Towards an All-Orders Flavor Formalism in the (geo)SM(EFT) & Beyond

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[2107.03951]

JHEP w/ M. Trott

+ future work!

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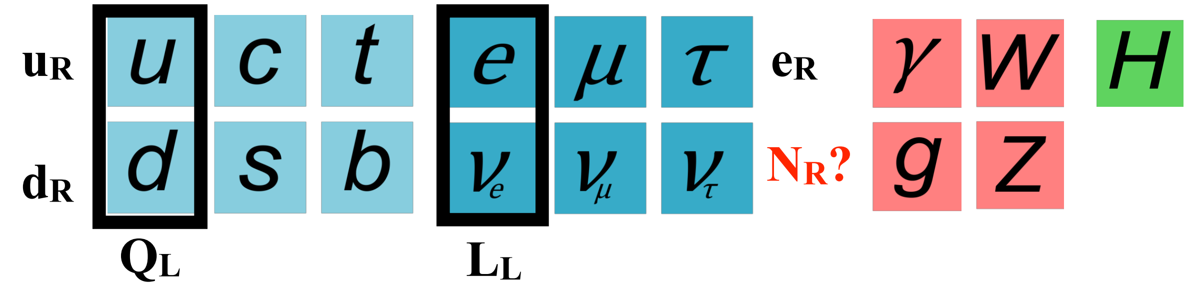
Flavor in the SM



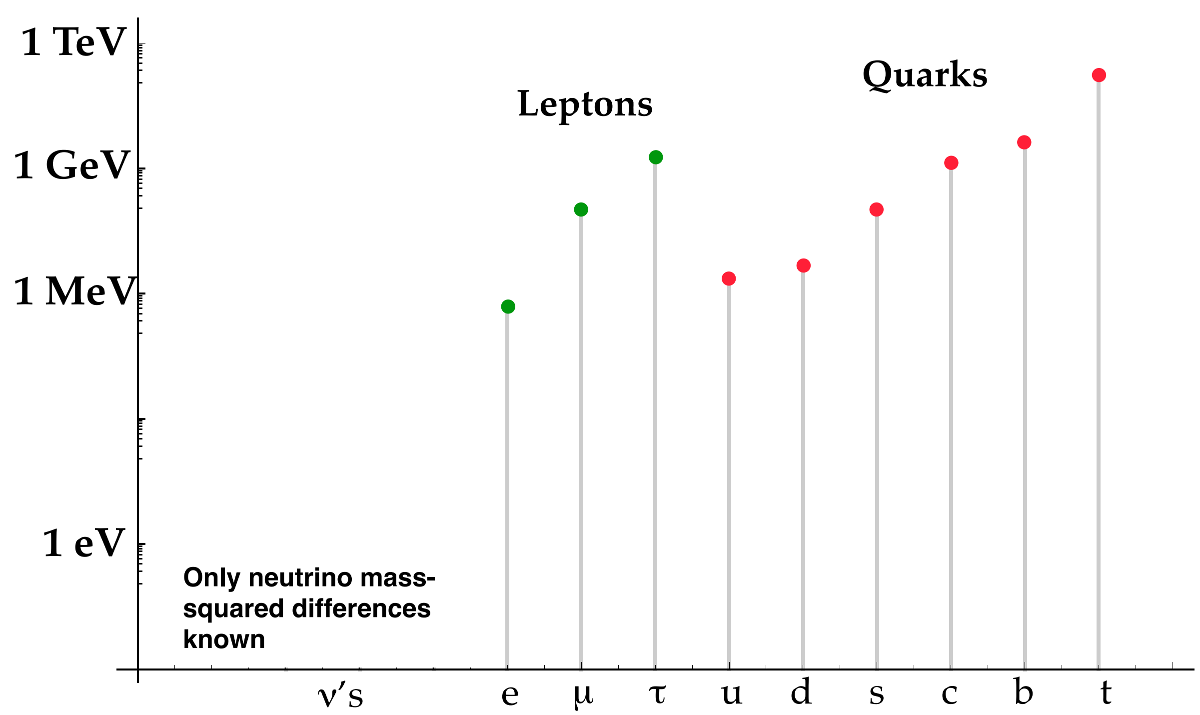
$$\mathcal{L}^{Y}_{SM} \supset Y^{u}_{pr}\, \overline{Q}_{L,p}\, \tilde{H}\, u_{R,r} + Y^{d}_{pr}\, \overline{Q}_{L,p}\, H\, d_{R,r} + Y^{e}_{pr}\, \overline{L}_{L,p}\, H\, e_{R,r} + \text{h.c.}$$

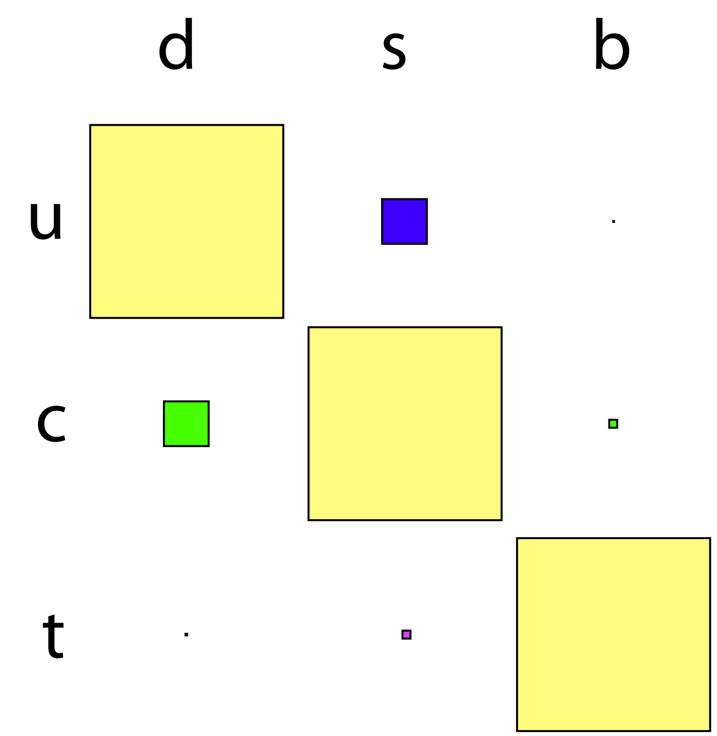
U(3)⁵

- From these (fundamental) Lagrangian terms one can use field redefinitions to show that only 9 masses, 3 mixing angles, and one CP-violating phase are needed for physical description.

$$[U^{\dagger}_{\psi L}]_{ir}\, [\mathcal{Y}^{\psi}]_{rp}\, [U_{\psi R}]_{pj} \equiv [D_{\psi}]_{ij} = \text{diag}\,(y_{\psi 1}, y_{\psi 2}, y_{\psi 3})$$

$$V_{CKM} \equiv U^{\dagger}_{u} U_{d} \equiv \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$





[1212.6374]

