OSAF Topical Lectures on Effective Field Theories

Application of EFTs for B Physics

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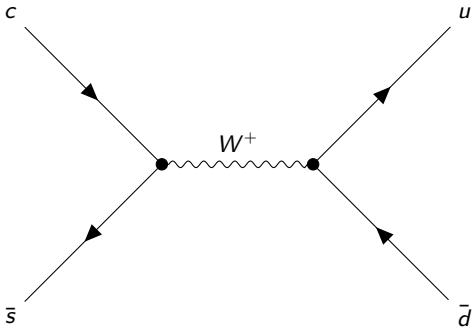
Groningen – April 13th, 2023

A Simple Weak Interaction

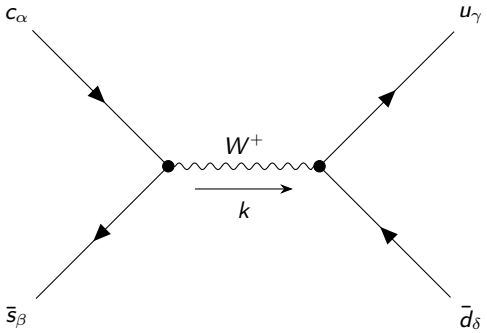


Energy Scales:

- Interaction involves 2 very distinct energy scales
- Mass of the W boson: 80.377 GeV
- Mass of the initial quarks
  - Charm: 1.27 GeV
  - Beauty: 4.18 GeV

$\Rightarrow$ Interaction is highly virtual

Let's have a closer look ...



Decay Amplitude:

$$A = \underbrace{(\bar{s}_\beta c_\alpha)_{\mathrm{V-A}}}_{\text{initial state}} \times \underbrace{\frac{g_{\mathrm{EW}} V_{cs}^*}{2\sqrt{2}} \delta_{\alpha\beta}}_{\text{vertex}} \times \underbrace{\frac{1}{k^2 - m_W^2}\left(g^{\mu\nu} - \frac{k^\mu k^\nu}{m_W^2}\right)}_{\text{propagator}} \times \underbrace{\frac{g_{\mathrm{EW}} V_{ud}}{2\sqrt{2}} \delta_{\gamma\delta}}_{\text{vertex}} \times \underbrace{(\bar{u}_\delta d_\gamma)_{\mathrm{V-A}}}_{\text{final state}} \tag{1}$$

Footnote on Notation

$$A = \underbrace{(\bar{s}_\beta c_\alpha)_{\mathrm{V-A}}}_{\text{initial state}} \times \underbrace{\frac{g_{\mathrm{EW}} V_{cs}^*}{2\sqrt{2}} \delta_{\alpha\beta}}_{\text{vertex}} \times \underbrace{\frac{1}{k^2 - m_W^2} \left(g^{\mu\nu} - \frac{k^\mu k^\nu}{m_W^2} \right)}_{\text{propagator}} \times \underbrace{\frac{g_{\mathrm{EW}} V_{ud}}{2\sqrt{2}} \delta_{\gamma\delta}}_{\text{vertex}} \times \underbrace{(\bar{u}_\delta d_\gamma)_{\mathrm{V-A}}}_{\text{final state}} \tag{2}$$

Explanations:

- $\alpha, \beta, \gamma, \delta$ are colour indices
- μ, ν are Lorentz indices
- $g^{\mu\nu}$ is the metric tensor
- k is the momentum transfer
- V−A represents the Vector–Axial-Vector Lorentz structure:

$$(\bar{q}p)_{\mathrm{V-A}} \equiv \bar{q}\gamma_\mu(1 - \gamma_5)p \tag{3}$$

Interaction Hamiltonian

- The Standard Model Hamiltonian for the $c\bar{s} \to u\bar{d}$ process

$$\mathcal{H} = \frac{g_{\mathrm{EW}}^2}{8} V_{cs}^* V_{ud} (\bar{s}_\alpha c_\alpha)_{\mathrm{V-A}} (\bar{u}_\beta d_\beta)_{\mathrm{V-A}} \frac{1}{k^2 - m_W^2} \left(g^{\mu\nu} - \frac{k^\mu k^\nu}{m_W^2} \right) \tag{4}$$

