Light scattering using a Markov Chain Monte Carlo

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Markov Chain Monte Carlo

- Sample from probability distribution
- Start in some initial state
- Propose a “small change” to the state
- If the probability becomes higher → accept
- Else accept with probability P_new / P_old

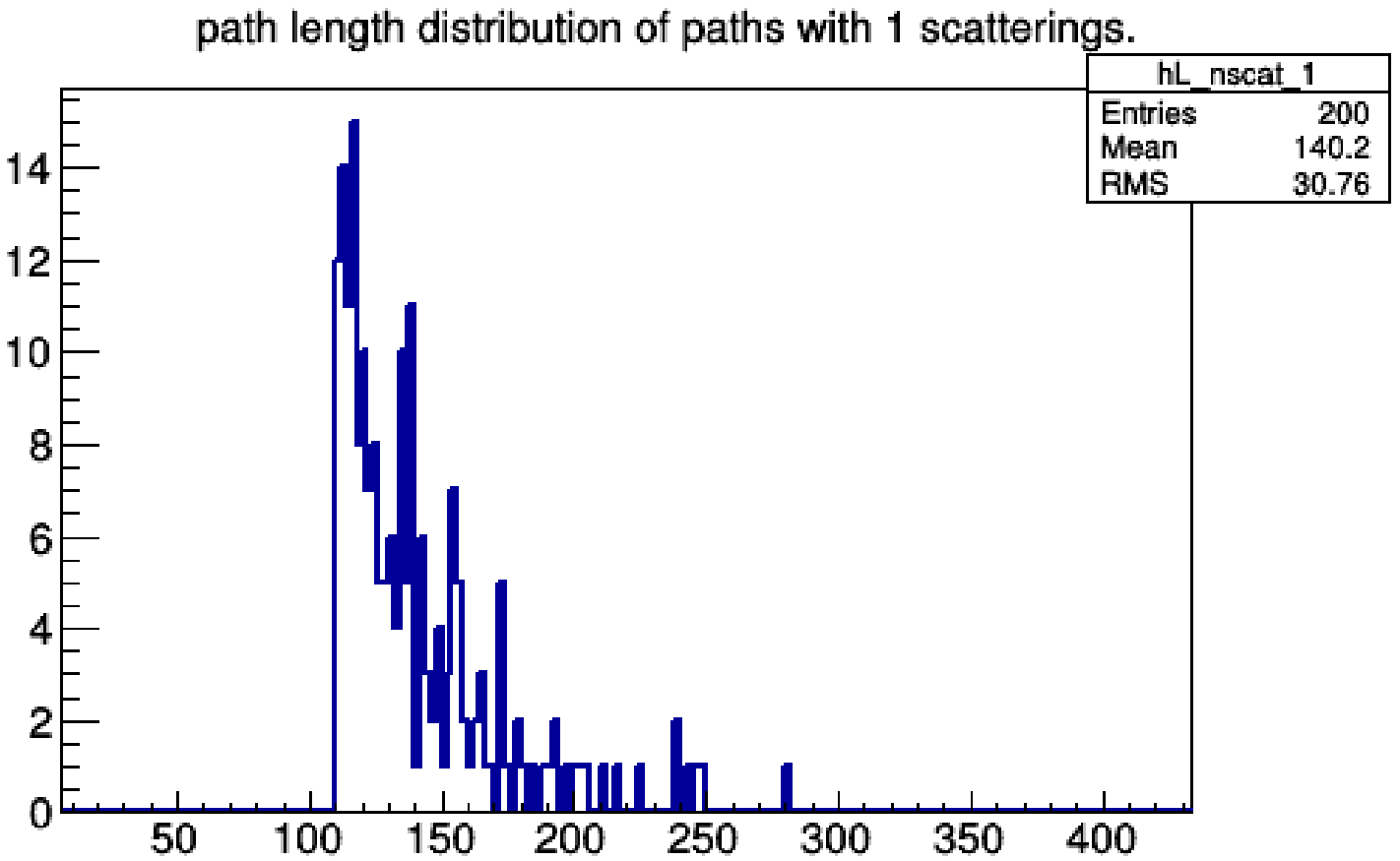
Probability Density

- Photon paths with fixed
  - Source
  - Target
  - Number of scatterings
- Degrees of freedom are the scattering vertices
-