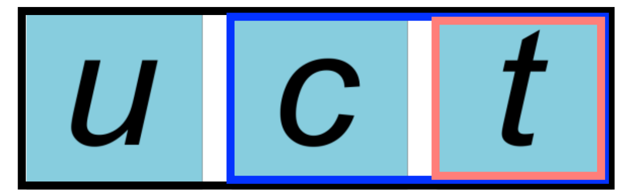
[1405.5495]

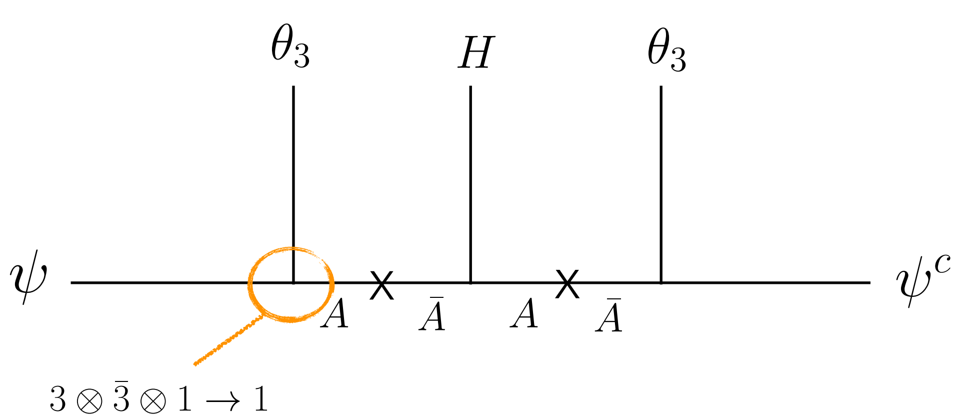
13 free and unexplained parameters exist in SM Yukawa sector

Flavor Beyond the SM

- BSM flavor physics tends to come in two forms. On the one hand, one might want to ***explain*** the patterns of mass and mixing in the SM…

$$\mathcal{G}_{BSM} \times SM \qquad \mathcal{R}\left(\mathcal{G}_{BSM}\right) \sim 3, \bar{3}, 2, 1, \ldots$$





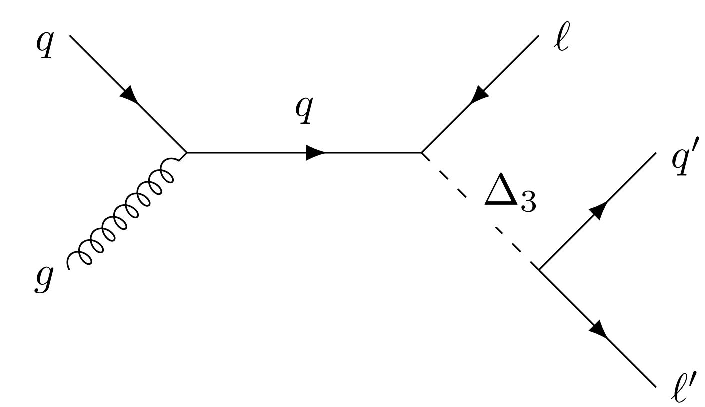
$$\mathcal{L}_{UV} \sim \psi\, \theta_3\, A + \bar{A}\, H\, A + \ldots \qquad \mathcal{L}_{IR} \sim \psi\, \theta_3\, H\, \theta_3\, \psi^c$$

$$\langle \theta_3 \rangle = v_3 \cdot (0, 0, 1) \quad \Rightarrow \quad \mathcal{M} \propto v_3^2 \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

- On the other hand, one might want to introduce a new flavored state in the IR spectrum, to account for new physics that can be tested experimentally. **Leptoquarks** (e.g.) are popular these days…

$$\Delta_3 \sim (\bar{\mathbf{3}}, \mathbf{3}, 1/3)$$

$$\mathcal{G}_{SM} \equiv SU(3)_C \times SU(2)_L \times U(1)_Y$$



- In either instance, one can parameterize the effects of said new physics into an OPE composed of SM fields and gauge symmetries, the so-called **SMEFT**:

$$\mathcal{L}_{SMEFT} = \mathcal{L}_{SM} + \sum_i \frac{C_i^{(d)}}{\Lambda^{d-4}} \mathcal{Q}_i^{(d)}$$

The (flavored) SMEFT at dim-6, briefly

- The SMEFT's operator basis can be expanded order by order in mass dimension. At **dim-5**, the 'Weinberg Operator' [PRL 43, '79] is the unique new-physics contribution (and accounts for neutrino masses!).

$$\mathcal{L}^{(5)} = \frac{c_{ij}^{(5)}}{2}\left(\ell_i^T \tilde{H}^\star\right) C \left(\tilde{H}^\dagger \ell_j\right) + \text{h.c.}$$

EWSB

$$\mathcal{L} \supset -\frac{m_{\nu,k}}{2}\overline{\nu_L^{c,k}}\nu_L^k + \text{h.c.}$$

- The 'Warsaw Basis' of [1008.4884] is a non-redundant, complete set of **dim-6** operators.

$\psi^2 X \varphi$		$\psi^2 \varphi^2 D$	
Q_{eW}	$(\bar{l}_p \sigma^{\mu\nu} e_r)\tau^I \varphi W_{\mu\nu}^I$	$Q_{\varphi l}^{(1)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar{l}_p \gamma^\mu l_r)$
Q_{eB}	$(\bar{l}_p \sigma^{\mu\nu} e_r)\varphi B_{\mu\nu}$	$Q_{\varphi l}^{(3)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu^I \varphi)(\bar{l}_p \tau^I \gamma^\mu l_r)$
Q_{uG}	$(\bar{q}_p \sigma^{\mu\nu} T^A u_r)\widetilde{\varphi}\, G_{\mu\nu}^A$	$Q_{\varphi e}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar{e}_p \gamma^\mu e_r)$
Q_{uW}	$(\bar{q}_p \sigma^{\mu\nu} u_r)\tau^I \widetilde{\varphi}\, W_{\mu\nu}^I$	$Q_{\varphi q}^{(1)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar{q}_p \gamma^\mu q_r)$
Q_{uB}	$(\bar{q}_p \sigma^{\mu\nu} u_r)\widetilde{\varphi}\, B_{\mu\nu}$	$Q_{\varphi q}^{(3)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu^I \varphi)(\bar{q}_p \tau^I \gamma^\mu q_r)$
Q_{dG}	$(\bar{q}_p \sigma^{\mu\nu} T^A d_r)\varphi\, G_{\mu\nu}^A$	$Q_{\varphi u}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar{u}_p \gamma^\mu u_r)$
Q_{dW}	$(\bar{q}_p \sigma^{\mu\nu} d_r)\tau^I \varphi\, W_{\mu\nu}^I$	$Q_{\varphi d}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar{d}_p \gamma^\mu d_r)$
Q_{dB}	$(\bar{q}_p \sigma^{\mu\nu} d_r)\varphi\, B_{\mu\nu}$	$Q_{\varphi ud}$	$i(\widetilde{\varphi}^\dagger D_\mu \varphi)(\bar{u}_p \gamma^\mu d_r)$

