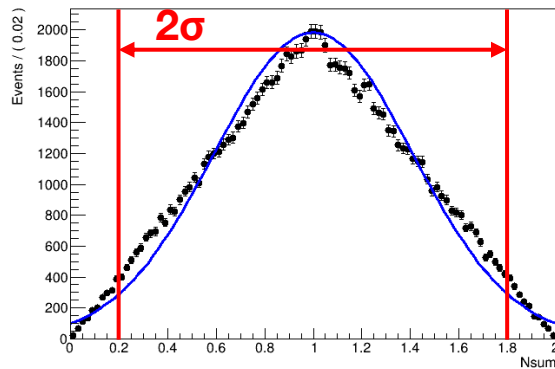


(Solutions - day 1)

Solution – Exercise 1

N=2

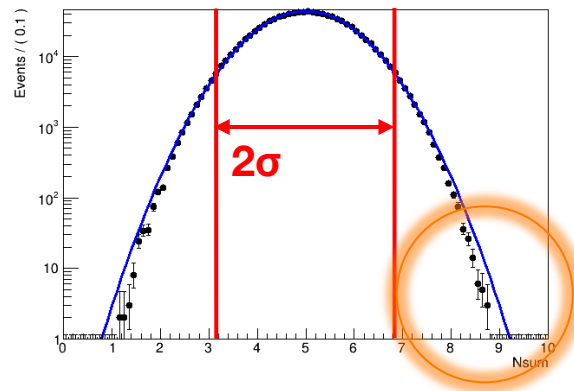
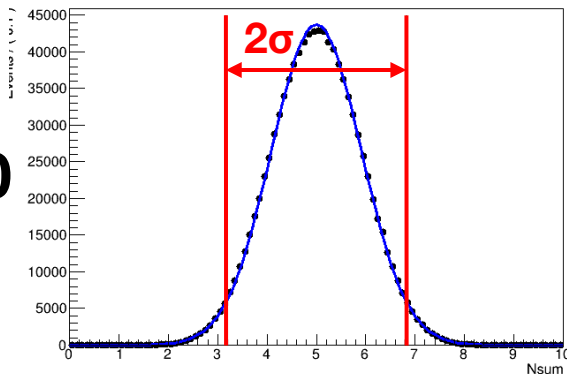


$$\begin{aligned}
 V(N=1) &= E[x^2] - E[x]^2 \\
 &= \int_0^1 x^2 dx - \left(\int_0^1 x \right)^2 \\
 &= \frac{1}{3} - \left(\frac{1}{2} \right)^2 = \frac{1}{12}
 \end{aligned}$$

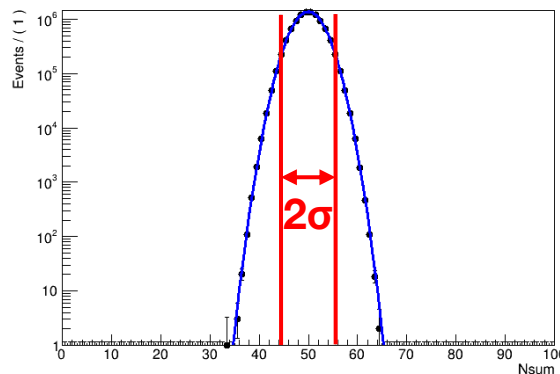
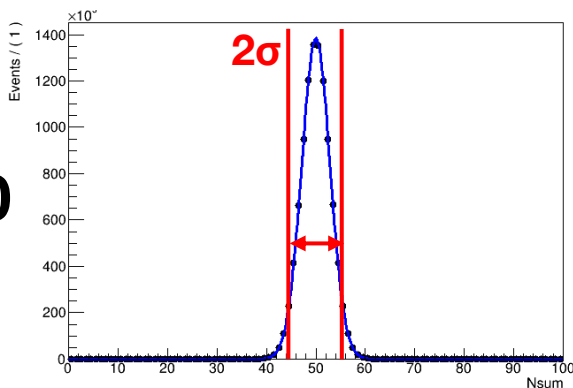


$$\begin{aligned}
 \sigma(N) &= \frac{\sigma(1)}{\sqrt{N}} N \\
 &= \sqrt{\frac{N}{12}}
 \end{aligned}$$

