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Energy scales in the Standard Model and beyond

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April 13, 2023

Standard Model of Elementary Particles

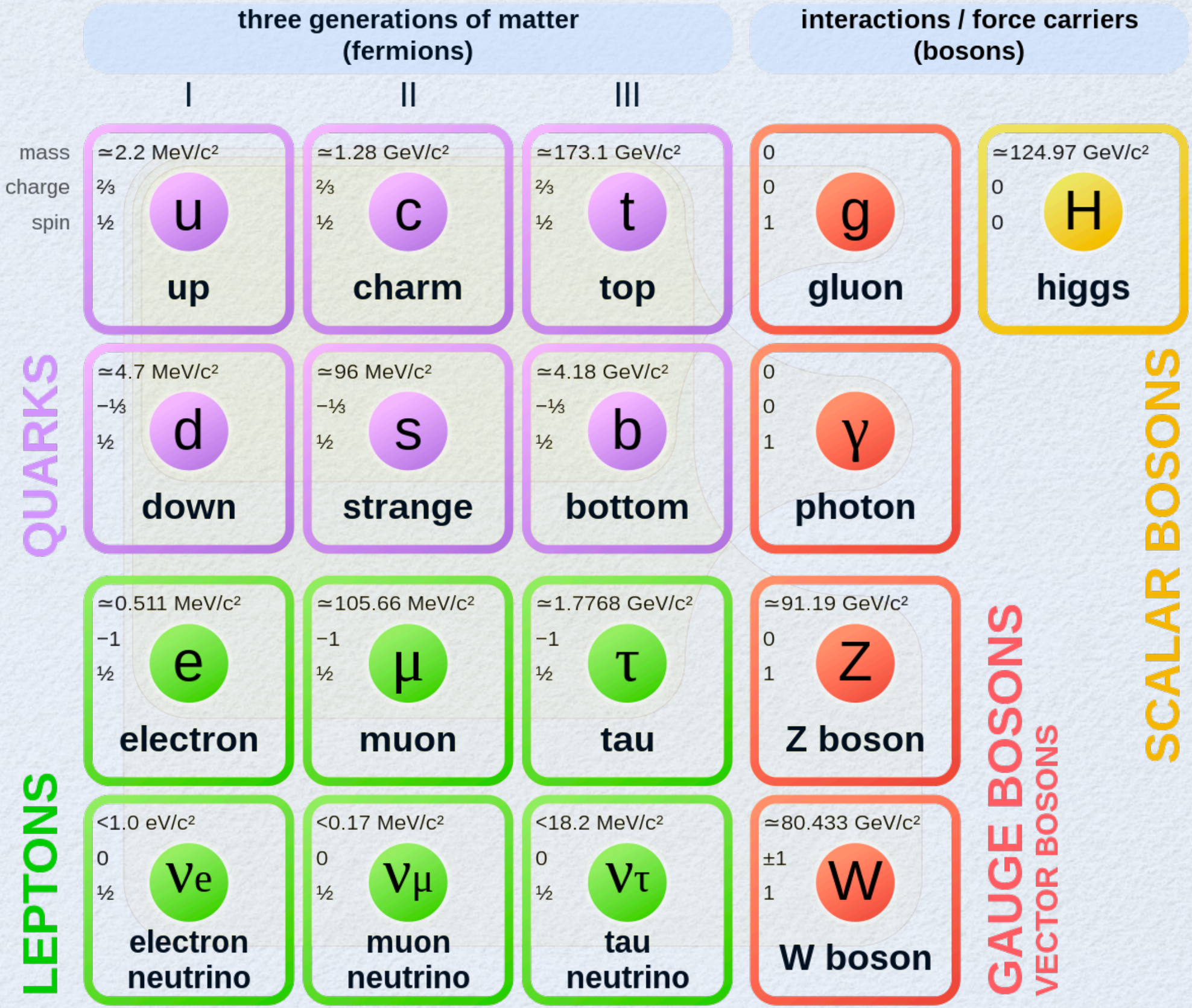


Figure from Wikipedia

The SM has two types of massless particles: the photon and the gluons

Let's start with the photon, the force carrier of the EM force

Question about photons: why can we see each other?

Classically photons do not interact, photons are electrically neutral

Maxwell equations are linear in E and B [in vacuum]

$$\mathcal{L} = \frac{1}{2}\left(\mathbf{E}^2 - \mathbf{B}^2\right)$$

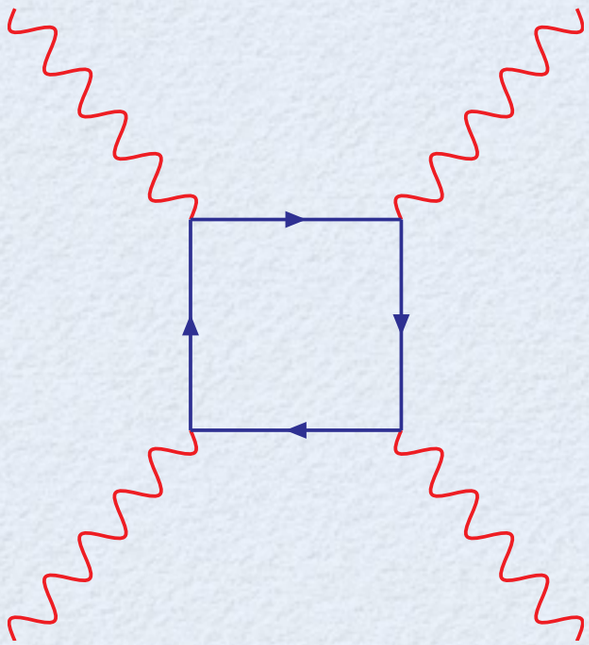
$$\mathcal{L}_{\mathrm{QED}} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \bar{\psi}(i\,\not{\partial} - e\,\not{A} - m)\psi$$

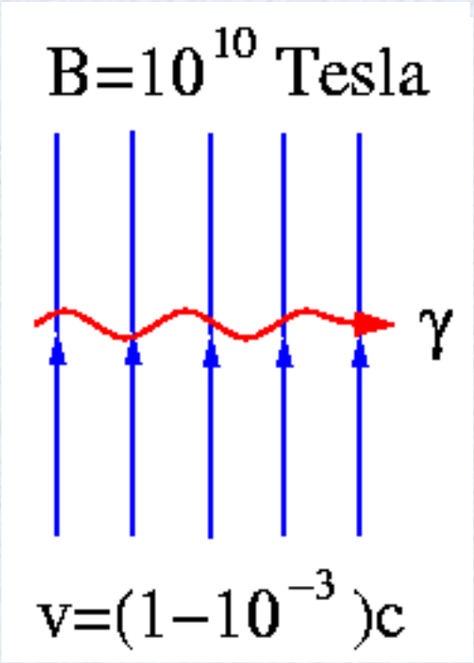
$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$$

But photons do couple to electrons, because those have charge (e)

Photons in the visible spectrum have O(2-3 eV) energy, but electrons have 0.5 MeV mass, so two visible light photons colliding can't produce electrons

In QED: photons interact via electrons and positrons that exist temporarily





This yields small nonlinear corrections to Maxwell equations

For comparison:
Neutron star ~ 10^7-10^8 Tesla

Could play a role in magnetars

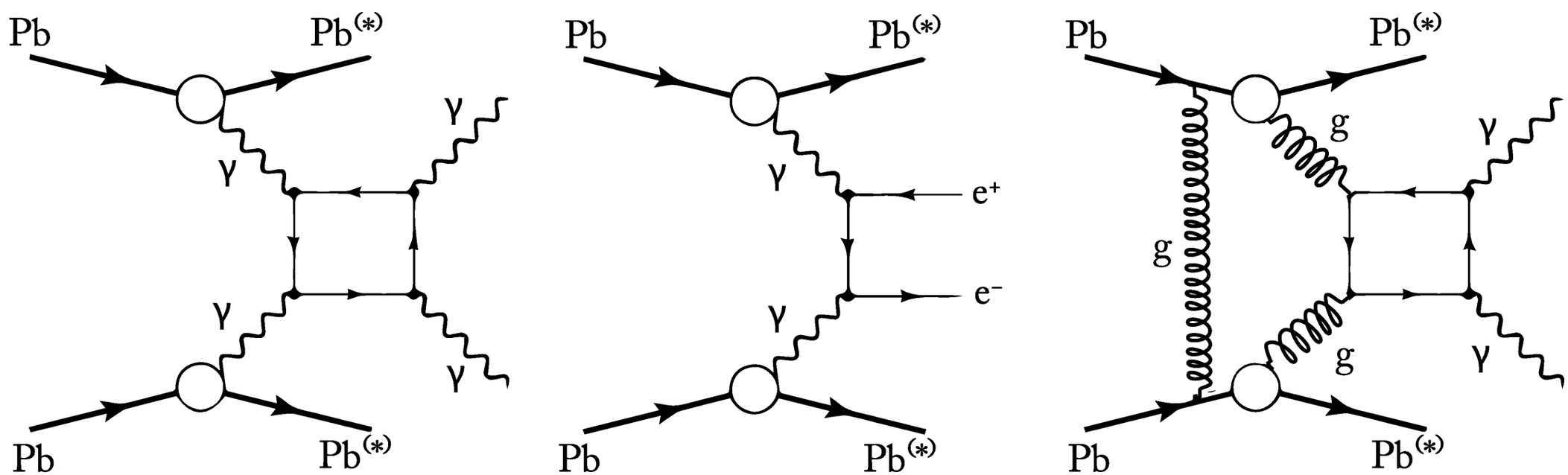
This light-by-light (LbL) scattering has never been observed at low energies, i.e. when $\hbar\omega$ is small compared to $m_e c^2$

$$\sigma_{\gamma\gamma\to\gamma\gamma} = \frac{1}{2\pi}\frac{139}{(90)^2}\frac{56}{11}\alpha^4\left(\frac{\omega}{m}\right)^6\frac{1}{m^2} \qquad \omega/m \ll 1$$

For visible light this is 10^{-60} cm^2 or 10^{-36} barn

$$\alpha_{SI} = \frac{1}{4\pi\epsilon_0}\frac{e^2}{\hbar c} \approx \frac{1}{137}$$

Light-by-light scattering at LHC



LbL scattering has been observed in high energy scattering of photons from Ultra-Peripheral Collisions (UPCs) of Pb ions at LHC

The c.o.m. energy of that LbL is 5+ GeV, so the intermediate particles are primarily heavy quarks and leptons (charm, bottom, tau), not the electron

At this energy also QCD reactions matter, it is not simply QED and m_e is not at all a relevant scale in the process

Figure from CMS, PLB 797 (2019) 134826

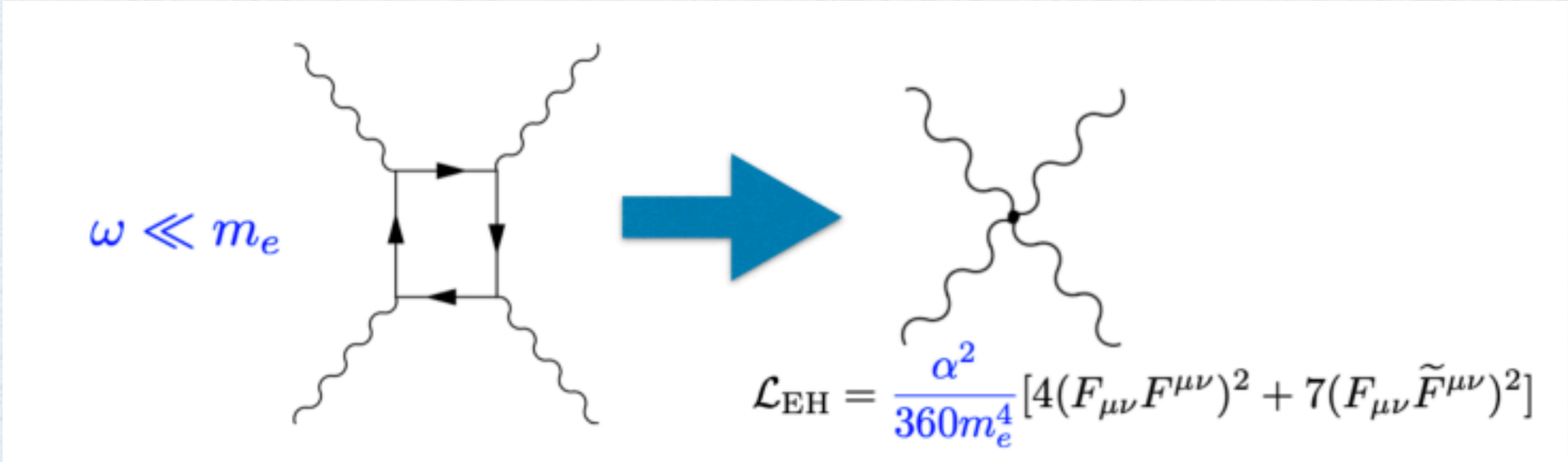


Figure from arXiv:2203.12714

$$\mathcal{L}_{\rm EH} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{\alpha^2}{90m^4}\left[(F^{\mu\nu}F_{\mu\nu})^2 + \frac{7}{4}\left(F^{\mu\nu}\tilde{F}_{\mu\nu}\right)^2\right] + \mathcal{O}(\alpha^4)$$

Additional terms in this effective lagrangian are all P- and CP-even

$$\tilde{F}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}F^{\alpha\beta}$$

$$\mathcal{L} = \frac{1}{2}\left(\mathbf{E}^2 - \mathbf{B}^2\right) + \frac{2\alpha^2}{45m^4}\left[\left(\mathbf{E}^2 - \mathbf{B}^2\right)^2 + 7(\mathbf{E}\cdot\mathbf{B})^2\right]$$

When valid? $\alpha \ll 1$ (basically always) and $\omega \ll m_e$

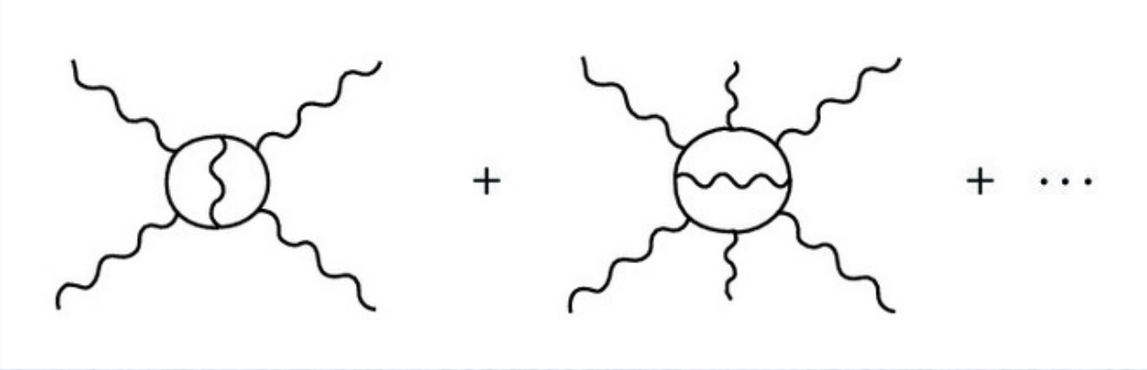
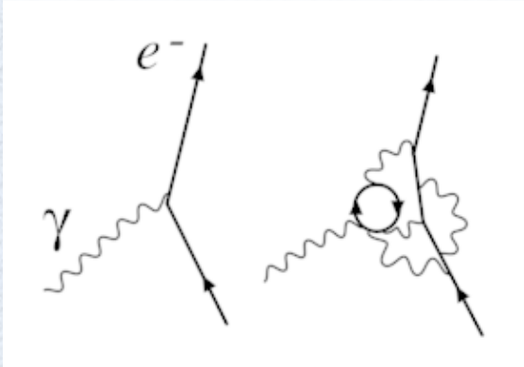


Figure from https://doi.org/10.1155/2017/6214341

As the energy approaches m_e more corrections are needed, until full QED is needed

Fine structure constant not constant due to quantum corrections

Effective structure constant as a function of energy/resolution scale



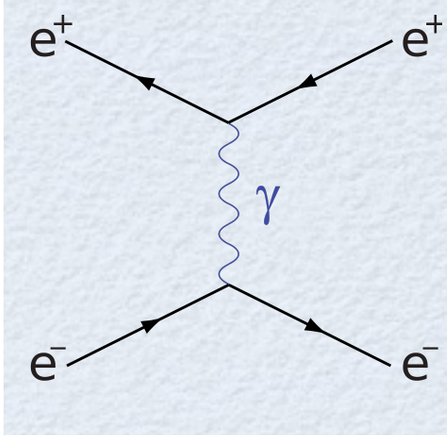
Recall that Bhabha scattering is $e^+e^- \to e^+e^-$, where at small angles the t-channel one photon exchange is dominant and $t = -s(1-\cos\theta)/2 \approx -s\theta^2/4$. The differential cross section is written as:

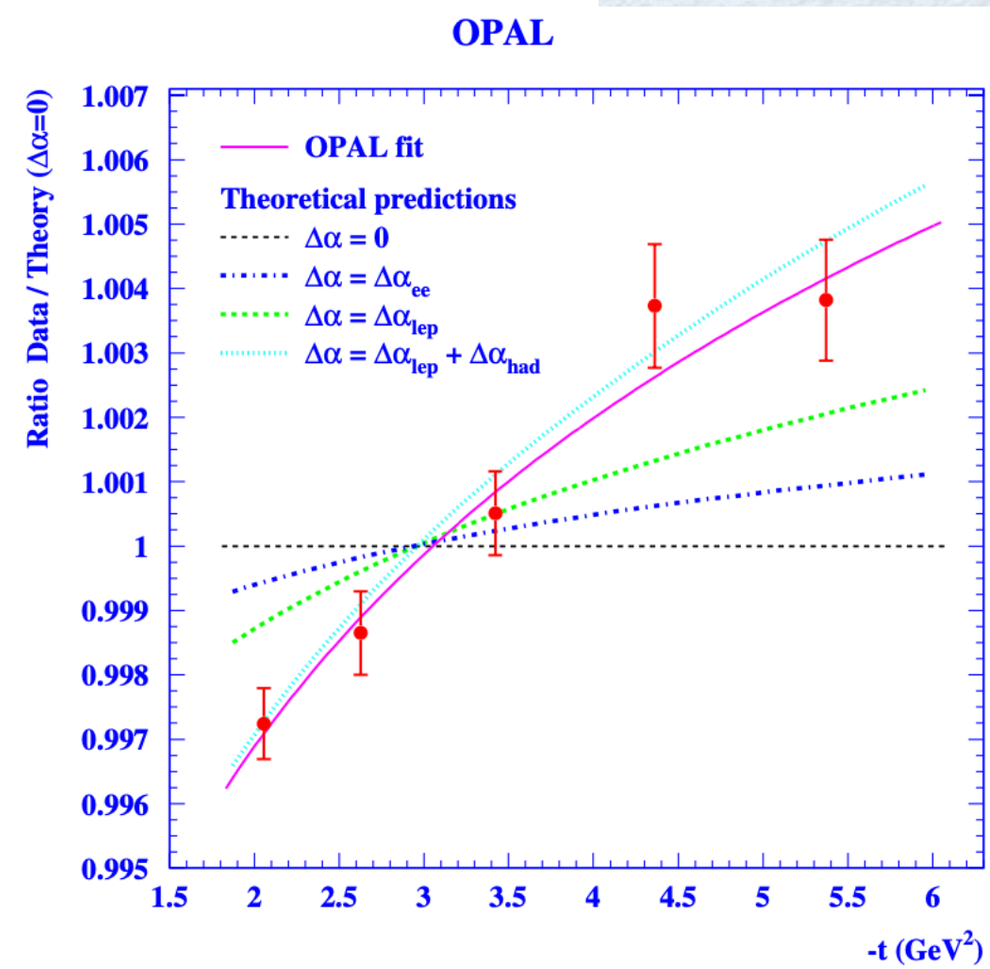
$$\frac{d\sigma}{dt} \approx \frac{d\sigma^{(0)}}{dt}\left(\frac{\alpha(t)}{\alpha_0}\right)^2 (1+\epsilon), \tag{5.2}$$

where $d\sigma^{(0)}/dt$ stands for the Born process $(4\pi\alpha_0^2/t^2)$ and ϵ stands for the radiative corrections to the t-channel one photon exchange diagram. In corrections from the s-channel annihilation diagrams for the have also been taken into account. Furthermore, $\Delta\alpha$ is defined

$$\alpha(q^2) = \frac{\alpha_0}{1-\Delta\alpha(q^2)},$$

where $\alpha_0 = \alpha(q^2=0) \approx 1/137.036$. It is stated in the article tl of the dominant uncertainties in the electroweak fits constraini





G. Abbiendi *et al.*, Eur. Phys. J. C **45** (2006) 1

Figure from Peskin & Schroeder

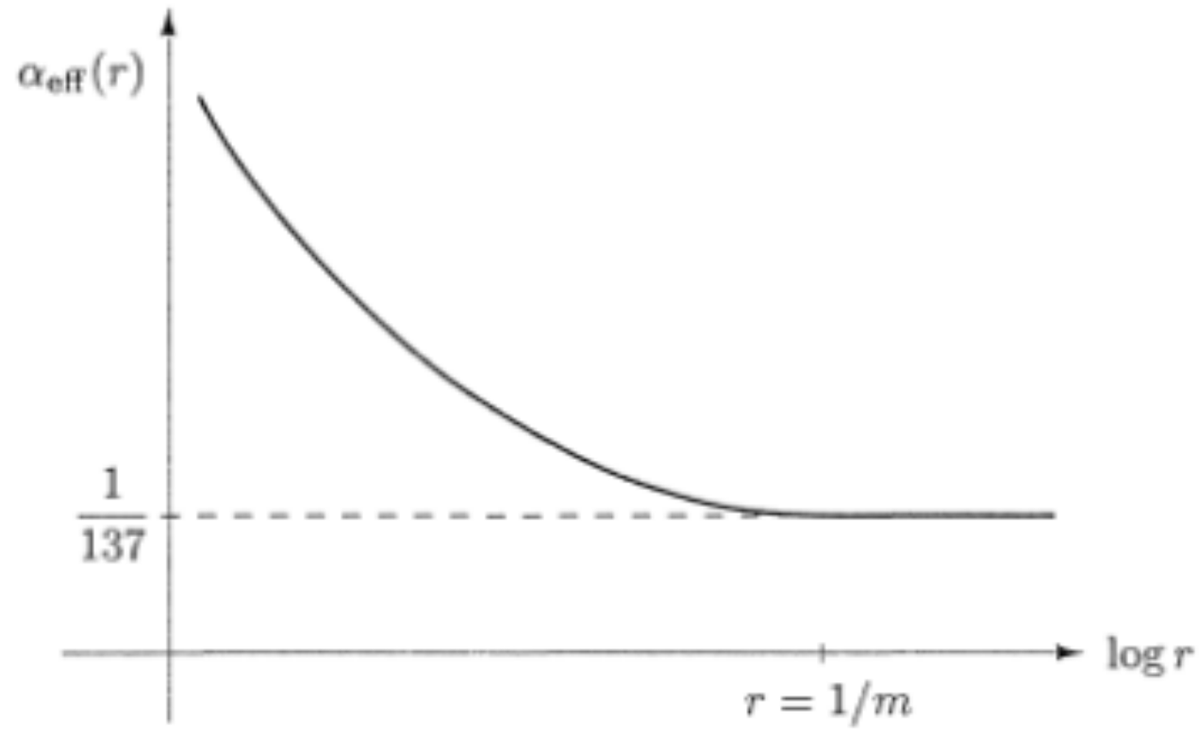


Figure 7.10. A qualitative sketch of the effective electromagnetic coupling constant generated by the one-loop vacuum polarization diagram, as a function of distance. The horizontal scale covers many orders of magnitude.

At low energies (= large distances) the fine structure constant does not run and equals 1/137

But for high energies it increases in strength: $\alpha(M_Z) \sim 1/128$

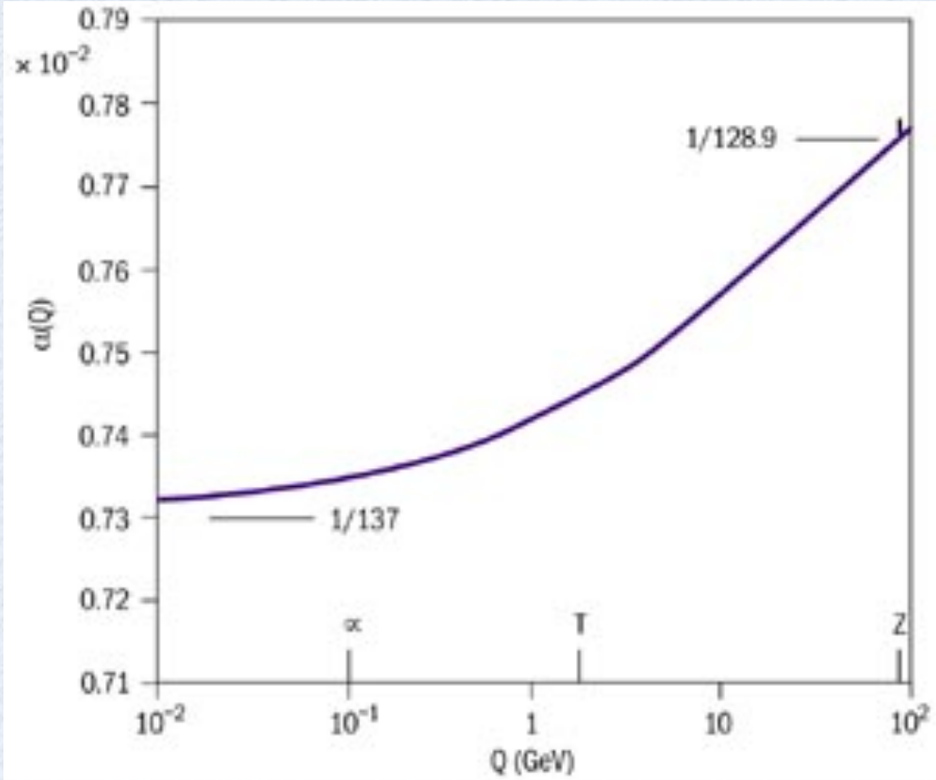
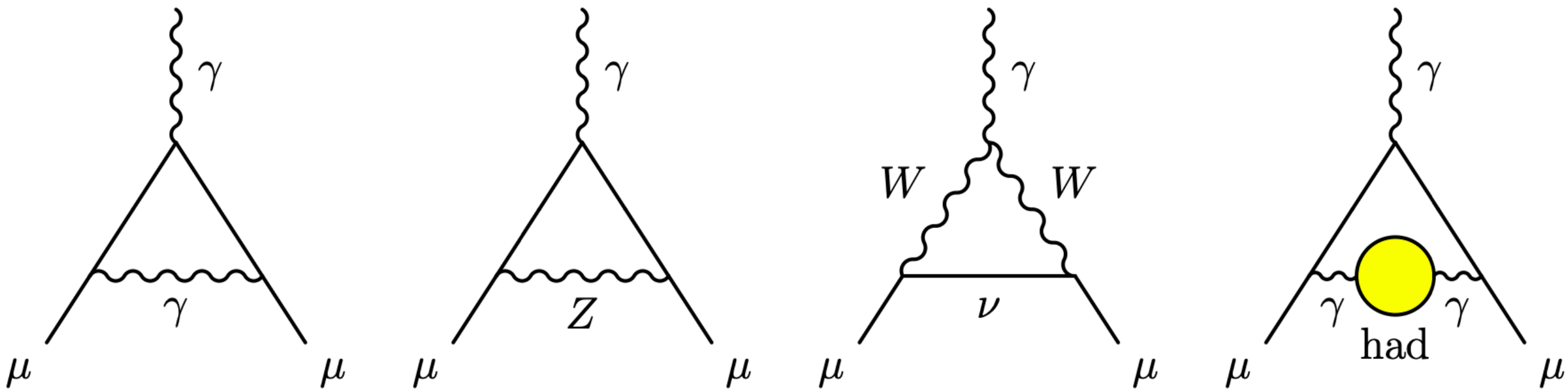


Figure from CERN Courier

At extremely high energies (far beyond the Planck scale of 10^{19} GeV, where α still $< 1/100$) it becomes infinitely large (Landau pole is around 10^{268} GeV) but far below it new d.o.f. will affect its running (first up will be muons and effects of light quarks, which are hard to calculate)

It not only depends on the energy scale one is probing (the mass of the new d.o.f.), but also on the precision with which one measures (indirect effect of new d.o.f.)

Figure from PDG



g-2 of the electron is measured at very low energies, still the measurement is so precise it is sensitive to heavy particles through loop corrections

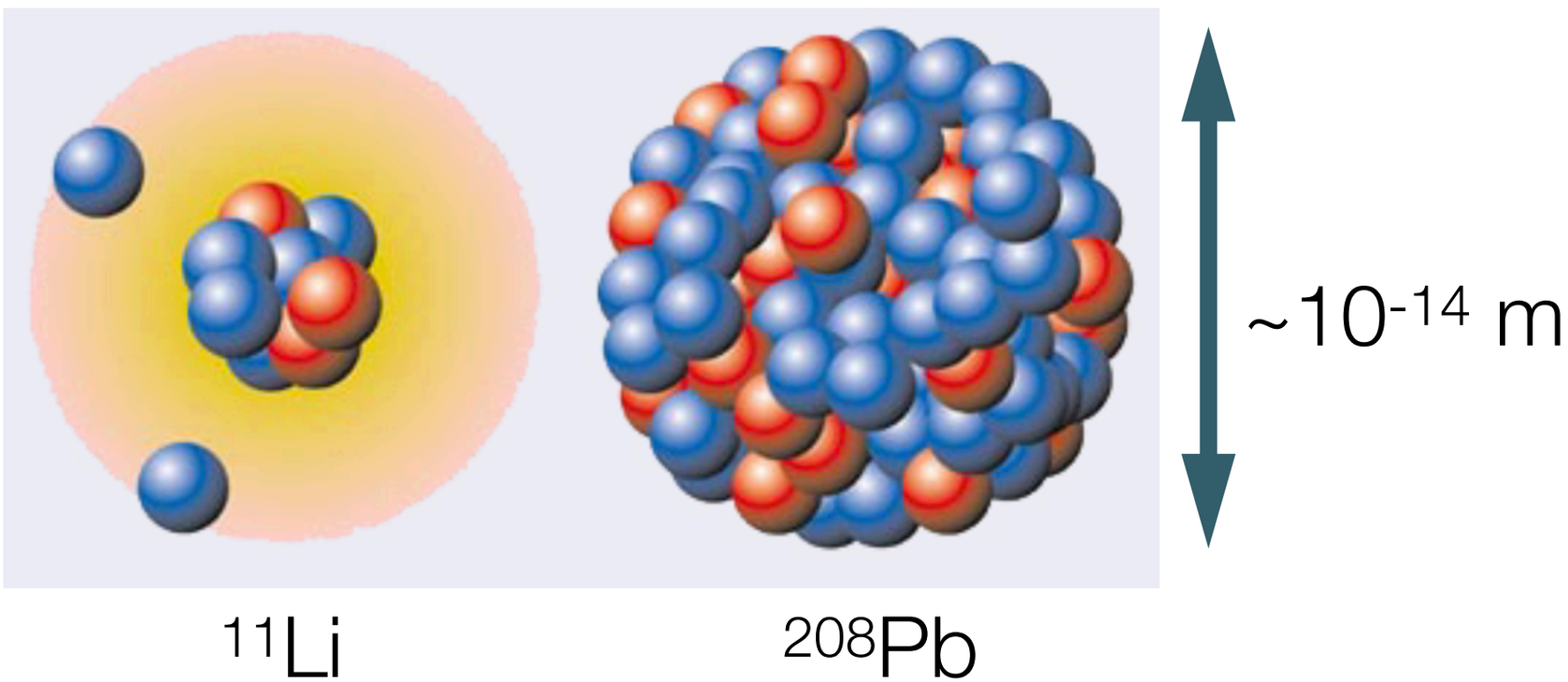
QED is then far from sufficient, QCD enters, but also the EW corrections from W, Z bosons and neutrino's, but eventually also BSM physics

Just like at LHC one can do direct searches (actual production of new particles, by going to the energy of the new physics) or indirect searches (e.g. in B decays)

Before turning to BSM, let's have a look at QCD

The strong nuclear force

The force binding protons and neutrons into nuclei of atoms



The force is extremely short range

The carriers of the strong force, the gluons, are massless, but lead to a short range force as if they are heavy

Figure from CERN Courier

QCD - the microscopic theory of quarks and gluons

$$\mathcal{L}_{\mathrm{QCD}} = -\frac{1}{4} F_a^{\mu\nu} F^a_{\mu\nu} + \bar{\psi}(i \not{\partial} - g \not{A}_a T^a - m)\psi$$

