(Solutions - day 2)

Solution – Exercise 6

- Normalized Exponential p.d.f $$f(x) = \tau e^{-\tau x}$$

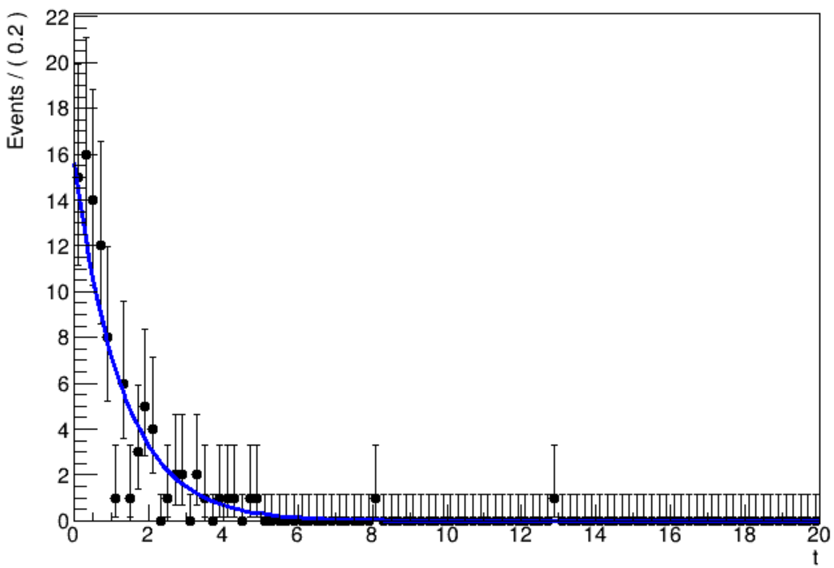
- Negative log-Likelihood (1 event) $$-\log\big(f(x)\big) = -\log(\tau) - \log(e^{-\tau x})$$ $$= -\log(\tau) + \tau x$$

- Negative log-Likelihood (N events) $$-\log(L) = \sum_{i=1}^{N} -\log(\tau) + \tau x_i$$ $$= -N\log(\tau) + \tau\sum_{i=1}^{n} x_i$$

- ML estimator for N events $0 = \dfrac{d\log L}{d\tau}$ $$0 = -N\frac{1}{\tau} + \sum_{i=1}^{n} x_i$$ $$\frac{N}{\tau} = \sum_{i=1}^{n} x_i \Rightarrow \frac{1}{\tau} = \frac{\sum_{i=1}^{n} x_i}{N} = \langle x \rangle$$

Solution – Exercise 6

- Analytical vs numeric calculation




```
...........
COVARIANCE MATRIX CALCULATED SUCCESSFULLY
FCN=124.735 FROM HESSE     STATUS=OK              5 CALLS          23 TOTAL
                     EDM=7.97722e-09    STRATEGY= 1      ERROR MATRIX ACCURATE
 EXT PARAMETER                                INTERNAL      INTERNAL
 NO.   NAME      VALUE            ERROR       STEP SIZE       VALUE
  1  tau        1.28062e+00   1.28048e-01   4.17606e-05  -1.07870e+00
                                ERR DEF= 0.5
EXTERNAL ERROR MATRIX.    NDIM=  25    NPAR=  1    ERR DEF=0.5
 1.640e-02
```

$$\frac{1}{\tau} = \frac{\sum_{i=1}^{n} x_i}{N} = \langle x \rangle \quad \tau = 1.28063$$

Agreement MINUIT/analytical within ~10^{-5} relative precision
Default MINUIT numerical precision (EDM 10^{-3}) ~4% of error → 5×10^{-3}