N=10



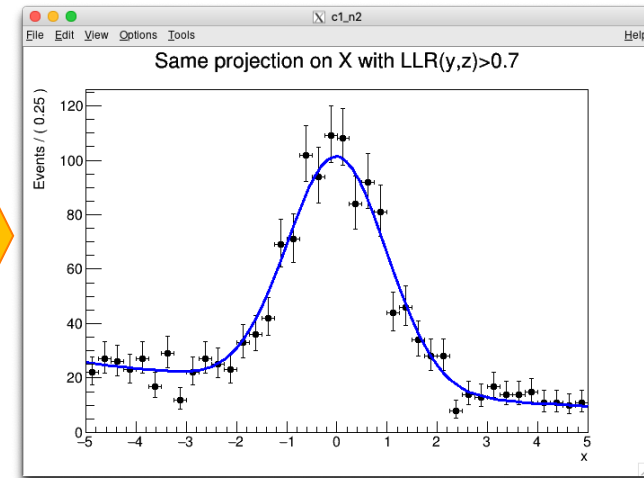
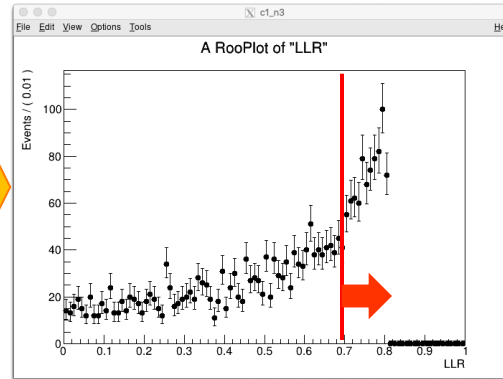
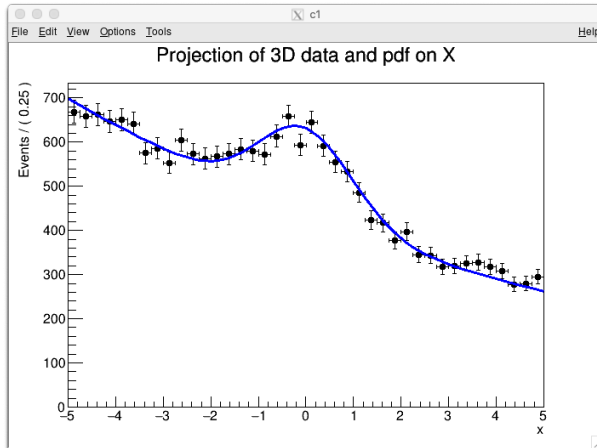
N=100



Solution – Exercise 3

- Analytical application of Neyman-Pearson lemma

'per-event signal purity
according to model(y,z)'



RooFitResult: minimized FCN value: 135474, estimated distance to minimum: 2.25574e-06
covariance matrix quality: Full, accurate covariance matrix
Status : MINIMIZE=0 HESSE=0

Floating Parameter	FinalValue +/- Error
fsig	9.7816e-02 +/- 2.78e-03

→ 35.2σ

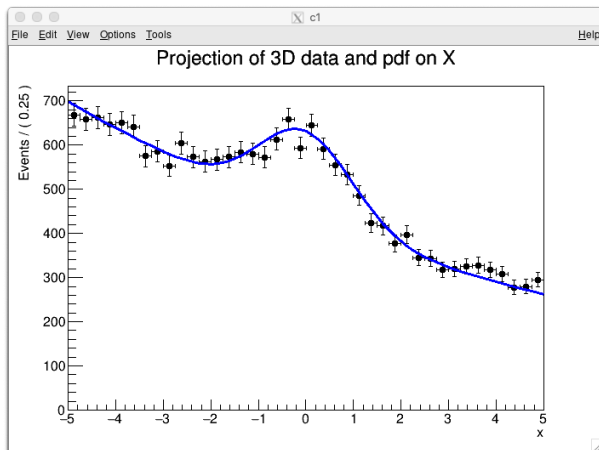
RooFitResult: minimized FCN value: 3096.86, estimated distance to minimum: 2.14652e-05
covariance matrix quality: Full, accurate covariance matrix
Status : MINIMIZE=0 HESSE=0

