Light propagation with Markov Chain Monte Carlo

Martijn Jongen
ANTARES/KM3NeT group meeting
4 April 2016

Model

- Scattering in 3D
- Completely random (no preferred direction)
- Scattering length = 70 m
- No absorption (but that is okay)
- Isotropic source at origin
- Spherical target on z-axis at 37 m

Note: my approach works **for any model**.
This is just a simple reference simulation.

Model

- Probability density defines model:

$$\rho_{\text{hit}}^{(n)}(\{\vec{x}_i\}) = \left\{ \sum_{i=0}^{n-1} \frac{1}{4\pi\lambda\ell_i^2} e^{-\ell_i/\lambda} \right\} \times \frac{1}{4\pi\ell_n^2} e^{-\ell_n/\lambda},$$

- **Probability density** for a photon to
  - be emitted at x_0
  - then scatter in infinitesimal volume dV_1 around x_1
  - then scatter in infinitesimal volume dV_2 around x_2
  - ...
  - then hit a spherical target of cross-section sigma at x_{n+1}
- Multiply by sigma and dV_1, dV_2, etc. to get a **probability**

Model

- Probability density defines model:

Scatter at the given distance (includes 1/lambda)

Do not scatter before hitting the target

$$\rho_{\mathrm{hit}}^{(n)}(\{\vec{x}_i\}) = \left\{ \sum_{i=0}^{n-1} \frac{1}{4\pi\lambda\ell_i^2} e^{-\ell_i/\lambda} \right\} \times \frac{1}{4\pi\ell_n^2} e^{-\ell_n/\lambda},$$

Scatters n times

Phase space to hit the target

Phase space to hit vertex n (does not include 1/lambda)

- **Probability density** for a photon to
  - be emitted at x_0
  - then scatter in infinitesimal volume dV_1 around x_1
  - then scatter in infinitesimal volume dV_2 around x_2
  - ...
  - then hit a spherical target of cross-section sigma at x_{n+1}
- Multiply by sigma and dV_1, dV_2, etc. to get a **probability**

Method

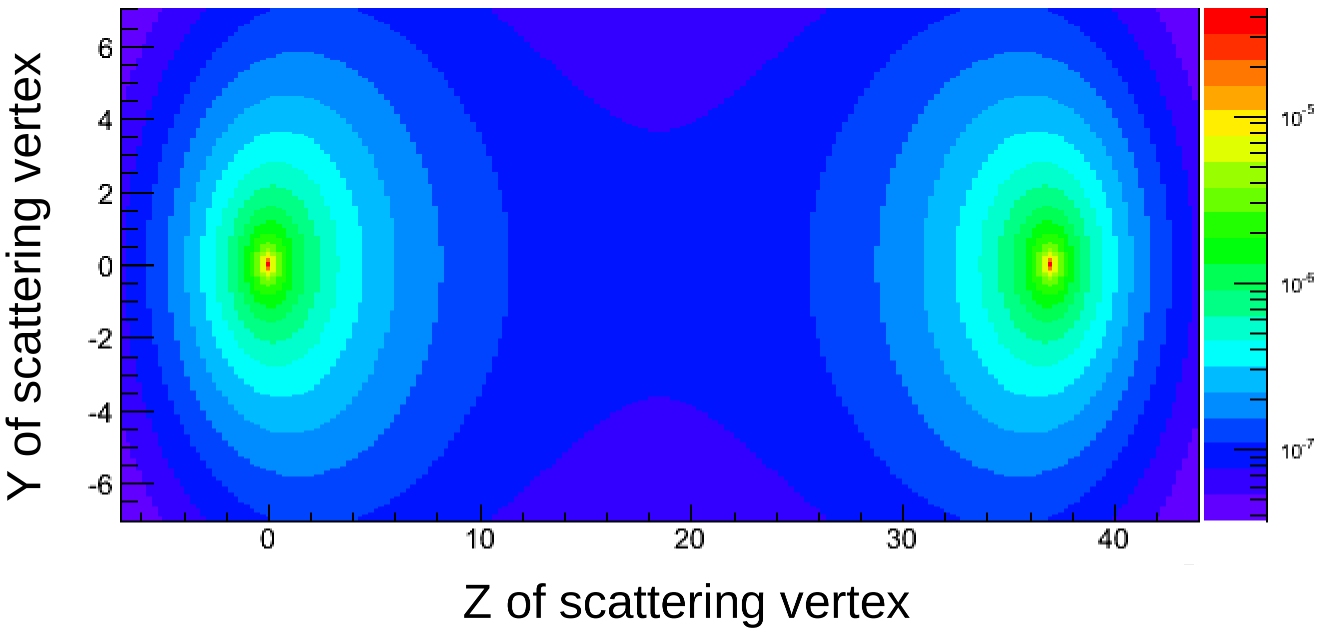
- Markov Chain Monte Carlo (MCMC)
- Weighted random walk through parameter space
- Allows to draw representative samples from probability distribution
- Works for every number of scatterings
- But does not give integral, i.e. the total probability to scatter n times”

Verification method

- “Brute-force” integration of single-scattering probability density

$$\rho_{\mathrm{hit}}^{(1)}(\vec{x}_1) = \frac{1}{16\pi^2\lambda\ell_0^2\ell_1^2}\exp\left(-\frac{\ell_0+\ell_1}{\lambda}\right),$$