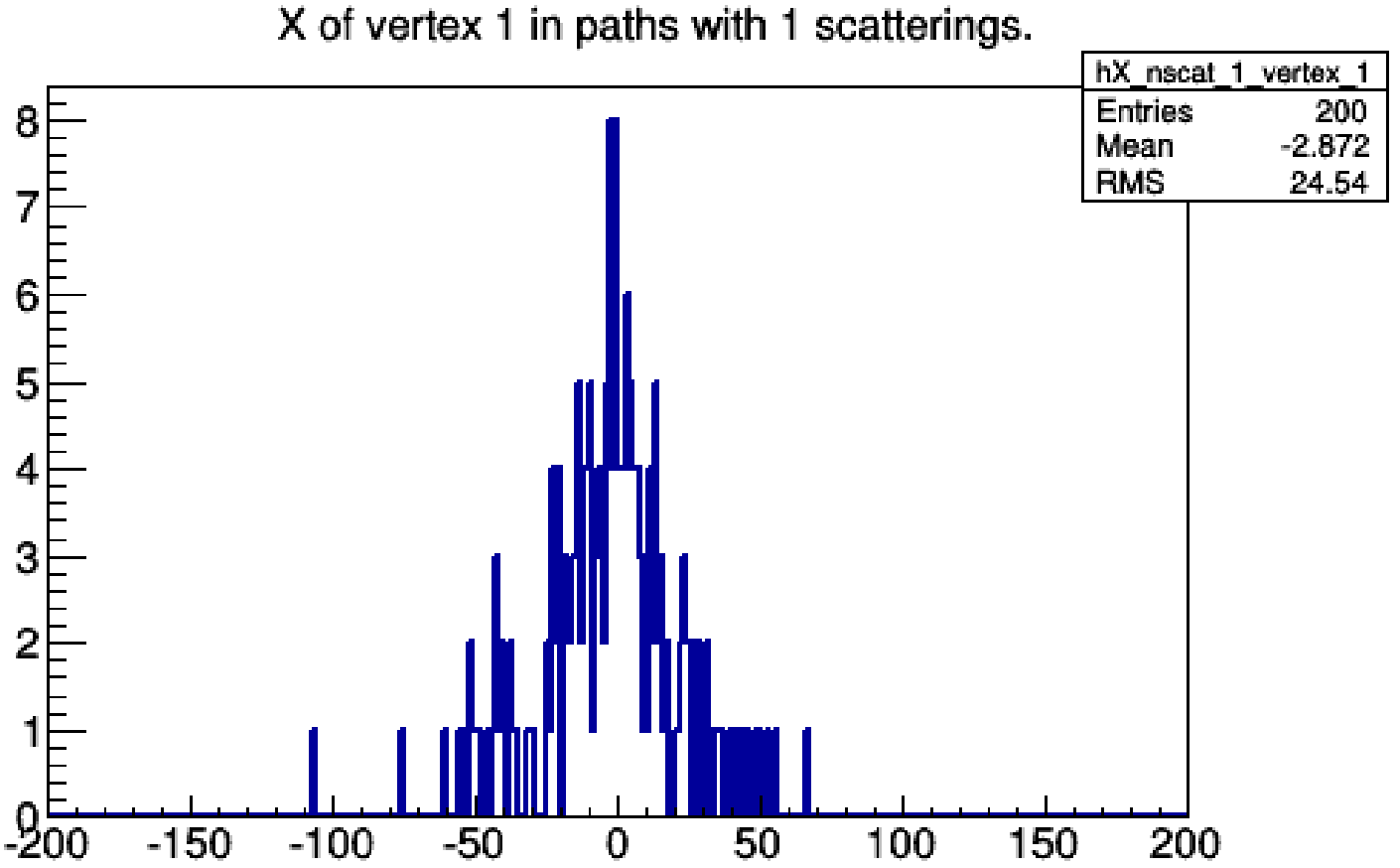
$$\rho_{\mathrm{hit}}^{(n)}(\{\vec{x}_i\}) = \left\{ \sum_{i=0}^{n-1} \frac{1}{4\pi\lambda\ell_i^2} e^{-\ell_i/\lambda} \right\} \times \frac{1}{4\pi\ell_n^2} e^{-\ell_n/\lambda},$$