- Use that $k = \mathcal{O}(m_q) \ll m_W$

$$\frac{1}{k^2 - m_W^2} \left(g^{\mu\nu} - \frac{k^\mu k^\nu}{m_W^2} \right) \xrightarrow{k \ll m_W} \frac{g^{\mu\nu}}{m_W^2} + \mathcal{O}\left(\frac{k^2}{m_W^2} \right) \tag{5}$$

- Introduce the Fermi constant

$$G_{\mathrm{F}} \equiv \frac{\sqrt{2} g_{\mathrm{EW}}^2}{8 m_W^2} \tag{6}$$

- End up with the leading order in a series expansion

$$\mathcal{H} = \frac{G_{\mathrm{F}}}{\sqrt{2}} V_{cs}^* V_{ud} (\bar{s}_\alpha c_\alpha)_{\mathrm{V-A}} (\bar{u}_\beta d_\beta)_{\mathrm{V-A}} + \mathcal{O}\left(\frac{k^2}{m_W^2} \right) \tag{7}$$

From Full to Effective Theory

- Full Standard Model theory

$$\mathcal{H} = \frac{G_F}{\sqrt{2}} V_{cs}^* V_{ud} (\bar{s}_\alpha c_\alpha)_{V-A} (\bar{u}_\beta d_\beta)_{V-A} + \mathcal{O}\left(\frac{k^2}{m_W^2}\right) \tag{8}$$

- Interpretation: Short-range exchange force approximately behaves point interaction
- Note: expression remains exact until you neglect $\mathcal{O}\left(\frac{k^2}{m_W^2}\right)$!

$$\Downarrow$$

$$\mathcal{H} = \frac{G_F}{\sqrt{2}} V_{cs}^* V_{ud} (\bar{s}_\alpha c_\alpha)_{V-A} (\bar{u}_\beta d_\beta)_{V-A} \tag{9}$$

- This has become an effective local four-fermion interaction
  (cfr. Fermi's 1933 theory of β^- decay)

Effective Hamiltonian

$$\mathcal{H} = \frac{G_F}{\sqrt{2}} V_{cs}^* V_{ud} (\bar{s}_\alpha c_\alpha)_{V-A} (\bar{u}_\beta d_\beta)_{V-A} \tag{10}$$

- Introduce the notation

$$\mathcal{O}_2 \equiv (\bar{s}_\alpha c_\alpha)_{V-A} (\bar{u}_\beta d_\beta)_{V-A} \qquad \text{and} \qquad C_2 = 1 \tag{11}$$

  - $\mathcal{O}_2$ is referred to as an operator
  - C_2 is referred to as a Wilson coefficient

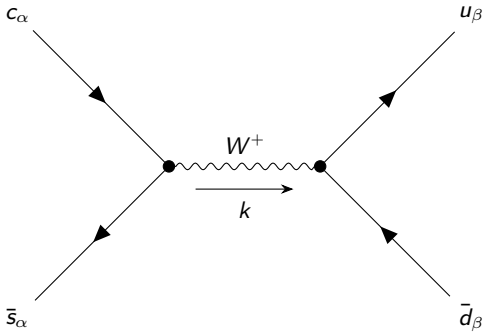
- End up with the formalism known as Operator Product Expansion

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{cs}^* V_{ud} C_2 \mathcal{O}_2 \tag{12}$$

- We have completely removed the W-dependence from the theory ... or "integrated out" the W

Towards an Effective Field Theory

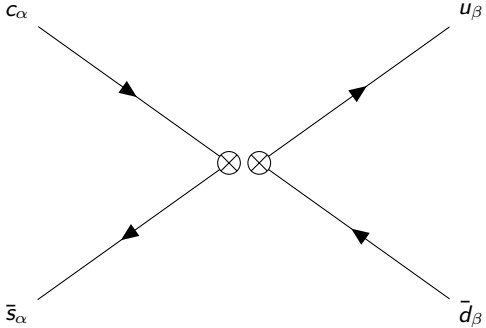
Full Standard Model Theory



- Explicit W-exchange
- Calculate interaction using

$$\mathcal{H} = \frac{g_{\mathrm{EW}}^2}{8} V_{cs}^* V_{ud} (\bar{s}_\alpha c_\alpha)_{\mathrm{V-A}} (\bar{u}_\beta d_\beta)_{\mathrm{V-A}} \frac{1}{k^2 - m_W^2} \left(g^{\mu\nu} - \frac{k^\mu k^\nu}{m_W^2} \right)$$