$(\bar{L}L)(\bar{L}L)$		$(\bar{R}R)(\bar{R}R)$		$(\bar{L}L)(\bar{R}R)$	
Q_{ll}	$(\bar{l}_p \gamma_\mu l_r)(\bar{l}_s \gamma^\mu l_t)$	Q_{ee}	$(\bar{e}_p \gamma_\mu e_r)(\bar{e}_s \gamma^\mu e_t)$	Q_{le}	$(\bar{l}_p \gamma_\mu l_r)(\bar{e}_s \gamma^\mu e_t)$
$Q_{qq}^{(1)}$	$(\bar{q}_p \gamma_\mu q_r)(\bar{q}_s \gamma^\mu q_t)$	Q_{uu}	$(\bar{u}_p \gamma_\mu u_r)(\bar{u}_s \gamma^\mu u_t)$	Q_{lu}	$(\bar{l}_p \gamma_\mu l_r)(\bar{u}_s \gamma^\mu u_t)$
$Q_{qq}^{(3)}$	$(\bar{q}_p \gamma_\mu \tau^I q_r)(\bar{q}_s \gamma^\mu \tau^I q_t)$	Q_{dd}	$(\bar{d}_p \gamma_\mu d_r)(\bar{d}_s \gamma^\mu d_t)$	Q_{ld}	$(\bar{l}_p \gamma_\mu l_r)(\bar{d}_s \gamma^\mu d_t)$
$Q_{lq}^{(1)}$	$(\bar{l}_p \gamma_\mu l_r)(\bar{q}_s \gamma^\mu q_t)$	Q_{eu}	$(\bar{e}_p \gamma_\mu e_r)(\bar{u}_s \gamma^\mu u_t)$	Q_{qe}	$(\bar{q}_p \gamma_\mu q_r)(\bar{e}_s \gamma^\mu e_t)$
$Q_{lq}^{(3)}$	$(\bar{l}_p \gamma_\mu \tau^I l_r)(\bar{q}_s \gamma^\mu \tau^I q_t)$	Q_{ed}	$(\bar{e}_p \gamma_\mu e_r)(\bar{d}_s \gamma^\mu d_t)$	$Q_{qu}^{(1)}$	$(\bar{q}_p \gamma_\mu q_r)(\bar{u}_s \gamma^\mu u_t)$
		$Q_{ud}^{(1)}$	$(\bar{u}_p \gamma_\mu u_r)(\bar{d}_s \gamma^\mu d_t)$	$Q_{qu}^{(8)}$	$(\bar{q}_p \gamma_\mu T^A q_r)(\bar{u}_s \gamma^\mu T^A u_t)$
		$Q_{ud}^{(8)}$	$(\bar{u}_p \gamma_\mu T^A u_r)(\bar{d}_s \gamma^\mu T^A d_t)$	$Q_{qd}^{(1)}$	$(\bar{q}_p \gamma_\mu q_r)(\bar{d}_s \gamma^\mu d_t)$
				$Q_{qd}^{(8)}$	$(\bar{q}_p \gamma_\mu T^A q_r)(\bar{d}_s \gamma^\mu T^A d_t)$

$(\bar{L}R)(\bar{R}L)$ and $(\bar{L}R)(\bar{L}R)$		B-violating	
Q_{ledq}	$(\bar{l}_p^j e_r)(\bar{d}_s q_t^j)$	Q_{duq}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(q_s^{\gamma j})^T C l_t^k\right]$
$Q_{quqd}^{(1)}$	$(\bar{q}_p^j u_r)\varepsilon_{jk}(\bar{q}_s^k d_t)$	Q_{qqu}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(u_s^\gamma)^T C e_t\right]$
$Q_{quqd}^{(8)}$	$(\bar{q}_p^j T^A u_r)\varepsilon_{jk}(\bar{q}_s^k T^A d_t)$	Q_{qqq}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jn}\varepsilon_{km}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(q_s^{\gamma m})^T C l_t^n\right]$
$Q_{lequ}^{(1)}$	$(\bar{l}_p^j e_r)\varepsilon_{jk}(\bar{q}_s^k u_t)$	Q_{duu}	$\varepsilon^{\alpha\beta\gamma}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(u_s^\gamma)^T C e_t\right]$
$Q_{lequ}^{(3)}$	$(\bar{l}_p^j \sigma_{\mu\nu} e_r)\varepsilon_{jk}(\bar{q}_s^k \sigma^{\mu\nu} u_t)$		

$\psi^2 \varphi^3$	
$Q_{e\varphi}$	$(\varphi^\dagger \varphi)(\bar{l}_p e_r \varphi)$
$Q_{u\varphi}$	$(\varphi^\dagger \varphi)(\bar{q}_p u_r \widetilde{\varphi})$
$Q_{d\varphi}$	$(\varphi^\dagger \varphi)(\bar{q}_p d_r \varphi)$

$$Y^{ij} \sim Y_{SM}^{ij} + \left[C_{\psi H}^{(6)}\right]^{ij} \cdot \Lambda^{-2}$$

Describing Flavor in the SM(EFT) & Beyond

Standard Model	EFTs of Flavor	SMEFT
$Y^{ij} \sim \begin{pmatrix} Y_{11} & Y_{12} & Y_{13} \\ Y_{21} & Y_{22} & Y_{23} \\ Y_{31} & Y_{32} & Y_{33} \end{pmatrix}$	$Y^{ij} \sim \sum_k \left[f_k\left(\langle\theta\rangle\right)\right]^{ij}_{\mathbf{1}}$	$Y^{ij} \sim Y^{ij}_{SM} + \left[C^{(6)}_{\psi H}\right]^{ij} \cdot \Lambda^{-2}$

