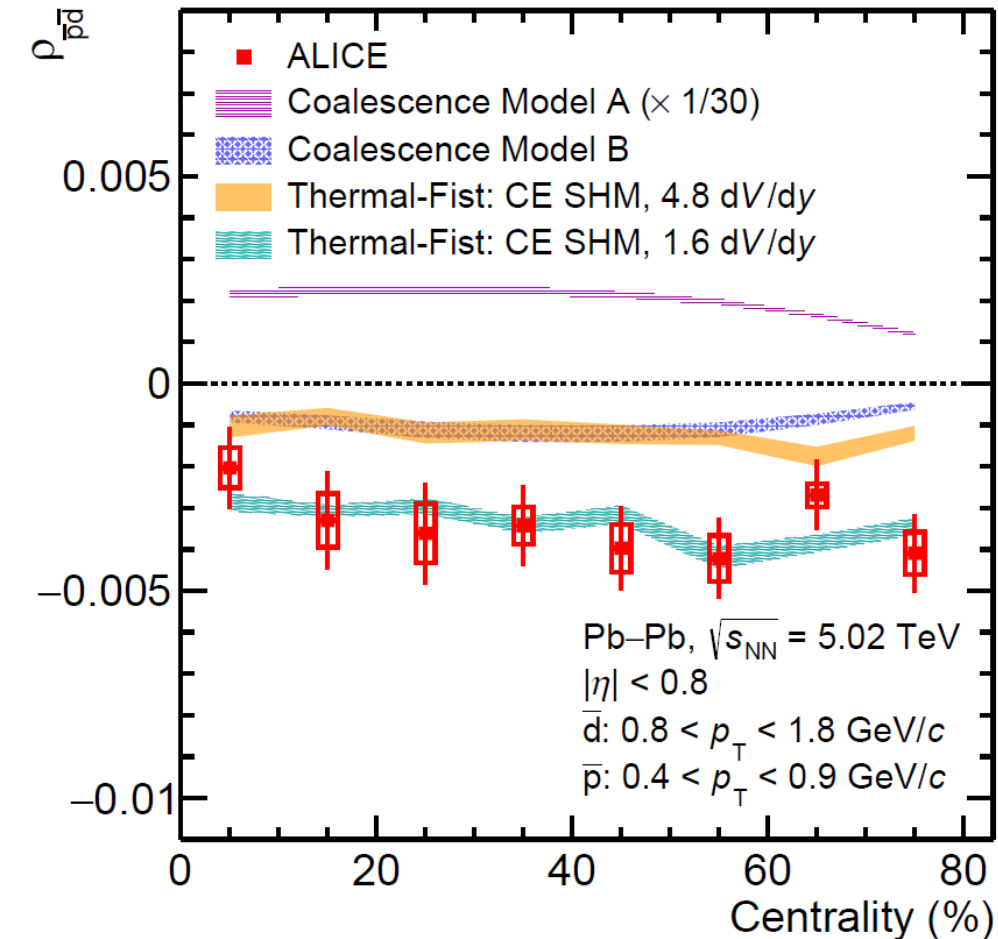
$$\kappa_m = \langle (n - \langle n \rangle)^m \rangle,$$

$$\rho_{ab} = \langle (n_a - \langle n_a \rangle)(n_b - \langle n_b \rangle) \rangle / \sqrt{\kappa_{2a} \kappa_{2b}},$$

- Ratio of the second to the first order cumulant for antideuteron is found to be consistent with unity within uncertainties as expected from a Poisson distribution
- Measurements consistent with statistical hadronisation models (SHM)
- Deviations from coalescence models

“First measurement of antideuteron number fluctuations at energies available at the Large Hadron Collider”, accepted by PRL, [2204.10166](https://arxiv.org/abs/2204.10166)

Antideuteron number fluctuations



Higher order cumulants and Pearson correlation coefficient

$$\kappa_1 = \langle n \rangle,$$

$$\kappa_m = \langle (n - \langle n \rangle)^m \rangle,$$

$$\rho_{ab} = \langle (n_a - \langle n_a \rangle)(n_b - \langle n_b \rangle) \rangle / \sqrt{\kappa_{2a} \kappa_{2b}},$$

- A significant negative correlation between antiprotons and antideuterons is observed in all collision centralities
- In events with at least one antideuteron there are ~0.1% less antiprotons observed than in an average event
- CE version of SHM with correlation volume for baryon number conservation of $V_c = 1.6$ dV/dy captures data

“First measurement of antideuteron number fluctuations at energies available at the Large Hadron Collider”, accepted by PRL, [2204.10166](https://arxiv.org/abs/2204.10166)

Net-baryon fluctuations

- Fluctuations of conserved charges are sensitive probes for the equation of state and are related to the thermodynamic susceptibilities – calculable in the framework of LQCD

$$\chi_{klmn}^{B,S,Q,C} = \frac{\partial^{(k+l+m+n)} (P(\hat{\mu}_B, \hat{\mu}_S, \hat{\mu}_Q, \hat{\mu}_C)/T^4)}{\partial \hat{\mu}_B^k \partial \hat{\mu}_S^l \partial \hat{\mu}_Q^m \partial \hat{\mu}_C^n} \Big|_{\vec{\mu}=0}$$