Weak Effective Theory

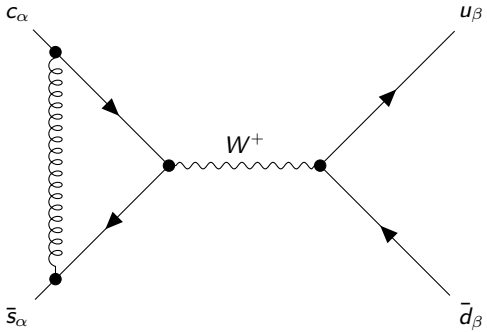


- Local 4-fermion interaction
- Calculate interaction using

$$\mathcal{H} = \frac{G_{\mathrm{F}}}{\sqrt{2}} V_{cs}^* V_{ud} C_2 \mathcal{O}_2$$

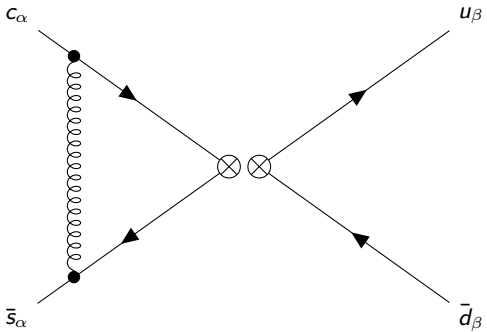
Next-to-Leading Order QCD Diagrams: Vertex Correction

Full Standard Model Theory



- Initial and final state have independent colour indices
- Same colour structure as LO diagram

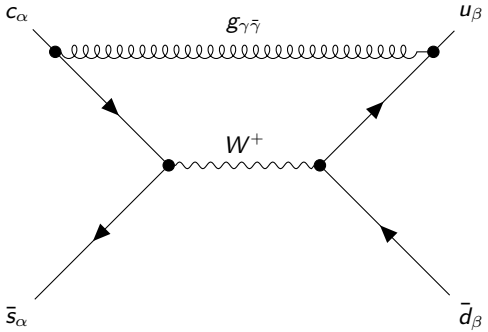
Weak Effective Theory



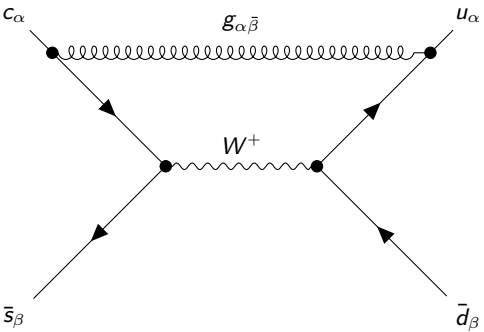
- Same operator $\mathcal{O}_2 = (\bar{s}_\alpha c_\alpha)_{\mathrm{V-A}}(\bar{u}_\beta d_\beta)_{\mathrm{V-A}}$
- Additional contribution to C_2

Next-to-Leading Order QCD Diagrams: Gluon Exchange I

Colour-Neutral Gluon

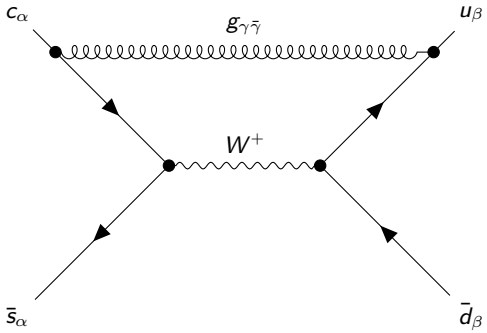


Coloured Gluon



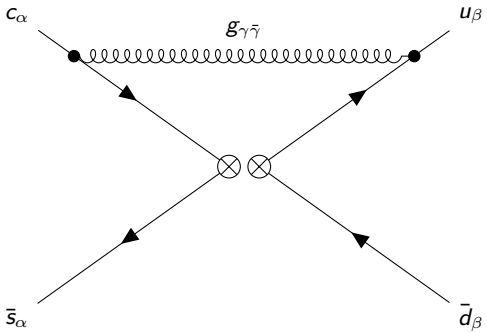
Next-to-Leading Order QCD Diagrams: Gluon Exchange Ia

Full Standard Model Theory



- Initial and final state have independent colour indices
- Same colour structure as LO diagram