Probability density for single scattering

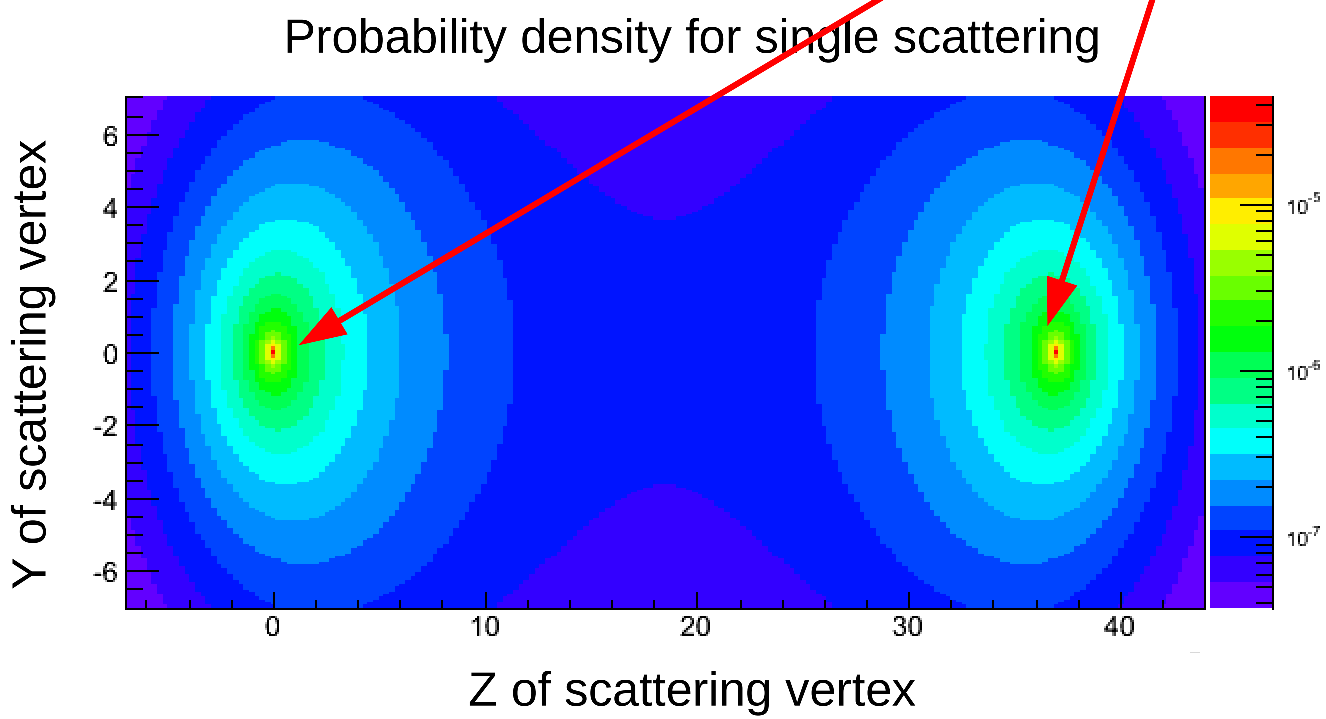


Verification method

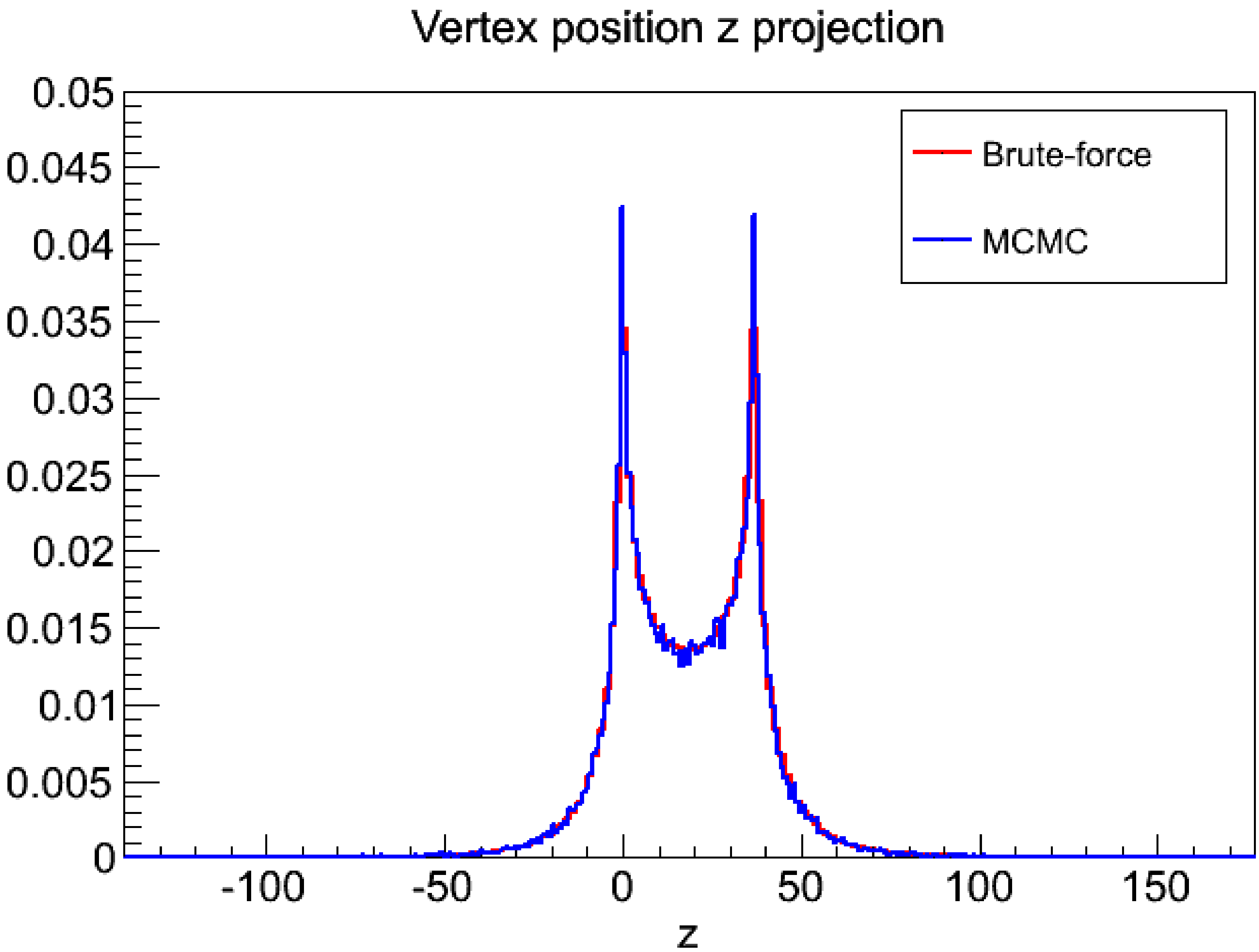
- “Brute-force” integration
  probability density

$$\rho_{\rm hit}^{(1)}(\vec{x}_1) = \frac{1}{16\pi^2\lambda\ell_0^2\ell_1^2}\exp\Big($$

