RENORMALIZATION GROUP FLOW

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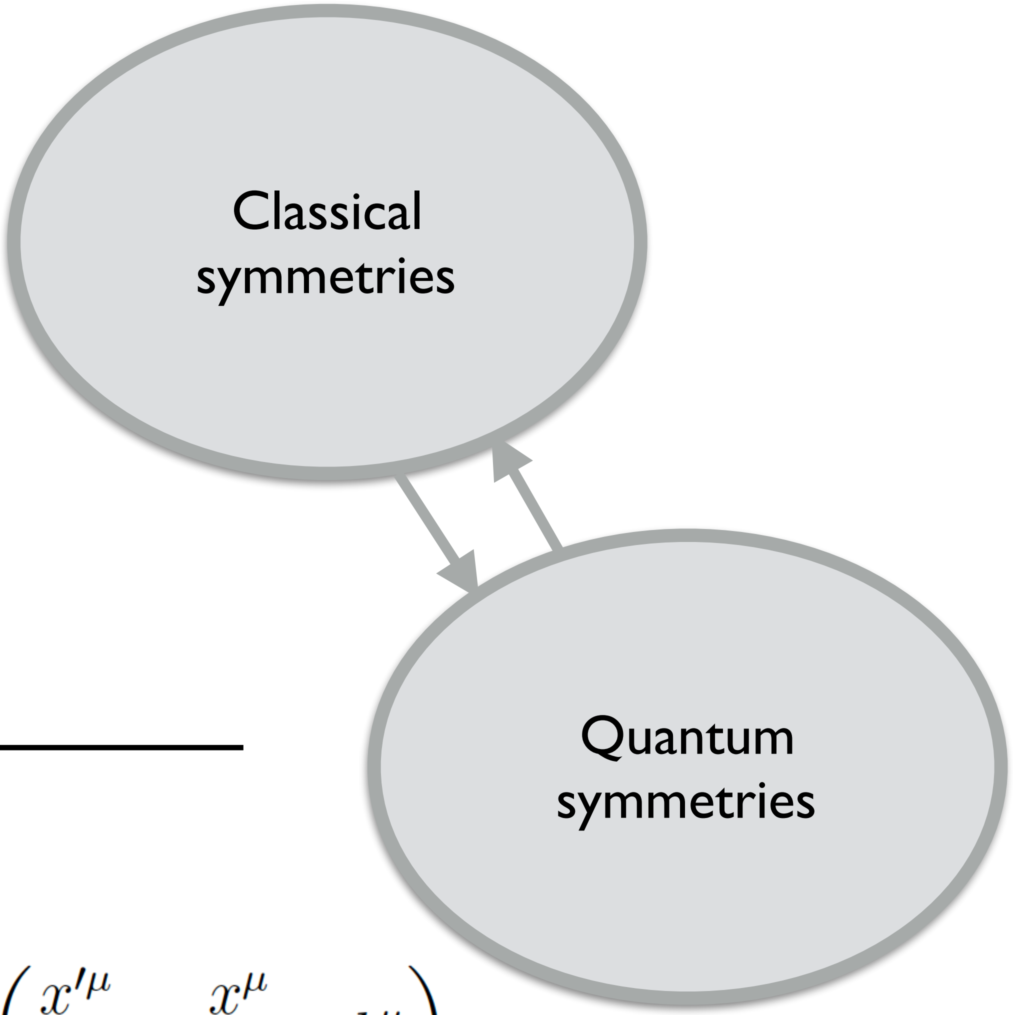
Topical Lectrures 2023, Groningen

Outline

- Symmetries and quantum effects
- Renormalization, what for? The “flow” from the UV to the IR
- One does it all: The Callan-Symanzik equation
- Hands on the flow: RG flow à la Wilson
- Effective field theory within the theory

Symmetries and quantum effects

The massless Lagrangian of the Standard Model is conformal invariant



Conformal group

Poincaré

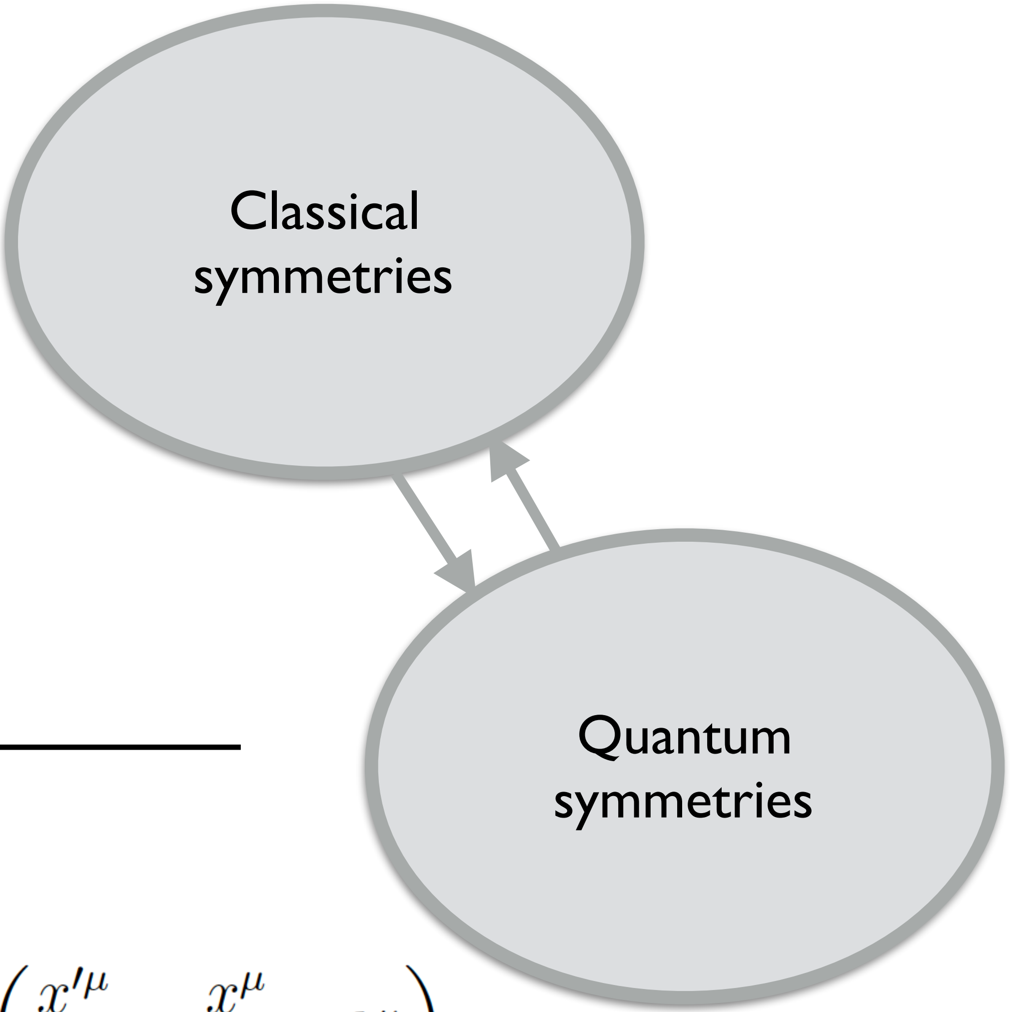
$$\begin{aligned} x \to x' &= x + a \\ x \to x' &= \Lambda x \quad (\Lambda^{\mu}_{\nu}) \end{aligned}$$