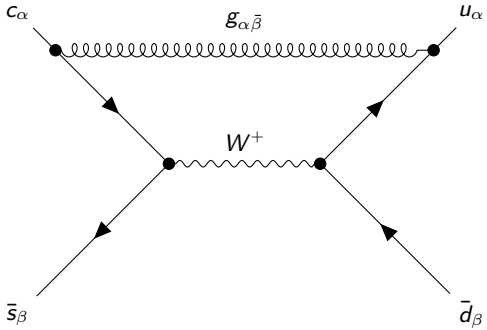
Weak Effective Theory



- Same operator $\mathcal{O}_2 = (\bar{s}_\alpha c_\alpha)_{\mathrm{V-A}}(\bar{u}_\beta d_\beta)_{\mathrm{V-A}}$
- Additional contribution to C_2

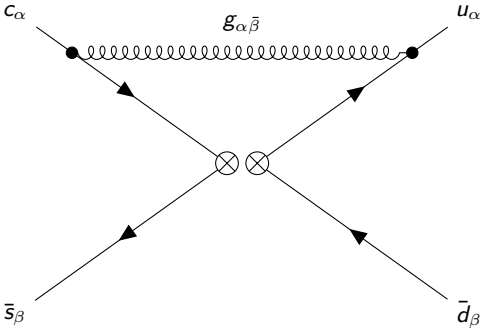
Next-to-Leading Order QCD Diagrams: Gluon Exchange Ib

Full Standard Model Theory



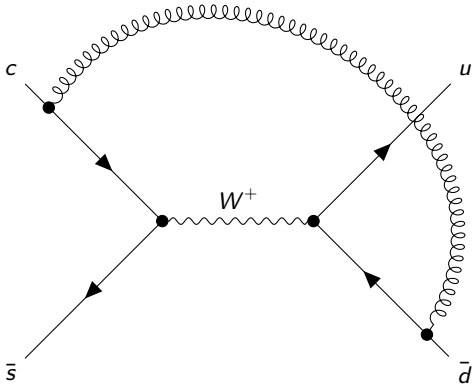
- Initial and final state mixed their colour indices
- Different colour structure as LO diagram

Weak Effective Theory



- New operator $\mathcal{O}_1 \equiv (\bar{s}_\beta c_\alpha)_{V-A}(\bar{u}_\alpha d_\beta)_{V-A}$
- New Wilson coefficient C_1

Next-to-Leading Order QCD Diagrams: Gluon Exchange II



- Contributes to both $\mathcal{O}_1$ and $\mathcal{O}_2$
- Additional contributions to both C_1 and C_2

An Effective Theory for the $c\bar{s} \to u\bar{d}$ Process

- The $c\bar{s} \to u\bar{d}$ process is described by effective Hamiltonian

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{cs}^* V_{ud} \left(C_1 \mathcal{O}_1 + C_2 \mathcal{O}_2 \right) \tag{13}$$

- C_j are the Wilson coefficients
- $\mathcal{O}_j$ are the current–current operators

$$\mathcal{O}_1 \equiv (\bar{s}_\beta c_\alpha)_{V-A} (\bar{u}_\alpha d_\beta)_{V-A} \qquad \mathcal{O}_2 \equiv (\bar{s}_\alpha c_\alpha)_{V-A} (\bar{u}_\beta d_\beta)_{V-A} \tag{14}$$

- They differ in their colour structure
- In the absence of QCD: $C_1 = 0$ and $C_2 = 1$

Weak Effective Theory or Operator Product Expansion

- For each weak decay process, you can write down an effective Hamiltonian

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{pq}^* V_{ab} \sum_j C_j \mathcal{O}_j \tag{15}$$

- Which set of operators to include depends on the process

→ Linked to the set of possible Feynman diagrams

Hadronic Matric Element: From Operator to Observable

- Example: Branching fraction calculation:

$$\mathcal{B}(B \to f) \quad \leftrightarrow \quad \Gamma(B \to f) \quad \leftrightarrow \quad |A(B \to f)|^2 \quad \leftrightarrow \quad |\langle f|\mathcal{H}_{\text{eff}}|B\rangle|^2 \quad \leftrightarrow \quad \langle f|\mathcal{O}_j|B\rangle$$

- $\langle f|\mathcal{O}_j|B\rangle$ is known as the Hadronic Matrix Element
- $\langle f|\mathcal{O}_j|B\rangle$ contains the long-distance physics, including hadronisation process
- Non-perturbative nature require additional tricks and/or approximations:
  Lattice QCD, LCSR, HQET, HQE, ...
- Different energy scales allow different approximations