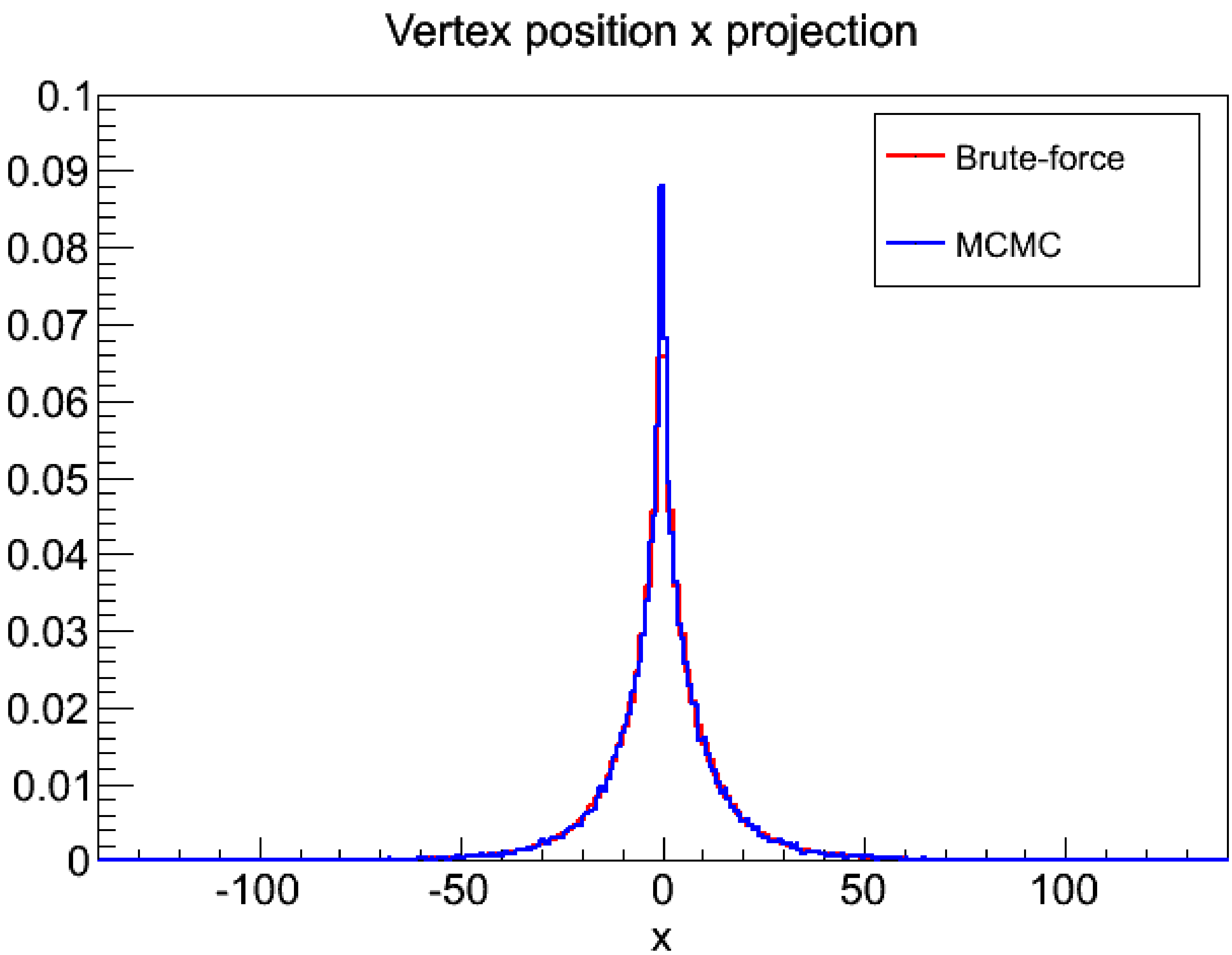
Scattering vertex prefers to be directly at the source or at the target



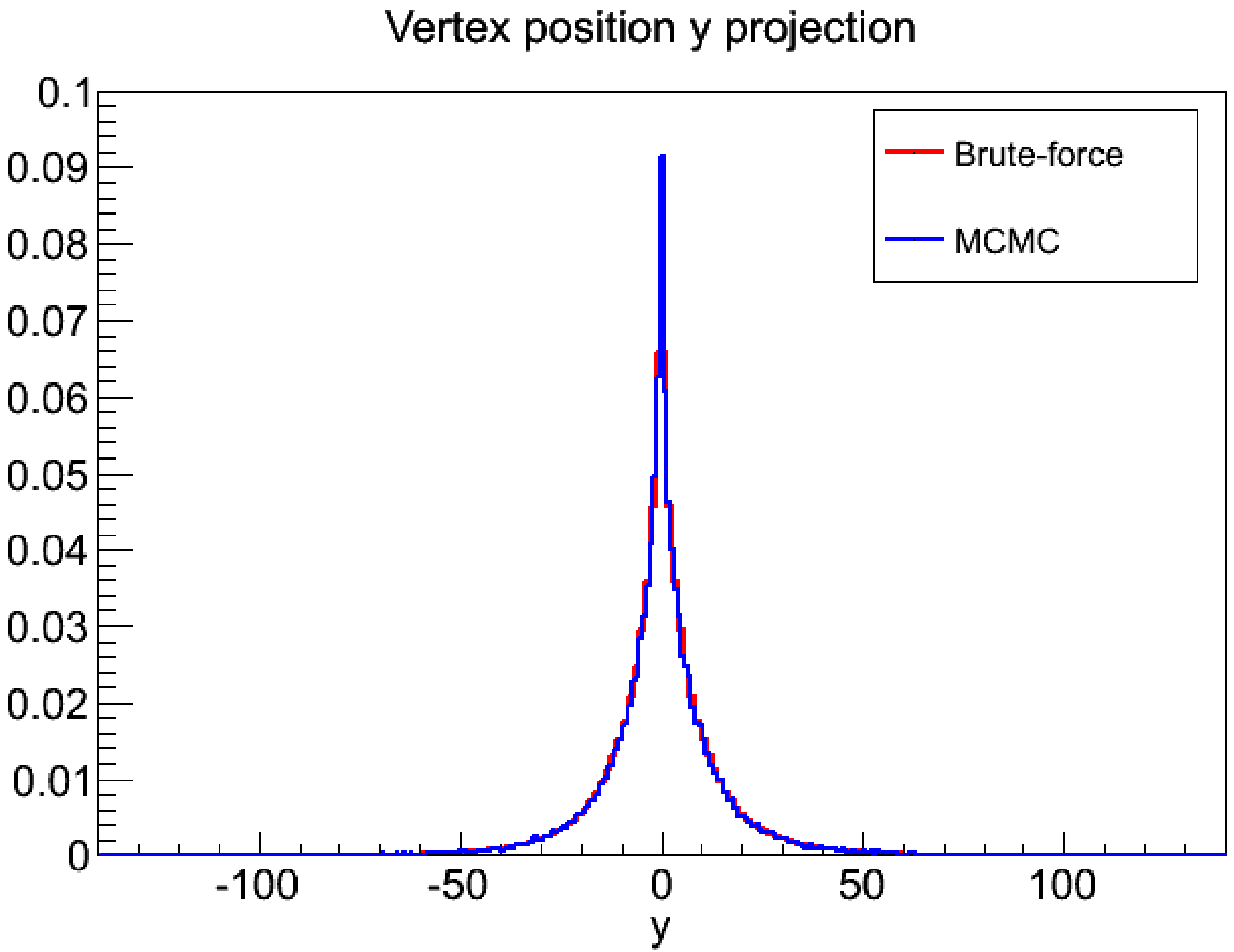
Results (single scattering)