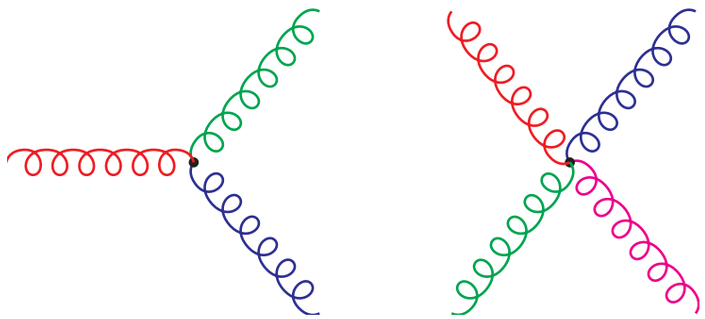
$$F_a^{\mu\nu} = \partial^\mu A_a^\nu - \partial^\nu A_a^\mu - g f_{abc} A_b^\mu A_c^\nu$$

a=1,...,8

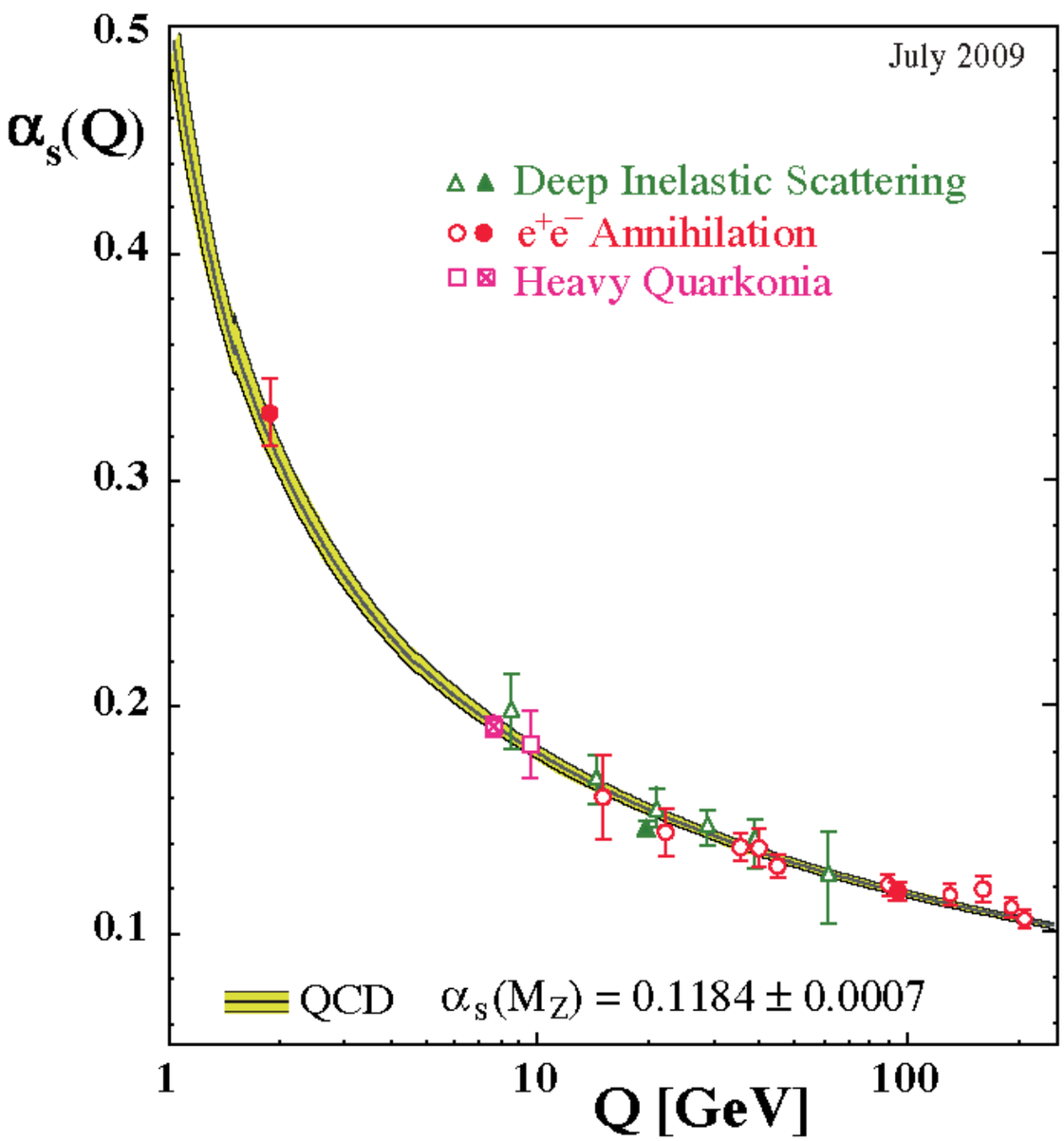
$SU(3)$ Non-Abelian gauge theory, highly nonlinear, already classically

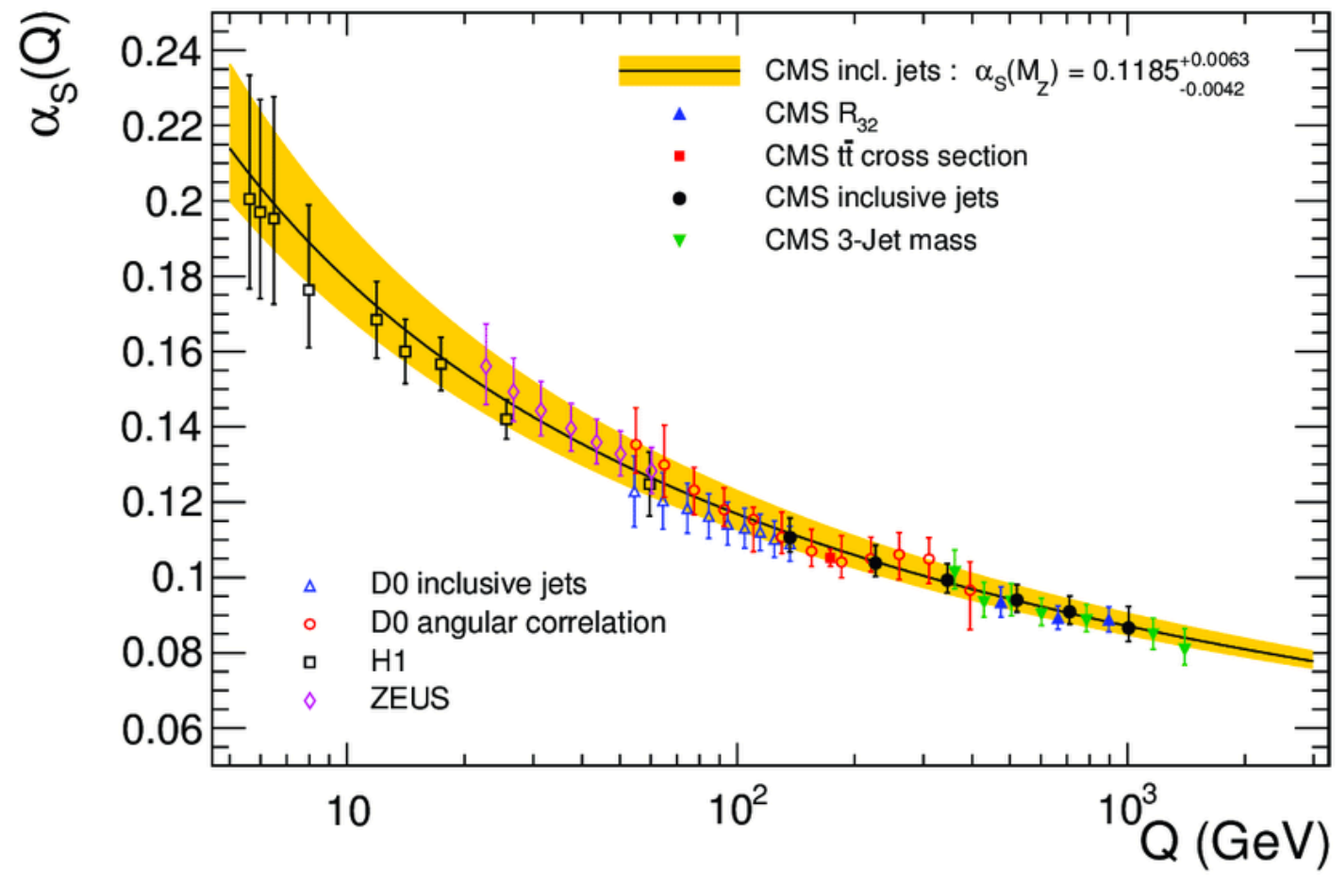
QCD has 8 “photons”, called gluons

They have color charge themselves and couple strongly to quarks and to each other



Asymptotic freedom

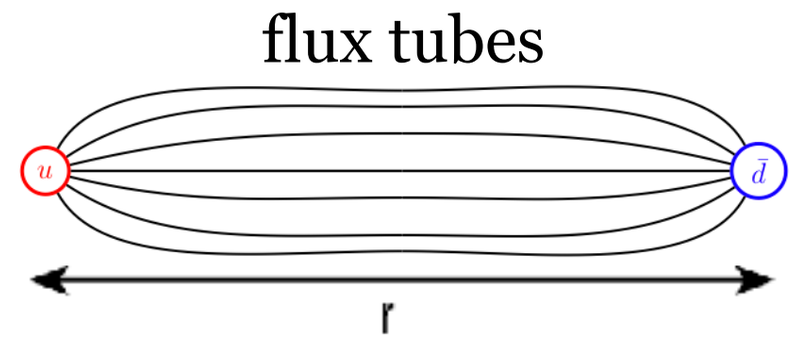




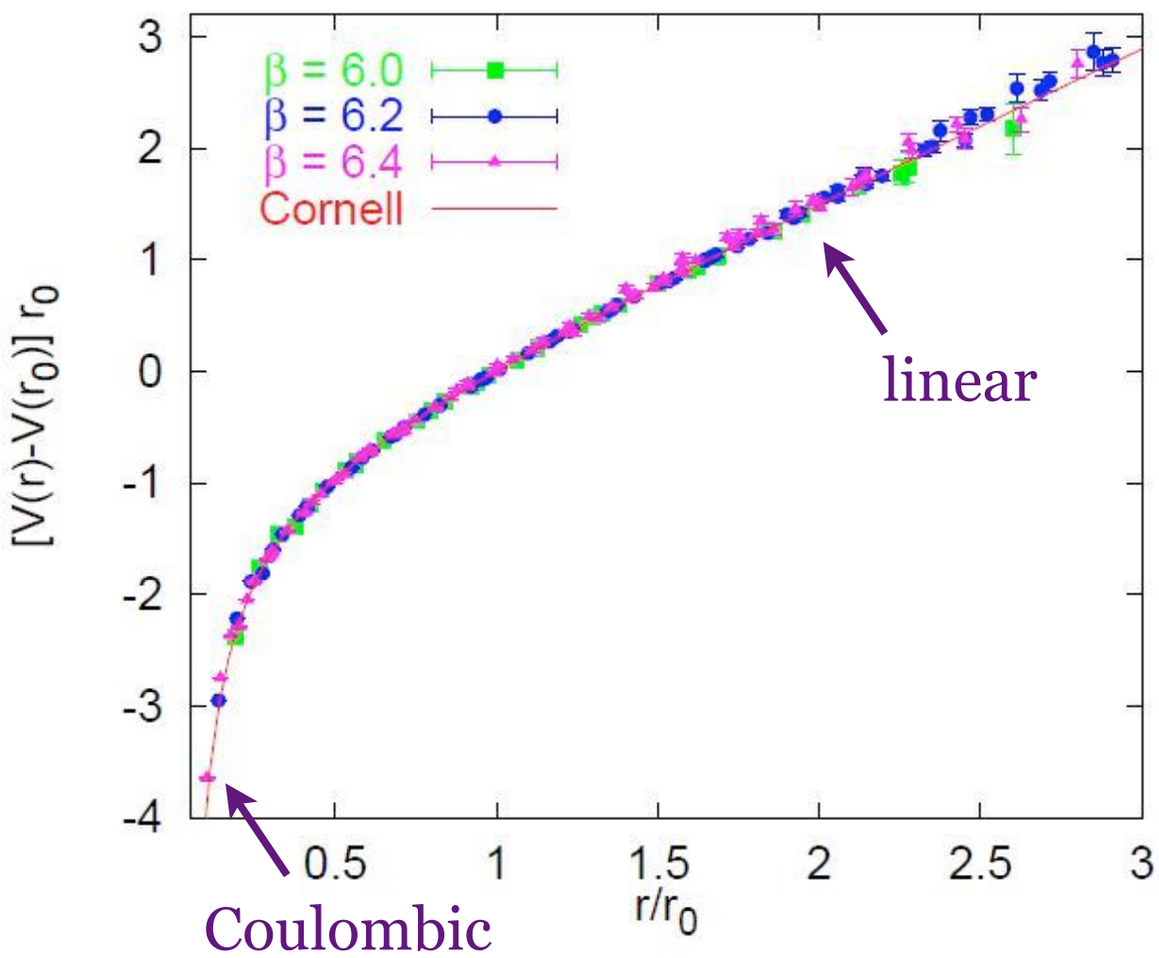
The strong force has a coupling strength that decreases with increasing energy

This allows for perturbative calculations, but when discussing hadrons (protons, pions, B mesons, etc) the nonperturbative part *always* enters

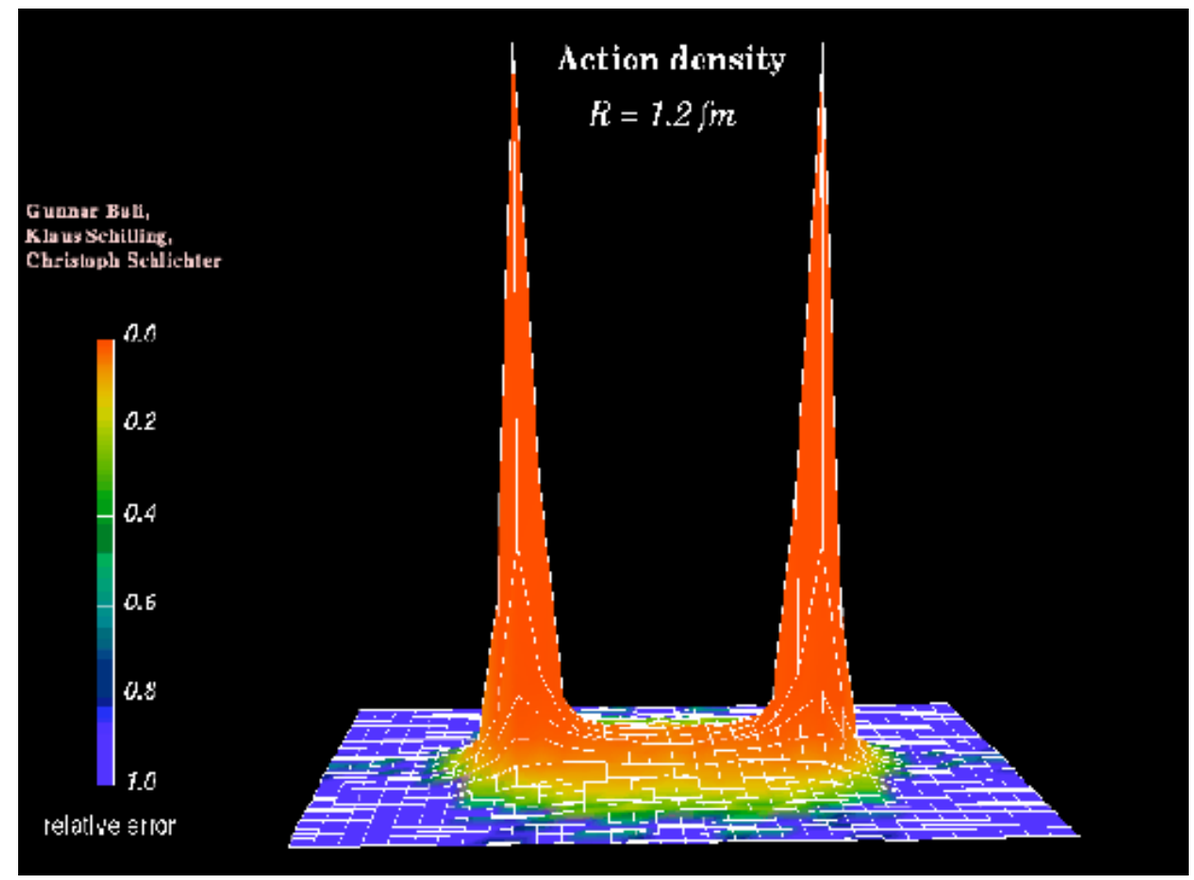
Confinement



Separating quarks costs increasingly more energy → quark confinement



From lattice QCD:



Bali, Schilling, Schlichter
Phys. Rev. D 51 (1995) 5165

Force carriers themselves subject to confinement → flux tubes

Strong coupling

The energy dependence of the strong coupling constant has the property that it becomes large at low energies, unlike the fine structure constant

The position of its Landau pole is the low energy scale $\Lambda_{QCD} \sim 200$ MeV
(below 1 GeV the perturbative calculation cannot be trusted anymore)

$$\alpha_s(Q^2) = \frac{1}{\beta_0 \ln\left[Q^2/\Lambda^2\right]} \qquad Q^2 \gg \Lambda^2$$

The magnitude of α_s can be determined at high energies, where perturbation theory *is* valid (>1 GeV)

However, its value depends on the order in perturbation theory one considers (and the renormalization scheme one chooses) and also on the energy range considered (which determines the number of active quark flavors)

Strong coupling

Λ_{QCD} is not a scale that appears in the Lagrangian of QCD

In a sense it is a scale that emerges from QCD, it sets the scale of nonperturbative physics

Λ_{QCD} is about 200 MeV

It is close to the mass of the lightest hadron m_π that is also an emergent scale

Gell-Mann, Oakes and Renner:

$$M_\pi^2 = (m_u + m_d)B + \mathcal{O}(m^2)$$
$$B = |\langle 0|\bar{u}u|0\rangle / f_\pi^2|_{m_q \to 0}$$