(for now: uniform source emission, uniform target acceptance and completely random scattering)

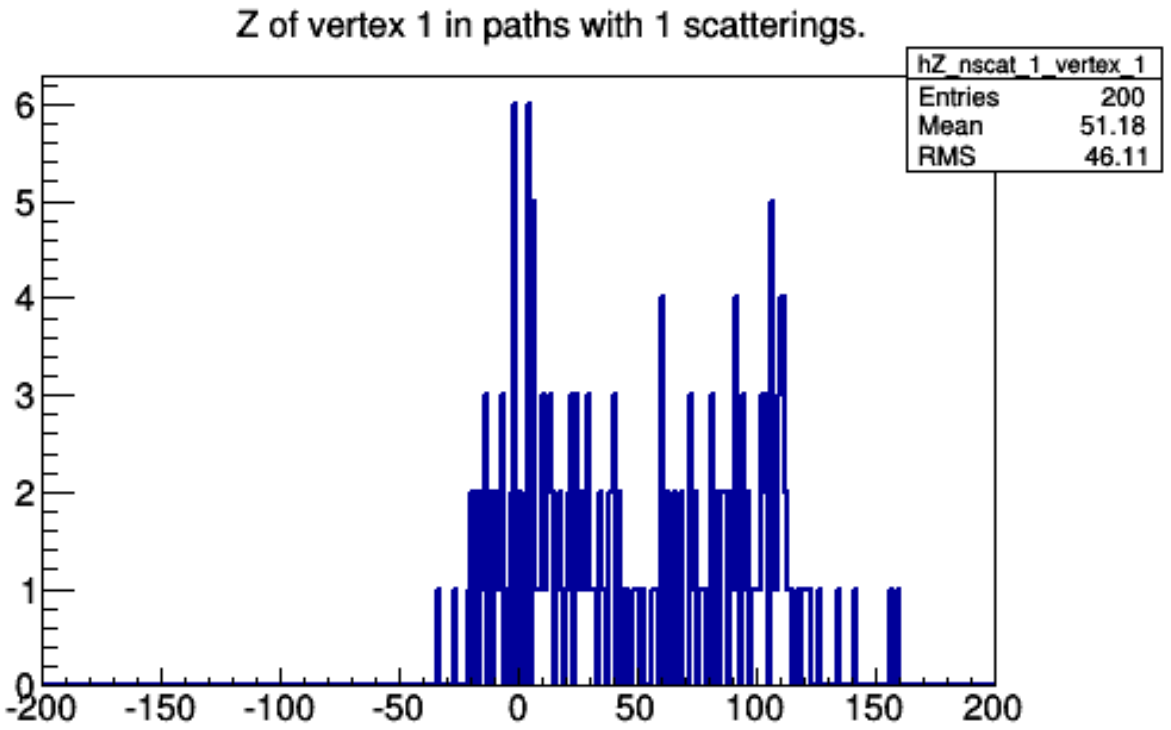
First results



Total path length for exactly 1 scattering. The target is at a distance of 111m, scattering length 70m. No absorption.