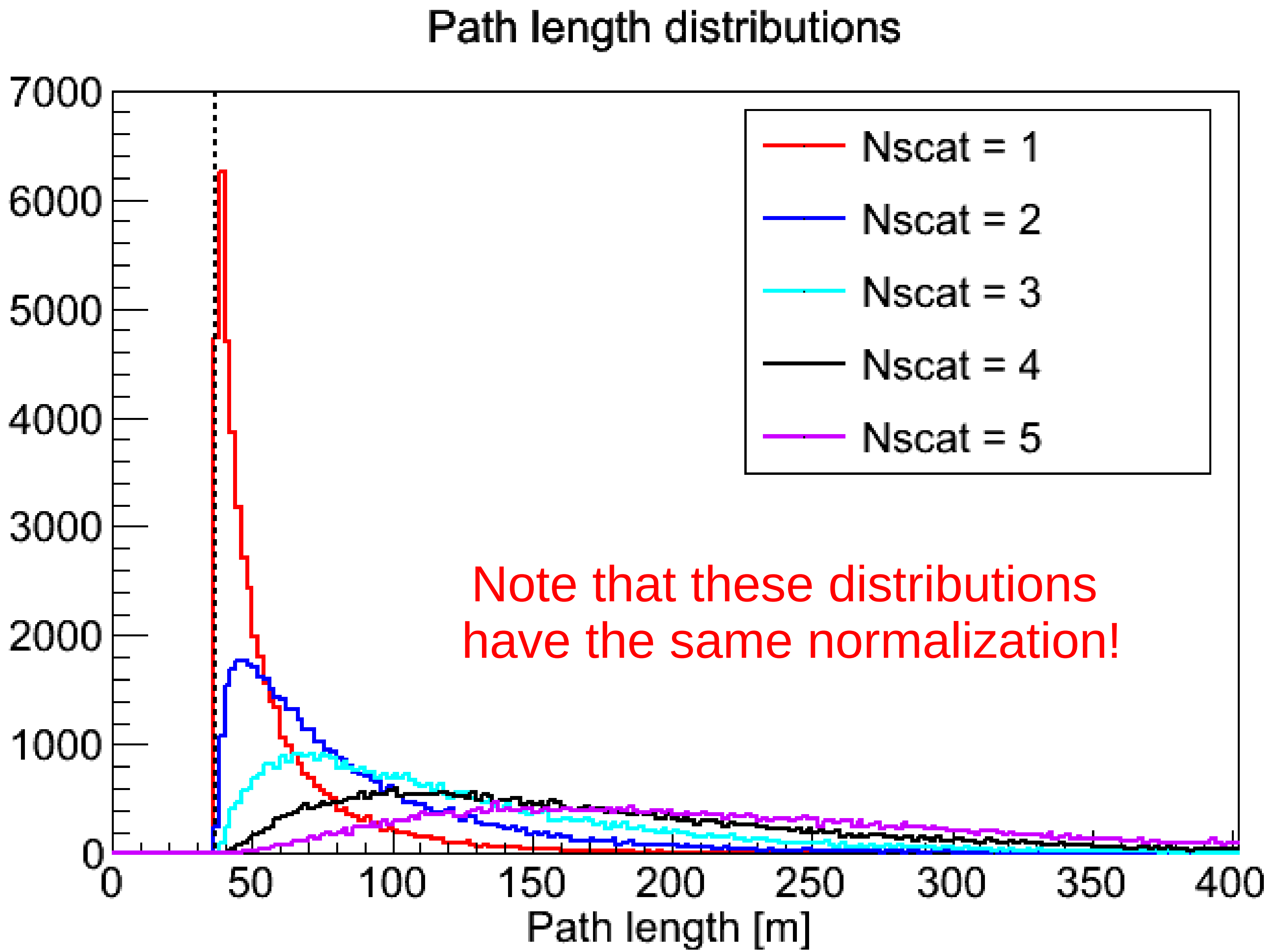
Results (single scattering)



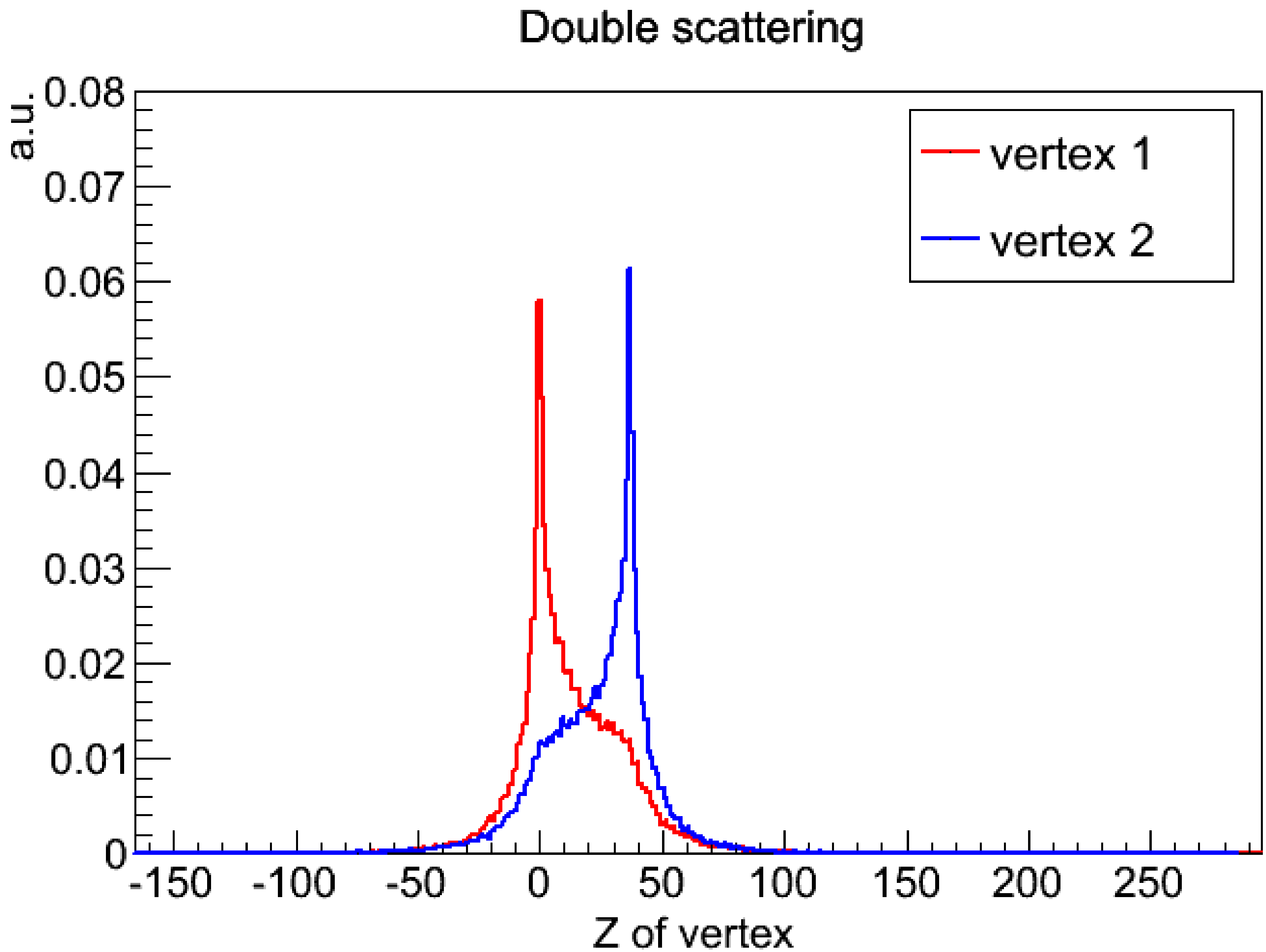
Results (single scattering)