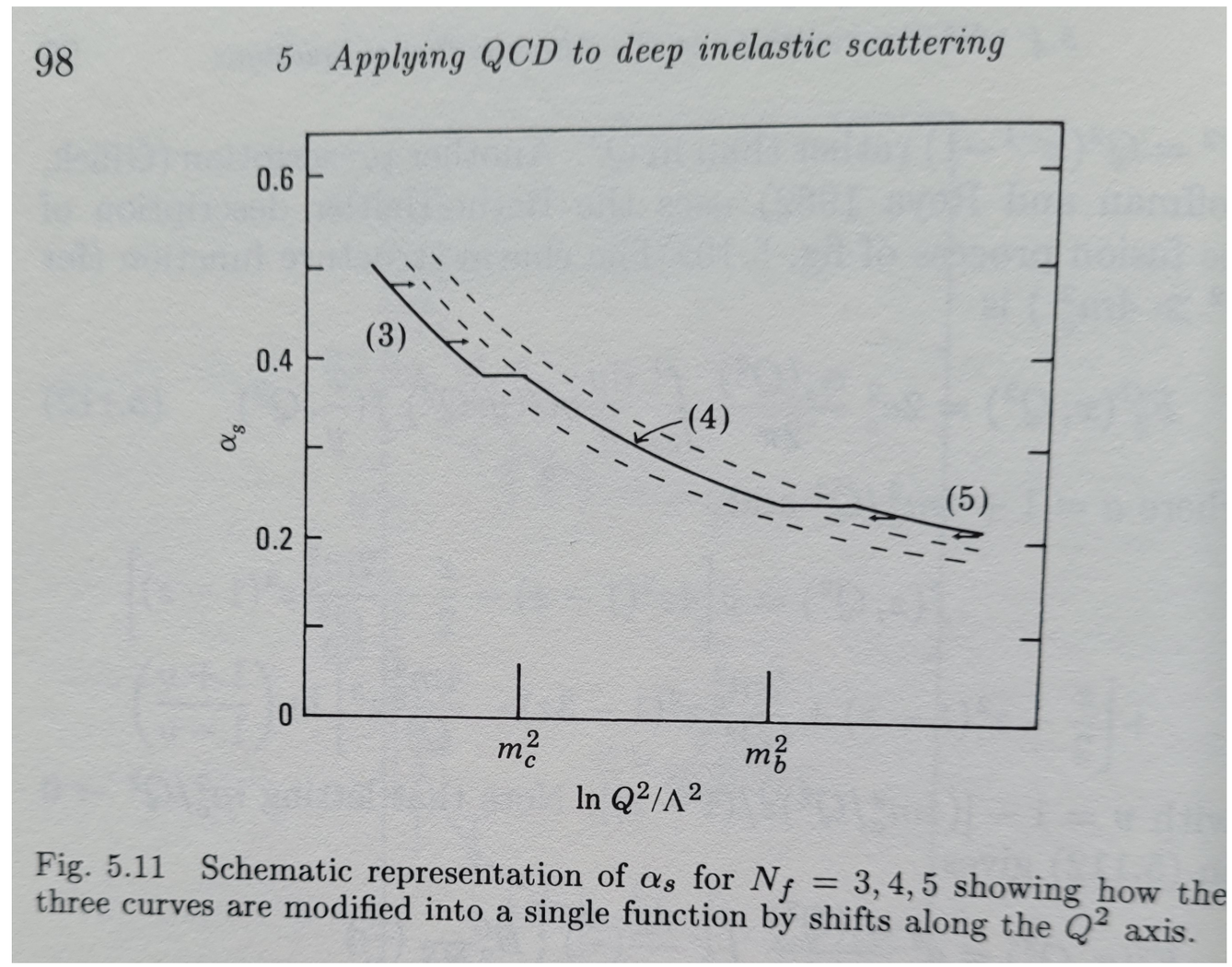


Figure from book by R.G. Roberts
"The Structure of the proton"

Hadronic EFTs

As soon as one deals with hadrons it is useful to see which d.o.f. are relevant and consider an EFT that describes these d.o.f.:

Hadrons with light quarks only: 2 and 3 flavor χPT (chiral perturbation theory, a p/Λ_χ and m/Λ_χ expansion, where the chiral symmetry breaking scale ~1.2 GeV)

Hadrons with heavy quarks: HQET (E/m_Q expansion), NRQCD (expansion in the (small) relative velocity v of the quark-antiquark inside a HQ meson), and related EFTs (see also lecture De Bruyn)

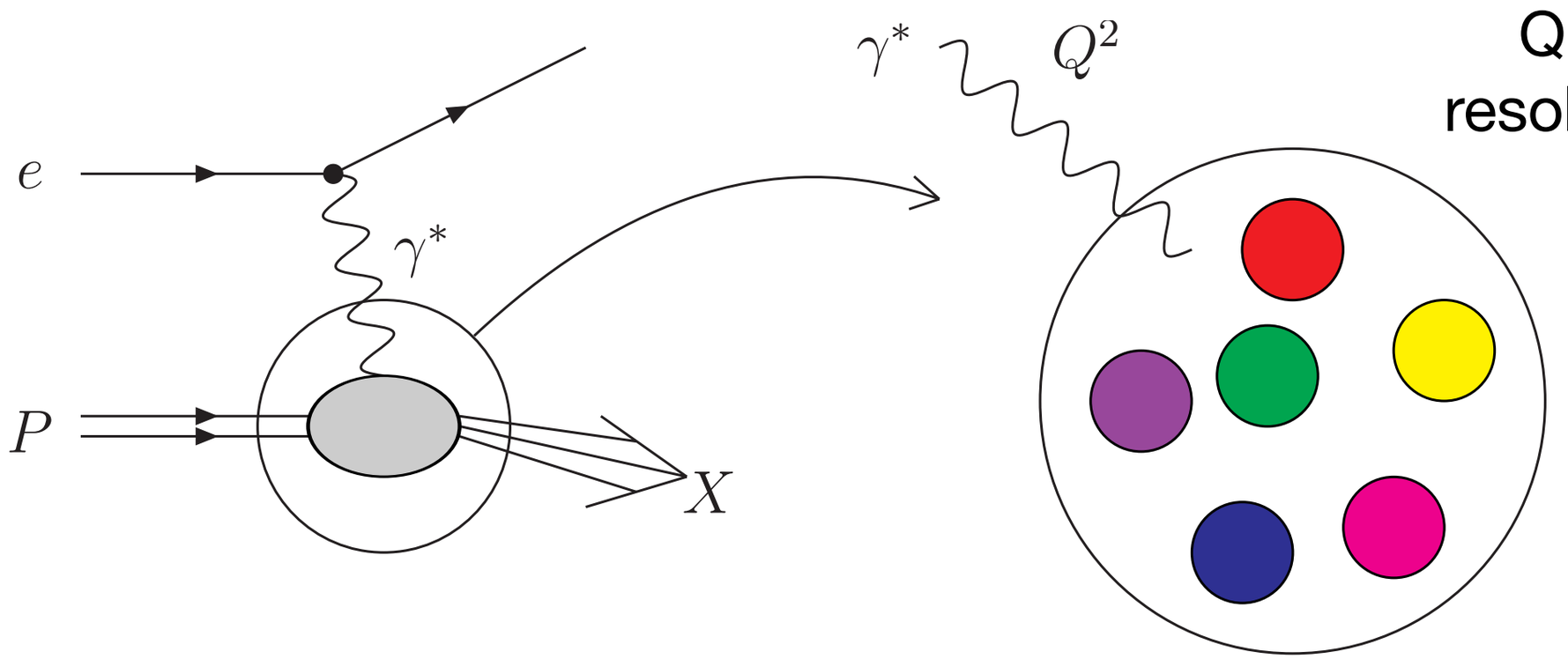
But QCD has one more emergent scale, which becomes apparent in scattering of hadrons (and nuclei) at high energy when there is a very high gluon density

Probing gluons

The confinement distance is of proton size (~10^{-15} m)

To look inside the proton requires energies larger than 200 MeV (~1 fm)

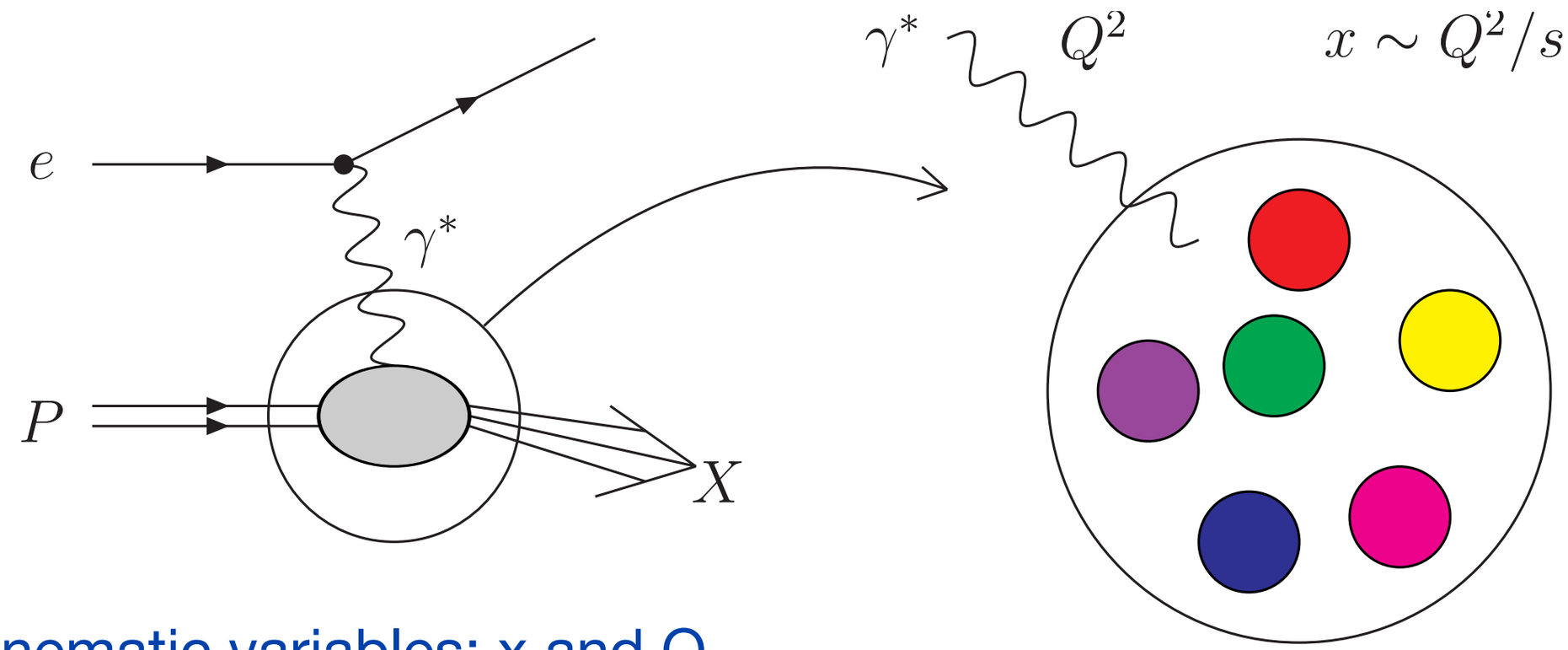
Electron-proton colliders have center of mass energies much larger than this, e.g. HERA had $\sqrt{s}$~320 GeV and EIC will have $\sqrt{s}$~20-140 GeV