Floating Parameter	FinalValue +/- Error
fsig	5.6477e-01 +/- 1.92e-02

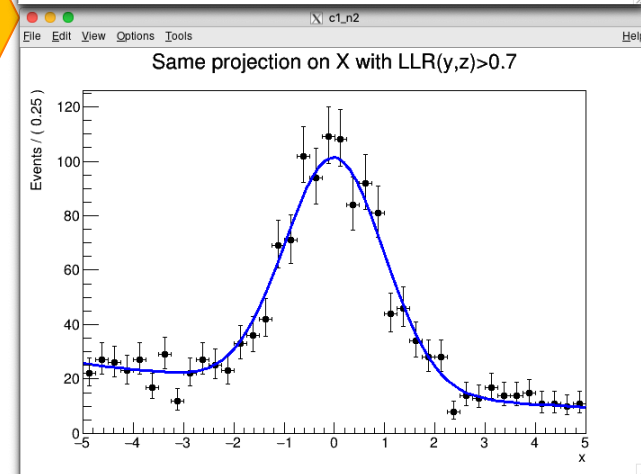
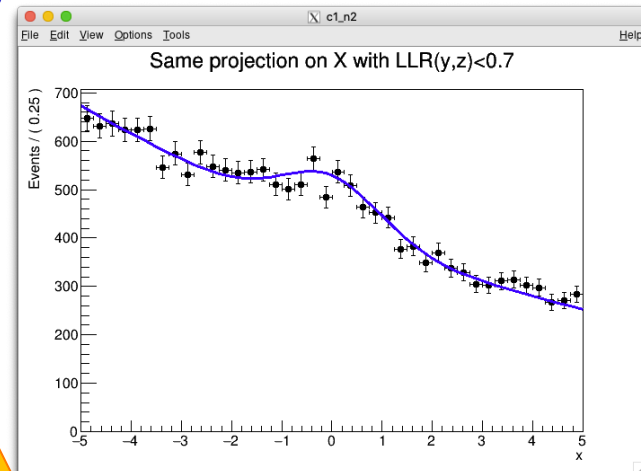
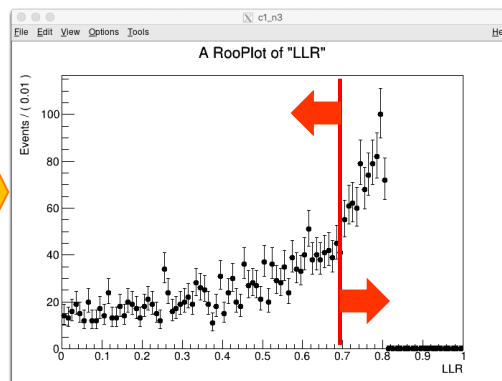
→ 29.4σ

- Note design choice to cut *only* on model $f(y,z)$ with NP-lemma
- Can use $f(x)$ to *measure* remaining background

Solution – Exercise 3



'per-event signal purity according to model(y,z)'



RooFitResult: minimized FCN value: 135474, estimated distance to minimum: 2.25574e-06
covariance matrix quality: Full, accurate covariance matrix
Status : MINIMIZE=0 HESSE=0

Floating Parameter	FinalValue +/- Error
fsig	9.7816e-02 +/- 2.78e-03

→ 35.2σ

RooFitResult: minimized FCN value: 3096.86, estimated distance to minimum: 2.14652e-05
covariance matrix quality: Full, accurate covariance matrix
Status : MINIMIZE=0 HESSE=0

Floating Parameter	FinalValue +/- Error
fsig	5.6477e-01 +/- 1.92e-02

→ 29.4σ

RooFitResult: minimized FCN value: 41908.6, estimated distance to minimum: 5.35272e-05
covariance matrix quality: Full, accurate covariance matrix
Status : MINIMIZE=0 HESSE=0