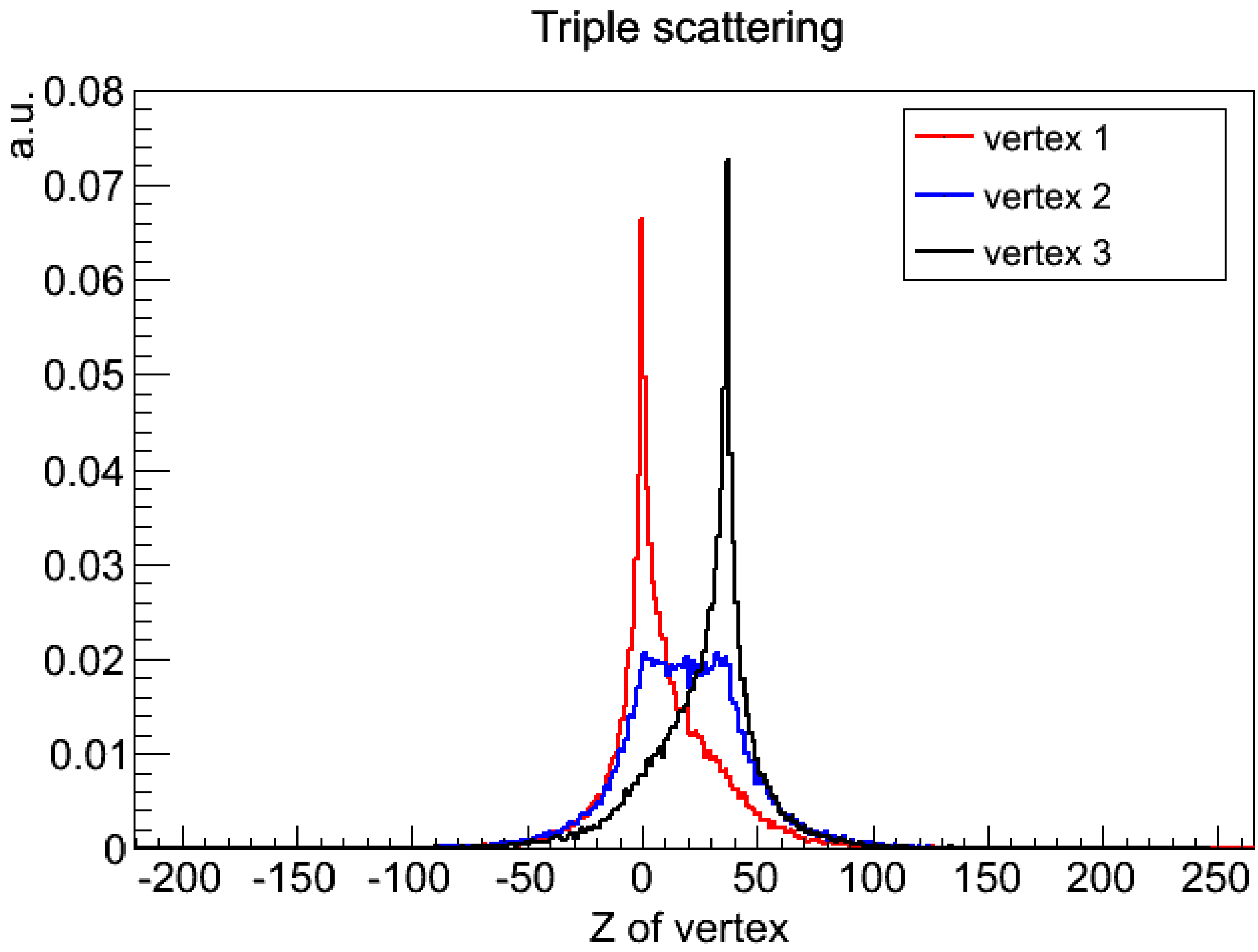
Results (MCMC)



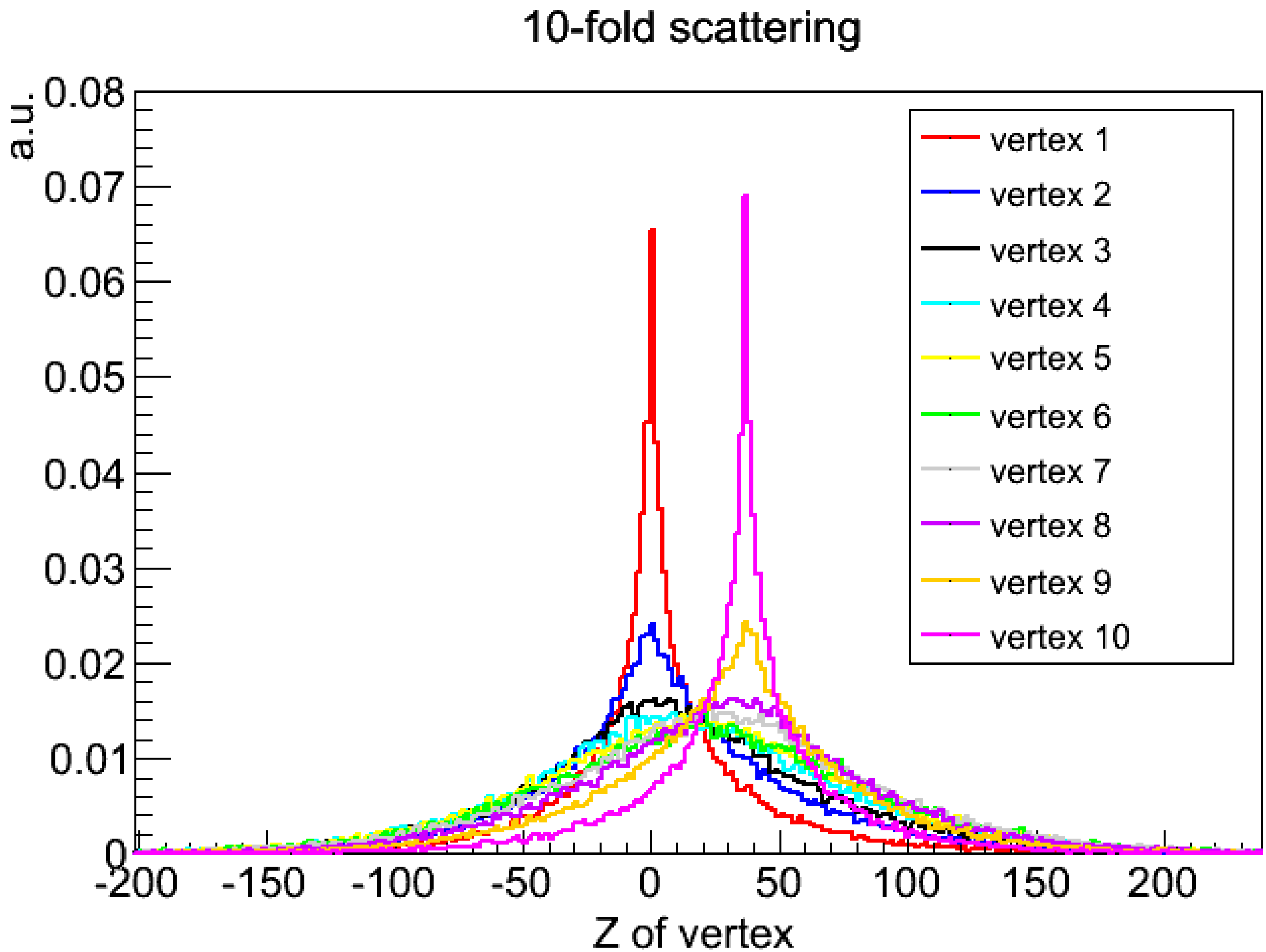
Results (MCMC)