Dilatations $x \to x' = \lambda x$

Special Conformal

$$x \to x' = \frac{x + bx^2}{1 + 2b \cdot x + b^2 x^2} \qquad \left(\frac{x'^{\mu}}{x'^2} = \frac{x^{\mu}}{x^2} + b^{\mu} \right)$$

Symmetries and quantum effects



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Conformal symmetry is lost due to quantum effects

Symmetries and quantum effects

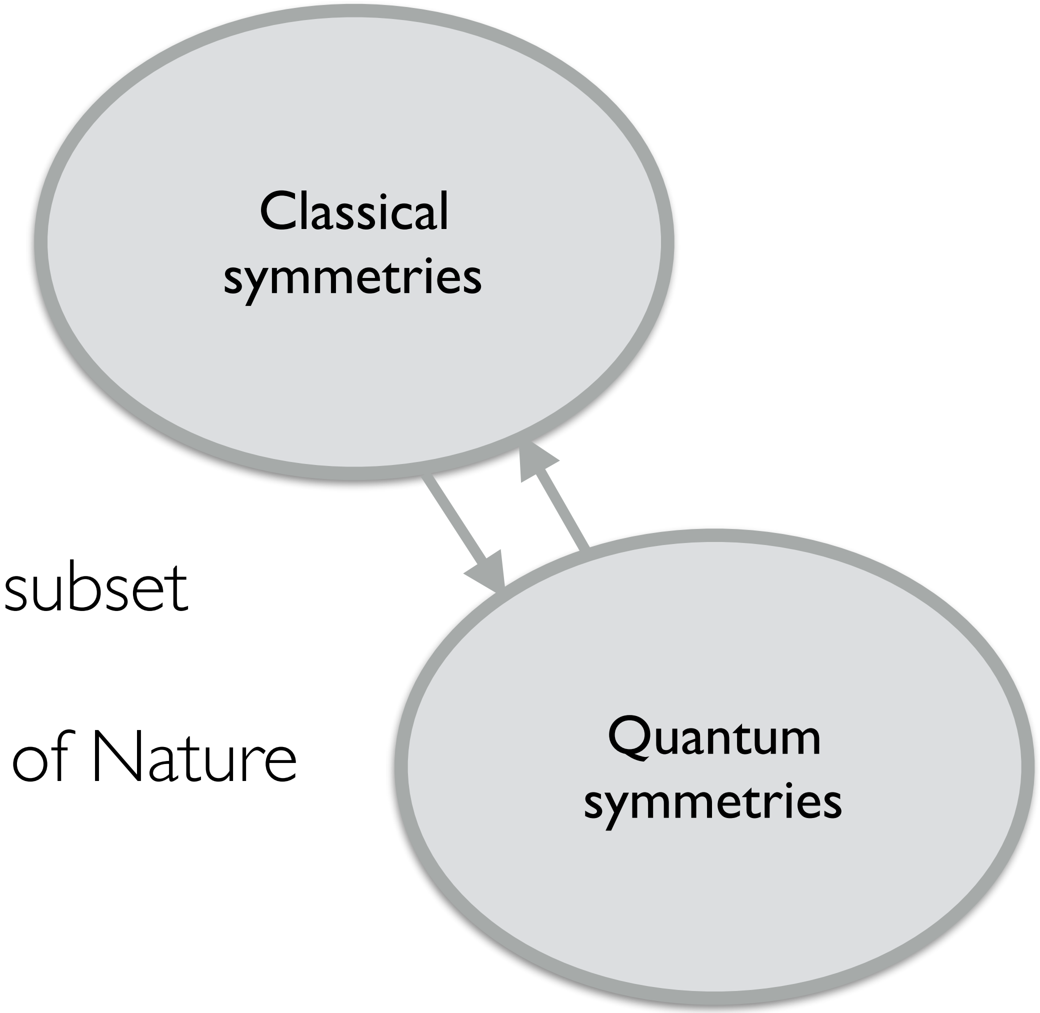
The massless propagator of a scalar field is conformal invariant

$$\begin{aligned} D(x) &= \frac{\Gamma(\frac{d}{2}-1)}{4\pi^{\frac{d}{2}}} \boxed{\frac{1}{(x^2)^{\frac{d}{2}-1}}} \\ &= \mathrm{FT}\left[\frac{1}{p^2}\right] = \int \frac{d^d p}{(2\pi)^d}\, e^{ipx}\, \frac{1}{p^2} \end{aligned}$$

The nonperturbative UV asymptotics of 2-point correlators in QCD (d=4) is not conformal invariant

$$\langle F^2(x) F^2(0) \rangle \sim \frac{48(N^2-1)}{\pi^4 \beta_0^2} \boxed{\frac{1}{x^8} \frac{1}{\log^2(\frac{1}{x^2 \Lambda_{QCD}^2})}}$$

Symmetries and quantum effects



Classical symmetries

Quantum symmetries

Conformal Field Theories (CFT) are a subset

of the Quantum Field Theories (QFT) of Nature

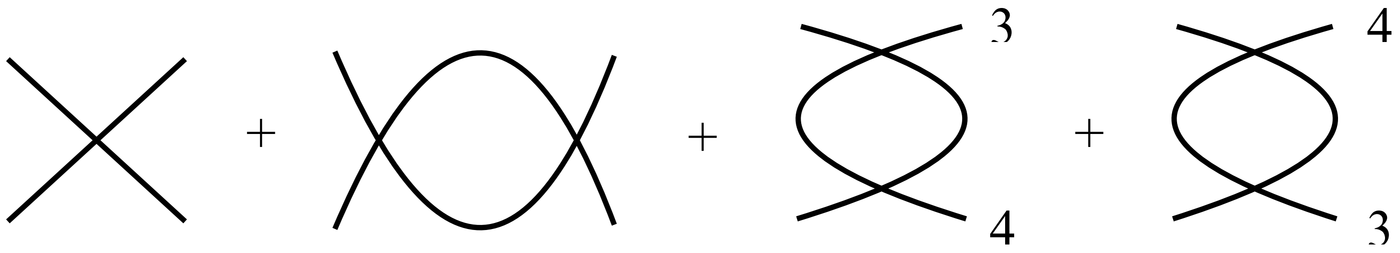
The renormalized (running) coupling

$$\mathcal{L} = \mathcal{L}_{\text{free}} + \frac{\lambda_0}{4!}\phi^4$$

Interaction

Scattering $\phi\phi \to \phi\phi$

$$\mathcal{A} =$$



Theory predicts…

$$\mathcal{A} = \lambda_0 + \lambda_0^2\, C \underbrace{\left(\log\frac{s}{\Lambda^2} + \log\frac{t}{\Lambda^2} + \log\frac{u}{\Lambda^2}\right)}_{\equiv L} + \cdots$$