→ Techniques are often specific to one problem

→ What works for b-hadron decays need not apply to kaon physics

Weak Effective Theory

- For each weak decay process, you can write down an effective Hamiltonian

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{pq}^* V_{ab} \sum_j C_j \mathcal{O}_j \tag{16}$$

- Which set of operators to include depends on the process

→ Linked to the set of possible Feynman diagrams

- OPE separates long-distance physics ($\mathcal{O}_j$) from short-distance physics (C_j)
- C_j only depend on the hard-scattering process and are thus universal
- Hadronic matrix elements are decay specific

⇒ The OPE makes calculations of weak decays possible

Weak Effective Theory or Operator Product Expansion

- Ideal tool to search for beyond the Standard Model physics

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{pq}^* V_{ab} \sum_j \left(C_j^{\text{SM}} + C_j^{\text{NP}} \right) \mathcal{O}_j \tag{17}$$

- Can calculate C_j^{SM} to any desired precision, control the hadronic matrix elements

$\Rightarrow$ C_j^{NP} are the only unknowns

- Fitting this OPE model to experimental data allows to constrain C_j^{NP}

  + Model-independent approach

  + Straightforward to combine multiple measurements (branching fraction & ratios, asymmetries, ...)

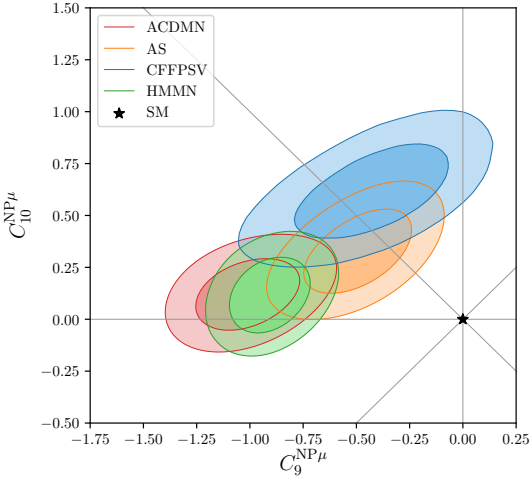
  − Does not reveal the underlying theory, only the allowed interactions

  − In general: Too many degrees of freedom

  $\rightarrow$ Need to be selective about the operators we do and do not include

  $\Rightarrow$ Difficult to interpret the significance

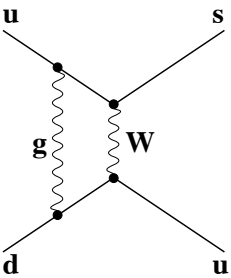
Example: Fit of the $b \to s\ell\ell$ Anomalies



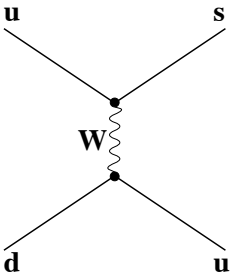
Source: LHCb Flavour Anomaly Workshop 2021

Current–Current Operators

arXiv:9512380[hep-ph]



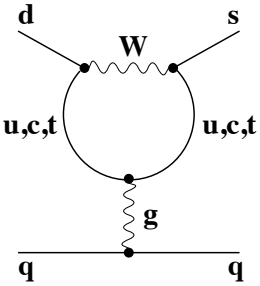
$$\mathcal{O}_1 = (\bar{s}_\alpha u_\beta)_{\mathrm{V-A}}(\bar{u}_\beta d_\alpha)_{\mathrm{V-A}}$$



$$\mathcal{O}_2 = (\bar{s}_\alpha u_\alpha)_{\mathrm{V-A}}(\bar{u}_\beta d_\beta)_{\mathrm{V-A}}$$

QCD Penguin Operators

arXiv:9512380[hep-ph]



$$\mathcal{O}_3 = (\bar{s}_\alpha d_\alpha)_{V-A} \sum_q (\bar{q}_\alpha q_\alpha)_{V-A}$$

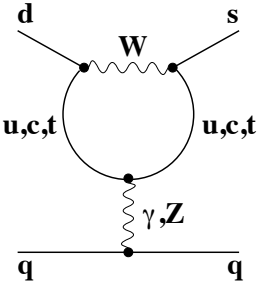
$$\mathcal{O}_4 = (\bar{s}_\alpha d_\beta)_{V-A} \sum_q (\bar{q}_\beta q_\alpha)_{V-A}$$