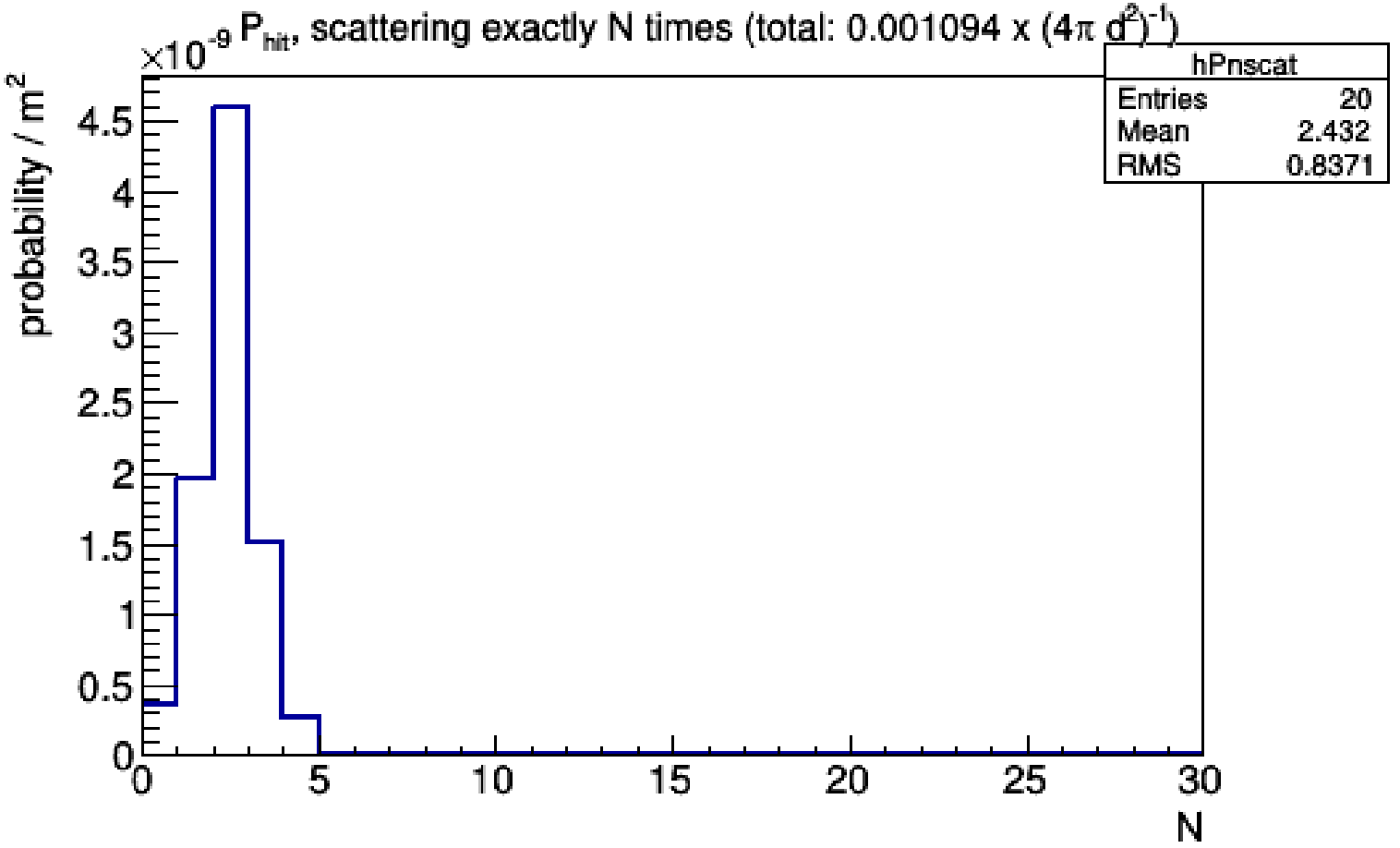
X-position of the scattering vertex (the source and target are on the z-axis)