UV cutoff

Experiment measures…

$$\mathcal{A} = \lambda \qquad @\,(\bar{s}, \bar{t}, \bar{u})$$

So that in terms of the measured coupling

$$\begin{aligned}\mathcal{A} &= \lambda + \lambda^2 C\,(L - \bar{L}) + \cdots \\ &= \lambda + \lambda^2 C\,\left(\log\frac{s}{\bar{s}} + \log\frac{t}{\bar{t}} + \log\frac{u}{\bar{u}}\right) + \cdots\end{aligned}$$

The UV cutoff dependence no longer appears in the physical i.e. renormalized amplitude

$$(s, t, u, \lambda_0, \Lambda) \longrightarrow (s, t, u, \lambda, \bar{s}, \bar{t}, \bar{u})$$

$\longrightarrow \mu$ renormalization scale

$$\lambda(\Lambda) \longrightarrow \lambda(\mu)$$

bare coupling	renormalized coupling

The dependence on a dimensionful parameter i.e. the energy scale μ stays as $\Lambda \to \infty$!

$$(s, t, u, \lambda_0, \Lambda) \longrightarrow (s, t, u, \lambda, \bar{s}, \bar{t}, \bar{u})$$

μ renormalization scale

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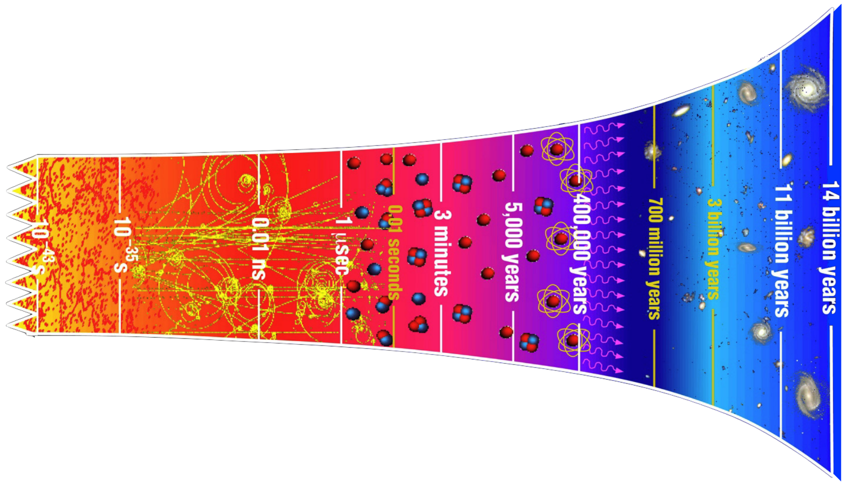
bare coupling	renormalized coupling

This dependence in a physical quantum theory measures the breaking of conformal symmetry



FUNDAMENTAL THEORY

UV $\xrightarrow{\text{FLOW}}$ **IR**

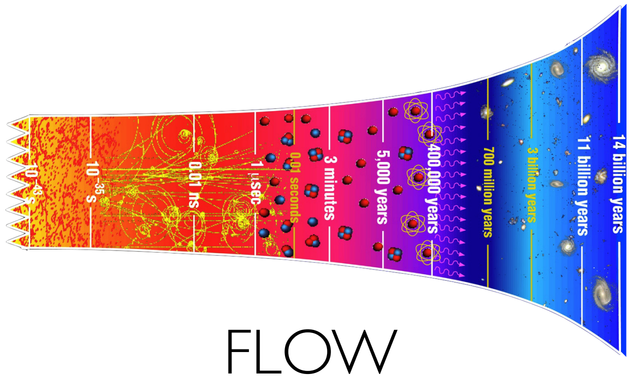
Information loss
(i.e. reduction of d.o.f.)

LOW ENERGY REMNANT

Decoupling: SM EW sector + Higgs mechanism (mass thresholds)

Non decoupling: (perturbatively) massless QCD

And QCD is UV complete !



FLOW

UV → IR

The Callan-Symanzik Equation

E.g. 2-point correlator of (multiplicatively renormalizable) composite operator O: $G^{(2)} = \langle O(x)O(0)\rangle$

Renormalized

Bare

$$G_0^{(2)}(x,\Lambda,g(\Lambda)) = Z_O^{-2}\left(\frac{\Lambda}{\mu}, g(\mu)\right) G^{(2)}(x,\mu,g(\mu))$$

Dimensionless
renormalization factor

CS eq states the μ independence of the bare correlator at fixed Λ

$$\boxed{\mu\frac{d}{d\mu}G_0^{(2)}\Big|_{\Lambda,g(\Lambda)} = 0}$$

Apply the chain rule for the total derivative to obtain

The Callan-Symanzik Equation

$$\left(\mu\frac{\partial}{\partial\mu}+\beta(g)\frac{\partial}{\partial g}+2\gamma_O(g)\right)G^{(2)}(x,\mu,g(\mu))=0$$

$$\beta(g)=\mu\frac{dg}{d\mu}\Big|_{\Lambda,g(\Lambda)}$$

Beta function

$$\gamma_O(g)=-\frac{d\log Z_O}{d\log\mu}\Big|_{\Lambda,g(\Lambda)}$$

Anomalous dimension

The Callan-Symanzik Equation

This UV nonperturbative asymptotics solves the CS equation

$$\langle F^2(x) F^2(0) \rangle \sim \frac{48(N^2-1)}{\pi^4 \beta_0^2} \frac{1}{x^8} \frac{1}{\log^2(\frac{1}{x^2 \Lambda_{QCD}^2})}$$

where for QCD (d=4)

$$\beta(g) = -\beta_0 g^3 - \beta_1 g^5 + \cdots$$

and

$$\gamma_{F^2}(g) = -2\beta_0 g^2 + \cdots$$

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UV to IR flow

and

$$\gamma_{F^2}(g) = -2\beta_0 g^2 + \cdots$$

Special points of the flow are the zeroes of the beta function

$$\beta(g_*) = 0$$

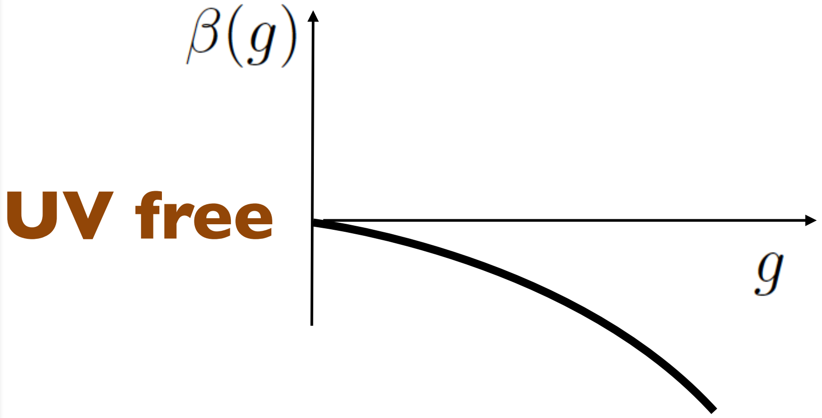
i.e. the derivative of g is zero given that $\beta(g) = \mu\frac{dg}{d\mu}|_{\Lambda,g(\Lambda)}$