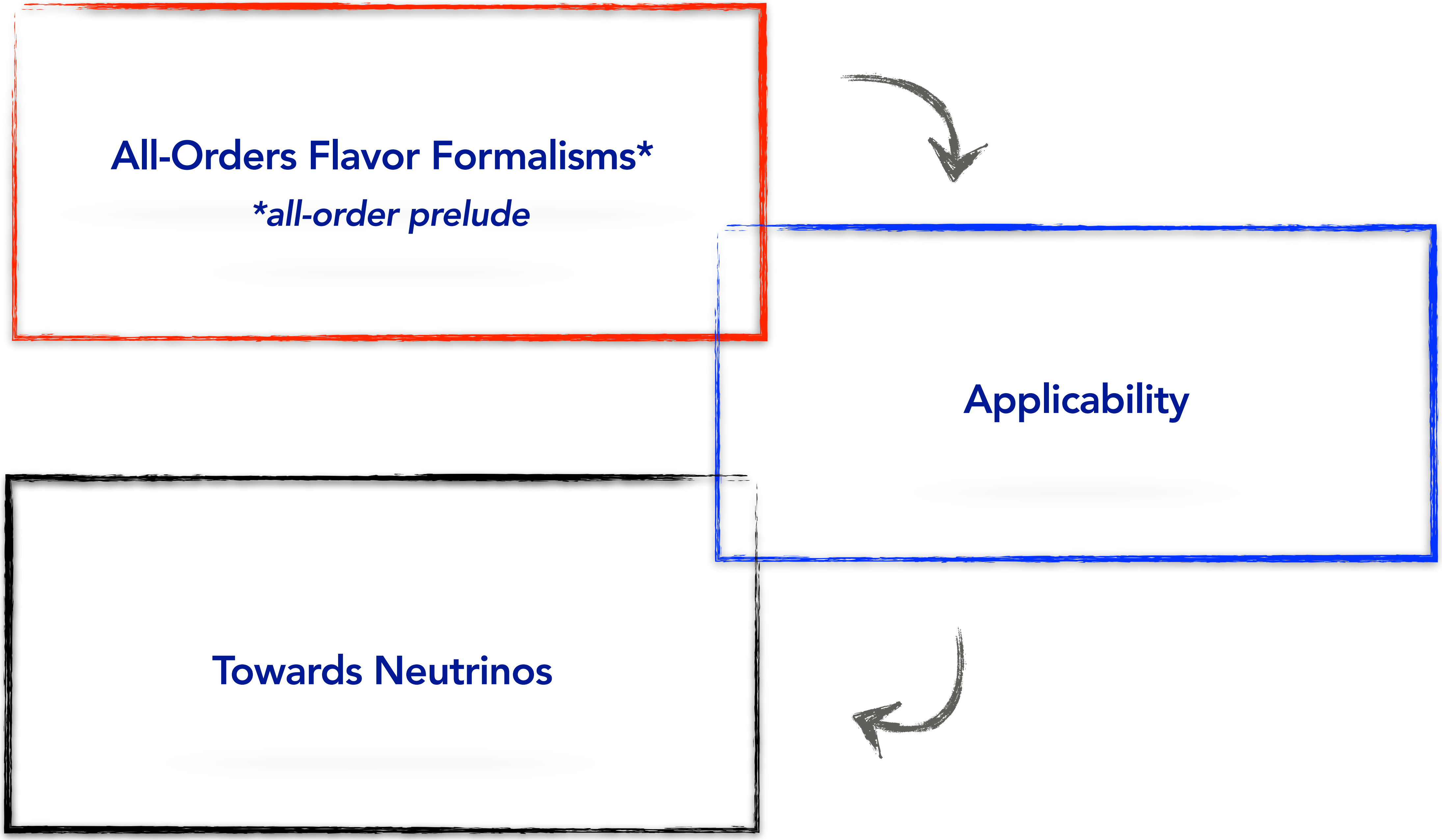
- Regardless of the formalism, physical predictions for flavored processes depend on the 9 parameters associated to mass eigenstates and their quantum mixings:

$$[U^{\dagger}_{\psi L}]_{ir}\,[\mathcal{Y}^{\psi}\mathcal{Y}^{\psi,\dagger}]_{rp}\,[U_{\psi L}]_{pj} = \text{diag}\left(y^2_{\psi 1}, y^2_{\psi 2}, y^2_{\psi 3}\right) \qquad V_{CKM} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}\,e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}\,e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}\,e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}\,e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}\,e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

Do you know how to write $y^2(Y)$, $\theta(Y)$, $\delta(Y)$?

It's not as easy as one might think (for three flavors)!

Outline


All-Orders Flavor Formalisms*
*all-order prelude
Applicability
Towards Neutrinos

The geoSMEFT, Intuited

[1605.03602]
[1803.08001]
[1909.08470]
[geoSMEFT,2001.01453] $+\ldots$

$$\mathcal{L}_{SMEFT} = \mathcal{L}_{SM} + \sum_i \frac{C_i^{(d)}}{\Lambda^{d-4}} \mathcal{Q}_i^{(d)}$$

$$\mathcal{L}_{\rm SMEFT} = \sum_i G_i\left(I, A, \phi, \ldots\right) f_i$$

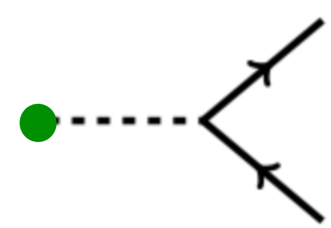
G: '*field space connections*' built from successive insertions of Higgs fields

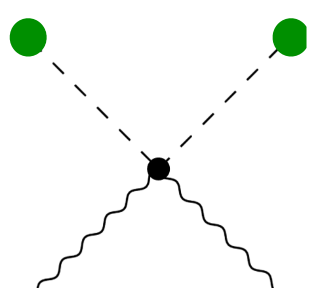
f: operator forms composed of Lorentz-index-carrying building blocks of the Lagrangian

$$H\left(\phi_I\right) = \frac{1}{\sqrt{2}} \begin{bmatrix} \phi_2 + i\phi_1 \\ \phi_4 - i\phi_3 \end{bmatrix}$$

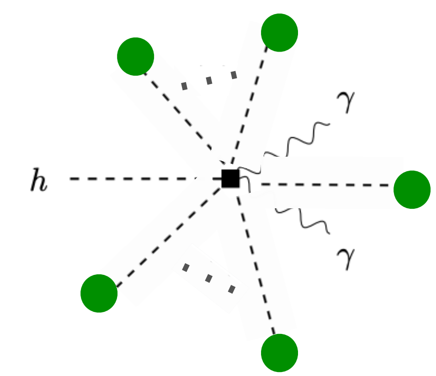
$$\bar{v}_T \equiv \sqrt{2\langle H^\dagger H\rangle}$$

$D \leq 4$





$D > 4$



[M. Trott KITP Talk]

vev -> fermion masses *-> boson masses* *-> geometries*

Gauge Field-Strength Terms at D=6 (e.g.)

$$\mathcal{W}^A = \{W_1, W_2, W_3, B\}$$

$$\mathcal{L}_{\rm WB} = -\frac{1}{4} W^a_{\mu\nu} W^{a,\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} + \frac{C_{HB}}{\Lambda^2} H^\dagger H B_{\mu\nu} B^{\mu\nu} + \frac{C_{HW}}{\Lambda^2} H^\dagger H W^a_{\mu\nu} W^{a,\mu\nu} + \frac{C_{HWB}}{\Lambda^2} H^\dagger \sigma^a H W^a_{\mu\nu} B^{\mu\nu},$$

$$\equiv -\frac{1}{4} g_{AB}(H) \mathcal{W}^A_{\mu\nu} \mathcal{W}^{B,\mu\nu}$$

$$g_{ab} = \left(1 - 4\frac{C_{HW}}{\Lambda^2} H^\dagger H\right) \delta_{ab}$$

$$g_{a4} = g_{4a} = -2\frac{C_{HWB}}{\Lambda^2} H^\dagger \sigma_a H.$$

$$g_{44} = 1 - 4\frac{C_{HB}}{\Lambda^2} H^\dagger H$$

*Connection amounts to **metric in field space**, whose degree of curvature depends on size of v/Λ. The **SM** is therefore a **FLAT** direction!*