Floating Parameter	FinalValue +/- Error
fsig	4.8966e-02 +/- 5.49e-03

→ 8.9σ

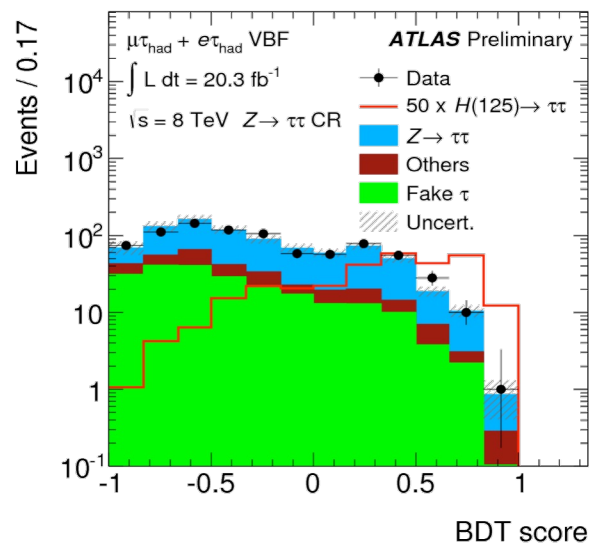
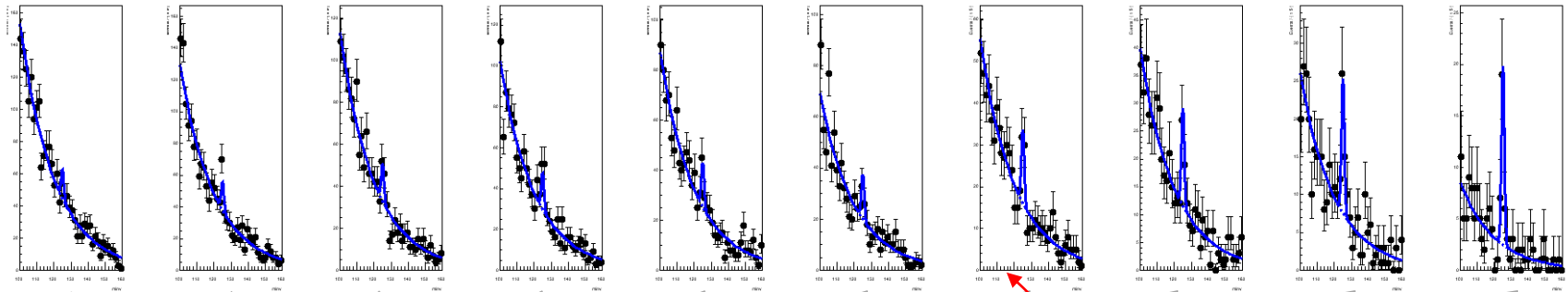
30.7σ

'not using prediction of event ratio over LLR cut'

NB: Feature!

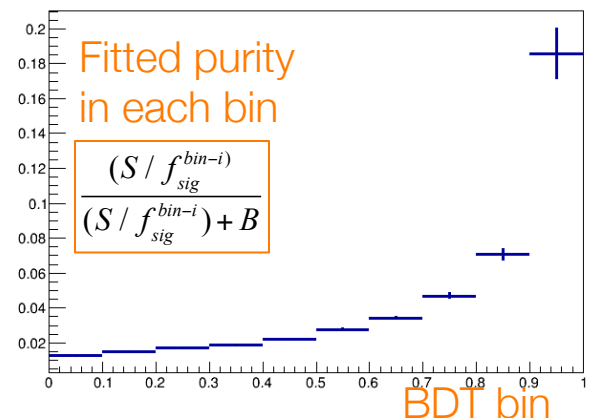
(Preview to day 3) → Can also split in >>2 regions

- In that case only exploit variation in purity, not distribution of LLR/BDT
→ *Analysis insensitive to mismodelling of LLR/BDT distribution!*



$$f_{bin-i}(m | S, B) = \frac{S}{f_{sig}^{bin-i}} f_S(m) + B_{bin-i} f_B(s)$$

Scale factor that ensures
that every bin interprets
S as the total signal yield



(Preview to day 3) → Can also split in >>2 regions

- In that case only exploit variation in purity, not distribution of LLR/BDT
→ Analysis insensitive to mismodelling of LLR/BDT distribution!