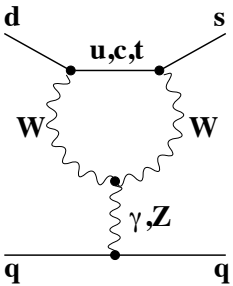
$$\mathcal{O}_5 = (\bar{s}_\alpha d_\alpha)_{V-A} \sum_q (\bar{q}_\alpha q_\alpha)_{V+A}$$

$$\mathcal{O}_6 = (\bar{s}_\alpha d_\beta)_{V-A} \sum_q (\bar{q}_\beta q_\alpha)_{V+A}$$

Electroweak Penguin Operators

arXiv:9512380[hep-ph]





$$\mathcal{O}_7 = \frac{3}{2}(\bar{s}_\alpha d_\alpha)_{V-A} \sum_q e_q (\bar{q}_\alpha q_\alpha)_{V+A}$$

$$\mathcal{O}_8 = \frac{3}{2}(\bar{s}_\alpha d_\beta)_{V-A} \sum_q e_q (\bar{q}_\beta q_\alpha)_{V+A}$$

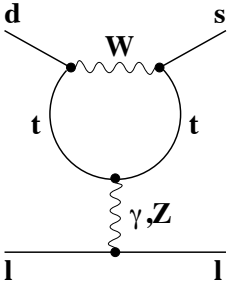
$$\mathcal{O}_9 = \frac{3}{2}(\bar{s}_\alpha d_\alpha)_{V-A} \sum_q e_q (\bar{q}_\alpha q_\alpha)_{V-A}$$

$$\mathcal{O}_{10} = \frac{3}{2}(\bar{s}_\alpha d_\beta)_{V-A} \sum_q e_q (\bar{q}_\beta q_\alpha)_{V-A}$$

Semileptonic & Other Operators

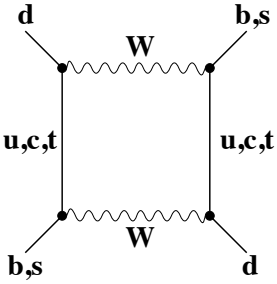
arXiv:9512380[hep-ph]

Semileptonic



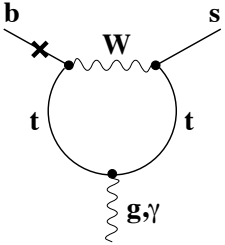
$$\mathcal{O}_{7V} = (\bar{s}_\alpha d_\alpha)_{V-A}(\bar{\ell}\ell)_V$$
$$\mathcal{O}_{7A} = (\bar{s}_\alpha d_\alpha)_{V-A}(\bar{\ell}\ell)_A$$
$$\mathcal{O}_{9} = (\bar{b}_\alpha s_\alpha)_{V-A}(\bar{\ell}\ell)_V$$
$$\mathcal{O}_{10} = (\bar{b}_\alpha s_\alpha)_{V-A}(\bar{\ell}\ell)_A$$

Mixing



$$\mathcal{O}_{\Delta B=2} = (\bar{b}_\alpha d_\alpha)_{V-A}(\bar{b}_\alpha d_\alpha)_{V-A}$$

Magnetic Penguins



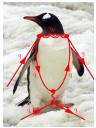
$$\mathcal{O}_{7\gamma} = \frac{e}{8\pi^2} m_b \bar{s}_\alpha \sigma^{\mu\nu}(1+\gamma_5) b_\alpha F_{\mu\nu}$$
$$\mathcal{O}_{8G} = \frac{g}{8\pi^2} m_b \bar{s}_\alpha \sigma^{\mu\nu}(1+\gamma_5) T^a_{\alpha\beta} b_\beta G^a_{\mu\nu}$$