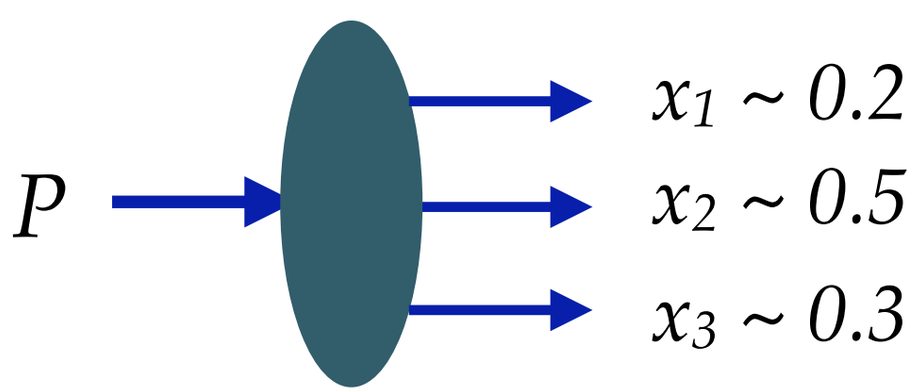
Q sets the resolution scale

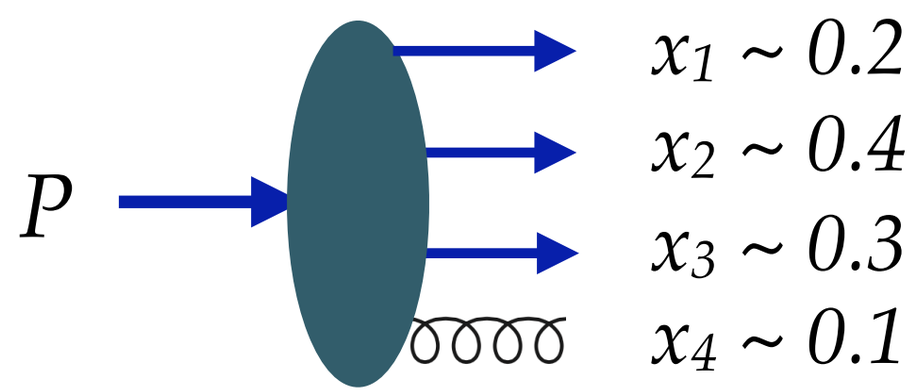
Deep inelastic scattering

Scattering off a proton at high energy = scattering off quarks and gluons



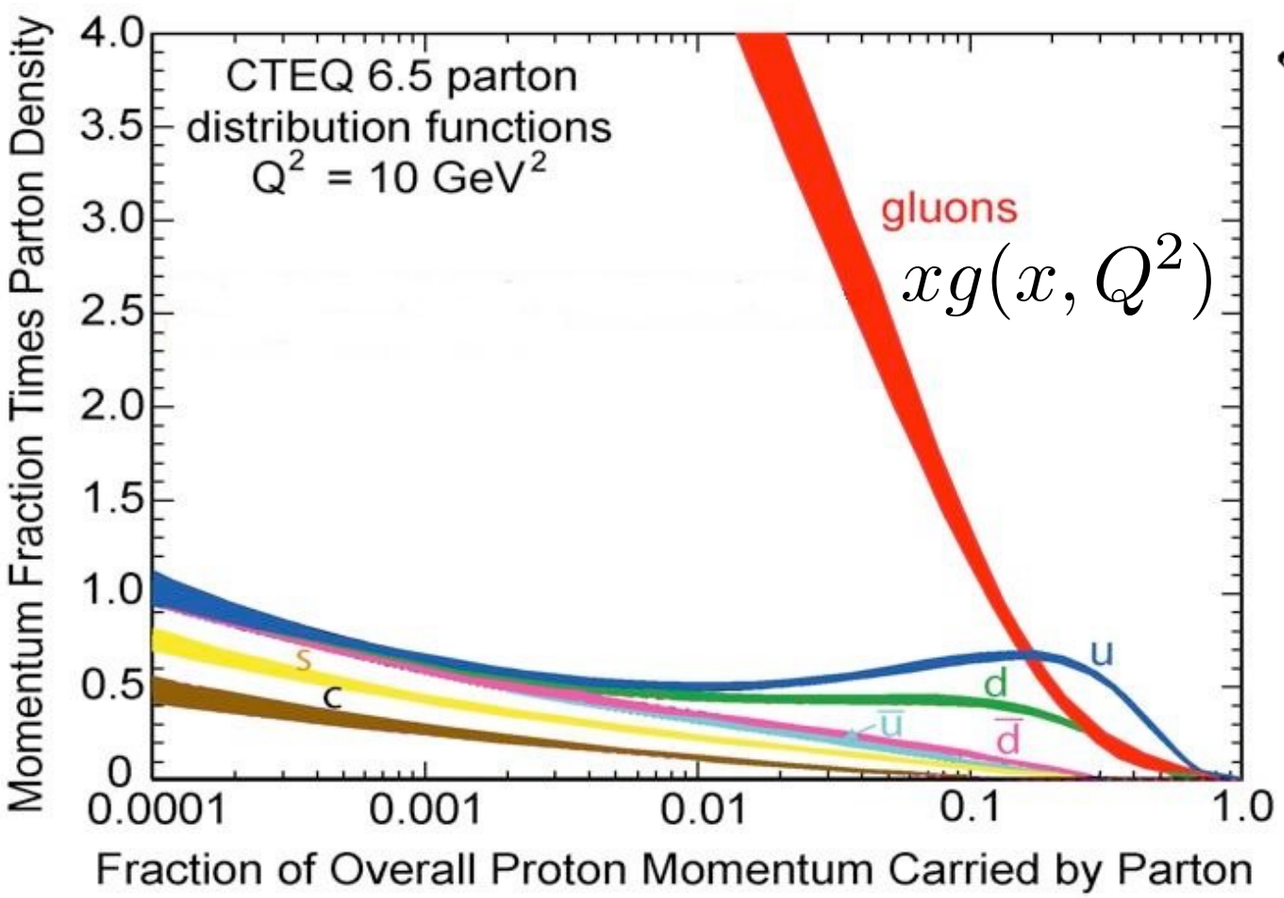
Two kinematic variables: x and Q

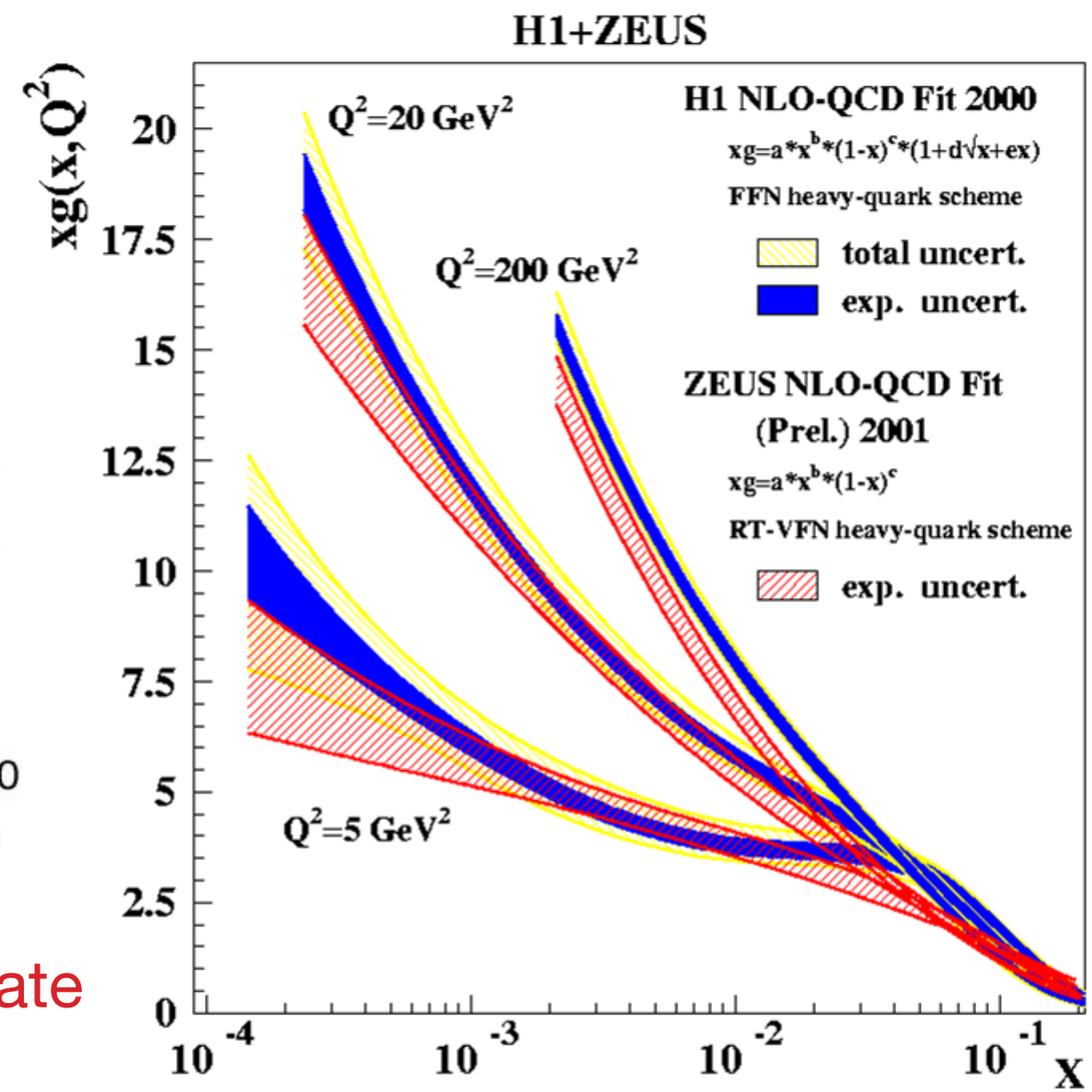




Gluon distribution

g(x,Q²) = probability of finding a gluon with momentum fraction x inside the proton at the energy scale Q

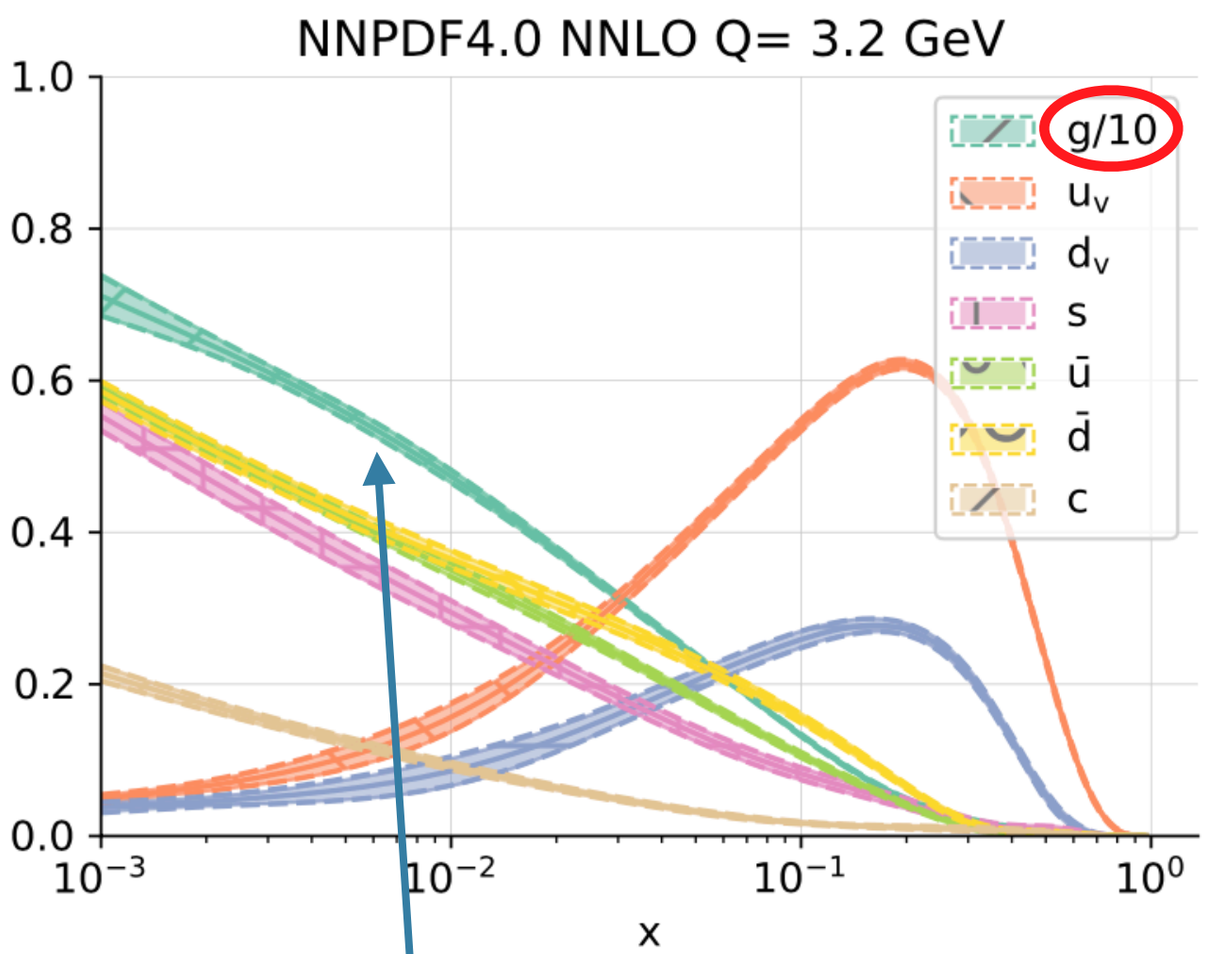


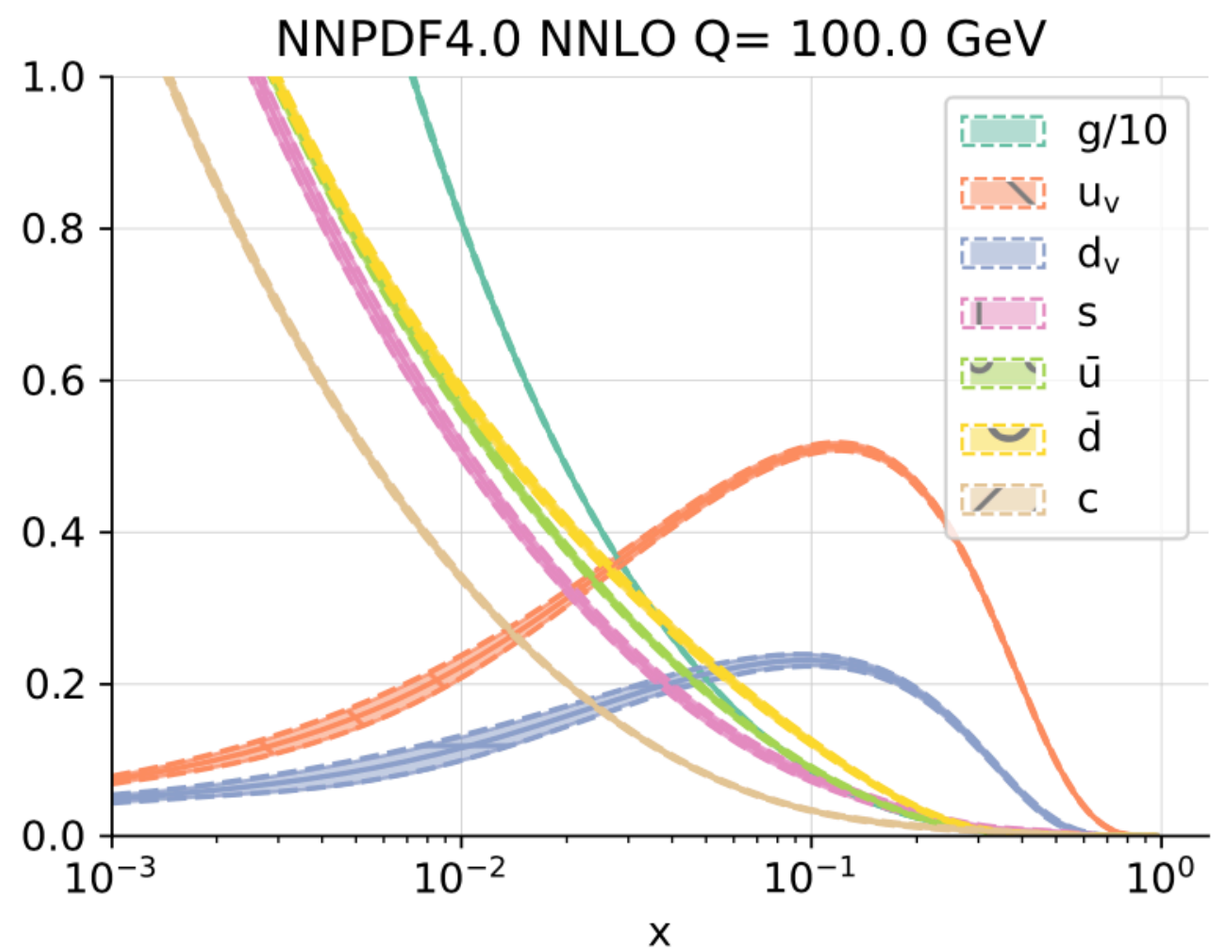


For small x values gluons by far dominate

→ very high gluon density

Parton densities (PDFs) of the proton anno 2023





Fraction of overall proton momentum carried by parton

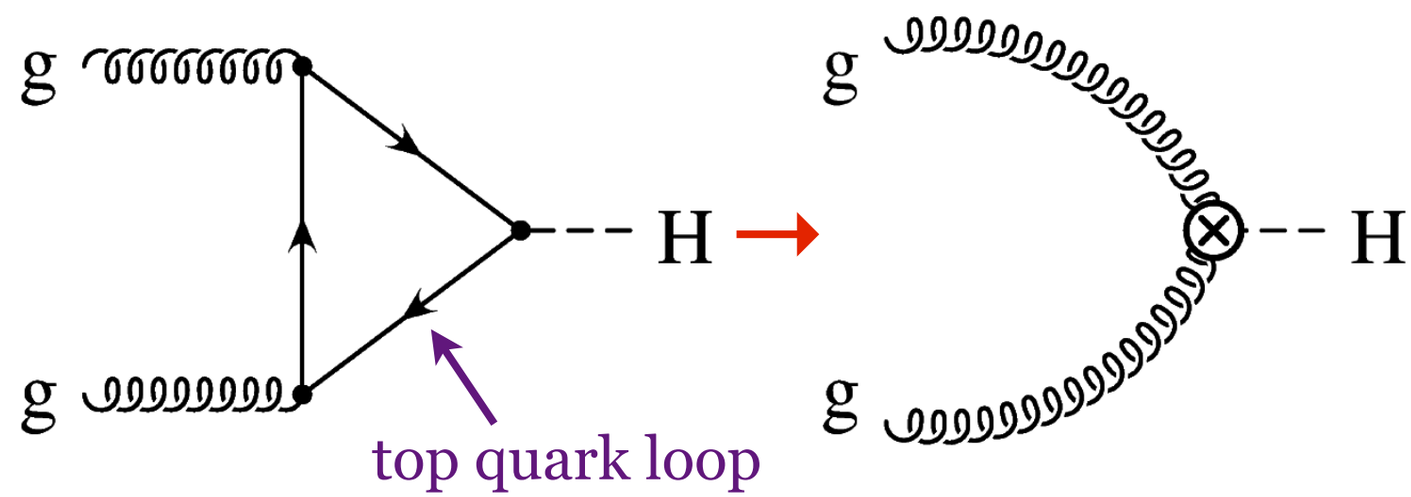
small momentum partons (x<0.1) are mostly gluons

At high energies collisions involve mostly gluons

Large x gluons

Gluon distributions matter for BSM searches (direct production)

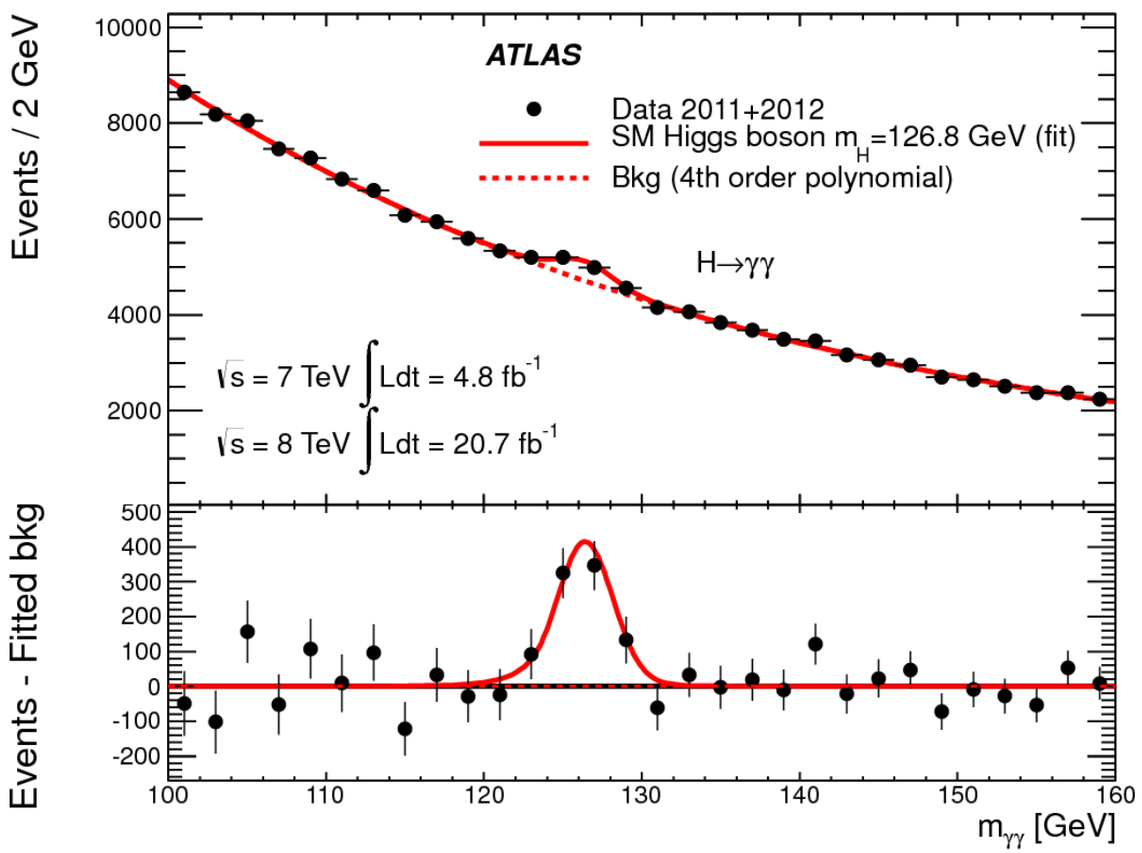
Higgs production at LHC happens predominantly via gg → H