These are the "fixed points" of the RG flow

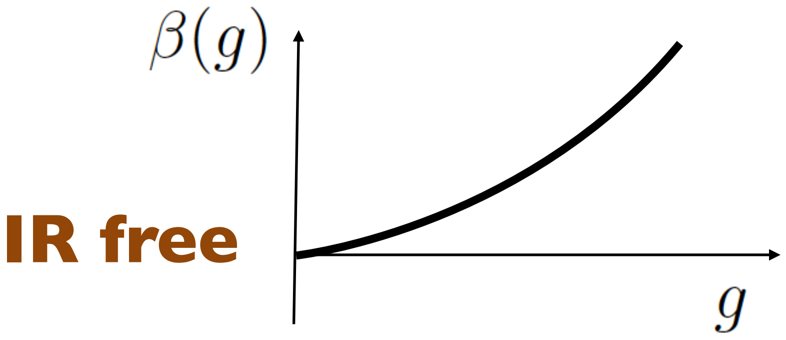
At a fixed point conformal symmetry (or scale invariance at least) is restored.

Types of UV to IR Flows

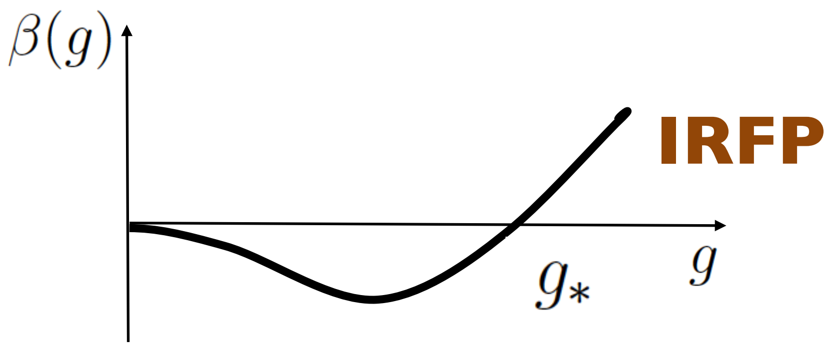
Asymptotic Freedom QCD



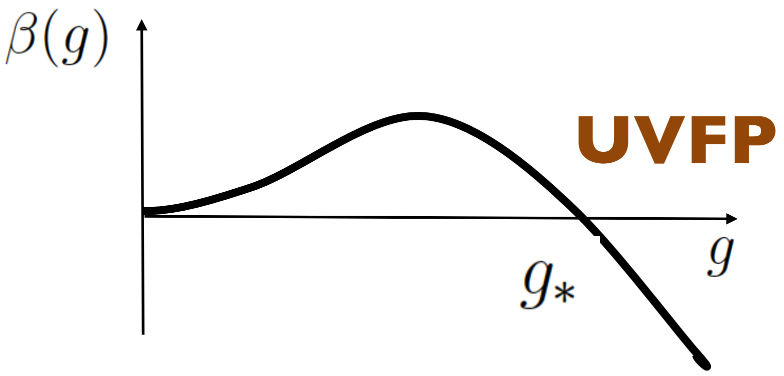
Infrared Freedom QED



Infrared Safety
(QCD conformal window)



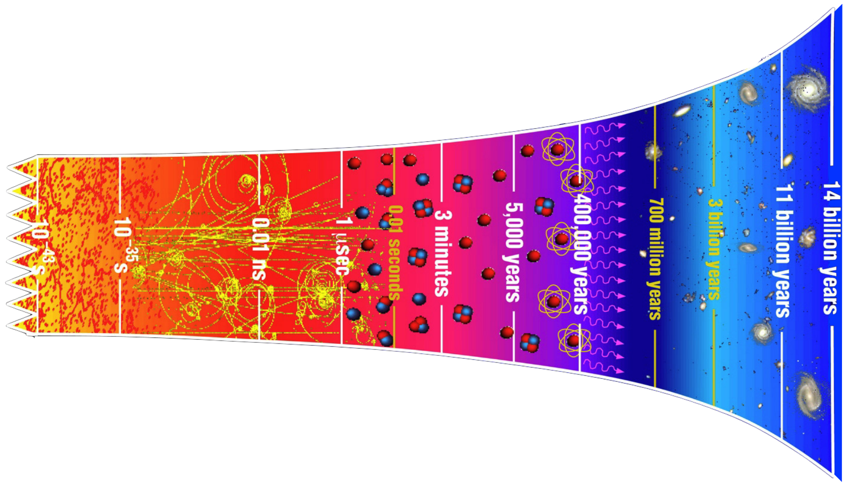
Asymptotic Safety
(gravity?)



RG Flow à la Wilson: Towards EFT

FUNDAMENTAL
THEORY

?



UV → **IR**

FLOW

Information loss
(i.e. reduction of d.o.f.)

LOW
ENERGY
REMNANT

RG Flow à la Wilson: Towards EFT

FUNDAMENTAL
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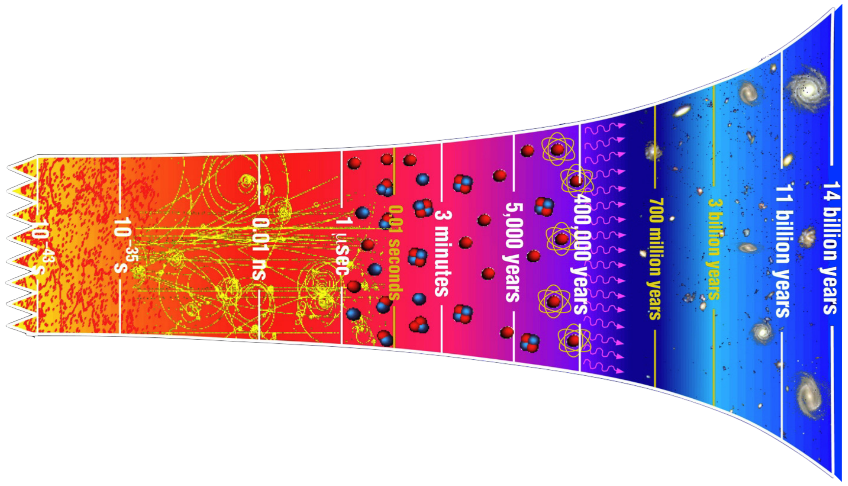


WE ARE HERE

FLOW

UV → **IR**

Information loss
(i.e. reduction of d.o.f.)

We need to exploit at its best the IR of the theory of the universe. How ?