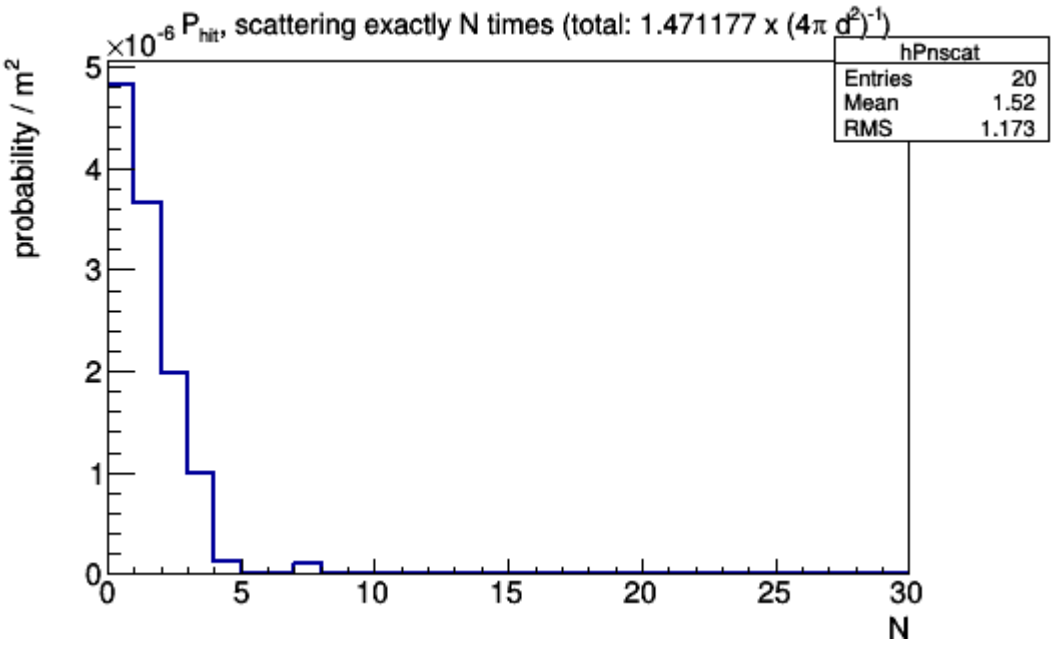
Z-position of the scattering vertex (the source and target are on the z-axis)

Combining different number of scatterings

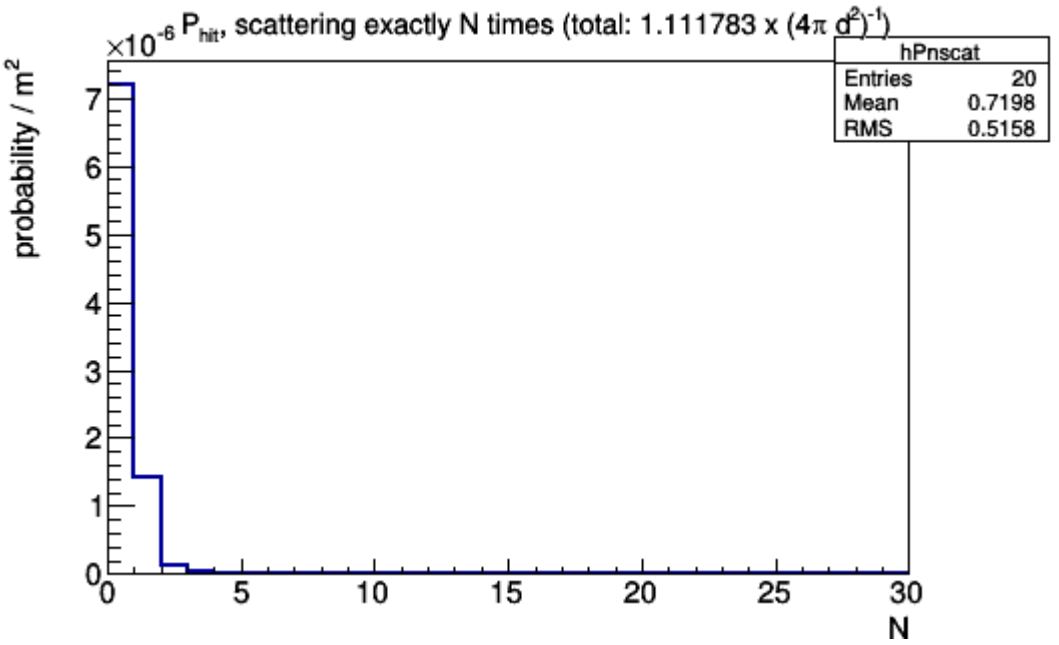
- Need to know the total probability for a given number of scatterings
- Requires to integrate probability density (very large-dimensional integrals)
- Now trying to use importance sampling
- Results not stable for some reason