Building Up the $g_{AB}(\phi)$ Metric

[2001.01453]
[2203.06771]

- Consider the higher-order operators that can connect two gauge field strengths:

Dim 6+

$$Q_{HB}^{(6+2n)} = (H^\dagger H)^{n+1} B^{\mu\nu} B_{\mu\nu},$$
$$Q_{HW}^{(6+2n)} = (H^\dagger H)^{n+1} W_a^{\mu\nu} W^a_{\mu\nu},$$
$$Q_{HWB}^{(6+2n)} = (H^\dagger H)^{n} (H^\dagger \sigma^a H) W_a^{\mu\nu} B_{\mu\nu}$$

Dim 8+

$$Q_{HW,2}^{(8+2n)} = (H^\dagger H)^{n} (H^\dagger \sigma^a H)(H^\dagger \sigma^b H) W_a^{\mu\nu} W_{b,\mu\nu}$$

That the operator forms saturate at all orders can be seen with **Hilbert Series** techniques:

	Mass Dimension				
Field space connection	6	8	10	12	14
$g_{AB}(\phi)\mathcal{W}^A_{\mu\nu}\mathcal{W}^{B,\mu\nu}$	3	4	4	4	4

- Expanding in terms of real scalar fields, and combining into a single gauge field (A,B = 1,2,3,4), one can write

$$H(\phi_I) = \frac{1}{\sqrt{2}} \begin{bmatrix} \phi_2 + i\phi_1 \\ \phi_4 - i\phi_3 \end{bmatrix}$$

$$\mathcal{W}^A = \{W_1, W_2, W_3, B\}$$

$$g_{AB}(\phi)\mathcal{W}^A_{\mu\nu}\mathcal{W}^{B,\mu\nu}$$

$$\begin{aligned} g_{AB}(\phi_I) = & \left[1 - 4\sum_{n=0}^{\infty}\left(C_{HW}^{(6+2n)}(1-\delta_{A4}) + C_{HB}^{(6+2n)}\delta_{A4}\right)\left(\frac{\phi^2}{2}\right)^{n+1}\right]\delta_{AB} \\ & + \sum_{n=0}^{\infty} C_{HW,2}^{(8+2n)} \left(\frac{\phi^2}{2}\right)^{n} \left(\phi_I \Gamma^I_{A,J}\phi^J\right)\left(\phi_L \Gamma^L_{B,K}\phi^K\right)(1-\delta_{A4})(1-\delta_{B4}) \\ & + \left[\sum_{n=0}^{\infty} C_{HWB}^{(6+2n)}\left(\frac{\phi^2}{2}\right)^{n}\right]\left(\phi_I \Gamma^I_{A,J}\phi^J\right)(1-\delta_{A4})\delta_{B4}, \end{aligned}$$

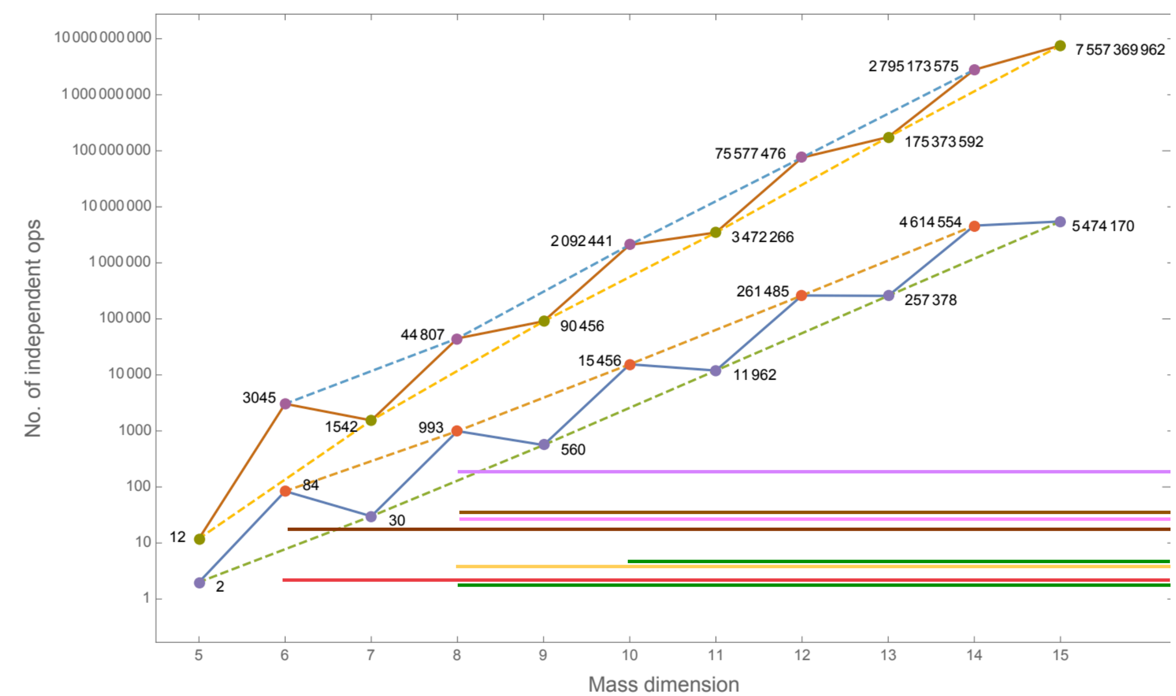
- This *field-space connection* is therefore valid at **all-orders in v/Λ**! In the Higgsed phase the connection reduces to a number + emissions of *h*.

The geoSMEFT at 2 & 3 pts

[2001.01453]

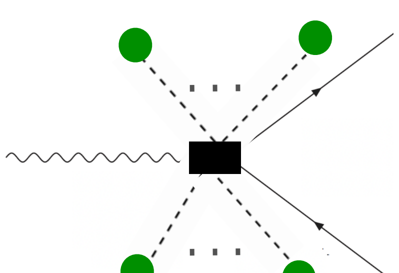
- EOM / Hilbert Series techniques allows for proof of ***all*** 2- and 3-pt field space connections!