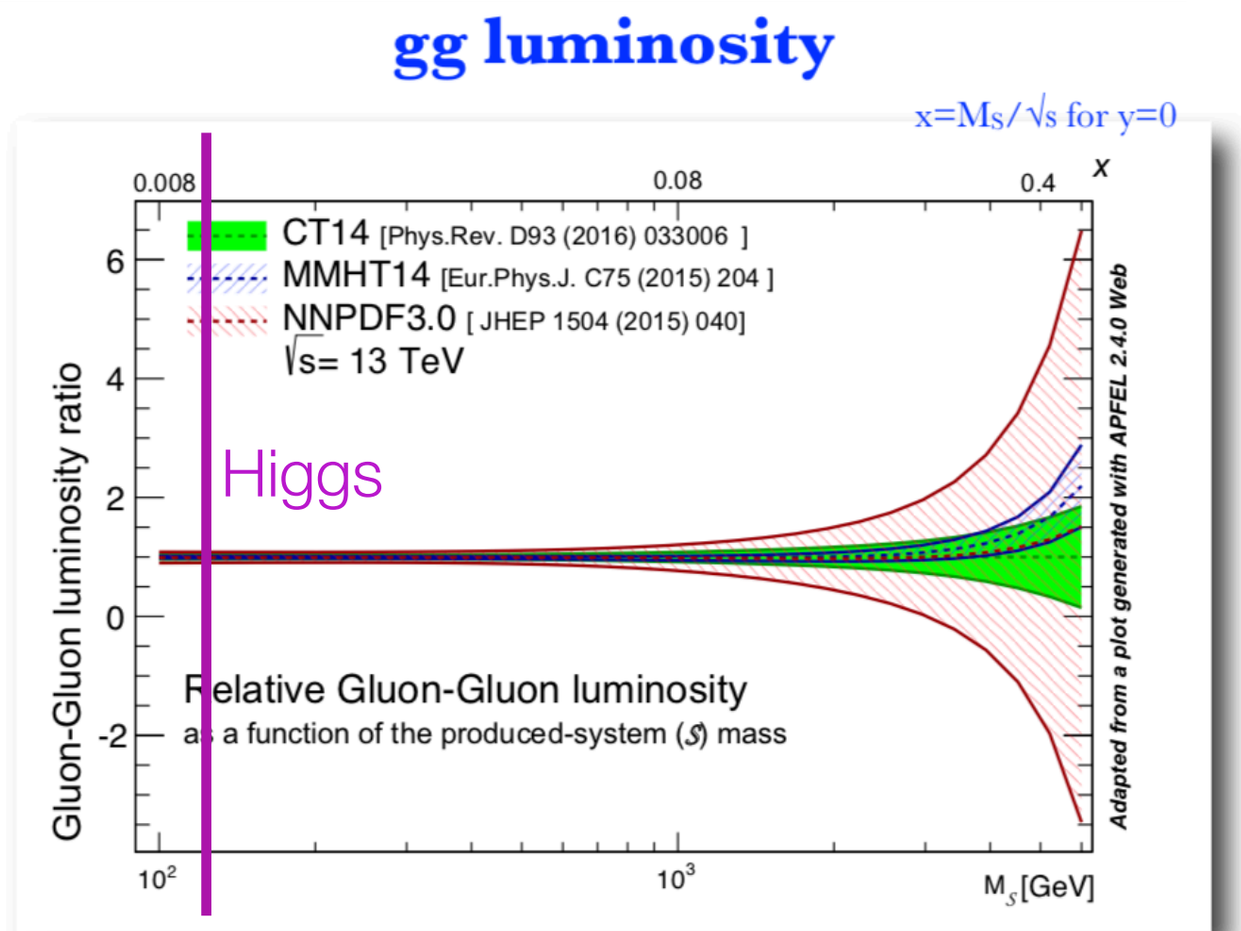


Gluons in Higgs production at LHC have x ~ 0.01 (in the well measured range)

Heavier states are produced from larger x partons

Large x gluons





$$\frac{\partial \mathcal{L}_{ab}}{\partial \tau} = \frac{1}{s}\int_{\tau}^{1} \frac{dx}{x} f_a(x, M_S^2)\, f_b(\tau/x, M_S^2), \qquad \tau = M_S^2/s$$

Discovery of new heavy particles (bumps) does not require knowledge on gluon distributions, but to compare to SM predictions and to extract the properties of the new particles does

From M. Echevarria, DIS2019 & 1807.00603

Will be improved by EIC, FT@LHC & HL-LHC experiments

Evolution

The gluon distribution cannot be calculated perturbatively, but its change with energy scale can be (for large Q^2 when $\alpha_s(Q^2) \ll 1$)

By means of evolution equation, such as the DGLAP equation:

$$\frac{\partial g(x,Q^2)}{\partial \ln Q^2} = \frac{\alpha_s(Q^2)}{2\pi} \int_x^1 dy P_{gg}\left(\frac{x}{y}\right) g(y,Q^2)$$

Gribov, Lipatov '72; Altarelli, Parisi '77; Dokshitzer '77

This describes the leading behavior in Q^2

Including higher order corrections all DIS data are well-described

Gluon distribution at small x

Fits to data show: $xg(x, Q^2) \sim \frac{1}{x^\lambda}$ with $\lambda \approx 0.4$

This leads cross sections to grow too fast with $s \sim Q^2/x$ in the limit $s \to \infty$

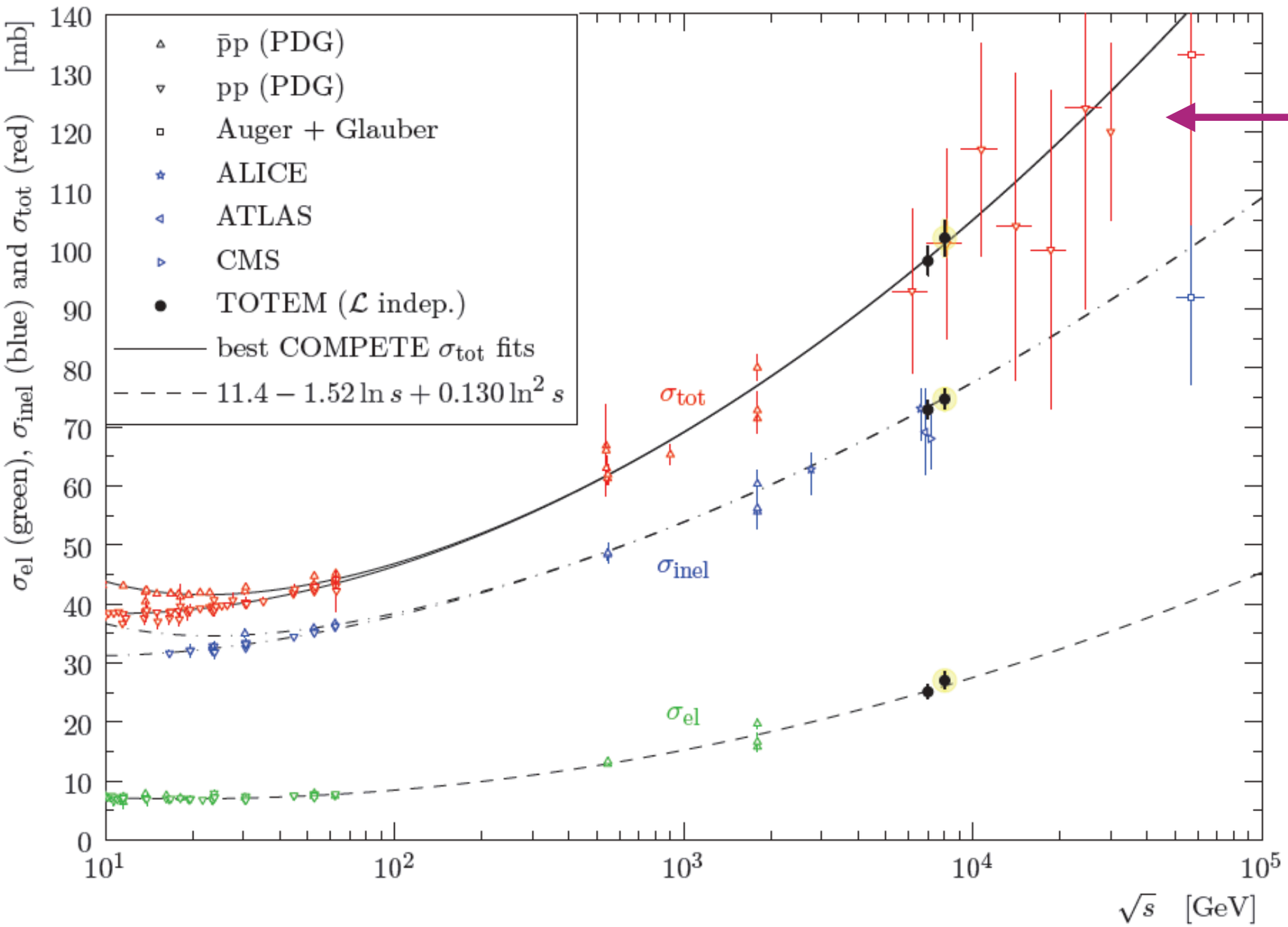
Eventually it leads to a violation of the so-called Froissart or unitarity bound:

$$\sigma_{pp,\mathrm{total}} \leq \frac{\pi}{m_\pi^2} (\ln s)^2$$

Froissart '61; Jin & Martin '64; Martin '66

Beyond this bound the probability to scatter becomes larger than 1

Total pp cross section



UHECRs

LHC: $\sigma_{tot} \sim 100$ mb

At LHC the Froissart bound is a few barn (1 barn=10^{-28} m^2), so no problem in practice

Gluon distribution at small x according to QCD

According to linear evolution equations, like DGLAP and BFKL, $g(x,Q^2)$ will grow too fast (exponentially) toward small x

Non-linear evolution equations can moderate the growth, e.g. GLR equation:

$$\frac{\partial^2 xg(x,Q^2)}{\partial \ln 1/x \partial \ln Q^2} = \frac{\alpha_s N_c}{\pi} xg(x,Q^2) - \frac{\alpha_s^2 N_c}{R^2 Q^2}[xg(x,Q^2)]^2$$

Gribov, Levin & Ryskin, 1983; Laenen, Levin, 1995

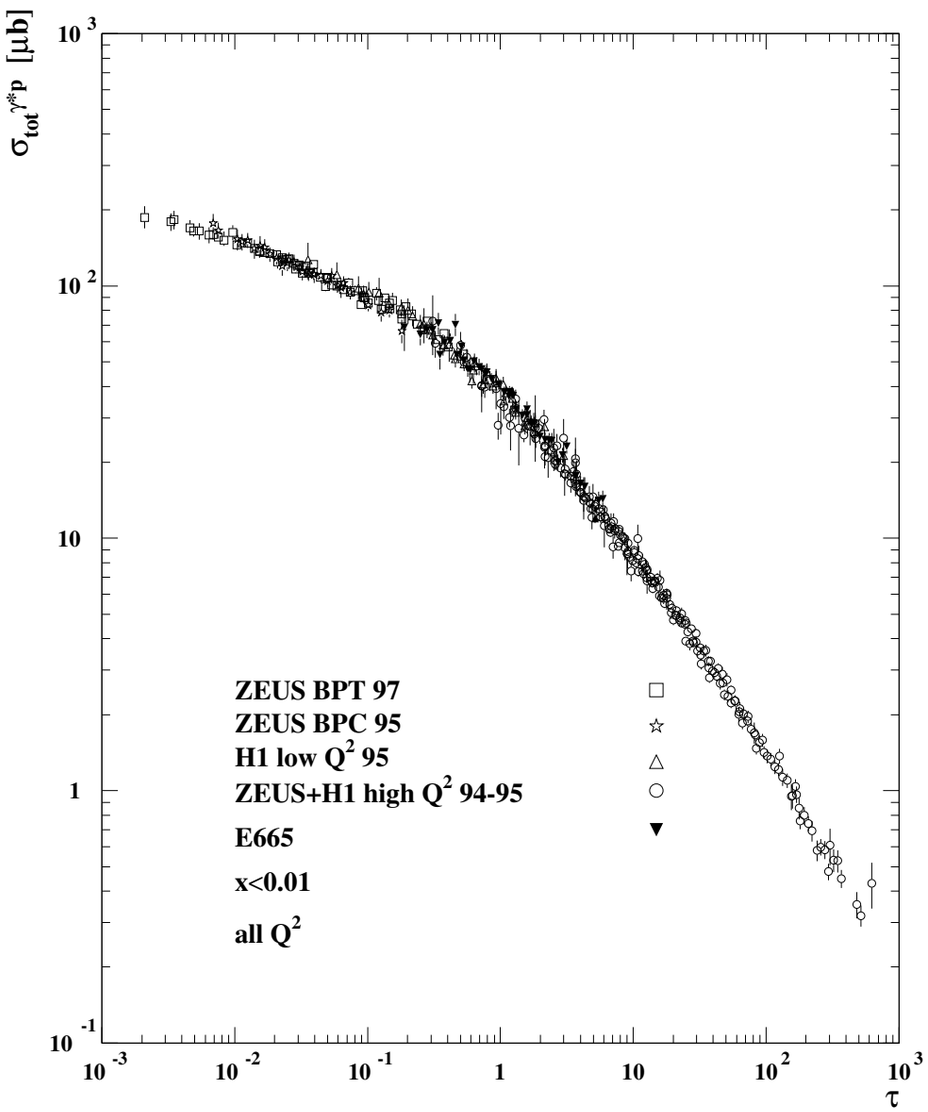
Generalizes to the non-linear BK equation and the JIMWLK equations

The nonlinear term(s) moderates the exponential growth of the gluon density

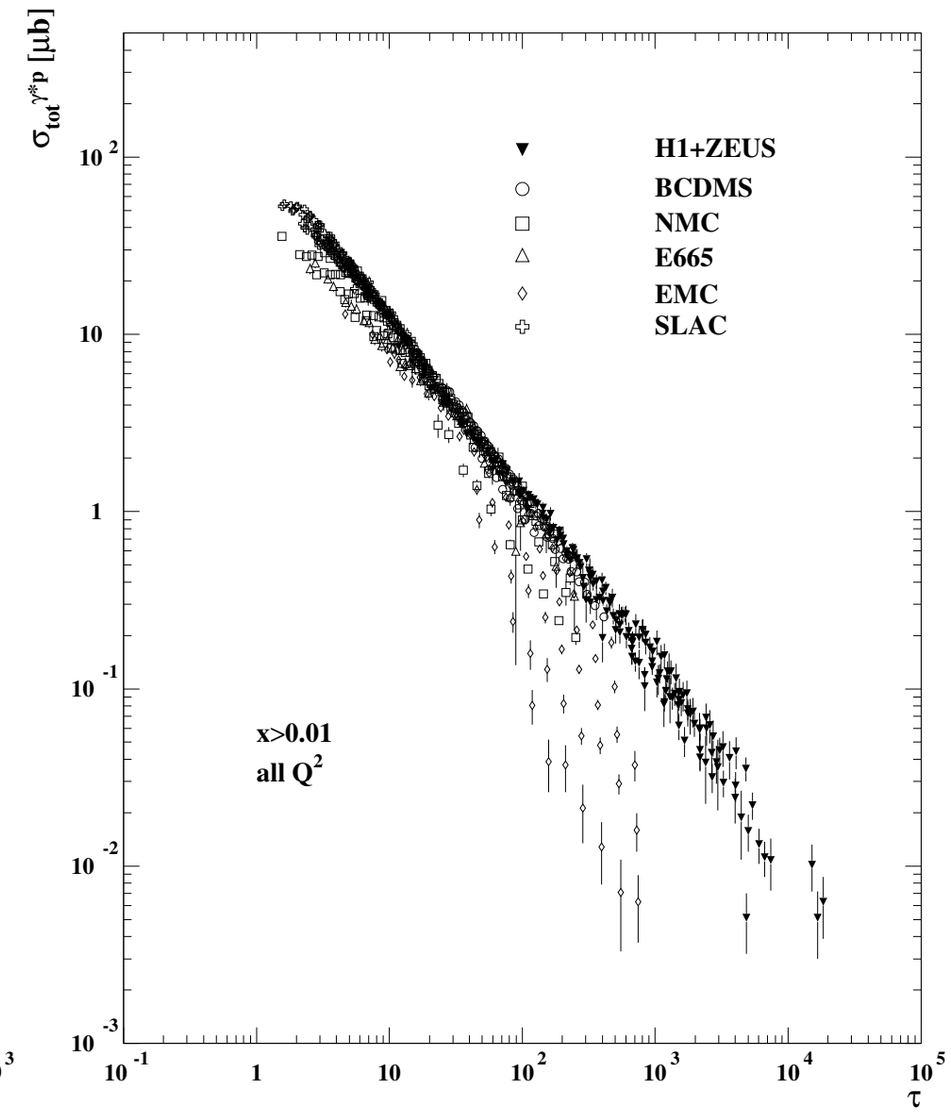
→ ultimately saturating into a state called the **Color Glass Condensate**