Joint PDF for
this model

$$f(m, n_{BDT} | S, \vec{B}) = \text{lookup}(n_{BDT})$$

$$f_{bin-0}(m | S, B_0) = \frac{S}{f_{sig}^{bin-0}} f_S(m) + B_{bin-0} f_B(s)$$

$$f_{bin-1}(m | S, B_1) = \frac{S}{f_{sig}^{bin-1}} f_S(m) + B_{bin-1} f_B(s)$$

$$f_{bin-2}(m | S, B_2) = \frac{S}{f_{sig}^{bin-2}} f_S(m) + B_{bin-2} f_B(s)$$

$$f_{bin-3}(m | S, B_3) = \frac{S}{f_{sig}^{bin-3}} f_S(m) + B_{bin-3} f_B(s)$$

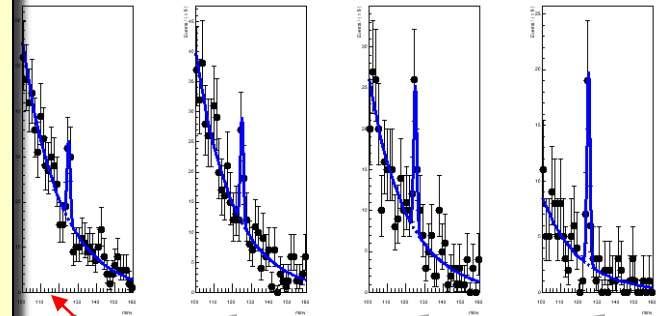
⋮

$$f_{bin-N}(m | S, B_N) = \frac{S}{f_{sig}^{bin-N}} f_S(m) + B_{bin-N} f_B(s)$$

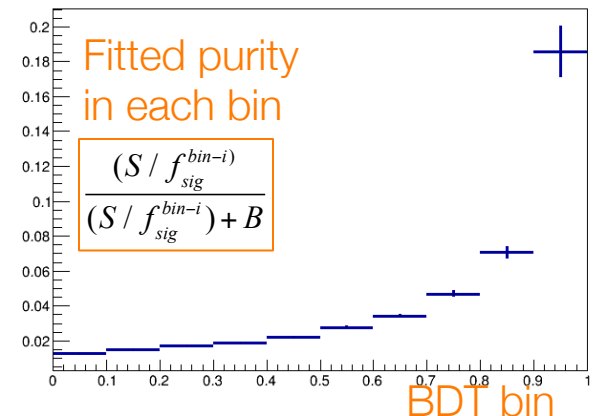
```
// Construct template model
w.factory("SUM::fit_template(prod(Nsig[30,0,100],frac[1])*sig1,
                               Nbkg[1000,0,10000]*bkg1)");

// Construct joint model from template clones
w.factory("SIMCLONE::fitmodel(fit_template,
                              $SplitParam({Nbkg,frac},bdtBin))");
```

BDT score

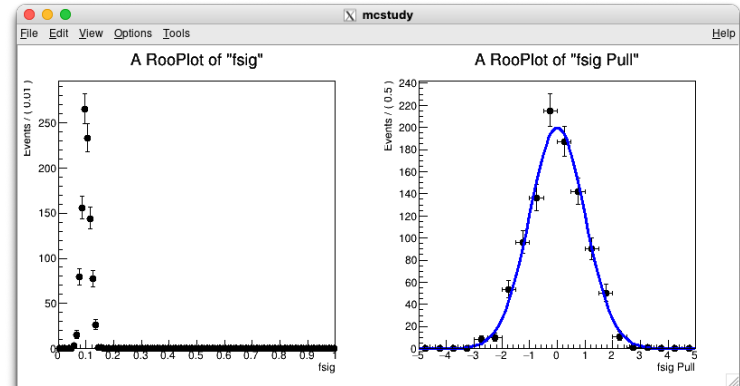
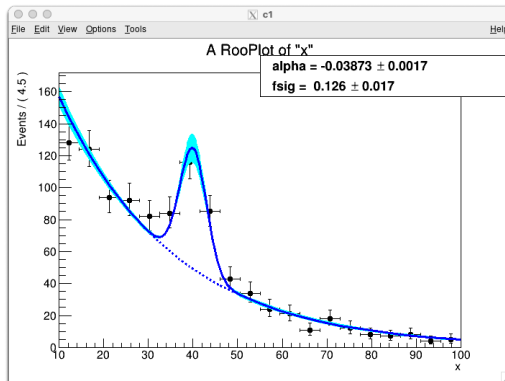