Field space connection	Mass Dimension 6	8	10	12	14
$h_{IJ}(\phi)(D_\mu\phi)^I(D^\mu\phi)^J$	2	2	2	2	2
$g_{AB}(\phi)\mathcal{W}^A_{\mu\nu}\mathcal{W}^{B,\mu\nu}$	3	4	4	4	4
$k_{IJA}(\phi)(D^\mu\phi)^I(D^\nu\phi)^J\mathcal{W}^A_{\mu\nu}$	0	3	4	4	4
$f_{ABC}(\phi)\mathcal{W}^A_{\mu\nu}\mathcal{W}^{B,\nu\rho}\mathcal{W}^{C,\mu}_\rho$	1	2	2	2	2
$Y^u_{pr}(\phi)\bar{Q}u+$ h.c.	$2\,N_f^2$	$2\,N_f^2$	$2\,N_f^2$	$2\,N_f^2$	$2\,N_f^2$
$Y^d_{pr}(\phi)\bar{Q}d+$ h.c.	$2\,N_f^2$	$2\,N_f^2$	$2\,N_f^2$	$2\,N_f^2$	$2\,N_f^2$
$Y^e_{pr}(\phi)\bar{L}e+$ h.c.	$2\,N_f^2$	$2\,N_f^2$	$2\,N_f^2$	$2\,N_f^2$	$2\,N_f^2$
$d^{e,pr}_A(\phi)\bar{L}\sigma_{\mu\nu}e\mathcal{W}^{\mu\nu}_A+$ h.c.	$4\,N_f^2$	$6\,N_f^2$	$6\,N_f^2$	$6\,N_f^2$	$6\,N_f^2$
$d^{u,pr}_A(\phi)\bar{Q}\sigma_{\mu\nu}u\mathcal{W}^{\mu\nu}_A+$ h.c.	$4\,N_f^2$	$6\,N_f^2$	$6\,N_f^2$	$6\,N_f^2$	$6\,N_f^2$
$d^{d,pr}_A(\phi)\bar{Q}\sigma_{\mu\nu}d\mathcal{W}^{\mu\nu}_A+$ h.c.	$4\,N_f^2$	$6\,N_f^2$	$6\,N_f^2$	$6\,N_f^2$	$6\,N_f^2$
$L^{\psi_R}_{pr,A}(\phi)(D^\mu\phi)^J(\bar{\psi}_{p,R}\gamma_\mu\sigma_A\psi_{r,R})$	N_f^2	N_f^2	N_f^2	N_f^2	N_f^2
$L^{\psi_L}_{pr,A}(\phi)(D^\mu\phi)^J(\bar{\psi}_{p,L}\gamma_\mu\sigma_A\psi_{r,L})$	$2\,N_f^2$	$4\,N_f^2$	$4\,N_f^2$	$4\,N_f^2$	$4\,N_f^2$



[M. Trott KITP Talk] [1512.03433]

- All-orders connections *field-redefinition invariant* & yield large reduction in operators (*at tree level*)!
- Lagrangian parameters & Feynman rules obtained at all v/Λ orders ***before*** physical amplitude calculated!
- This is more than reorganization. It allows for all-orders amplitudes of fundamental processes:



$$\bar{\Gamma}_{Z\to\bar{\psi}\psi} = \sum_\psi \frac{N_c^\psi}{24\pi}\sqrt{\bar{m}_Z^2}|g_{\rm eff}^{Z,\psi}|^2\left(1-\frac{4\bar{M}_\psi^2}{\bar{m}_Z^2}\right)^{3/2}$$

$$g_{\rm eff}^{Z,\psi} = \frac{\bar{g}_Z}{2}\left[(2s^2_{\theta_Z}Q_\psi - \sigma_3)\delta_{pr} + \bar{v}_T\langle L^{\psi,pr}_{3,4}\rangle + \sigma_3\bar{v}_T\langle L^{\psi,pr}_{3,3}\rangle\right]$$

defined at all orders in v/Λ !!

geoSMEFT Pheno @ dim-8: [2007.00565][2107.07470][2102.02819][2203.11976]; (**tadpole**)[2106.10284]

Compliments other work on dim-8 SMEFT: [2110.06929] [2201.09887] [2205.01561] + …

Flavoring the geoSMEFT

[2107.03951]
[2001.01453]

- Yukawa-like operators of the SMEFT are given by

$$Q^{6+2n}_{\substack{\psi H\\ pr}} = \left(H^\dagger H\right)^{n+1}\left(\bar\psi_{L,p}\psi_{R,r}H\right) \quad \text{with} \quad n \geq 0$$

- In the geoSMEFT formalism this all-order tower in v/Λ is captured by Yukawa field space connections:

$$Y(\phi)\overline{\psi}_1\psi_2$$

$$Y^{\psi_1}_{pr}(\phi_I) = \left.\frac{\delta\mathcal{L}_{\rm SMEFT}}{\delta(\overline{\psi}^I_{2,p}\psi_{1,r})}\right|_{\mathcal{L}(\alpha,\beta,\dots)\to 0}$$

Field space connection	Mass Dimension 6	8	10	12	14
$Y^u_{pr}(\phi)\bar{Q}u+$ h.c.	$2\,N_f^2$	$2\,N_f^2$	$2\,N_f^2$	$2\,N_f^2$	$2\,N_f^2$
$Y^d_{pr}(\phi)\bar{Q}d+$ h.c.	$2\,N_f^2$	$2\,N_f^2$	$2\,N_f^2$	$2\,N_f^2$	$2\,N_f^2$
$Y^e_{pr}(\phi)\bar{L}e+$ h.c.	$2\,N_f^2$	$2\,N_f^2$	$2\,N_f^2$	$2\,N_f^2$	$2\,N_f^2$

$$Y^{\psi}_{pr}(\phi_I) = -H(\phi_I)\left[Y_\psi\right]^\dagger_{pr} + H(\phi_I)\sum_{n=0}^{\infty} C^{(6+2n)}_{\substack{\psi H\\ pr}}\left(\frac{\phi^2}{2}\right)^n$$

- From this one can immediately derive the all-orders effective Yukawa interactions, in terms of SM and SMEFT contributions:

$$[\mathcal{Y}^\psi]_{rp} = \left.\frac{\delta(Y^\psi_{pr})^\dagger}{\delta h}\right|_{\phi_i\to 0} = \frac{\sqrt{h}^{44}}{\sqrt{2}}\left([Y_\psi]_{rp} - \sum_{n=3}^{\infty}\frac{2n-3}{2^{n-2}}\tilde{C}^{(2n),\star}_{\substack{\psi H\\ pr}}\right)$$

flavor not taken any further in 2001.01453!

$$[M_\psi]_{rp} = \langle (Y^\psi_{pr})^\dagger\rangle$$

What about actual mass eigenstates and mixing parameters?