BSM Operators

$$
\begin{aligned}
\mathcal{O}_{9'} &= (\bar{b}_\alpha s_\alpha)_{\mathrm{V+A}}(\bar{\ell}\ell)_{\mathrm{V}} \\
\mathcal{O}_{10'} &= (\bar{b}_\alpha s_\alpha)_{\mathrm{V+A}}(\bar{\ell}\ell)_{\mathrm{A}} \\
\mathcal{O}_{S} &= (\bar{b}_\alpha (1+\gamma_5) s_\alpha)(\bar{\ell}\ell) \\
\mathcal{O}_{S'} &= (\bar{b}_\alpha (1-\gamma_5) s_\alpha)(\bar{\ell}\ell) \\
\mathcal{O}_{P} &= (\bar{b}_\alpha (1+\gamma_5) s_\alpha)(\bar{\ell}\gamma_5\ell) \\
\mathcal{O}_{P'} &= (\bar{b}_\alpha (1-\gamma_5) s_\alpha)(\bar{\ell}\gamma_5\ell) \\
\mathcal{O}_{T} &= (\bar{b}_\alpha \sigma^{\mu\nu} s_\alpha)(\bar{\ell}\sigma_{\mu\nu}\ell) \\
\mathcal{O}_{T5} &= (\bar{b}_\alpha \sigma^{\mu\nu} (1-\gamma_5) s_\alpha)(\bar{\ell}\sigma_{\mu\nu}\gamma_5\ell)
\end{aligned}
$$

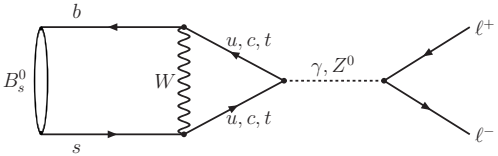
Simplest Case: Purely Leptonic Decays

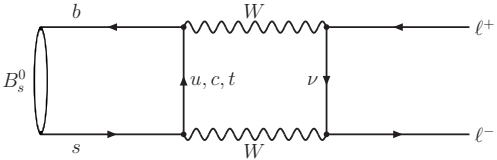
- Let's consider the decay $B_s^0 \to \mu^+\mu^-$



Penguin Topologies

Box Topologies





- In the Standard Model both diagrams correspond to operator $\mathcal{O}_{10} = (\bar{b}_\alpha s_\alpha)_{V-A}(\bar{\ell}\ell)_A$

Decay Constant: Linking Operator to Observable

- Left part of diagram contains quarks but no leptons: annihilation of the B_s meson into the vacuum state
- Right part of diagram contains leptons but no quarks: creation of a lepton pair from the vacuum state

$\Rightarrow$ Can factorise the calculation of hadronic matrix element into two parts

$$\langle \ell^+\ell^- | \mathcal{O}_{10} | B_s^0 \rangle = \langle \ell^+\ell^- | (\bar{\ell}\ell)_{\mathrm{A}} | 0 \rangle \times \langle 0 | (\bar{b}_\alpha s_\alpha)_{\mathrm{V-A}} | B_s^0 \rangle \tag{18}$$

- For a pseudo-scalar P, second term is parametrised by the decay constant source

$$\langle 0 | (\bar{q} q')_{\mathrm{V-A}} | P(p) \rangle = -i f_P p_\mu \tag{19}$$

- The decay constant can be calculated on the lattice: See *Flavour Lattice Averaging Group* (FLAG)
- Footnote: Similar expressions exist for scalar or vector states

Simplest Case: Purely Leptonic Decays

- Standard Model expression for the branching fraction arXiv:1303.3820

$$\mathcal{B}(B_s \to \mu^+\mu^-)\big|_{\text{SM}} = \frac{\tau_{B_s} G_F^2 m_W^4 \sin^4\theta_W}{4\pi^5} \times \left(\frac{G_F}{\sqrt{2}} \left|V_{ts}V_{tb}^*\right| \left|C_{10}^{\text{SM}}\right| f_{B_s}\right)^2 \times m_{B_s} m_\mu^2 \sqrt{1 - \frac{4m_\mu^2}{m_{B_s}^2}} \tag{20}$$

- Can still recognise the OPE using Wilson coefficients and operators

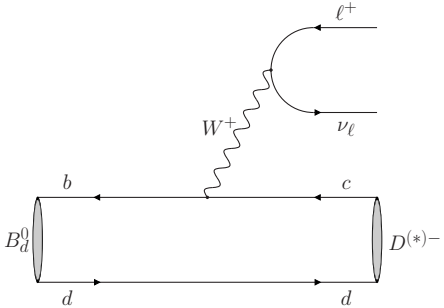
- Footnote: Expression for C_{10}^{SM}

$$C_{10}^{\text{SM}} = \frac{\eta_Y Y_0(x_t)}{\sin^2\theta_W} \tag{21}$$

- where Y_0 is one of the Inami-Lim functions

$$Y_0(x_t) = \frac{x_t}{8}\left[\frac{4 - x_t}{1 - x_t} + \frac{3x_t}{(1 - x_t)^2}\ln x_t\right] \quad , \quad x_t \equiv \left[\frac{\bar{m}_t(\bar{m}_t)}{m_W}\right]^2 \tag{22}$$