Results (MCMC)



Results (MCMC)



Total hit probability

- For each given number of scatterings N
- 3N-dimensional integral
- MC integration: importance sampling
- Sample distributions from the Markov Chain MC
- X, Y and Z of are sampled from different distributions for each vertex of each N

Total hit probability

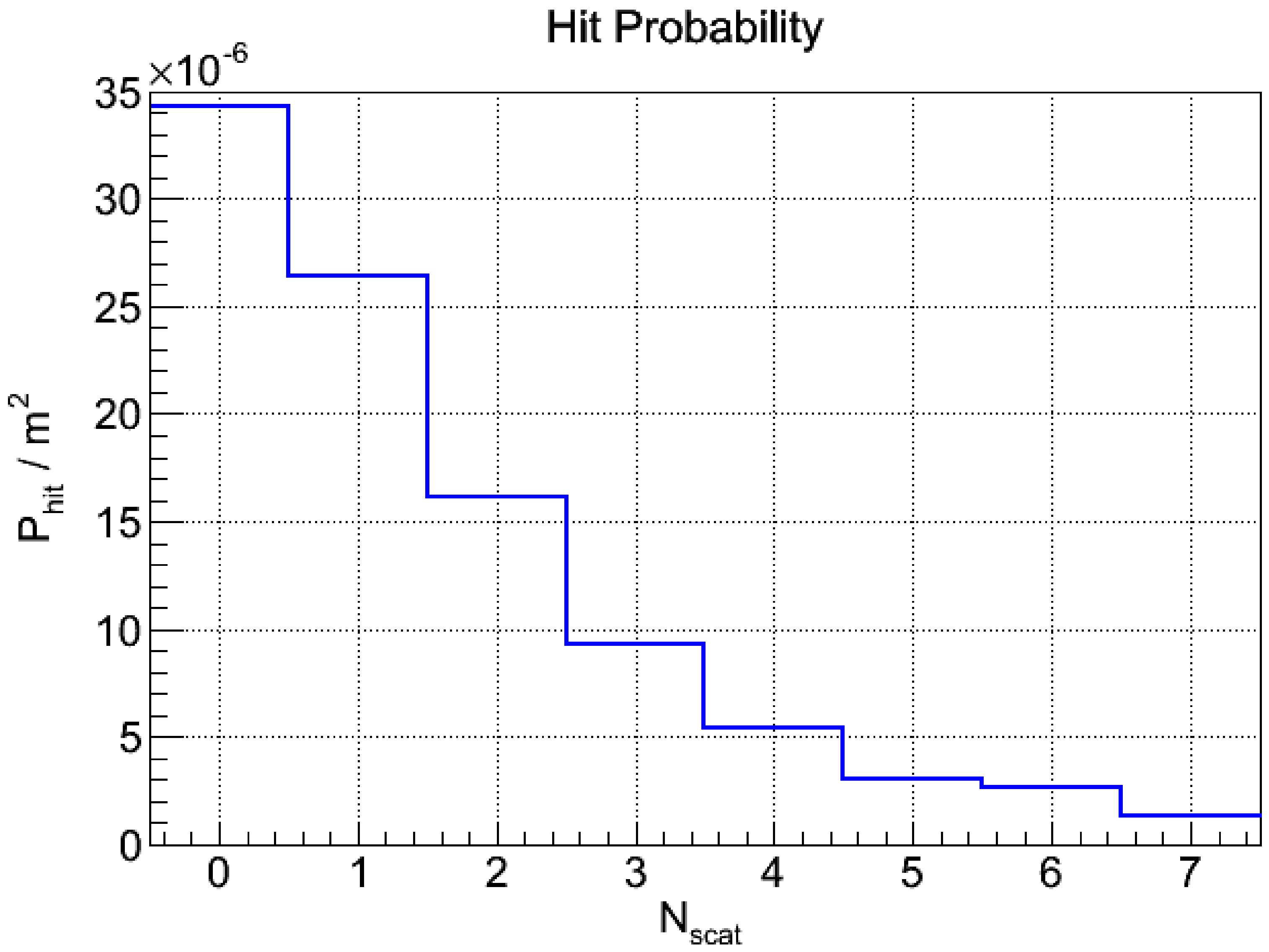
- Given in [m^-2] (multiply by target cross-section to get hit probability)
- no scattering (analytical)
  - P_0 = exp(-d/lambda) / (4 pi d^2)
    = 3.426 x 10-5
    = 0.589 / (4 pi d^2)
- single scattering (brute-force)
  - P_1 = 2.650e-5
    = 0.4559 / (4 pi d^2)
- single scattering (MCMC, 5M samples)
  - P_1 = 2.646e-5
    = 0.4552 / (4 pi d^2)