The scale at which nonlinear QCD effects become important is called Q_s

Geometric scaling in DIS



$\tau = Q^2/Q_s^2(x)$



Stasto, Golec-Biernat, Kwiecinski, 2001

$$Q_s^2(x) = Q_0^2 \left(\frac{x_0}{x}\right)^{\lambda}$$

Geometric Scaling: asymptotic solutions of nonlinear evolution depend on x and Q through the combination $Q^2/Q_s^2(x)$ only, where Q_s is the saturation scale

Remarkably this feature is seen in the data for $x<0.01$, far from asymptotically small

Nonperturbative but small coupling

$\alpha_s(Q_s)$ is small, but to describe the CGC (nonlinear QCD) effects one needs all orders

Recall α is always small, but to describe the ground state of the hydrogen atom (=bound state of the Coulomb potential) one needs all orders in α:

$$\psi(r) = \frac{1}{\sqrt{\pi a_0^3}} \exp\left(-r/a_0\right) \qquad a_0 = \frac{1}{\alpha m_e}$$

Bound states, resonances, collective states (CGC) usually are all-order expressions, sometimes even non-analytic ones (solitons, instantons, sphalerons, etc):

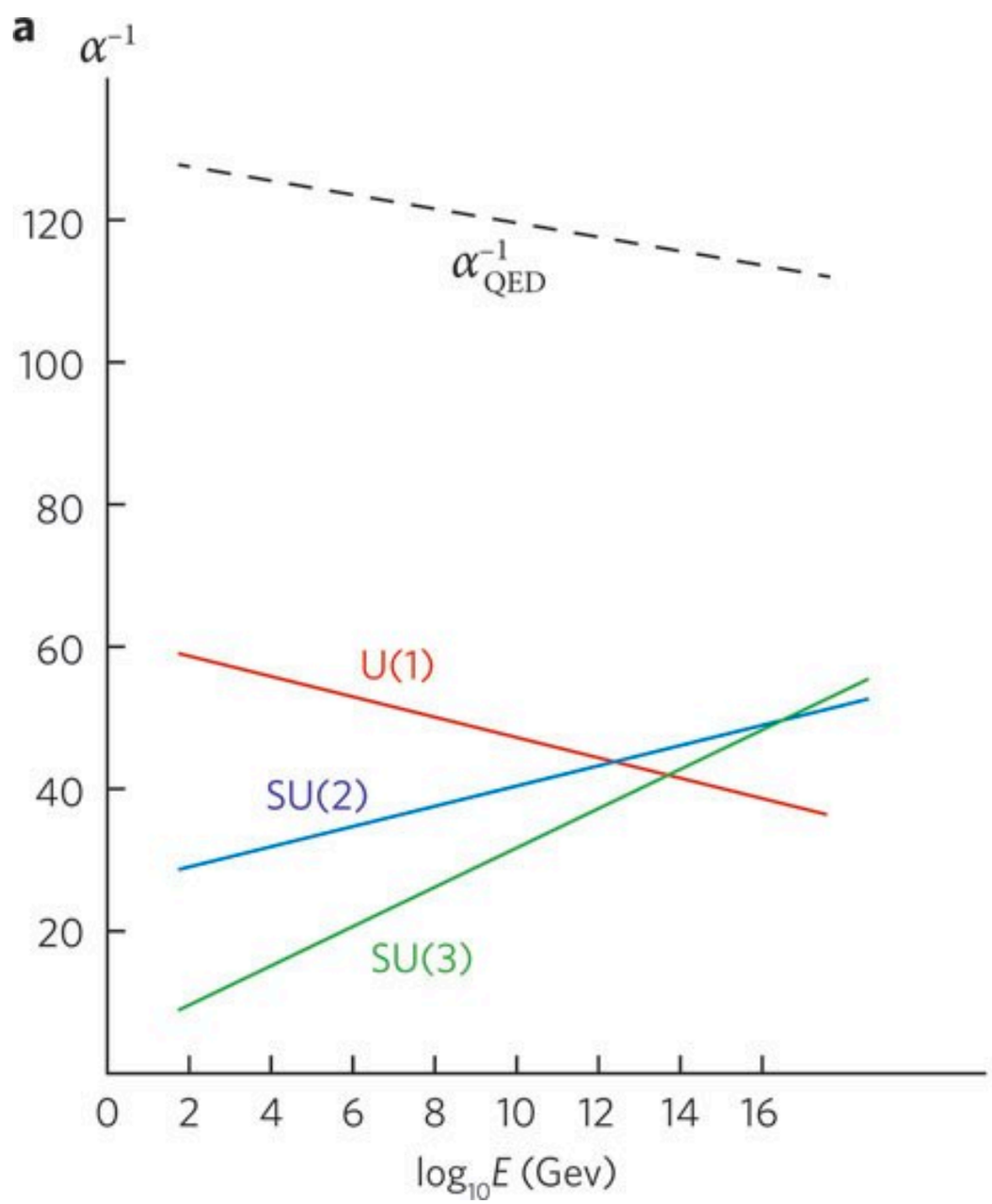
$$\exp\left(-S/\alpha\right)$$

Back to the running of the couplings of the SM

and onto BSM scales

BSM

Running is logarithmic so straight lines in log plot of α^{-1}:



EW symmetry restored at ~250 GeV

New d.o.f. & symmetry will modify the running

Coupling constants of the three interactions are roughly of the same magnitude in the region of $10^{12\text{-}16}$ GeV

This suggests unification of forces, be it MSSM or more elaborate versions of it, or GUTs (which also come in many varieties)

Figure from https://www.nature.com/articles/nphys2066

Supersymmetric extensions of the SM

MSSM running

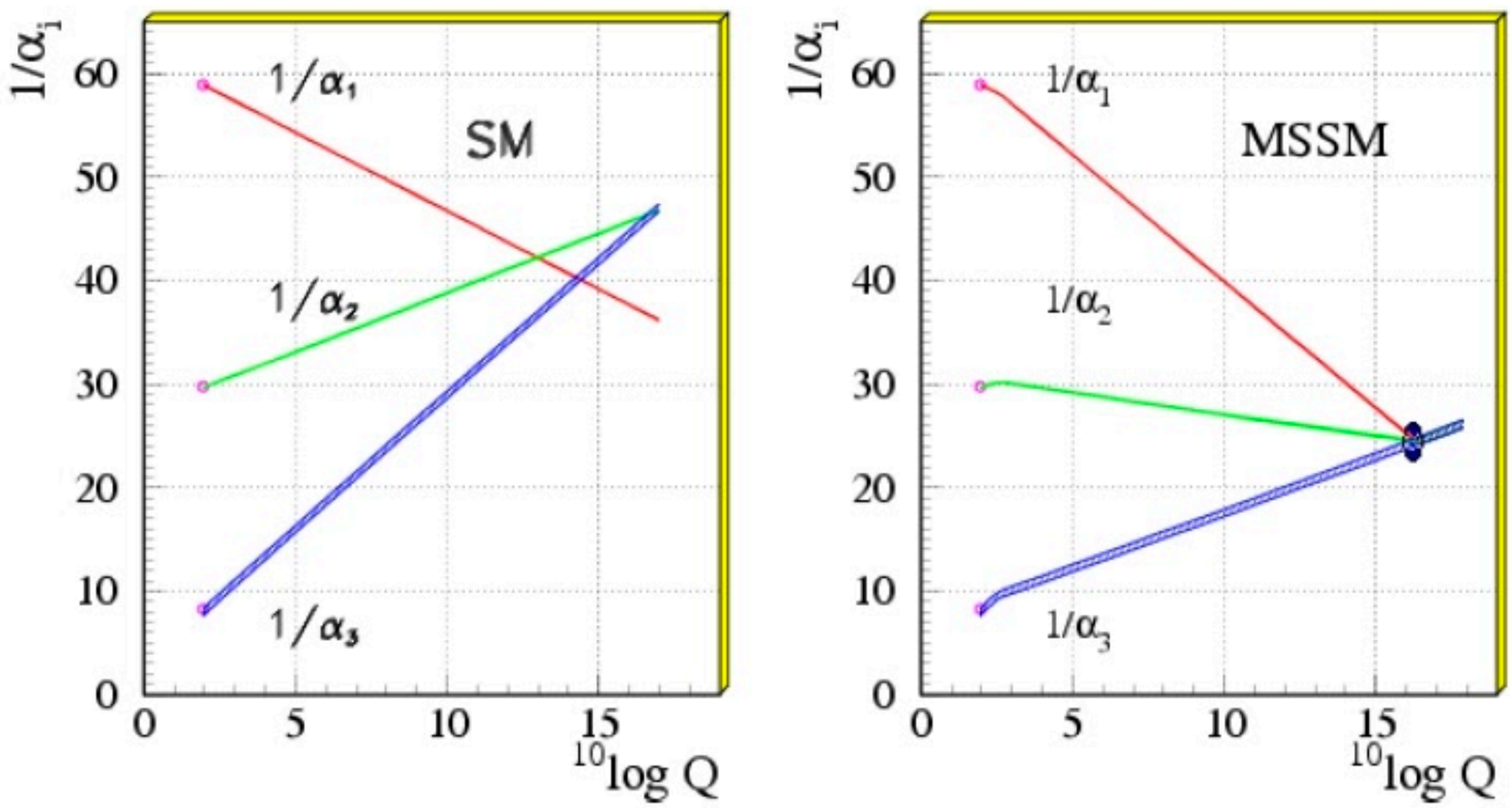


Figure from D.I. Kazakov, hep-ph/0012288

SO(10) GUT

Example of an SO(10) GUT scenario

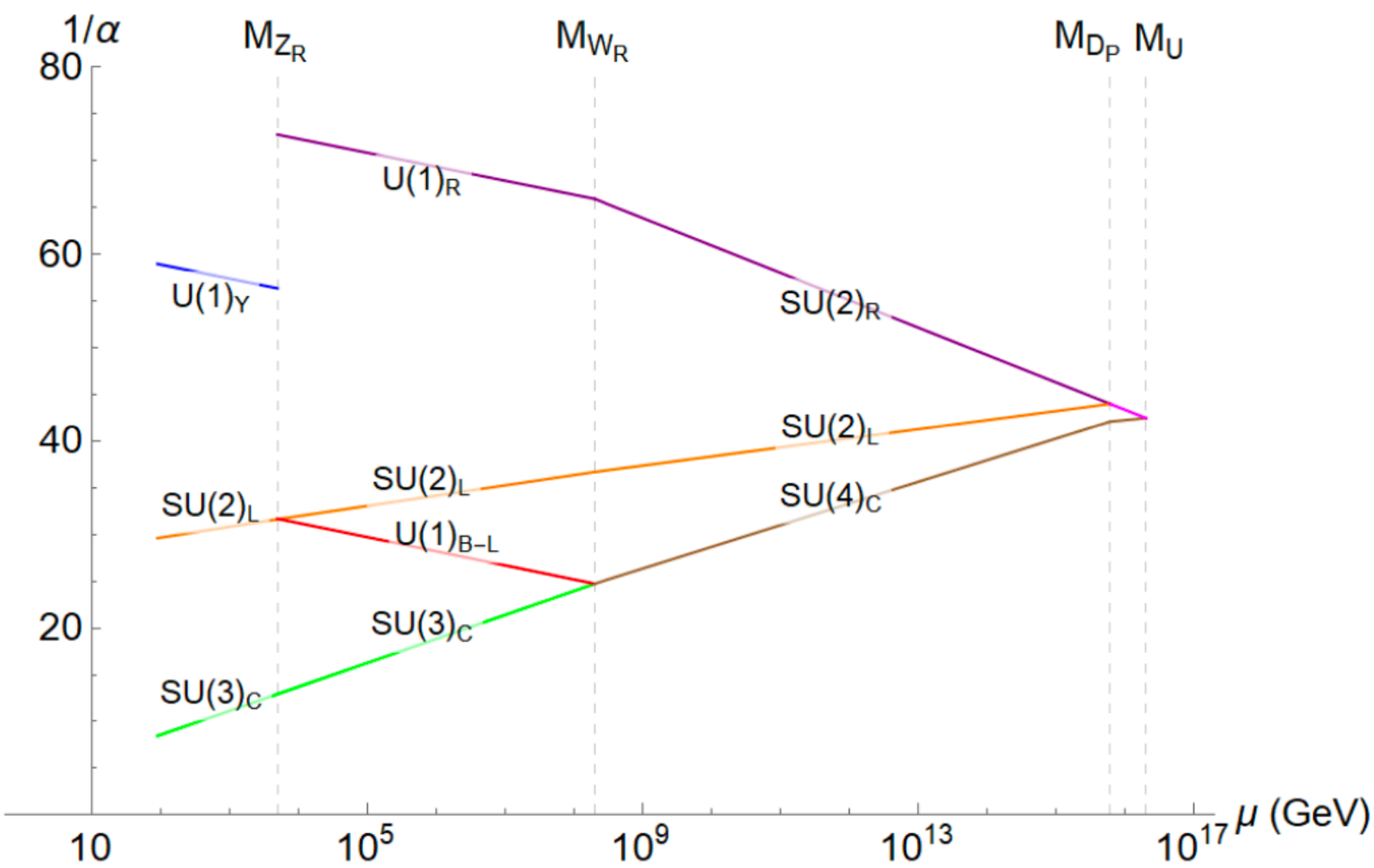


Figure by Gefré Wortelboer

There can be intermediate stages between the SM and the SO(10) GUT

It has additional righthanded W and Z bosons, but also leptoquarks (LQs)

These can be either scalar or vector LQs

LHC searches for LQs, which can be 5 TeV (still hope) but also 10^5 TeV (hopeless)

Direct production

PDG review on LQs:

Collider experiments provide direct limits on the leptoquark states through limits on the pair- and single-production cross sections. The leading-order cross sections of the parton processes

$$
\begin{aligned}
q + \bar{q} &\to \mathrm{LQ} + \overline{\mathrm{LQ}} \\
g + g &\to \mathrm{LQ} + \overline{\mathrm{LQ}} \\
e + q &\to \mathrm{LQ}
\end{aligned}
\tag{94.1}
$$

may be written as [20]

$$
\begin{aligned}
\hat{\sigma}_{\mathrm{LO}}\left[q\bar{q} \to \mathrm{LQ} + \overline{\mathrm{LQ}}\right] &= \frac{2\alpha_s^2\pi}{27\hat{s}}\beta^3, \\
\hat{\sigma}_{\mathrm{LO}}\left[gg \to \mathrm{LQ} + \overline{\mathrm{LQ}}\right] &= \frac{\alpha_s^2\pi}{96\hat{s}} \times \left[\beta(41 - 31\beta^2) + (18\beta^2 - \beta^4 - 17)\log\frac{1+\beta}{1-\beta}\right], \\
\hat{\sigma}_{\mathrm{LO}}\left[eq \to \mathrm{LQ}\right] &= \frac{\pi\lambda^2}{4}\delta(\hat{s} - M_{\mathrm{LQ}}^2)
\end{aligned}
\tag{94.2}
$$

for a scalar leptoquark. Here $\sqrt{\hat{s}}$ is the invariant energy of the parton subprocess, and $\beta \equiv \sqrt{1 - 4M_{\mathrm{LQ}}^2/\hat{s}}$. The leptoquark Yukawa coupling is given by λ. Leptoquarks are also produced

Indirect effects through 4-fermi interactions

Revisiting the vector leptoquark explanation of the B-physics anomalies

Claudia Cornella, Javier Fuentes-Martín and Gino Isidor, JHEP07(2019)168

By integrating out the vector leptoquark at tree level, we obtain the following high-scale ($\mu \sim M_U$) effective Lagrangian:

$$\mathcal{L}_{\text{eff}} = -\frac{2C_U}{v^2}\Big[-2\,(\beta_L^{i\alpha})^*\beta_R^{l\beta}(\bar{\ell}_L^{\alpha}e_R^{\beta})(\bar{d}_R^{l}q_L^{i}) + \text{h.c.} + \beta_R^{i\alpha}(\beta_R^{l\beta})^*(\bar{e}_R^{\beta}\gamma_\mu e_R^{\alpha})(\bar{d}_R^{i}\gamma^\mu d_R^{l}) \\ + \frac{1}{2}\beta_L^{i\alpha}(\beta_L^{l\beta})^*(\bar{\ell}_L^{\beta}\gamma_\mu \ell_L^{\alpha})(\bar{q}_L^{i}\gamma^\mu q_L^{l}) + \frac{1}{2}\beta_L^{i\alpha}(\beta_L^{l\beta})^*(\bar{\ell}_L^{\beta}\sigma^a\gamma_\mu \ell_L^{\alpha})(\bar{q}_L^{i}\sigma^a\gamma^\mu q_L^{l})\Big], \tag{2.5}$$

where $C_U \equiv g_U^2 v^2/(4M_U^2)$ and $v = (\sqrt{2}\,G_F)^{-1/2} \approx 246\,\text{GeV}$ is the SM Higgs vacuum expectation value (vev).