RG Flow à la Wilson: Towards EFT

$$Z = \int D\phi_\Lambda \, e^{-\int d^dx \, \mathcal{L}}$$

$$\mathcal{L} = \frac{1}{2}(\partial_\mu\phi)^2 + \frac{1}{2}m^2\phi^2 + \frac{\lambda}{4!}\phi^4$$

$$= \int D\phi_\Lambda \, e^{-\int d^dx \, \frac{1}{2}(\partial_\mu\phi)^2 + \text{perturbations}}$$

Divide in momentum shells

$$\phi(k) = \phi_{\text{low}}(k) + \phi_{\text{high}}(k) \qquad \boxed{b < 1}$$

$$\boxed{k \leqslant b\Lambda} \qquad \boxed{b\Lambda < k \leqslant \Lambda}$$

Integrate "out" the high momentum modes to obtain

$$Z \;=\; \int D\phi_{b\Lambda}\, e^{-\int d^dx\, \mathcal{L}_{eff}}$$

Rescale momenta and coordinates $k' = \frac{k}{b} \qquad x' = bx$

$$Z \;=\; \int D\phi'_{\Lambda}\, e^{-\int d^dx'\; \frac{1}{2}(\partial'_\mu \phi')^2 + \text{perturbations'}}$$

The perturbations in the effective Lagrangian now are

$$\text{perturbations'} = \frac{1}{2}m'^2\phi'^2 + \frac{\lambda'}{4!}\phi'^4 + C'\phi'^6 + \cdots$$

In the vicinity of the free-field point the new parameters depend linearly on the parameters before the iteration

$m'^2 = m^2\, b^{-2}$	Relevant
$\lambda' = \lambda\, b^{d-4}$	Marginal for d=4
$C' = C\, b^{d}$	Irrelevant

$b < 1$

The flow to lower momenta is dictated by the (canonical) dimensions. In general

$$C'_{N,M} = b^{-[C_{N,M}]}\, C_{N,M}$$

for a term with N fields and M derivatives

$$\begin{aligned} [C_{N,M}] &= d - [O_{N,M}] \\ &= d - N[\phi] - M \\ &= d - N\frac{d-2}{2} - M \end{aligned}$$

Effective theory within the theory

Apply to the EW sector of the Standard Model as EFT

$$\mathcal{L}_{eff} = \mathcal{L}_{\mathrm{SM}} + \sum_{n=1}^{\infty} \frac{c_n}{\Lambda^n} O_n$$