All-Orders Flavor Formalisms

Back to Basics: Two-Flavor Approximations

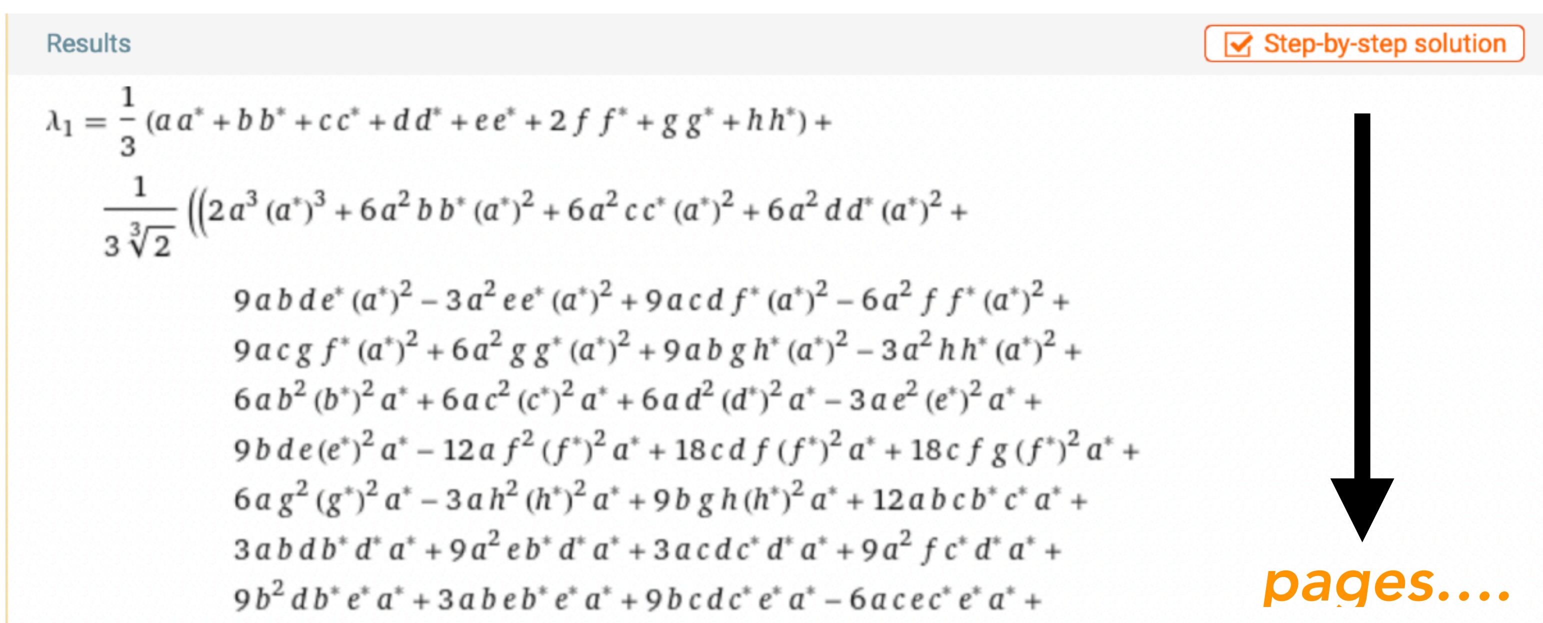
Do you know how to write $y^2(Y)$, $\theta(Y)$, $\delta(Y)$?

- In 2D, one can straightforwardly diagonalize the associated Yukawa couplings:

$$\mathcal{Y} = \frac{\sqrt{h}^{44}}{\sqrt{2}}\left[\begin{pmatrix} Y_{11} & Y_{22} \\ Y_{21} & Y_{22} \end{pmatrix} - \sum_{n=3}^{\infty} \frac{2n-3}{2^{n-2}} \begin{pmatrix} \tilde{C}_{11}^{(2n),\star} & \tilde{C}_{21}^{(2n),\star} \\ \tilde{C}_{12}^{(2n),\star} & \tilde{C}_{22}^{(2n),\star} \end{pmatrix}\right] \qquad |\mathcal{Y}\mathcal{Y}^{\dagger}| \equiv \begin{pmatrix} |\mathtt{y}_{11}| & |\mathtt{y}_{12}| \\ |\mathtt{y}_{12}| & |\mathtt{y}_{22}| \end{pmatrix} \quad \Longrightarrow U = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

$$y_{i,j}^2 = \frac{1}{2}\left(\mathtt{y}_{11} + \mathtt{y}_{22} \mp \sqrt{\mathtt{y}_{11}^2 + 4\mathtt{y}_{12}^2 - 2\mathtt{y}_{11}\mathtt{y}_{22} + \mathtt{y}_{22}^2}\right), \qquad t_{2\theta} = \frac{2\,|\mathtt{y}_{12}|}{(|\mathtt{y}_{22}| - |\mathtt{y}_{11}|)}$$

- However, results are **basis-dependent**, and at $N_f = 3$ one finds that standard techniques are …



- A different method needs to be found. Answer: use **flavor invariants**!

Flavor Invariants

- Flavor invariants are objects that are unchanged under flavor symmetry transformations.
- The most famous such invariant is the ***Jarlskog Invariant***: [Z.Phys.C 29 ('85)] [PRL 55 ('85)]

$$\left[M_u M_u^\dagger, M_d M_d^\dagger\right] = i\,C$$

$$\det C = -2\,T(m_u^2)\cdot B(m_d^2)\cdot \mathcal{J}$$

$$T(m_u^2) = \left(m_t^2 - m_u^2\right)\left(m_t^2 - m_c^2\right)\left(m_c^2 - m_u^2\right)$$
$$B(m_d^2) = \left(m_b^2 - m_d^2\right)\left(m_b^2 - m_s^2\right)\left(m_s^2 - m_d^2\right)$$

$$\mathcal{J} = \mathrm{Im}\left[V_{ij}V_{lk}V_{ik}^\star V_{lj}^\star\right] = s_{13}c_{13}^2 s_{12}c_{12}s_{23}c_{23}\sin\delta$$

$$V_{CKM} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}\,e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}\,e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}\,e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}\,e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}\,e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

- This object completely controls CP violation in the Standard Model <> det C vanishes *if and only if* there is no CP-violation present.
- **Original (bad) idea:** extract (all-orders) mass and mixing parameter, then eventually calculate *J*
- **Winning idea:** extract (all-orders) mass and mixing parameter *from* C (and other independent invariants)!

Can we find independent invariants for all Dirac parameters?

Approach with Invariants

[…]
[0706.4313]
[0907.4763]

- The answer is ***Yes!***. A group ring ***C[x]^G*** of polynomials ***x*** invariant under symmetry ***G*** can be rewritten in terms of polynomials of invariants ***I(x)***, which can be finitely generated.

$$\mathbb{C}[x_1,\ldots,x_n]^G \subseteq \mathbb{C}[x_1,\ldots,x_n] \qquad f(x_1,\ldots,x_n) = g(I_1,\ldots,I_n)$$

for us, the relevant polynomials are Yukawa couplings!