distance to target
d = 37 m

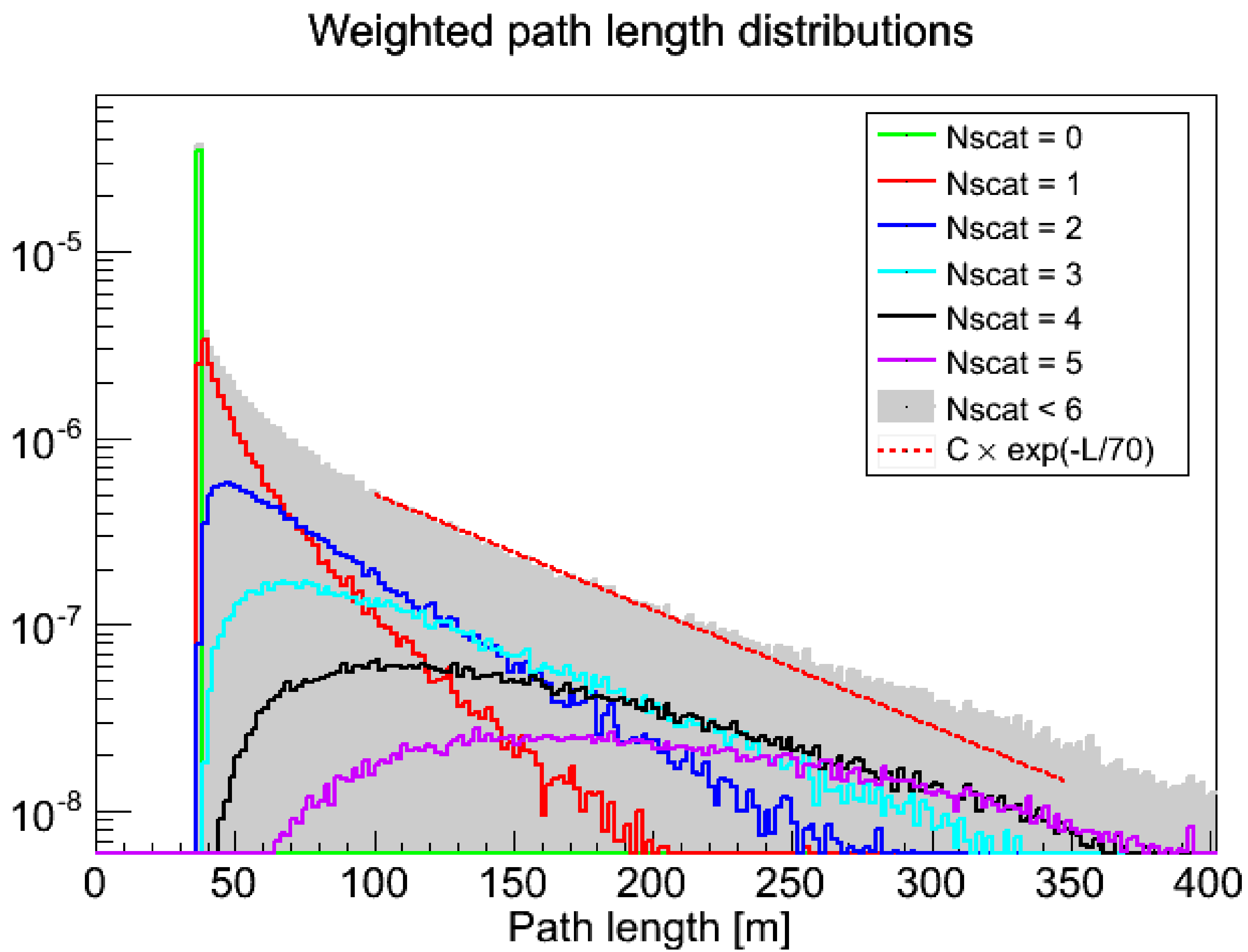
scattering length
lambda = 70 m

Less than 2 promille
relative error

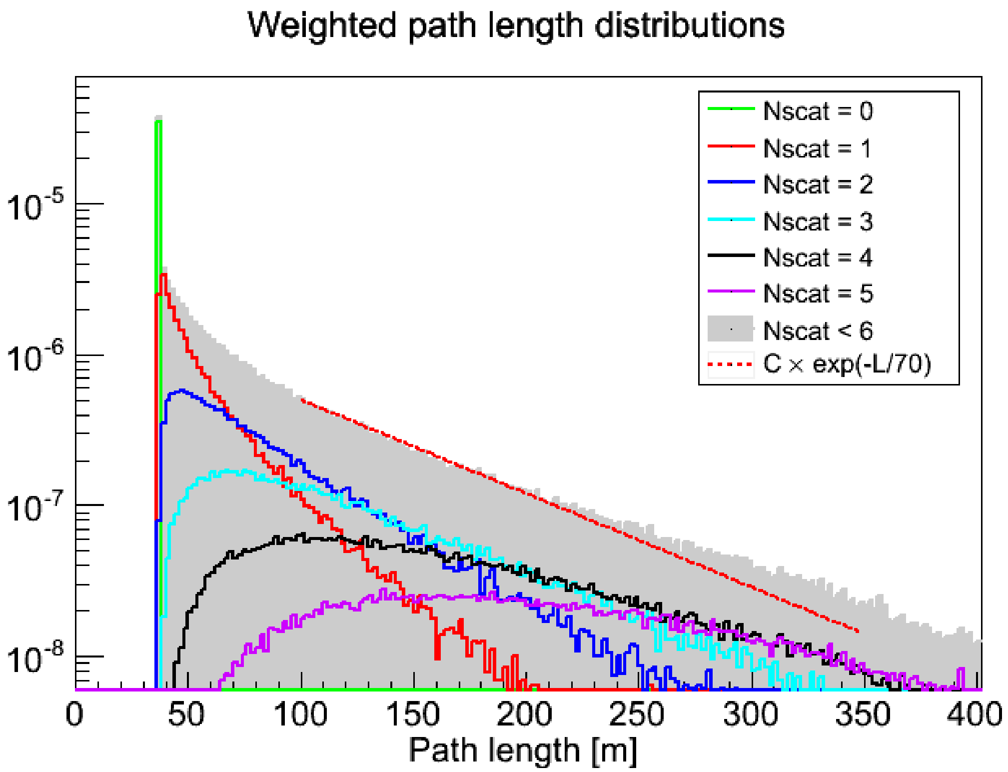
Total hit probability



Weighted path length distribution

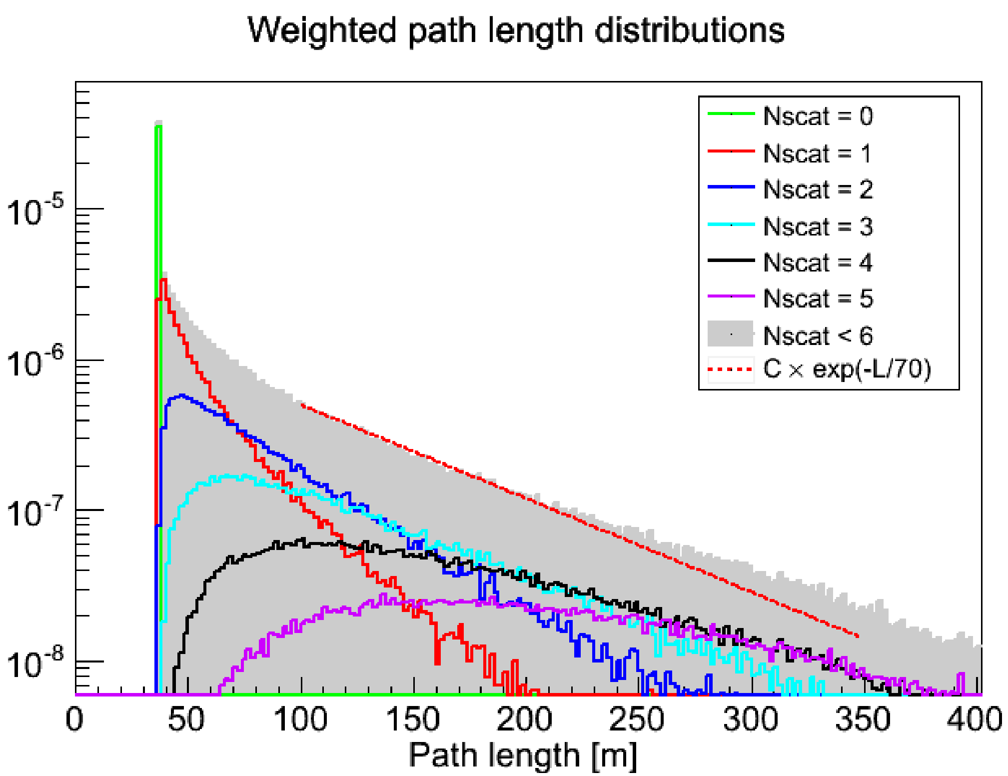


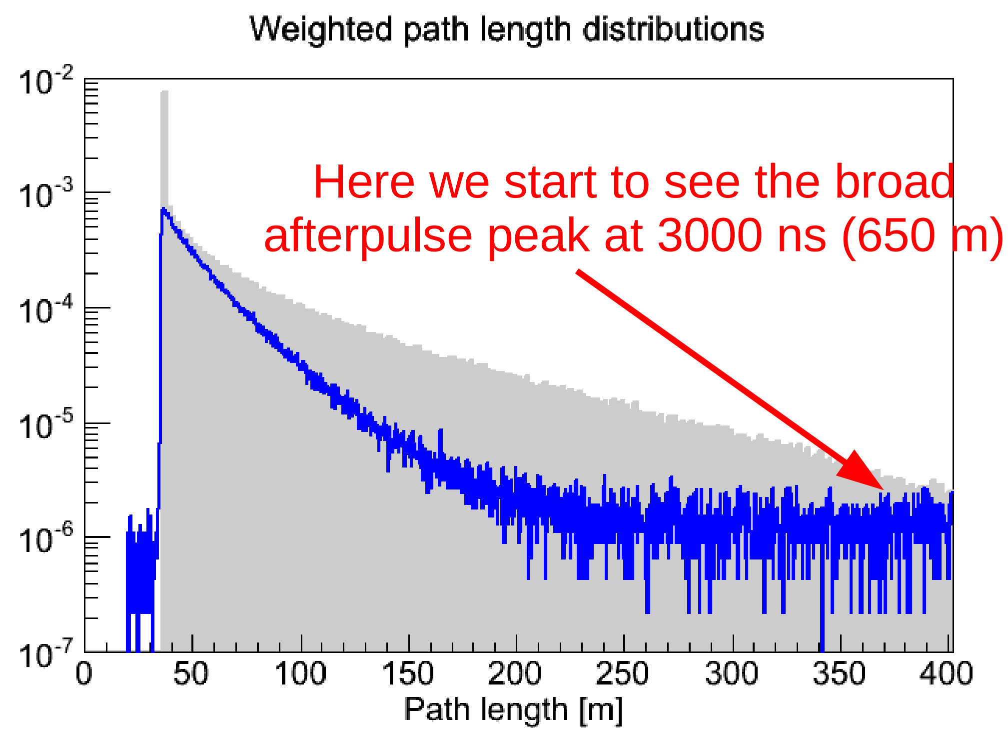
Weighted path length distribution



- Multiple scattering dominates at high path lengths
- Exponential tail with decay length ~70 m from scattering only
- Same order of magnitude as absorption length

Weighted path length distribution