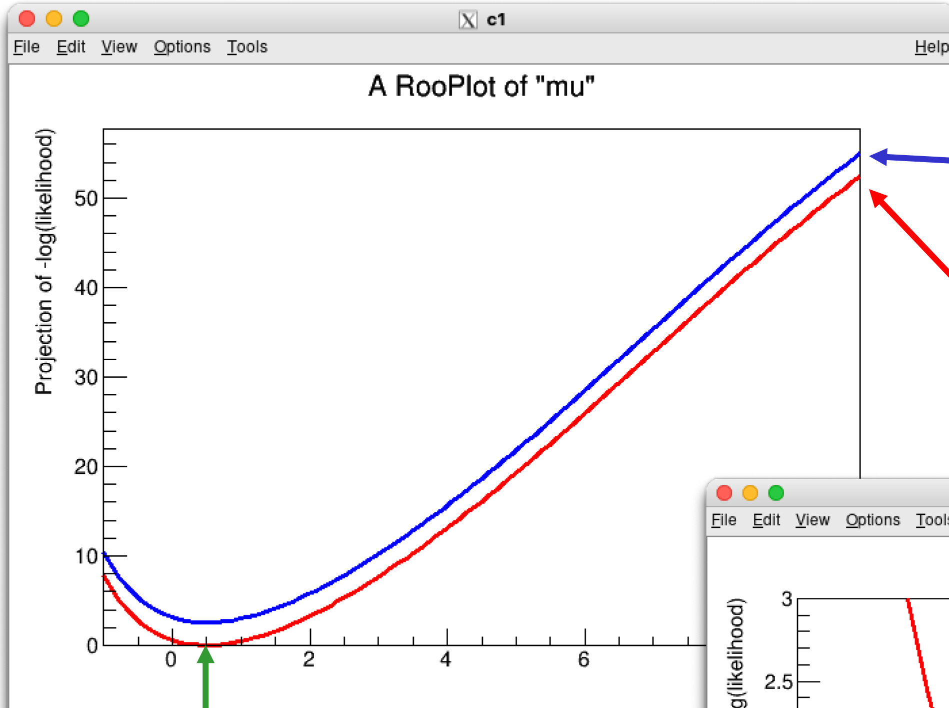
Solution – Exercise 9

$$P(N|\mu) = \text{Poisson}(N|\mu S+B)$$
$$= \text{Poisson}(N|\mu 10+20)$$

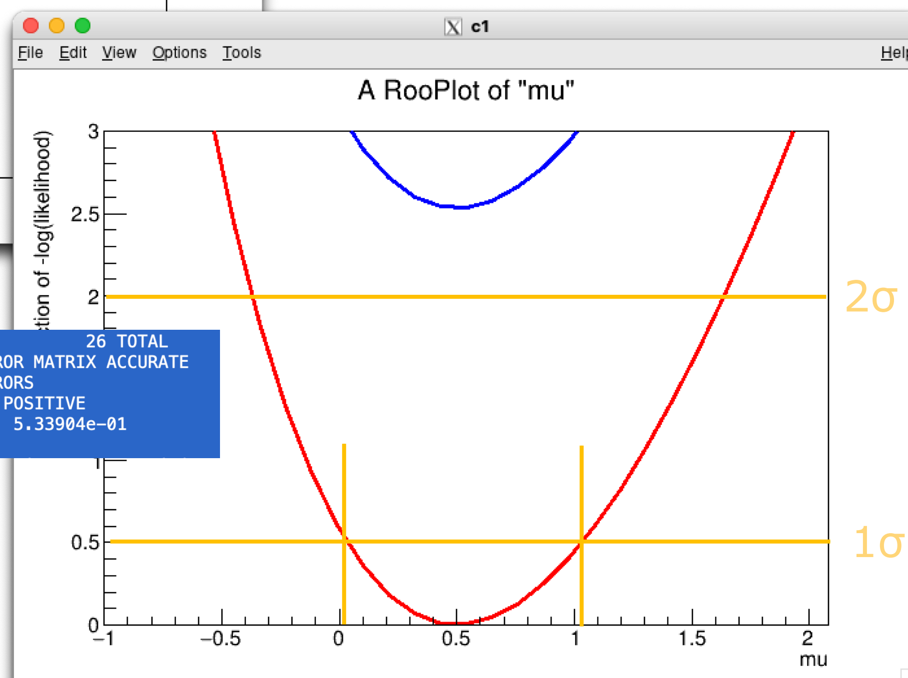
$L(25|\ \mu)=\text{Poisson}(25|\ \mu 10+20)$