- Transition from chiral crossover to a second-order transition – signs of criticality expected to show up starting only with the 6th order cumulants of net-charge distributions
- Currently available in terms of statistics: 2nd and 3rd order cumulants of net-proton distributions

$$\Delta N_B = X = N_B - N_{\bar{B}} \quad \kappa_n \rightarrow \text{cumulants (i.e. } \kappa_2 \equiv \langle X^2 \rangle - \langle X \rangle^2)$$

$$\hat{\chi}_2^B = \frac{\kappa_2(\Delta N_B)}{VT^3} \rightarrow \frac{\kappa_4(\Delta N_B)}{\kappa_2(\Delta N_B)} = \frac{\hat{\chi}_4^B}{\hat{\chi}_2^B}$$

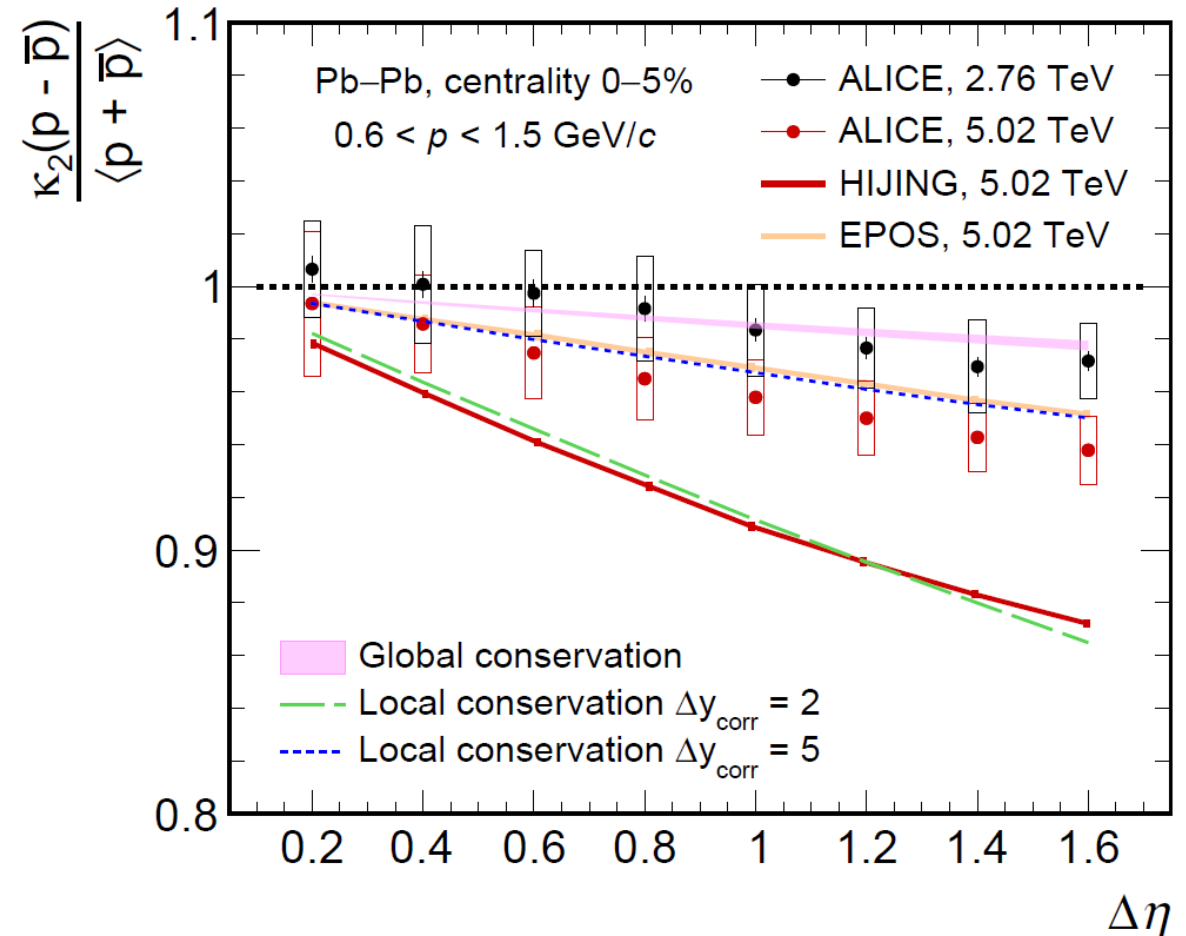
“Closing in on critical net-baryon fluctuations at LHC energies: cumulants up to third order in Pb–Pb collisions”,
 Physics Letters B, 2022, 137545, [2206.03343](https://doi.org/10.1016/j.phlet.2022.137545)

Net-baryon fluctuations

- Deviation from Skellam baseline (i.e. statistically independent Poisson limit) is due to baryon number conservation

$$\kappa_n^{\text{Skellam}}(p - \bar{p}) = \langle p \rangle + (-1)^n \langle \bar{p} \rangle$$

- Normalized 2nd order cumulants of net-protons are independent of centrality
- As a function of the width of the pseudorapidity interval, the fluctuations are increasingly reduced
 - Larger interval – increasing relevance of baryon number conservation
 - Narrowest interval – statistically independent Poisson fluctuations