$$f(m | S, B) = \frac{S}{f_{sig}^{bin-i}} f_S(m) + B_{bin-i} f_B(s)$$

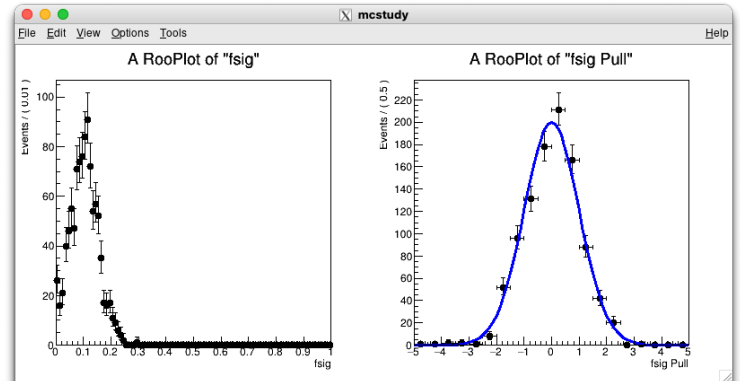
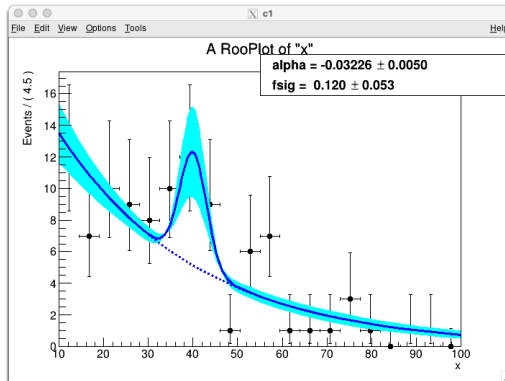


Solution – Exercise 5

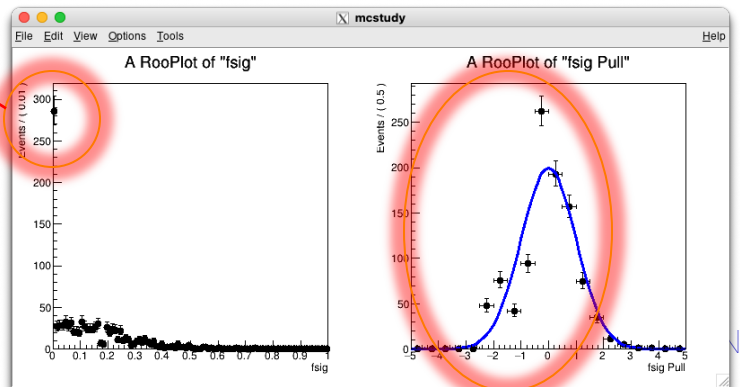
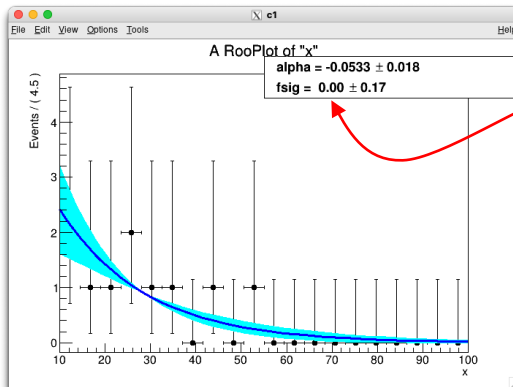
N=1000