$\hat{\mu}=0.5$

$\lambda(\mu) = L(25|\mu)/L(25|0.5)$



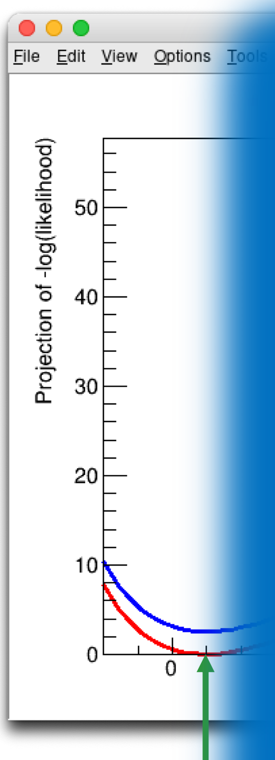

```
FCN=2.53171 FROM MINOS     STATUS=SUCCESSFUL      5 CALLS          26 TOTAL
                     EDM=3.72415e-09    STRATEGY= 1      ERROR MATRIX ACCURATE
 EXT PARAMETER                PARABOLIC         MINOS ERRORS
 NO.   NAME      VALUE            ERROR      NEGATIVE      POSITIVE
  1  mu           5.00031e-01   4.98543e-01  -4.67268e-01   5.33904e-01
                          ERR DEF= 0.5
```



2σ (95%)

1σ (68%)