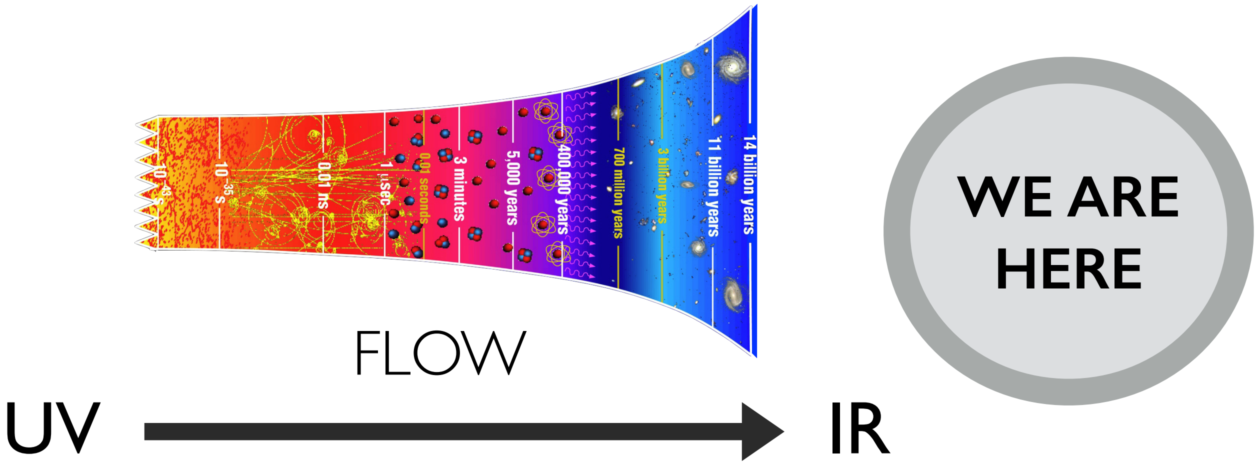
$$[\mathcal{L}_{eff}] = d$$

$$[O_n] = d + n$$

$$[c_n] = 0$$

The tiny perturbations are the IR remnant of the UV physics

EFT relevant whenever a finite UV scale exists that acquires physical meaning



FLOW

UV → IR

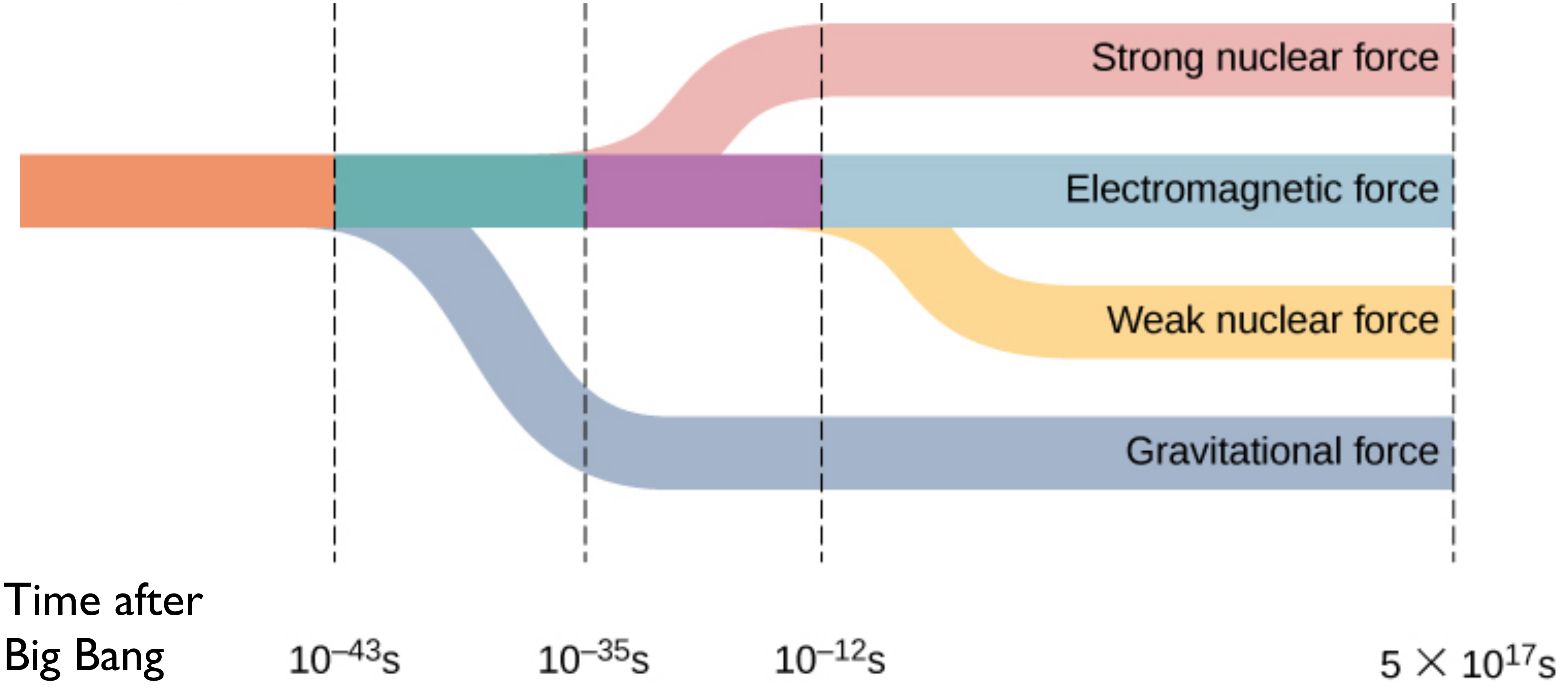
WE ARE HERE

Backup slides

A simplified view of particle physics

The TOE and GUT paradigms as pedagogical examples:

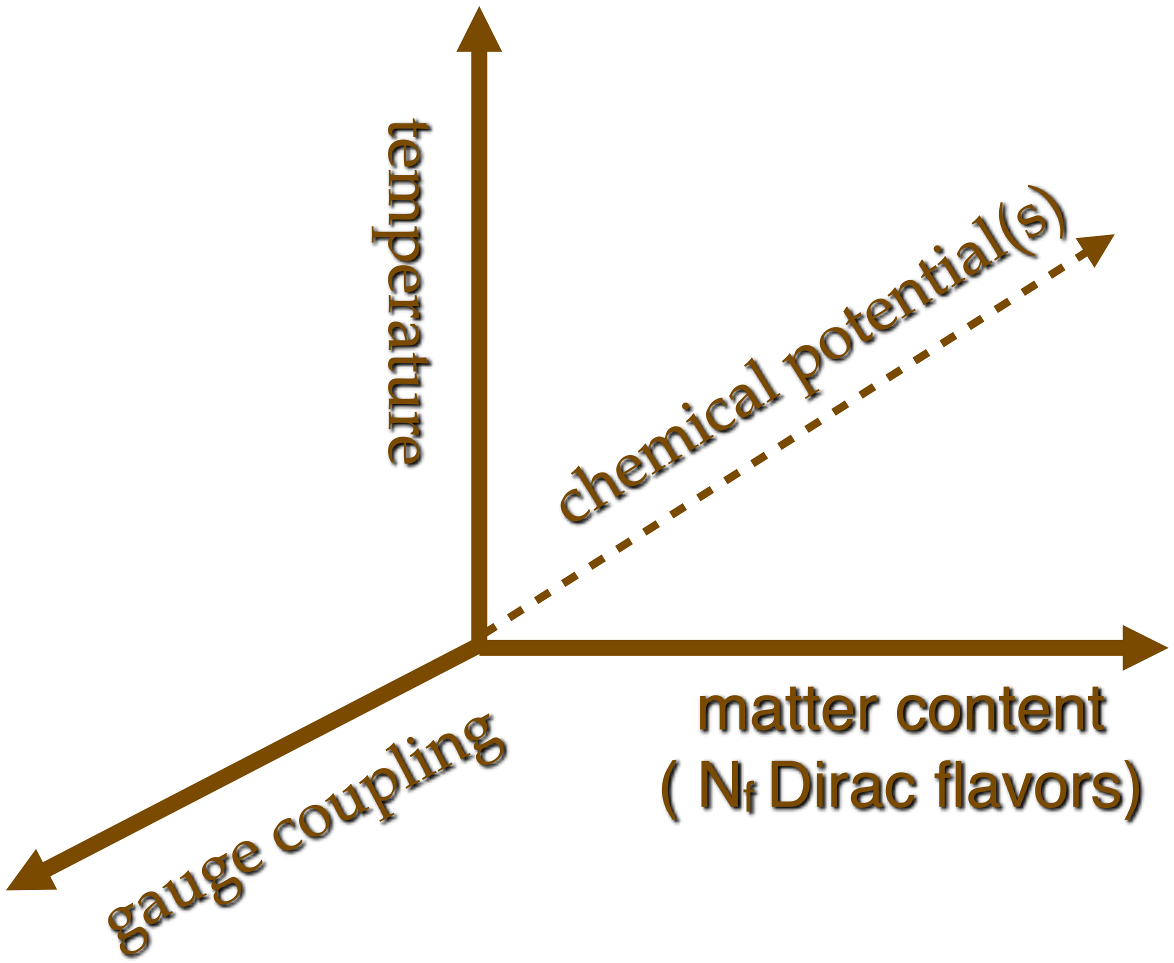
$$G_{TOE} \to G_{GUT} \to SU(3)_c \times SU(2)_L \times U(1)_Y \to SU(3)_c \times U(1)_{em}$$