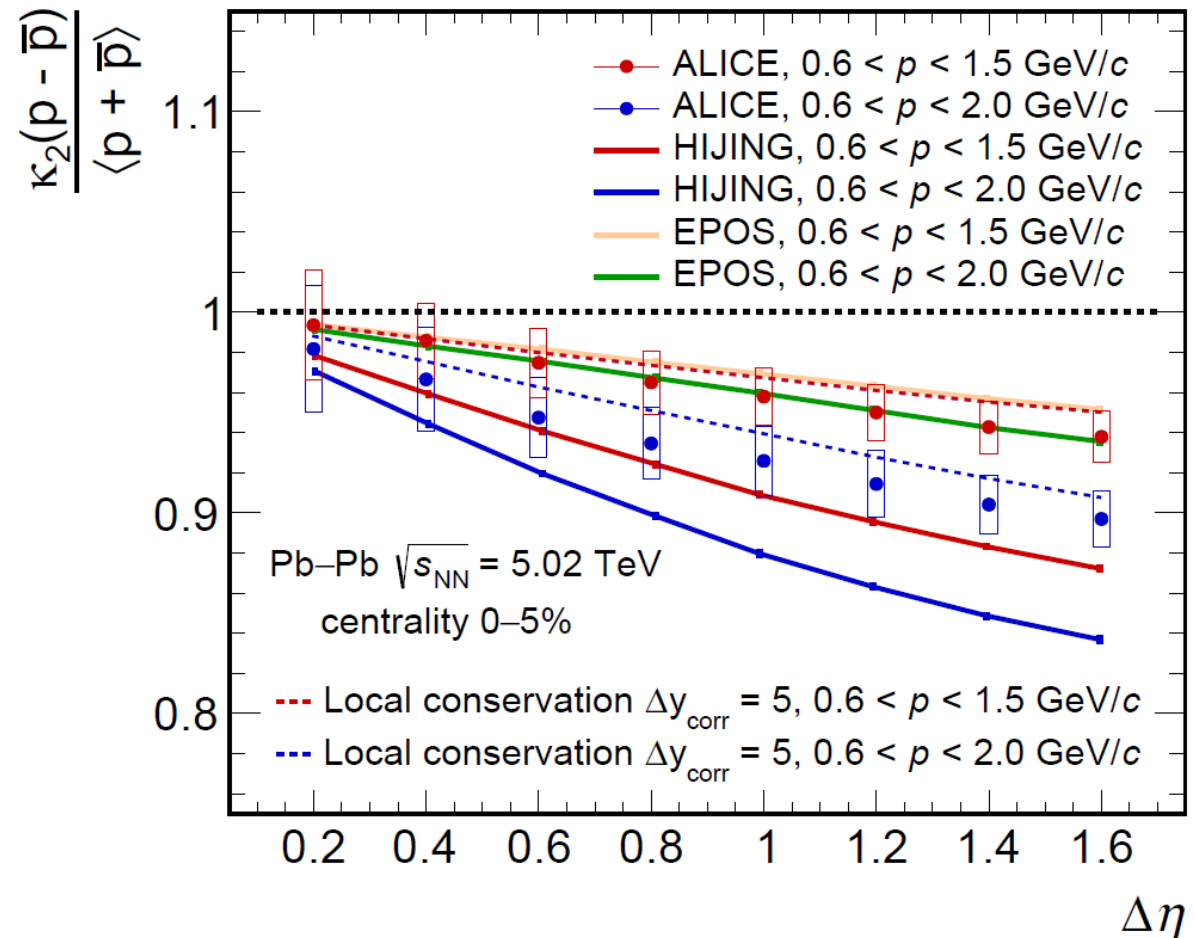
“Closing in on critical net-baryon fluctuations at LHC energies: cumulants up to third order in Pb-Pb collisions”,
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Net-baryon fluctuations

- Comparison to models: EPOS and HIJING
- ALICE data suggest long range correlations, $\Delta y = \pm 2.5$ unit or longer \rightarrow earlier in time