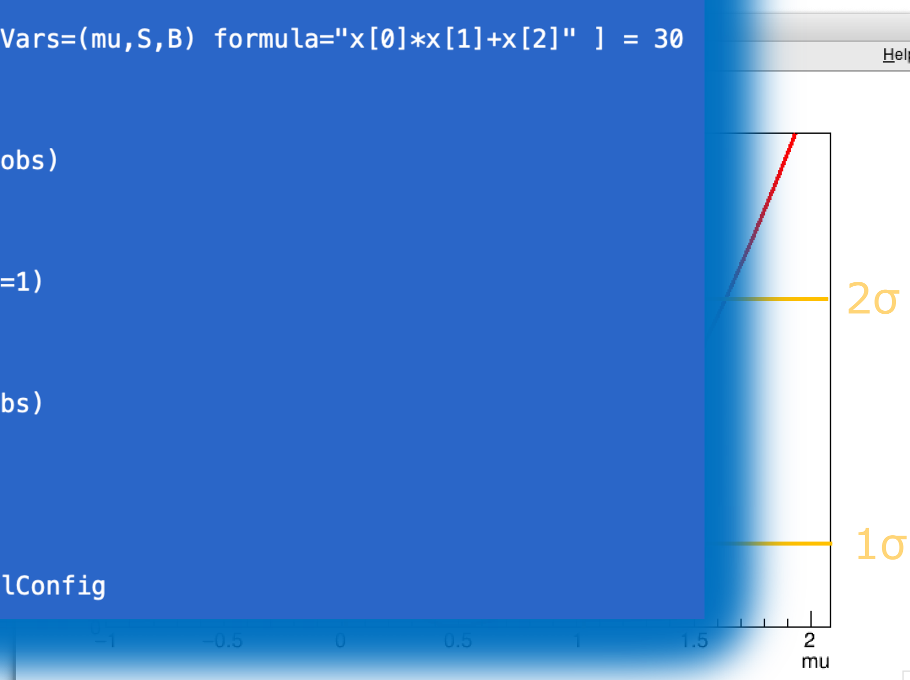
Solution – Exercise 9



FCN=2.53171 FROM
EXT PARAMETER
NO. NAME
1 mu

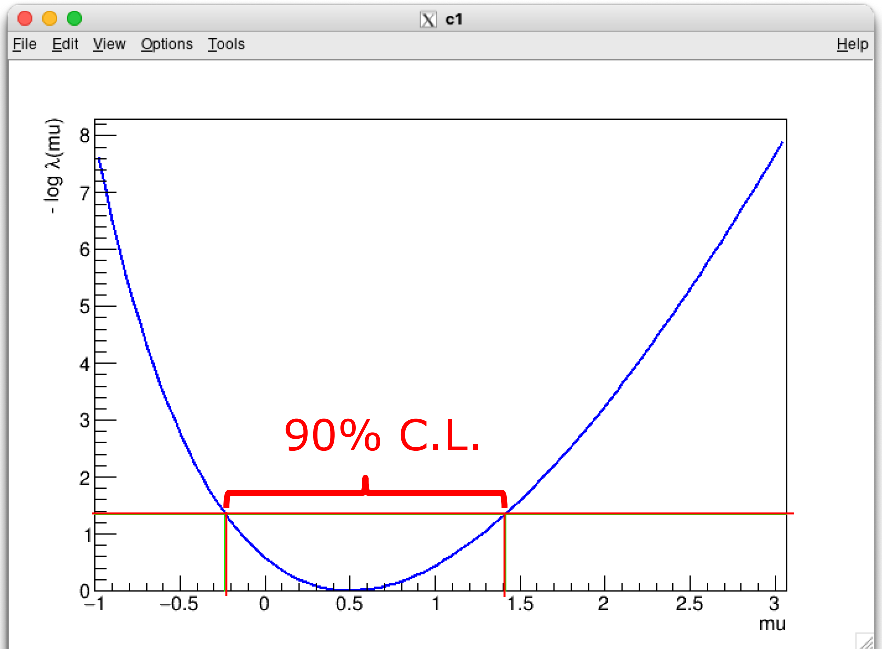
```
root [3] w->Print("v")

RooWorkspace(w) w contents

variables
---------
(B,Nobs,S,mu)

p.d.f.s
-------
RooPoisson::model[ x=Nobs mean=Nexp ] = 0.0511153

functions
--------
RooFormulaVar::Nexp[ actualVars=(mu,S,B) formula="x[0]*x[1]+x[2]" ] = 30

datasets
--------
RooDataSet::observed_data(Nobs)

parameter snapshots
-------------------
ModelConfig__snapshot = (mu=1)

named sets
----------
ModelConfig_Observables:(Nobs)
ModelConfig_POI:(mu)
ModelConfig__snapshot:(mu)

generic objects
---------------
RooStats::ModelConfig::ModelConfig
```

(N|μS+B)
n(N|μ10+20)
(25| μ10+20)
25|0.5)