$$R_D \approx R_D^{\text{SM}}\left[1 + 2C_U\,\text{Re}\left\{\left(1 - 1.5\,\eta_S\,(\beta_R^{b\tau})^*\right)\left(1 + \frac{V_{cs}}{V_{cb}}\beta_L^{s\tau} + \frac{V_{cd}}{V_{cb}}\beta_L^{d\tau}\right)\right\}\right],$$

$$R_{D^*} \approx R_{D^*}^{\text{SM}}\left[1 + 2C_U\,\text{Re}\left\{\left(1 - 0.14\,\eta_S\,(\beta_R^{b\tau})^*\right)\left(1 + \frac{V_{cs}}{V_{cb}}\beta_L^{s\tau} + \frac{V_{cd}}{V_{cb}}\beta_L^{d\tau}\right)\right\}\right]$$

Searches for LQs through the effect of such heavy new particles on SM fermion interactions in the form of four fermi interactions terms → SMEFT (Rojo & Giani)

Conclusions

- The SM contains particles that are relevant at many different energy scales
- Which d.o.f. are relevant depends on the observables *and* on the precision
- There are also emergent scales not present in the Lagrangian (bound states, vacuum expectation values, scales where the description changes qualitatively)
- The running of the coupling constants also point to scales at which new d.o.f.'s or new phenomena (unification) can occur, but no certainty, of course

Final thoughts

We still need to explain why three families and why the quark and lepton sectors are so distinct (CKM vs PMNS)

Leptoquarks, LR & family symmetry are to be expected at some point

Fine tuning issues will affect almost any BSM theory

Much clarification on these topics is needed still, but SM could also be valid up to the Planck scale (depends on the Higgs and top quark masses) and then we won't get any further understanding

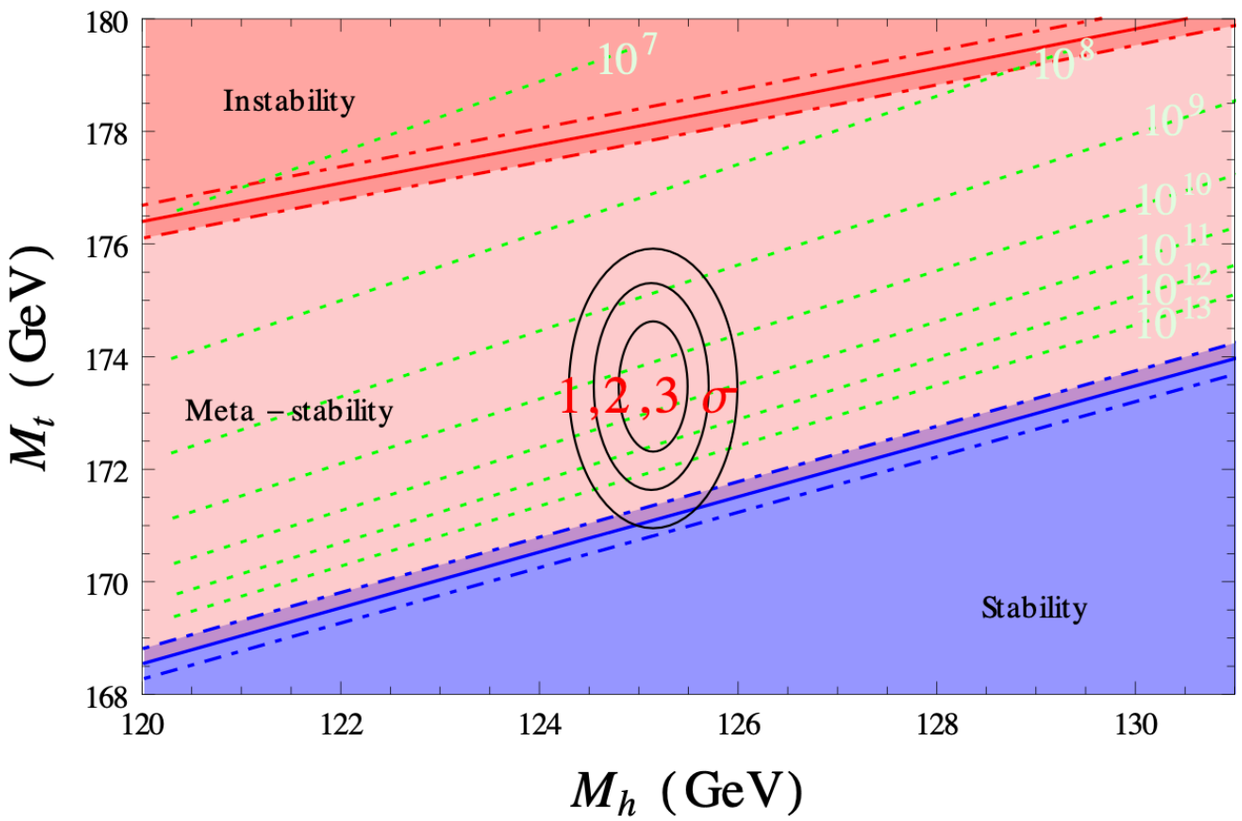
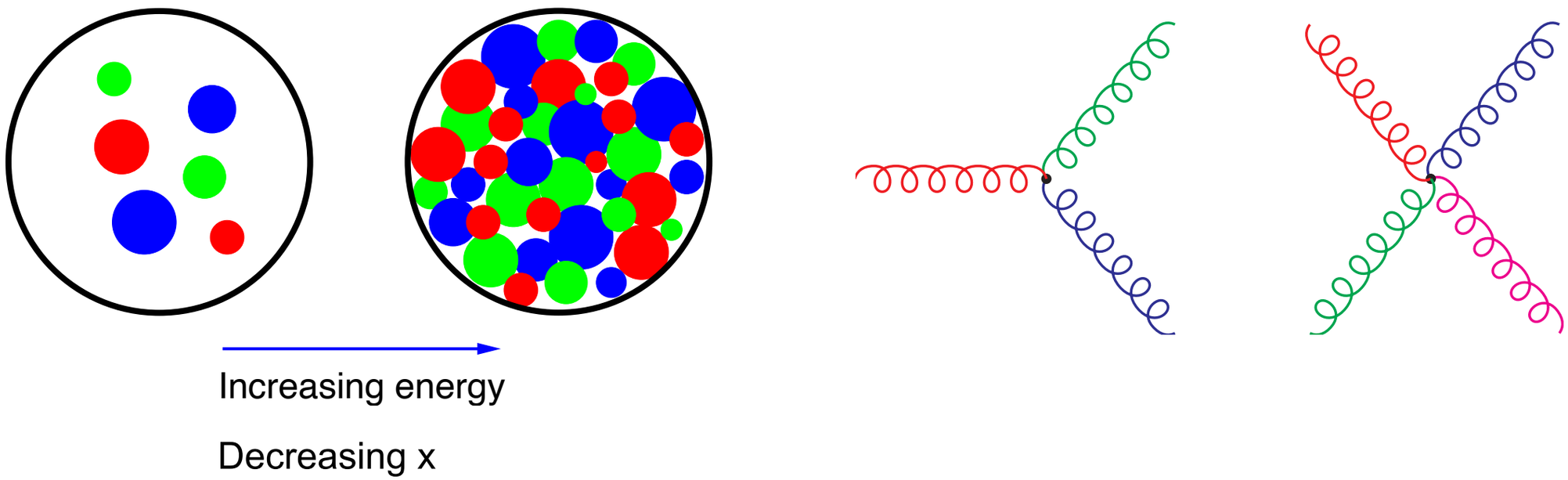


Figure from https://inspirehep.net/literature/1828369

Back-up slides

High gluon density

When x decreases, the density of gluons (*n*) increases

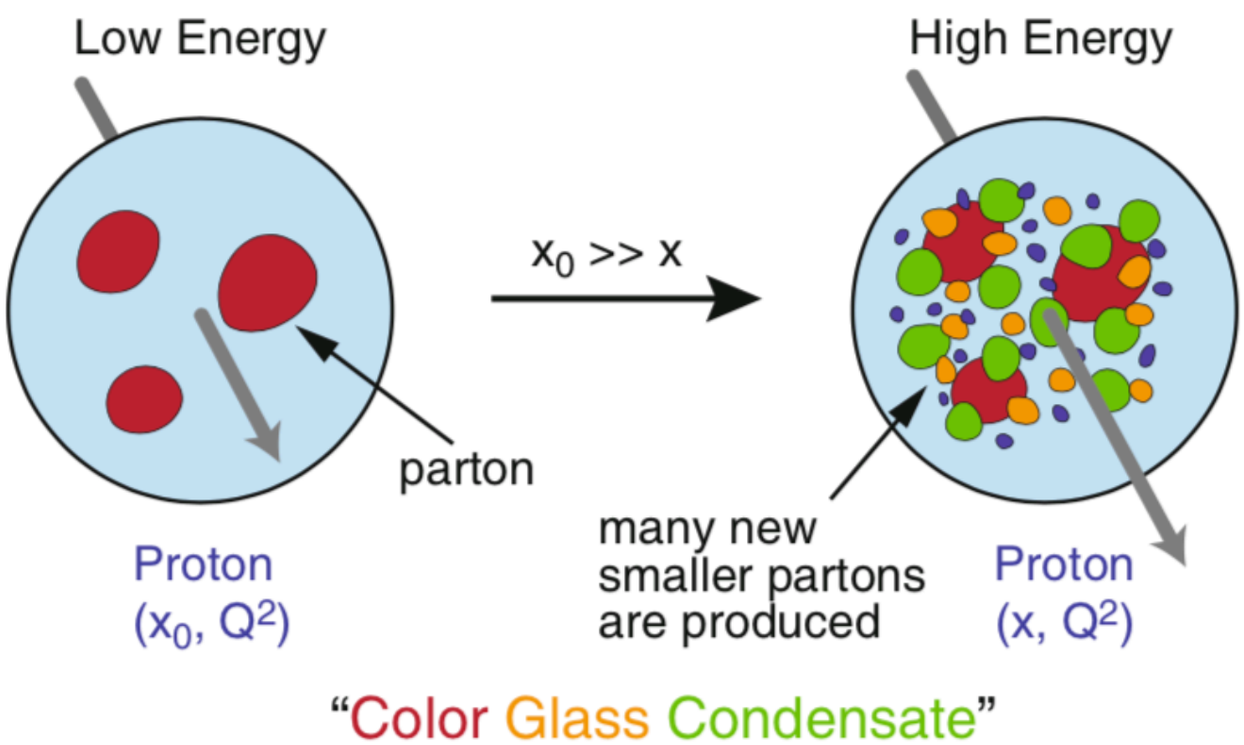


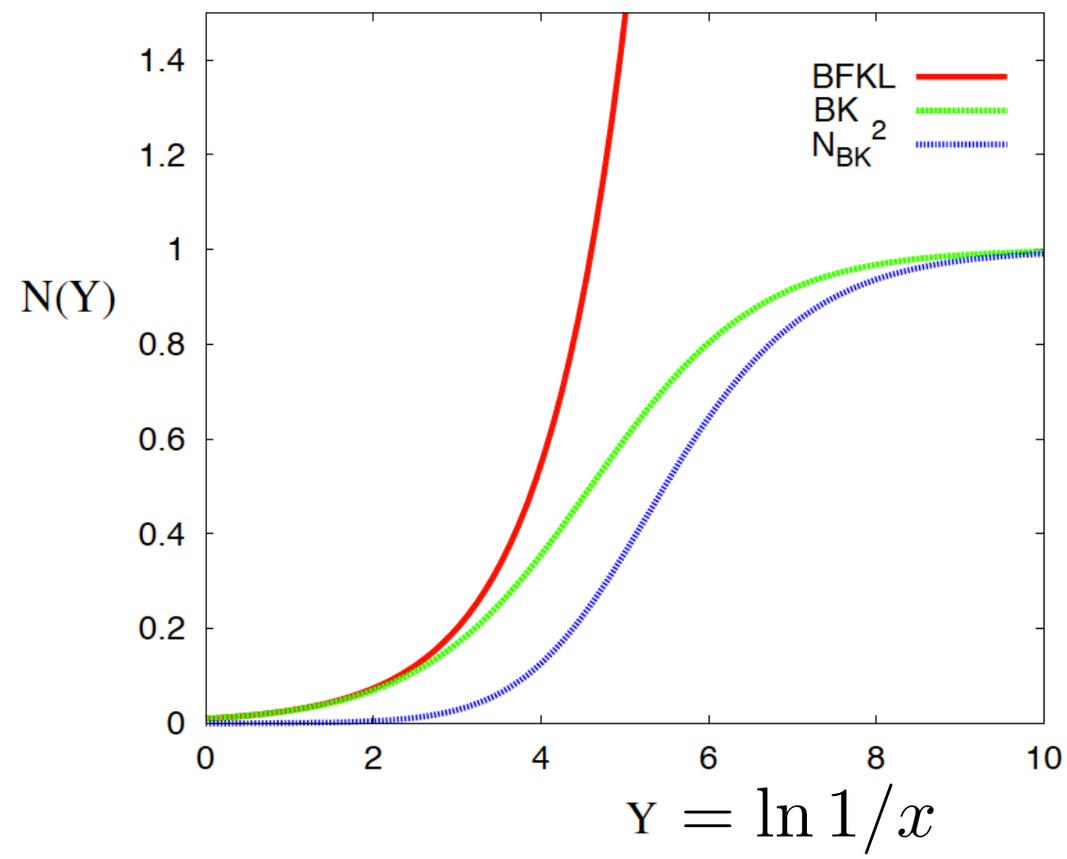
At some point *n* becomes so large ($n \to O(1/\alpha_s)$) that the probability for gluons to interact approaches 1 ($n \times \sigma_{gg} \to 1$) [No such effect arises for photons]

Scattering off a proton becomes scattering off multiple gluons simultaneously

Leads to nonlinear evolution and multi-gluon distributions

Gluon saturation



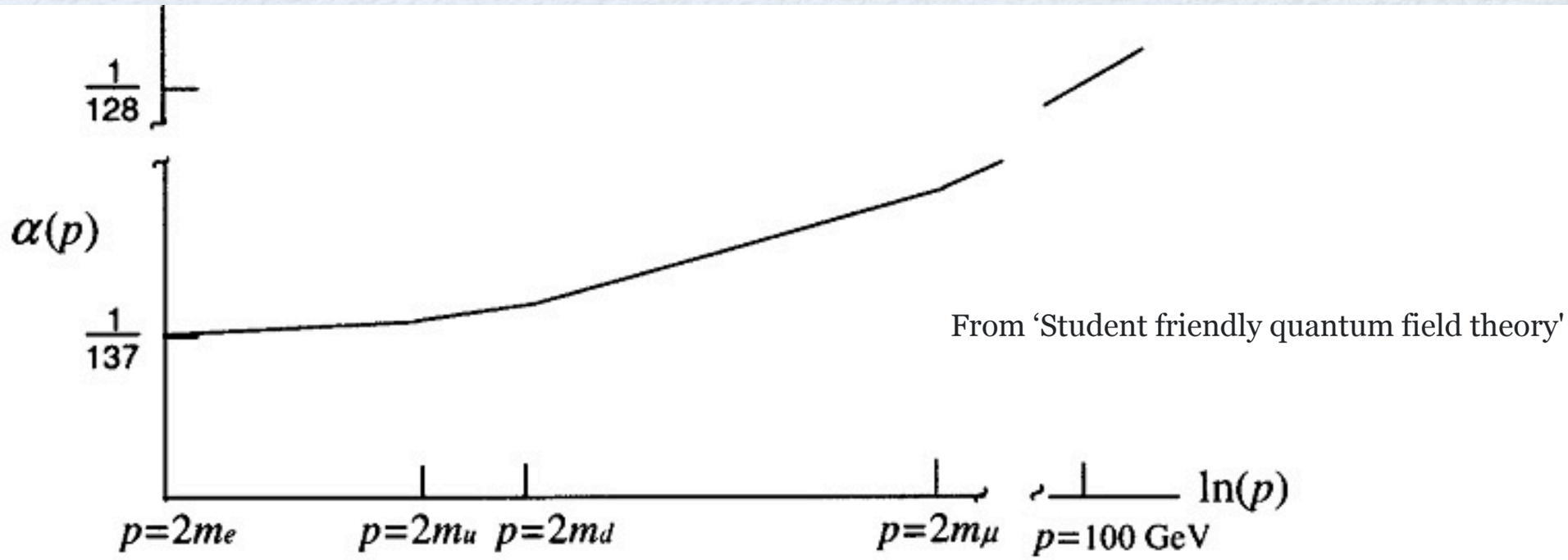


Scatter off multiple gluons simultaneously will probe their collective effect

This effect is expected to moderate the exponential growth of the gluon density

→ ultimately saturating into a state called the **Color Glass Condensate**

The scale at which nonlinear QCD effects become important is called Q_s



From 'Student friendly quantum field theory'

Figure 12-4. QED Coupling Constant Dependence on Log of Interaction Energy