A. Dumitru et al., Nucl. Phys. A 810 (2008) 91

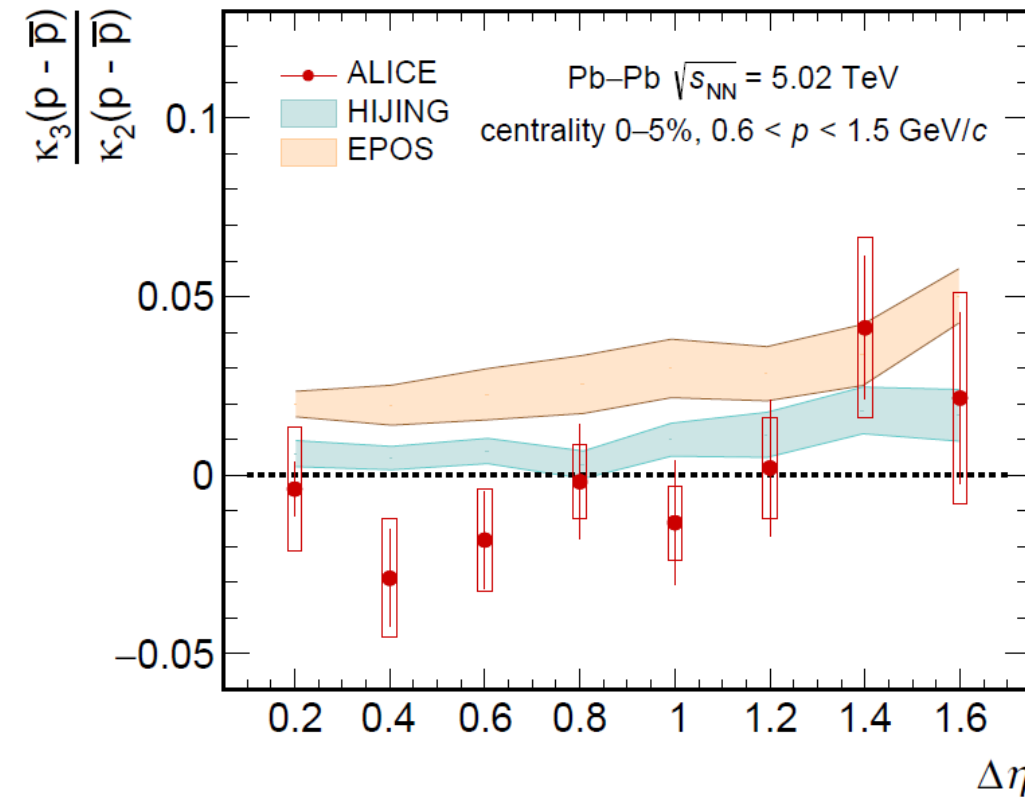
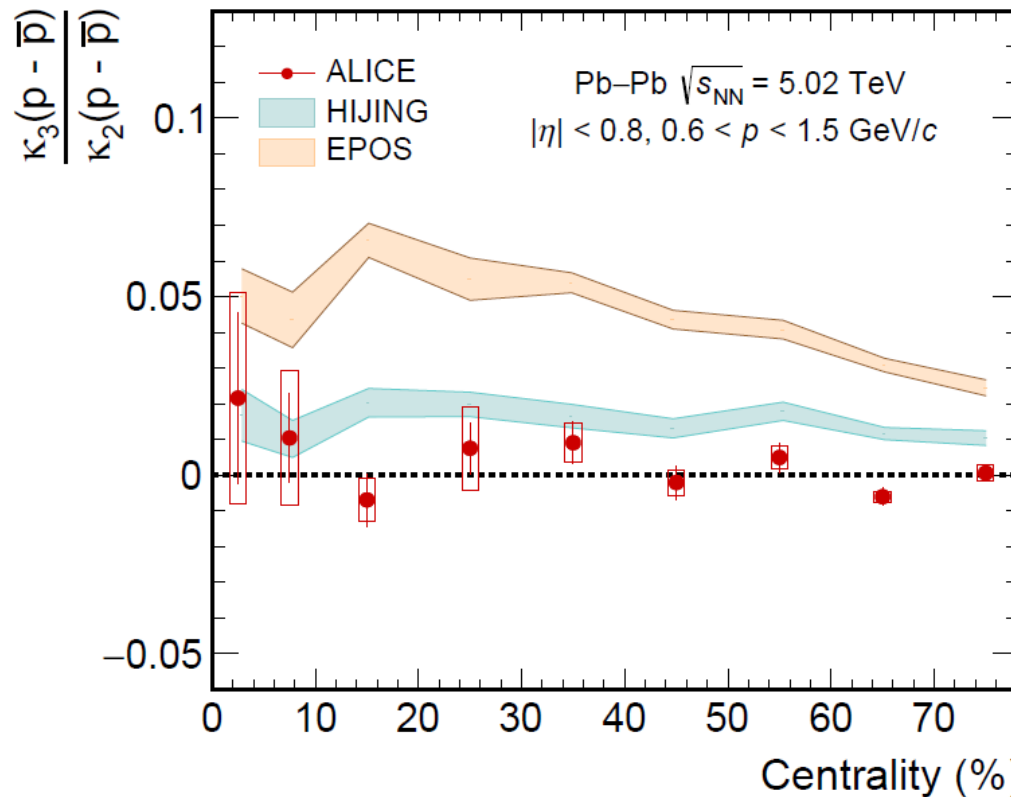
- EPOS agrees with ALICE data but HIJING deviates significantly
- EPOS Event generators based on string fragmentation (HIJING) conserve baryon number over $\Delta y = \pm 1$ unit



“Closing in on critical net-baryon fluctuations at LHC energies: cumulants up to third order in Pb-Pb collisions”,
Physics Letters B, 2022, 137545, [2206.03343](https://doi.org/10.1016/j.phlet.2022.137545)