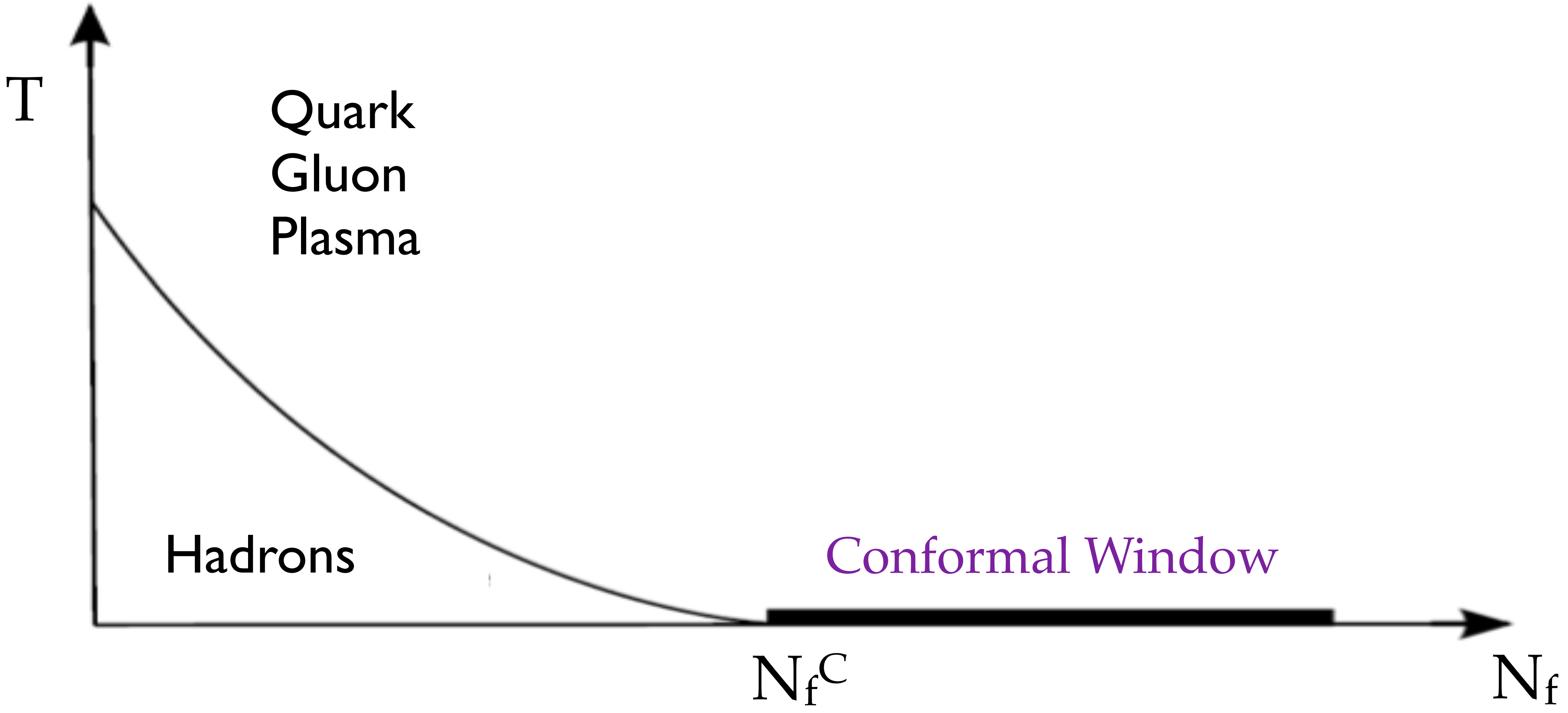
The Lagrangian

$$\mathcal{L} = -\frac{1}{4}\mathrm{Tr}F_{\mu\nu}F^{\mu\nu} + \sum_{f=1}^{N_f} \bar{\psi}_f^i \left(i\gamma^\mu D_\mu^{ij} - m_f \delta^{ij} \right) \psi_f^j$$

The parameters



QCD phase diagram: Temperature-Flavor (μ_B=0)



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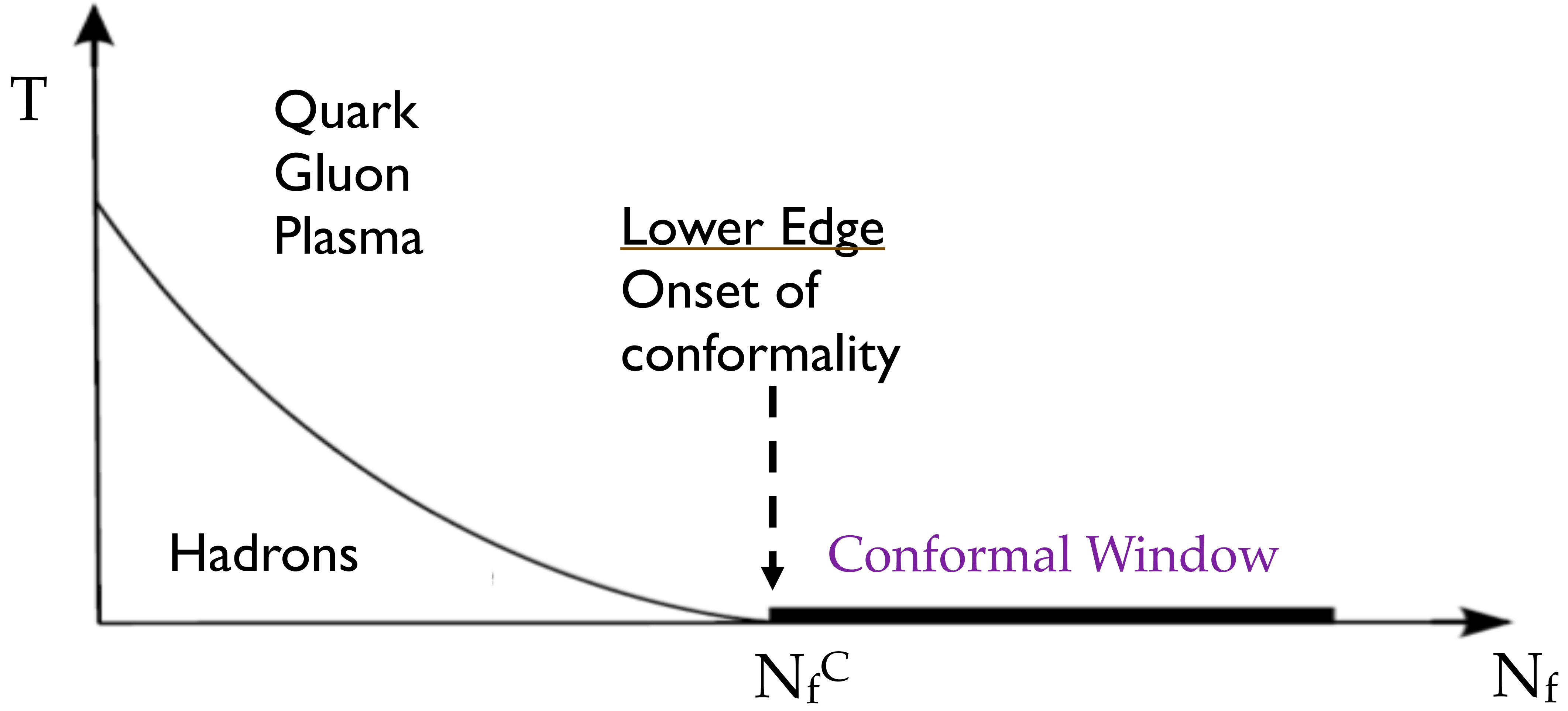