N=100

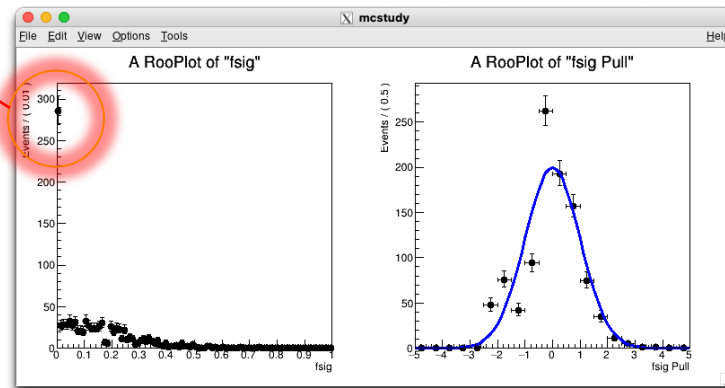
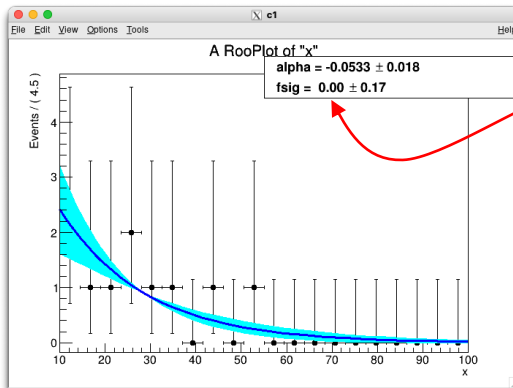


N=10

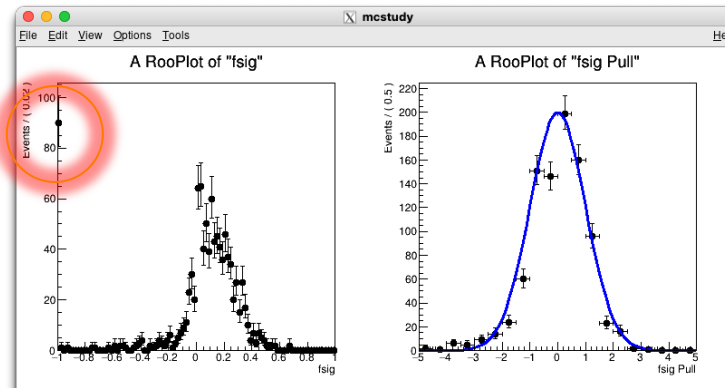
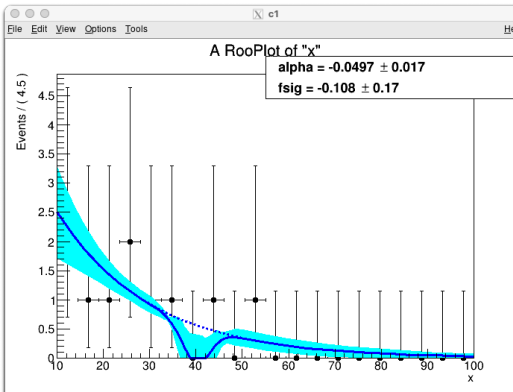


Solution – Exercise 5

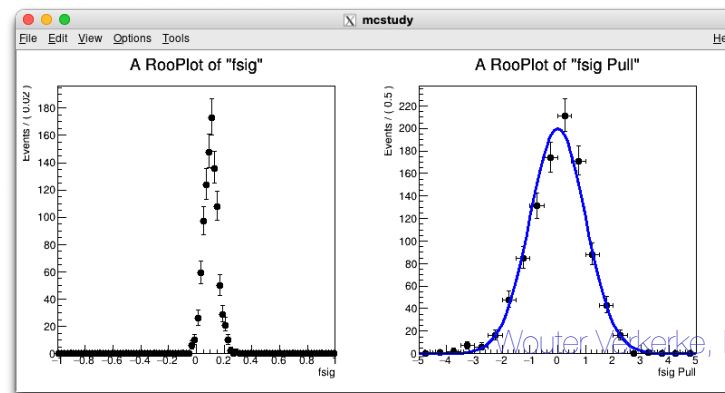
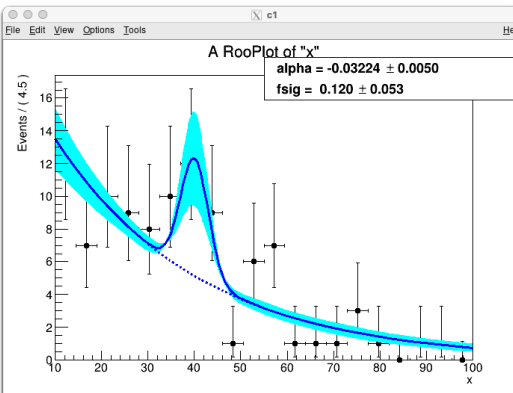
N=10