Net-baryon fluctuations

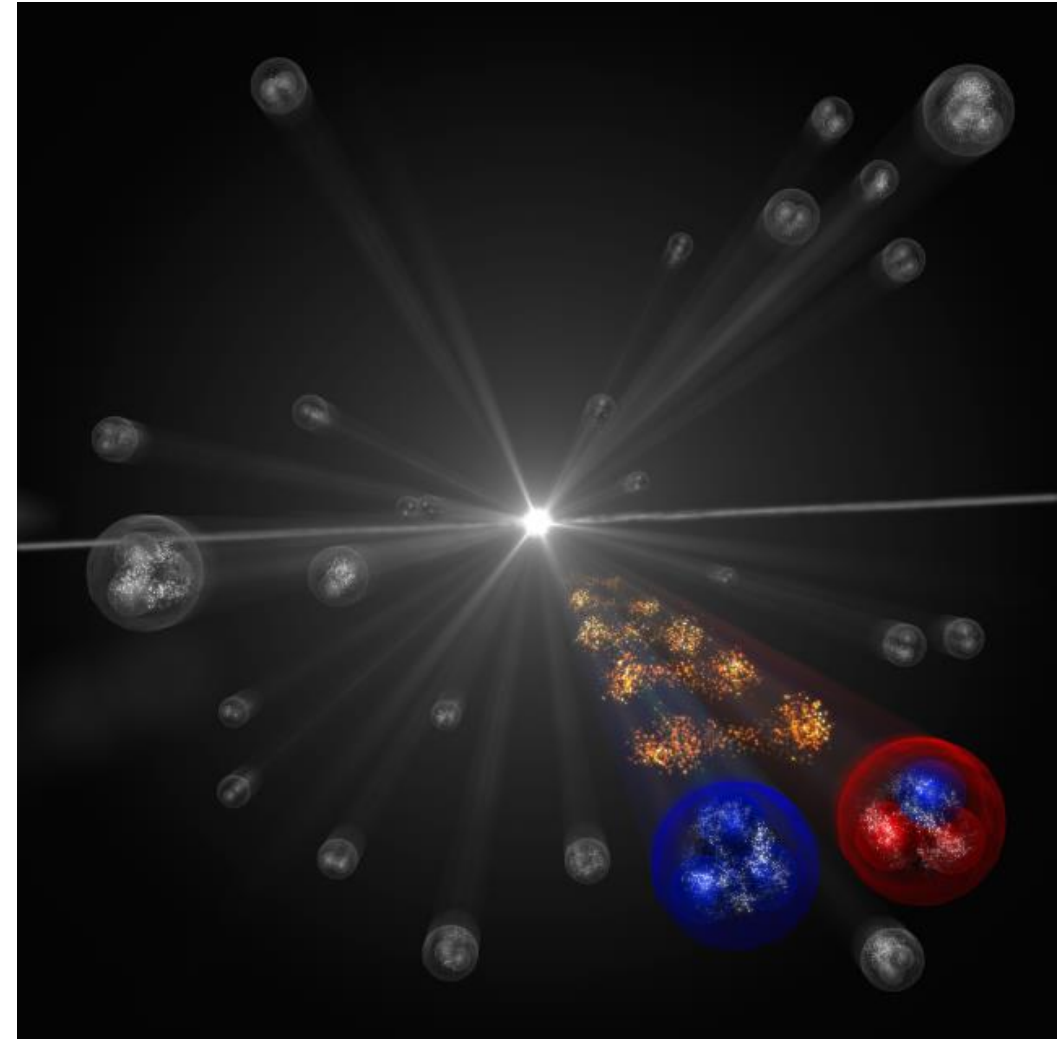
- First results for 3rd order cumulants of net protons



“Closing in on critical net-baryon fluctuations at LHC energies: cumulants up to third order in Pb–Pb collisions”,
Physics Letters B, 2022, 137545, [2206.03343](https://doi.org/10.1016/j.phlet.2022.137545)

Heavy-ion collisions

- Collective phenomena
- EbyE physics and criticality
- **Femtoscopy**



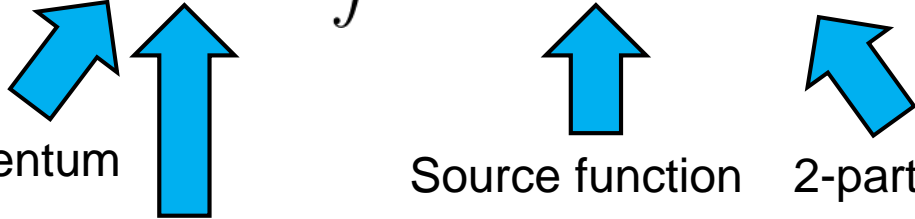
Femtoscscopy: Analysis technique

- 2-particle correlation function:

$$C(\mathbf{p}_1, \mathbf{p}_2) = \frac{N(\mathbf{p}_1, \mathbf{p}_2)}{N(\mathbf{p}_1)N(\mathbf{p}_2)} = \frac{E_1 E_2 dN / (d^3 p_1 d^3 p_2)}{(E_1 dN / d^3 p_1)(E_2 dN / d^3 p_2)}$$

- Rewritten as:

$$C(\mathbf{P}, \mathbf{k}) = \int d^3 r^* S_{\mathbf{P}}(r^*) |\phi(r^*, \mathbf{k})|^2$$