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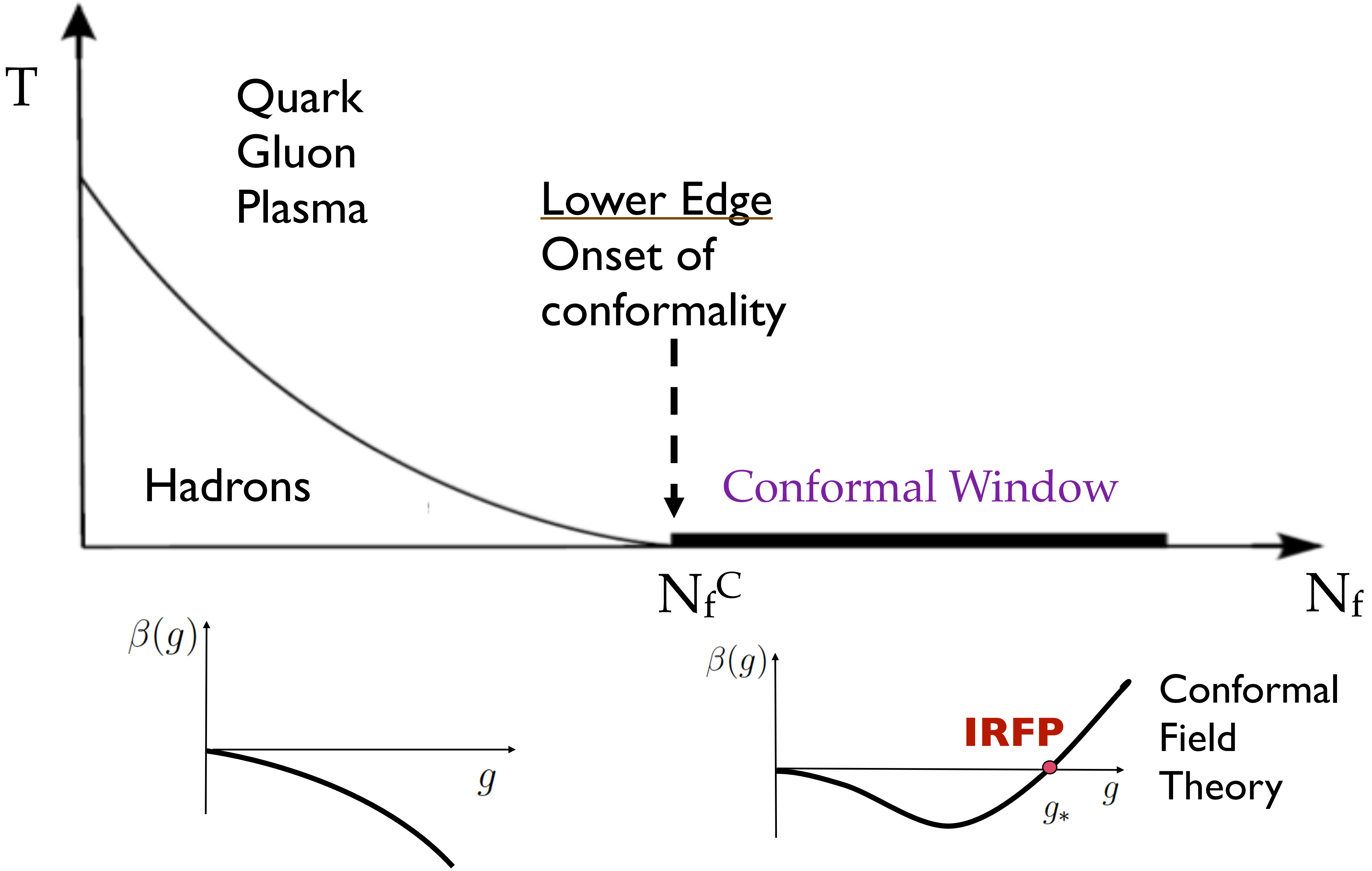


Quark
Gluon
Plasma

Lower Edge
Onset of
conformality

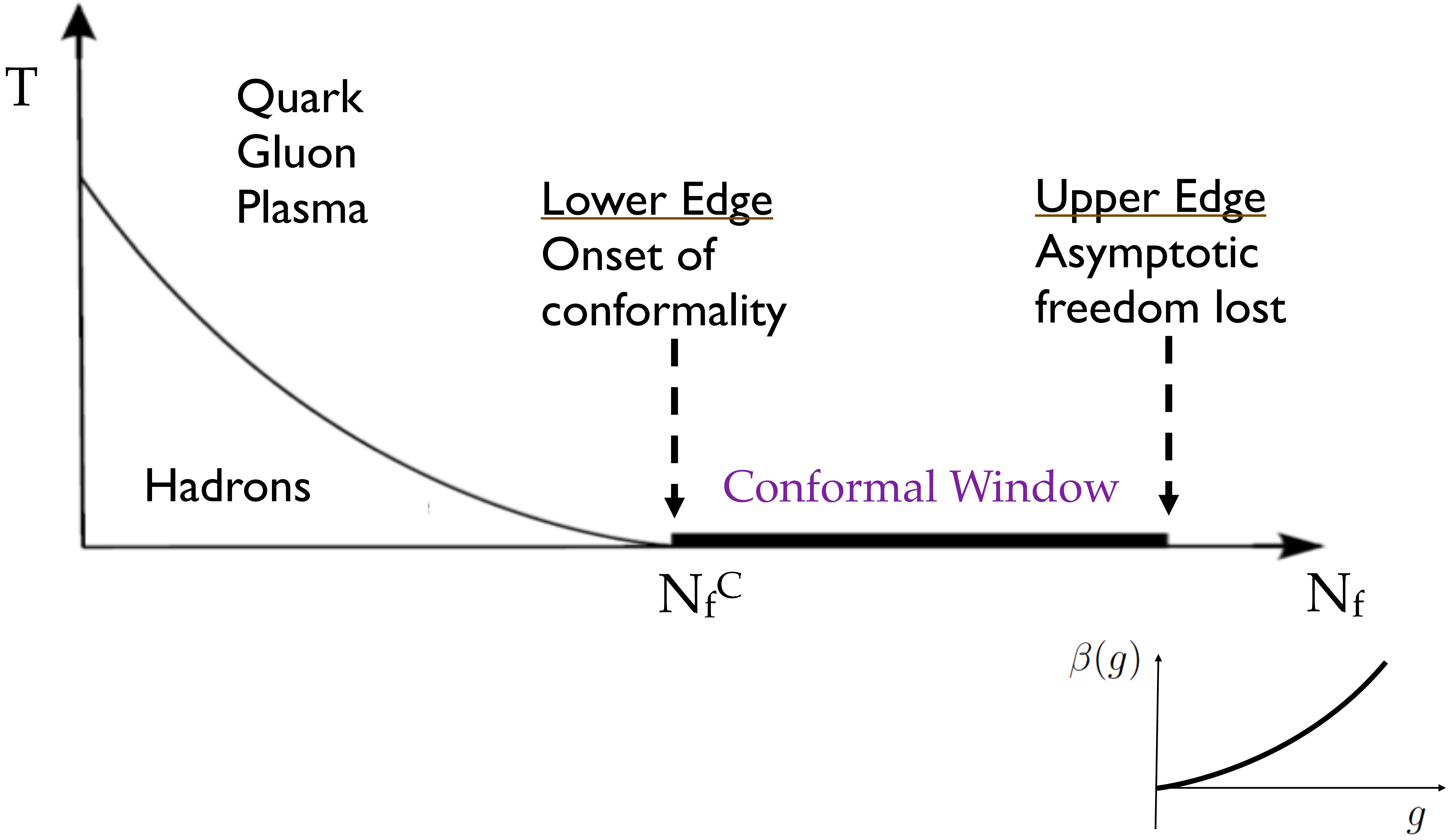
Hadrons

Conformal Window

IRFP

Conformal
Field
Theory

QCD phase diagram: Temperature-Flavor (μ_B =0)



Quark
Gluon
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Hadrons

Lower Edge
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Upper Edge
Asymptotic
freedom lost

Conformal Window