More Complex Case: Semi-Leptonic Decays



- Top part of diagram contains leptons but no quarks: creation of a lepton pair from the vacuum state
- Bottom part of diagram is transition from one quark state to another quark state

$\Rightarrow$ Can factorise the calculation of hadronic matrix element into two parts

$$\langle D^- \ell^+ \nu_\ell | \mathcal{O} | B_d^0 \rangle = \langle \ell^+ \nu_\ell | (\bar{\ell} \nu) | 0 \rangle \times \langle D^- | (\bar{b}_\alpha c_\alpha) | B_d^0 \rangle \tag{23}$$

Form Factor: Linking Operator to Observable

- The hadron-to-hadron transition is described by a Form Factor
- For a Pseudo-scalar to Pseudo-scalar transition source

$$\langle P'(p')|(\bar{q}\gamma_\mu q|P(p)\rangle = \left[(p+p')_\mu - \frac{m_P^2 - m_{P'}^2}{q^2} q_\mu\right] f^+_{P\to P'}(q^2) + \left[\frac{m_P^2 - m_{P'}^2}{q^2} q_\mu\right] f^0_{P\to P'}(q^2) \tag{24}$$

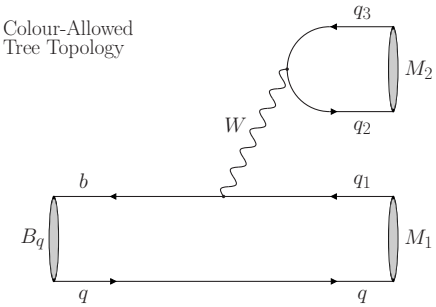
- $q_\mu \equiv p_\mu - p'_\mu$

- Form factors can be calculated on the lattice (high q^2) or using LCSR (low q^2)
- Need to extrapolate to other q^2 regions (large uncertainty!)
- q^2-dependence can be fitted to experimental data, for example $\mathrm{d}\mathcal{B}/\mathrm{d}q^2$ spectra
- Normalisation factor $f^{+/0}_{P\to P'}(0)$ needs to come from theory calculations

- Footnote: Similar expressions exist for pseudo-scalar to vector transitions

The Most Complex Case: Non-Leptonic Decays

The Ideal Case:



- Imagine velocity of q_2 and q_3 much larger than velocity of q_1

$\Rightarrow$ q_2 and q_3 will separate from q_1 before hadronisation takes place

- Expect no long-distance interactions between M_1 and M_2

$\Rightarrow$ Can factorise the calculation of hadronic matrix element into decay constant and form factors

$$\langle M_1 M_2 | \mathcal{O} | B_q \rangle = \langle M_2 | (\bar{q}_2 q_1) | 0 \rangle \times \langle M_1 | (\bar{b} q_1 | B_q \rangle = f_{M_2} \times f^{+/0}_{B_q \to M_1}(q^2) \tag{25}$$

The Most Complex Case: Non-Leptonic Decays

Every Other Case:

1. Explicitly account for non-factorisable corrections
2. Avoid observables that require absolute normalisation (i.e. branching fractions)

Use Clever Tricks!

- Exploit ratios (CP asymmetries, branching ratios, ...)
- Exploit flavour symmetries

When Does Factorisation Work?

Expected to Work Well

- Colour-allowed tree diagrams with $m_{M_1} \gg m_{M_2}$
- Examples: $B_d^0 \to D^-\pi^+$, $B_d^0 \to D^-K^+$, $B_s^0 \to D_s^-\pi^+$, $B_s^0 \to D_s^-K^+$

May or may not Work

- Colour-allowed tree diagrams with $m_{M_1} \approx m_{M_2}$
- Required separation still takes place, but q_1 is now too fast to hadronise with the spectator quark
- Additional gluon exchanges expected to dissipate energy
- Examples: $B_d^0 \to \pi^-\pi^+$, $B_s^0 \to K^-K^+$

Not Expected to Work

- Colour-allowed tree diagrams with $m_{M_1} \ll m_{M_2}$
- Example: $B_s^0 \to K^-D^+$
- Other decay topologies, like colour-suppressed tree diagrams, etc.
- Example: $B_s^0 \to \bar{K}^0 D^0$, $B_d^0 \to K^0 J/\psi$