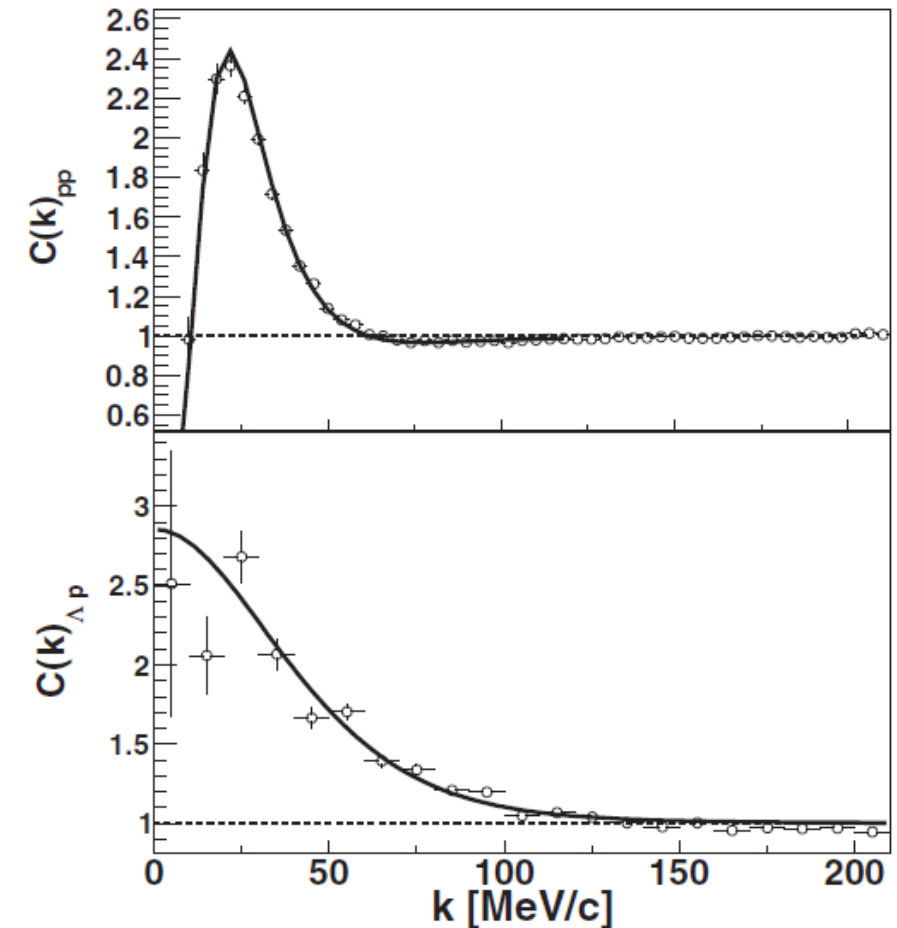


Total pair momentum Source function 2-particle wave function
 Relative pair momentum

- r^* is relative distance in a pair rest frame

Contributions to correlations

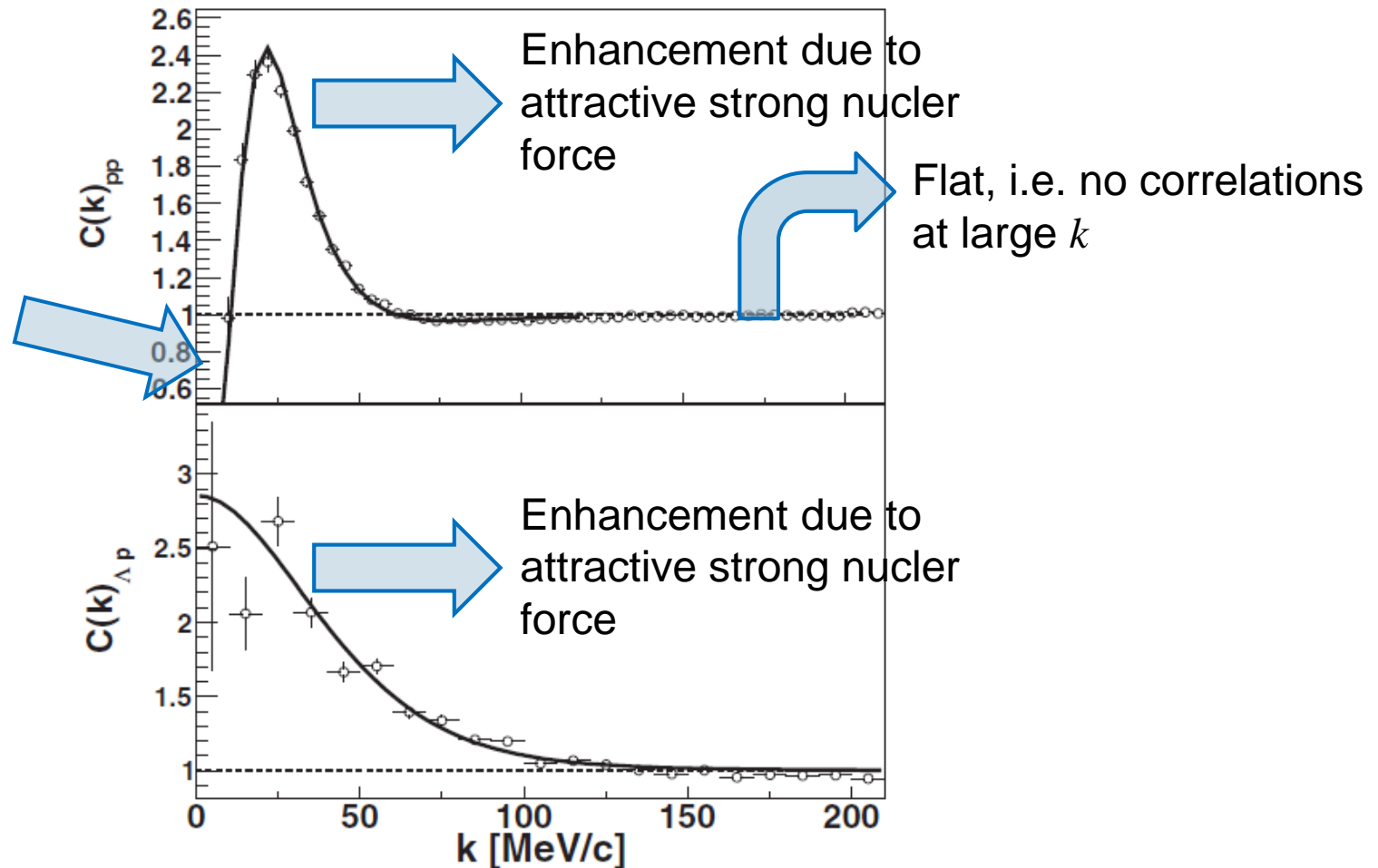
- Quantum Statistics (QS)
 - Relevant only for the pairs of identical particles
 - Depending on their spin, the wave function needs to be symmetrized (bosons) or anti-symmetrized (fermions)
- Final State Interactions (FSI)
 - Coulomb interaction
 - Strong nuclear force
- Energy and momentum conservation
- Typically diluted with inverse of total number of particles
- Jet-like correlations
- Detector artifacts (e.g. track splitting, merging, etc.)
- ...