fsig[-1,1]
N=10

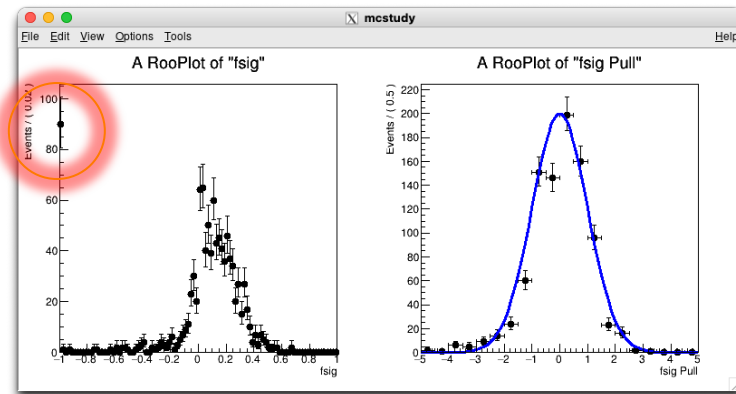
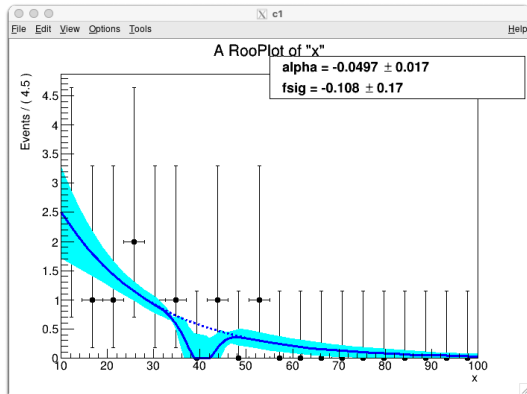


fsig[-1,1]
N=100

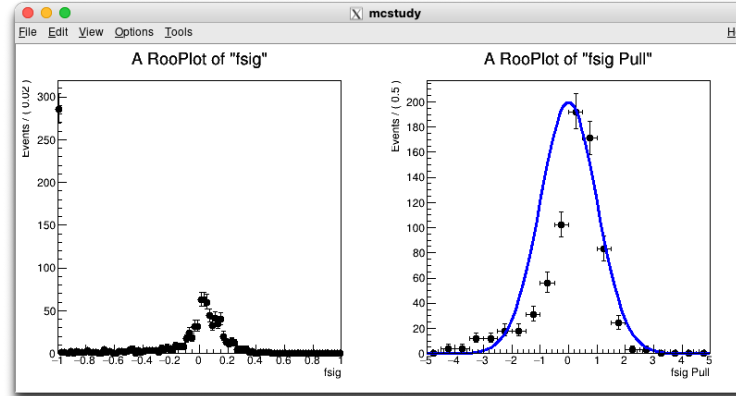
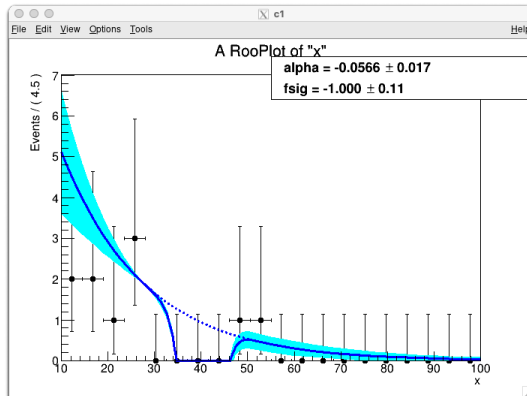


Solution – Exercise 5

fsig[-1,1]
N=10



fsig=0
N=10



fsig=0
N=100