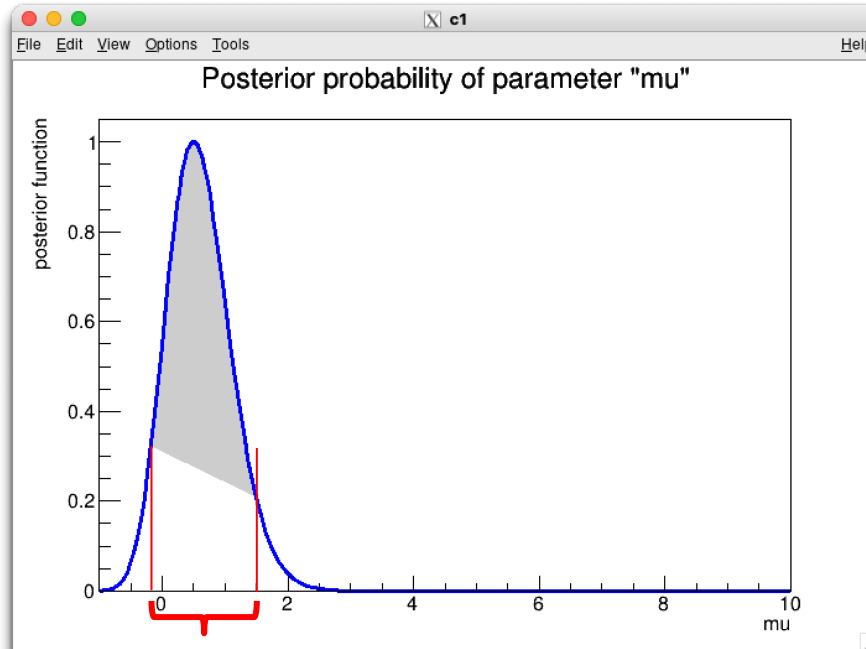


Solution – Exercise 10

PLR interval



Bayesian interval



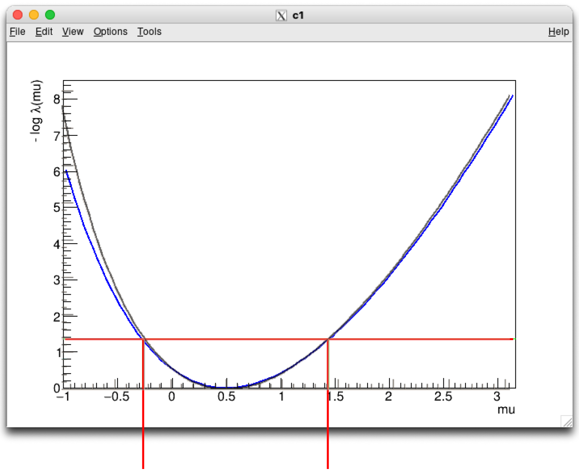
90% C.L.

```
RESULT: 90% interval is : [-0.234825, 1.41498]
```

```
INFO:Eval -- BayesianCalculator::GetInterval -
found a valid interval : [-0.178229 , 1.49173 ]
```

Solution – Exercise 11

ex11/ex09



$P(N_{SR},N_{CR}|\mu) = \text{Poisson}(N_{SR}|\mu S+B)\text{Poisson}(N_{CR}|\tau B)$

```
root [1] w->Print("t")

RooWorkspace(w) w contents

variables
---------
(B,Nobs_CR,Nobs_SR,S,mu,tau)

p.d.f.s
-------
RooProdPdf::model[ model_SR * model_CR ] = 0.00144134
  RooPoisson::model_SR[ x=Nobs_SR mean=Nexp_SR ] = 0.0511153
    RooFormulaVar::Nexp_SR[ actualVars=(mu,S,B) formula="x[0]*x[1]+x[2]" ] = 30
  RooPoisson::model_CR[ x=Nobs_CR mean=Nexp_CR ] = 0.0281977
    RooFormulaVar::Nexp_CR[ actualVars=(tau,B) formula="x[0]*x[1]" ] = 200

datasets
```

with N_{SR}=25, N_{CR}=200, τ=10 → B=20, μ=0.5

$L_{ex11}(25,200|\mu,B) = \text{Poisson}(25|\mu 10+B)\text{Poisson}(200|10B)$

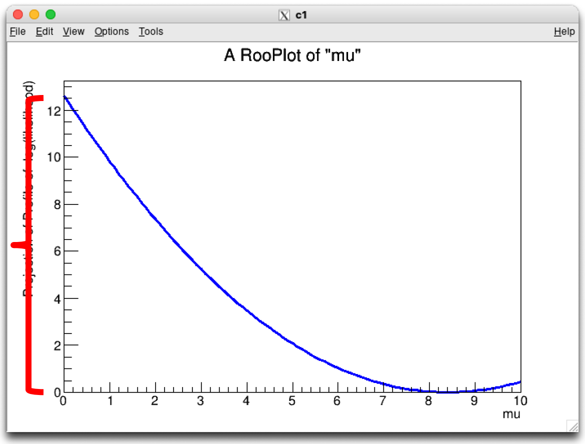
similar to ex09

$L_{ex09}(25|\mu)=\text{Poisson}(25|\mu 10+20)$

however in ex11, bkg is *not* presumed exactly known
slight broadening of interval

Solution – Exercise 11

$$P(N_{SR},N_{CR}|\mu) = \text{Poisson}(N_{SR}|\mu S+B)\text{Poisson}(N_{CR}|\tau B)$$




```
root [1] w->Print("t")

RooWorkspace(w) w contents

variables
---------
(B,Nobs_CR,Nobs_SR,S,mu,tau)

p.d.f.s
-------
RooProdPdf::model[ model_SR * model_CR ] = 0.00144134
  RooPoisson::model_SR[ x=Nobs_SR mean=Nexp_SR ] = 0.0511153
    RooFormulaVar::Nexp_SR[ actualVars=(mu,S,B) formula="x[0]*x[1]+x[2]" ] = 30
  RooPoisson::model_CR[ x=Nobs_CR mean=Nexp_CR ] = 0.0281977
    RooFormulaVar::Nexp_CR[ actualVars=(tau,B) formula="x[0]*x[1]" ] = 200

datasets
```

$t^{\mu}(0) = 0.5Z^2$

$5\sigma \rightarrow t^{\mu}(0) = 12.5$

with $N_{SR}=184$, $N_{CR}=100$, $\tau=1 \rightarrow B=100$, $\mu=8.4$

$$L_{ex11}(184,100|\mu,B) = \text{Poisson}(184|\mu 10+B)\text{Poisson}(100|B)$$