Contributions to correlations: Anatomy

- Example: Correlations function for pp and p Λ pairs

Depletion due to:
1) Coulomb repulsion
2) Quantum Stat.



Femtoscscopy in a nutshell

- Experimentally we measure correlation function via the ratio:

$$C(k) = \mathcal{N} \frac{A(k)}{B(k)}$$

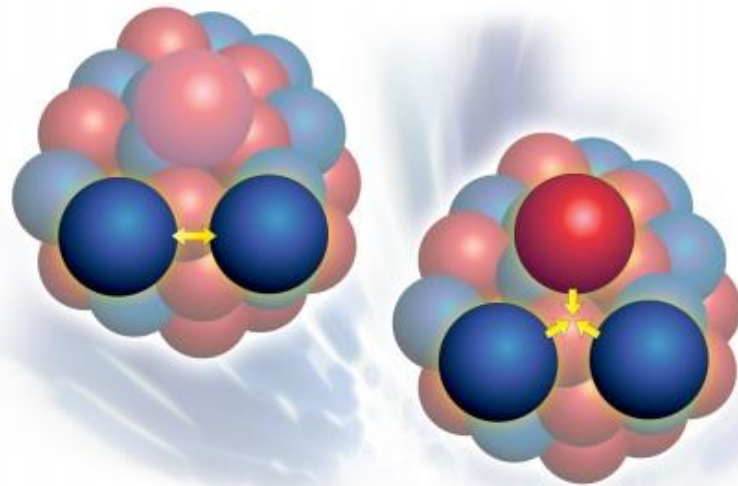
- Those results are then compared to the theoretical result:

$$C(k) = \int d^3 r^* S(r^*) |\phi(r^*, k)|^2$$

- With measured correlation function, we can then constrain:
 - Source function, if interaction is known
 - Interaction, if source function is known

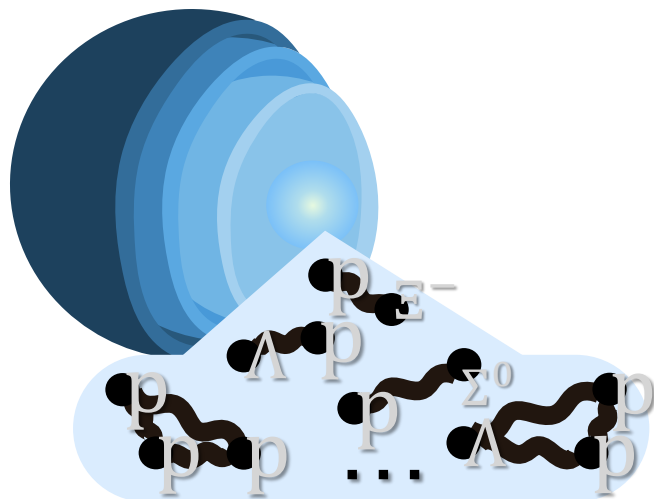
What is happening in the neutron stars?

- The densest stars observed in the Universe
- Recent observations of very massive neutron stars are proving to be a serious challenge: Onset of hyperons?



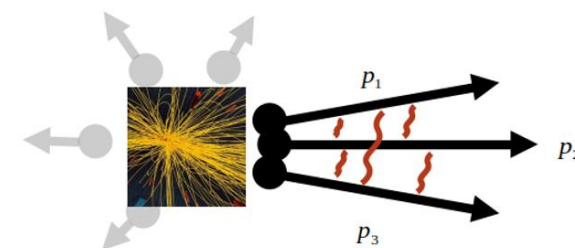
What is the nature of 2-body hyperon-nucleon force?
Genuine 3-body hyperon-nucleon-nucleon force?

Nuclear matter in neutron stars

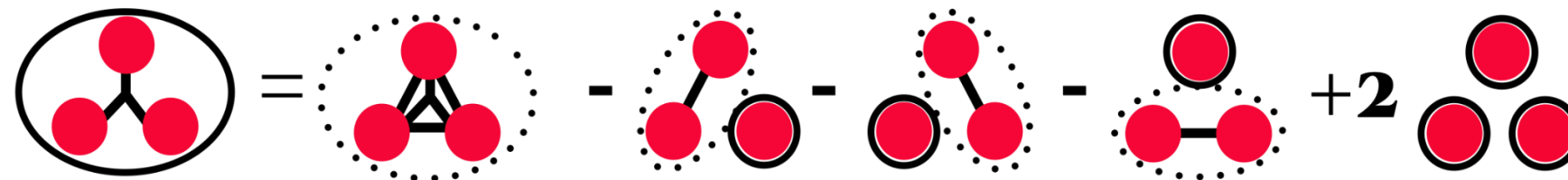


- Properties of nuclei and hypernuclei cannot be described satisfactorily with two-body forces only
L.E. Marcucci et al., Front. Phys. 8:69 (2020)
- N-N-N and N-N- Λ interactions: fundamental ingredients for the Equation of State (EoS) of neutron stars
D. Lonardoni et al., PRL 114, 092301 (2015)