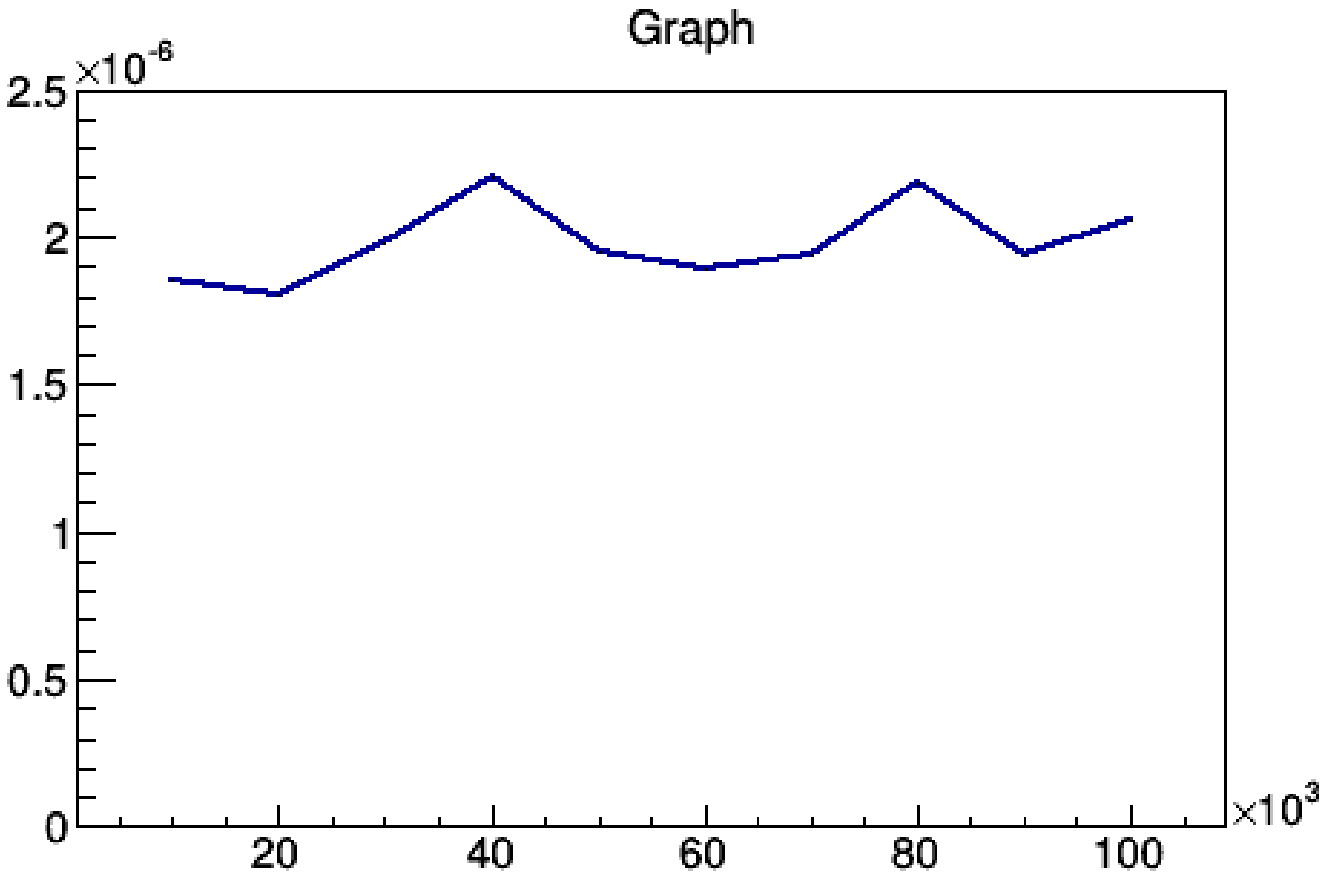
Example: probability to scatter a given number of times (scattering length=10m, target at 100m)