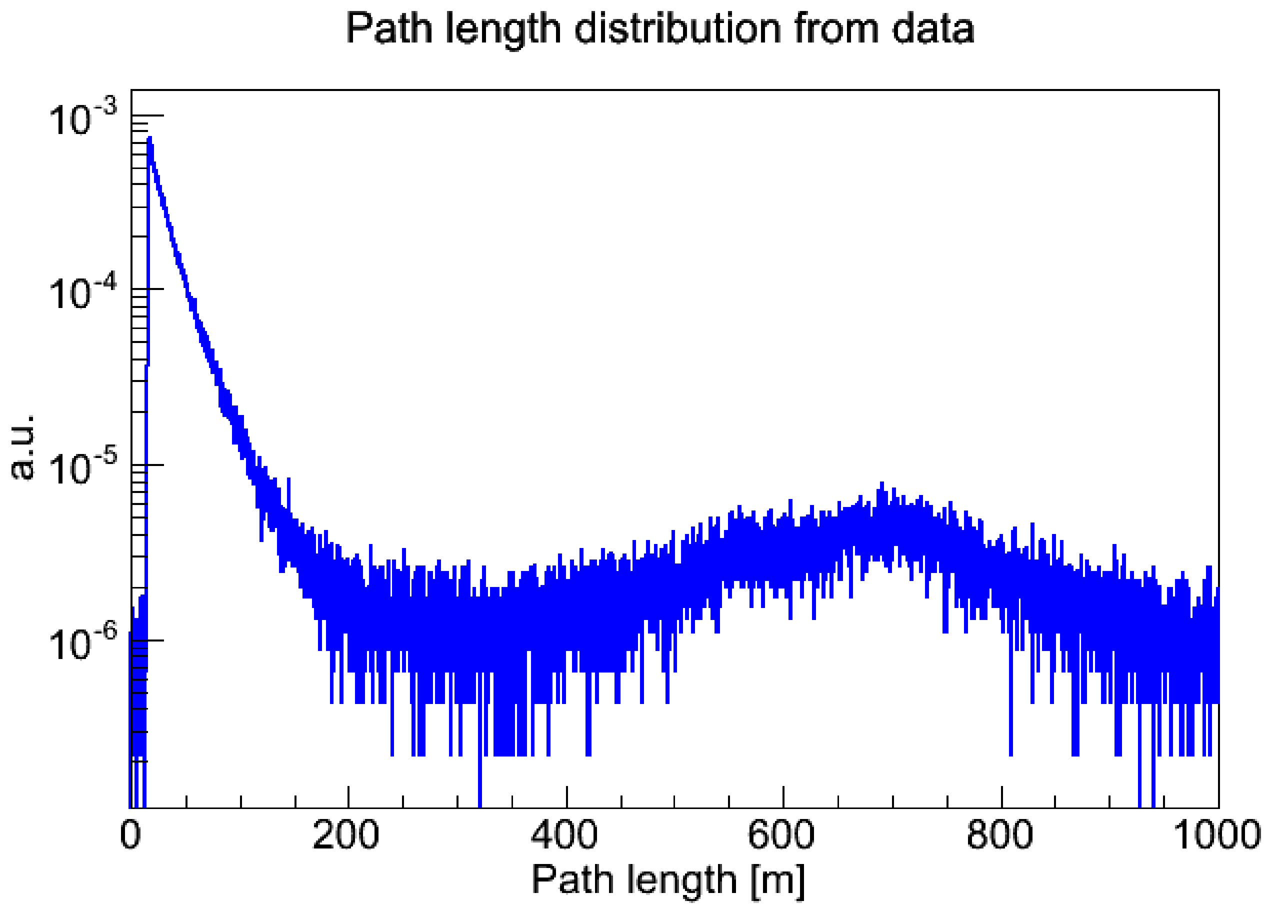


- Data from run 2621. NB on DOM1, distribution DOM2 DAQ Ch. 0 (upward looking)
- It has been converted using c_w = 0.217449 m/ns and shifted
- In the right figure, the grey fill area has been rescaled

Performance

- Path generation
  - Current results based on 50,000 paths, up to 20 scatterings
  - Takes ~2 hours on a single core
- Integral calculation
  - takes ~1 minute

Backup



Path length distribution from data
a.u.
10^{-3}
10^{-4}
10^{-5}
10^{-6}
0
200
400
600
800
1000
Path length [m]