- Genuine multiparticle correlations, or cumulants, can be isolated from the measured multiparticle correlations with the Kubo's formalism:



Femtoscopic technique

$$\text{Genuine 3-body correlation} = \text{3-body correlation} - \text{2-body correlation + permutations} + 2 \times \text{Non-interacting particles}$$


Genuine 3-body correlation

3-body correlation

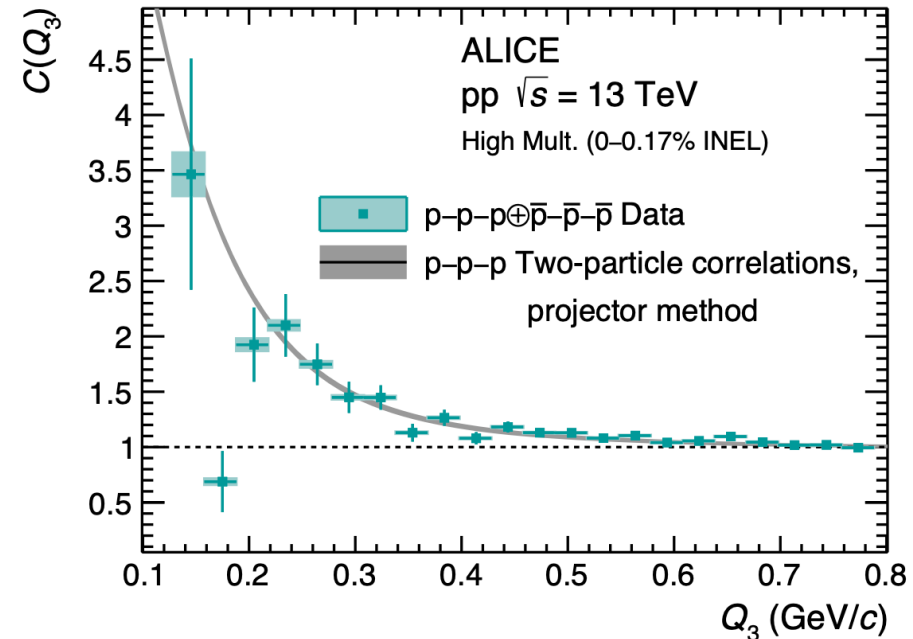
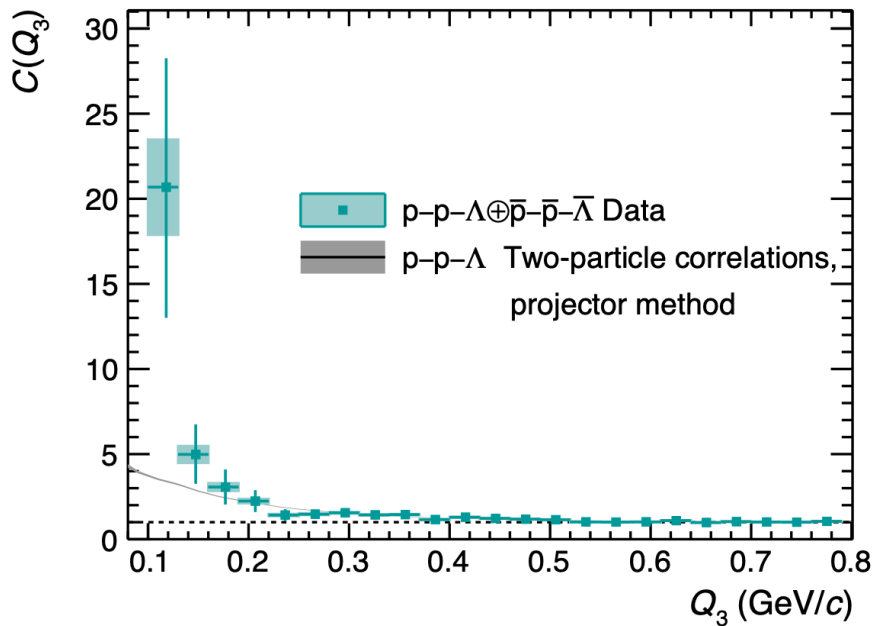
2-body correlation + permutations

Non-interacting particles

R. Kubo, J. Phys. Soc. Jpn. **17**(7), 1100 (1962)

3-body correlation functions

- Measured three-particle correlation functions deviate from lower-order contributions, hinting to three-body effects



Statistical significance:

$$Q_3 = \sqrt{-q_{ij}^2 - q_{jk}^2 - q_{ki}^2}$$

p-p- Λ : $n_\sigma = 0.8$ for $Q_3 < 0.4$ GeV/c

p-p-p: $n_\sigma = 6.7$ for $Q_3 < 0.4$ GeV/c

“Towards the understanding of the genuine three-body interaction for p-p-p and p-p- Λ ”
(ALICE, [arXiv:2206.03344](https://arxiv.org/abs/2206.03344))

Thanks!

Backup slides