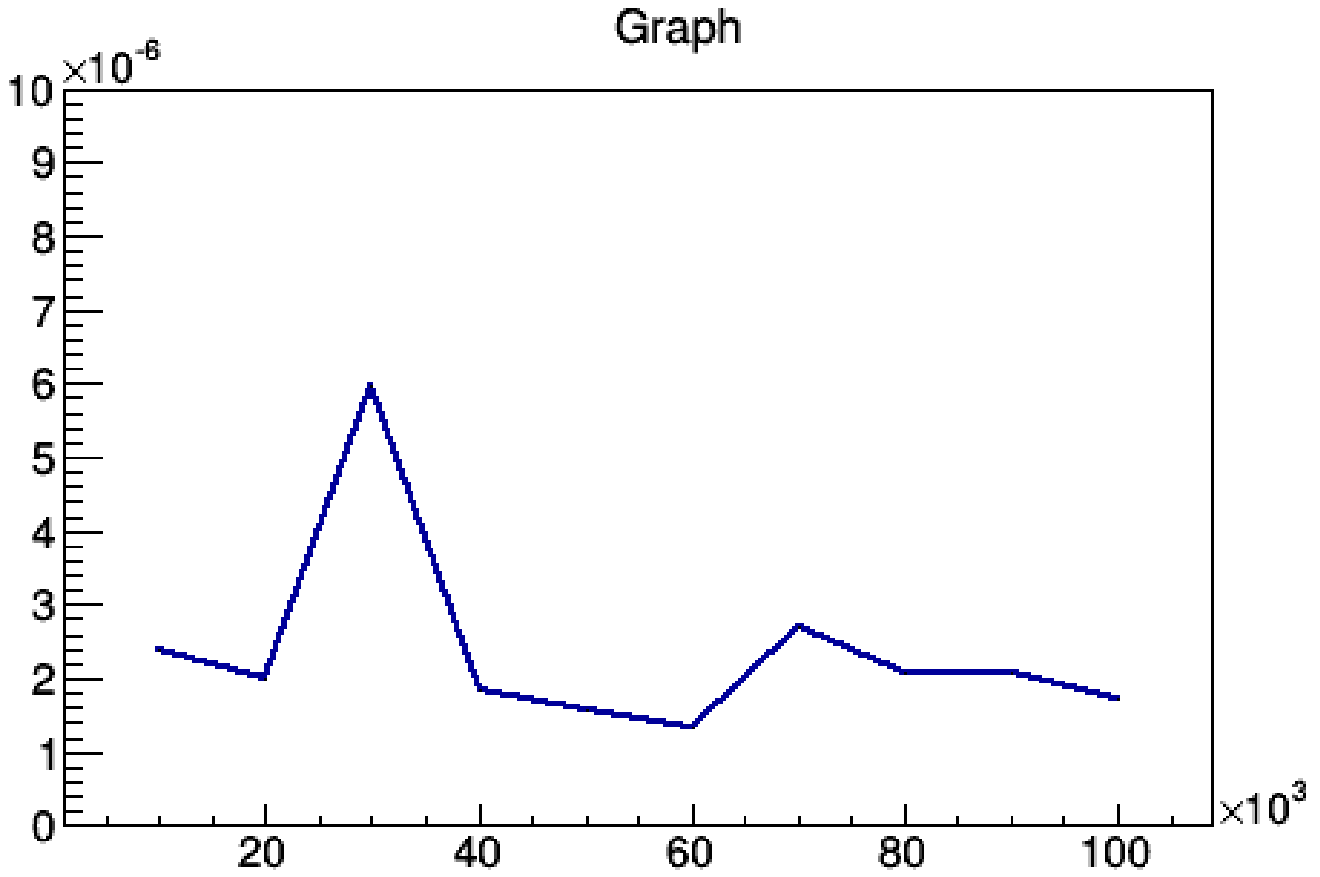
Example: probability to scatter a given number of times (scattering length=200m, target at 100m)



Example: probability to scatter a given number of times (scattering length=1000m, target at 100m)



Estimated probability to scatter exactly once in the given model.
The value of the integral is estimated with importance sampling and on the horizontal axis is the number of MC-generated points.
==>Not very stable.



Same as previous but now showing the probability to scatter exactly twice.
==> looks very unstable