- Basis of invariants captured by ***Hilbert series***, which has special properties for semi-simple groups:

$$H(q) = \sum_{r=0}^{\infty} c_r q^r \quad \Rightarrow \quad H(q) = \frac{N(q)}{D(q)}$$

semi-simple Lie groups

$$N(q) = 1 + c_1 q + \ldots c_{d_N-1} q^{d_N-1} + q^{d_N}$$

$$D(q) = \prod_{r=1}^{p} (1 - q^{d_r})$$

p = # free parameters

- Consider a **toy model** with two mass parameters and an Abelian flavor symmetry:

$$G = U(1)\times U(1) \quad \rightarrow \quad m_1 \to e^{i\phi_1} m_1, \qquad m_2 \to e^{i\phi_2} m_2 \quad \rightarrow \quad \mathbb{C}[m_1, m_1^*, m_2, m_2^*]^{U(1)\times U(1)}$$

$$\underset{I_1}{(m_1 m_1^*)^{r_1}} \quad \underset{I_2}{(m_2 m_2^*)^{r_2}}$$

$$H(q) = 1 + 2q^2 + 3q^4 + 4q^6 + 5q^8 + \ldots = \sum_{n=0}^{\infty} (n+1) q^{2n} = \frac{1}{(1-q^2)^2}$$

$$N(q) = 1,\ d_1 = d_2 = 2$$

A Complete Basis for Quarks

[0907.4763]
[1507.00328]

For us, the relevant polynomials are Yukawa couplings!

$$Y^{\psi}Y^{\psi\dagger} \to U^{\dagger}\,Y^{\psi}Y^{\psi\dagger}\,U \qquad U(3)_{Q_L}$$

$$H(q) \;=\; h(q,q) = \frac{1+q^{12}}{(1-q^2)^2(1-q^4)^3(1-q^6)^4(1-q^8)}$$

- A set of 11 invariants can be found to fully parameterize the theory, including six ‘unmixed’ *I*

$$YY^{\dagger} \equiv \mathbb{Y}$$

$$I_1 \equiv \mathrm{tr}\left(\mathbb{Y}_u\right)\,, \qquad \hat{I}_3 \equiv \mathrm{tr}\left(\mathrm{adj}\,\mathbb{Y}_u\right)\,, \qquad \hat{I}_6 \equiv \mathrm{tr}\left(\mathbb{Y}_u\,\mathrm{adj}\,\mathbb{Y}_u\right) = 3\det\mathbb{Y}_u$$

$$I_2 \equiv \mathrm{tr}\left(\mathbb{Y}_d\right)\,, \qquad \hat{I}_4 \equiv \mathrm{tr}\left(\mathrm{adj}\,\mathbb{Y}_d\right)\,, \qquad \hat{I}_8 \equiv \mathrm{tr}\left(\mathbb{Y}_d\,\mathrm{adj}\,\mathbb{Y}_d\right) = 3\det\mathbb{Y}_d$$

- as well as four ‘mixed’ *I*, relevant for extracting information about the CKM (overlap) matrix

$$\hat{I}_5 \equiv \mathrm{tr}\left(\mathbb{Y}_u\,\mathbb{Y}_d\right), \qquad \hat{I}_7 \equiv \mathrm{tr}\left(\mathrm{adj}\,\mathbb{Y}_u\,\mathbb{Y}_d\right), \qquad \hat{I}_9 \equiv \mathrm{tr}\left(\mathbb{Y}_u\,\mathrm{adj}\,\mathbb{Y}_d\right), \qquad \hat{I}_{10} \equiv \mathrm{tr}\left(\mathrm{adj}\,\mathbb{Y}_u\,\mathrm{adj}\,\mathbb{Y}_d\right)$$

- and finally one mixed, CP-odd invariant relevant to pinning down the overall sign of CP violation:

$$I_{11}^{-} = -\frac{3i}{8}\det\left[\mathbb{Y}_u, \mathbb{Y}_d\right]$$

proportional to to the Jarlskog Invariant J!

- The fundamental geoSMEFT object we can construct at all-orders is then given by

$$\mathbb{Y}_{rp} = \frac{\hbar}{2}\left(Y_{ri}Y^{\star}_{pi} - \sum_{n'}^{\infty} f(n')Y_{ri}\tilde{C}^{(2n')}_{ip} - \sum_{n}^{\infty} f(n)\tilde{C}^{(2n),\star}_{ir}Y^{\star}_{pi} + \sum_{n,n'}^{\infty} f(n)f(n')\tilde{C}^{(2n),\star}_{ir}\tilde{C}^{(2n')}_{ip}\right)$$

All-Orders Formulae: Masses

[2107.03951]

- Unmixed invariants can be solved to obtain exact formulae for Yukawa couplings / masses:

$$y_i^2 = \frac{(-2)^{1/3}}{3\,\psi_u}\left(I_1^2 - 3\,\hat{I}_3 + (-2)^{-1/3}\; I_1\,\psi_u + (-2)^{-2/3}\;\psi_u^2\right)\,,$$

$$y_{j,k}^2 = \frac{1}{12\psi_u}((-2)^{4/3}\; I_1^2 - 3\cdot(-2)^{4/3}\;\hat{I}_3 + 4\,I_1\,\psi_u$$

$$\mp\,\psi_u\sqrt{24\left(I_1^2 - 3\,\hat{I}_3\right) + \frac{6\cdot(-2)^{5/3}\left(I_1^2 - 3\,\hat{I}_3\right)^2}{\psi_u^2} - 3\cdot(-2)^{4/3}\,\psi_u^2 + (-2)^{2/3}\,\psi_u^2})$$

Valid for up-quark masses. Send $I_{1,3,6}$ to $I_{2,4,8}$ for down quark masses.

$$\psi_u = \left(-2\,I_1^3 + 9\,I_1\hat{I}_3 - 9\,\hat{I}_6 + 3\sqrt{-3\,I_1^2\hat{I}_3^2 + 12\,\hat{I}_3^3 + 4\,I_1^3\hat{I}_6 - 18\,I_1\hat{I}_3\hat{I}_6 + 9\,\hat{I}_6^2}\right)^{1/3}$$

- Of course, the only distinction between fermions of the same family are their (measured) mass eigenvalues….

$$y_u^2 \equiv \min\{y_i^2, y_j^2, y_k^2\}\,, \qquad y_c^2 \equiv \mathrm{mid}\{y_i^2, y_j^2, y_k^2\} \qquad y_t^2 \equiv \max\{y_i^2, y_j^2, y_k^2\}$$

All-Orders Formulae: Mixings & CP

[2107.03951]

- Similarly, the mixed invariants give predictions for (CKM) mixing angles:

$$s_{13} = \left[\frac{-\hat{I}_{10} - y_b^2\left(\hat{I}_7 - \Delta^+_{ds}\Delta^+_{uc}\Delta^+_{ut}\right) - y_u^2\left(\hat{I}_9 + y_b^2\left(\hat{I}_5 - y_b^2\Delta^+_{ct}\right) - y_d^2\, y_s^2\, \Delta^+_{ct}\right)}{\Delta^-_{bd}\Delta^-_{bs}\Delta^-_{cu}\Delta^-_{ut}}\right]^{1/2} \qquad \Delta^\pm_{ij} \equiv y_i^2 \pm y_j^2$$

$$s_{23} = \left[\frac{\Delta^-_{tu}\left(-\hat{I}_{10} + y_c^2\left(-\hat{I}_9 + (y_b^4 + y_d^2 y_s^2)\,\Delta^+_{ut}\right) + y_b^2\left(-\hat{I}_7 + y_c^2\left(-\hat{I}_5 + \Delta^+_{ct}\Delta^+_{ds}\right) + y_u^2\Delta^+_{ct}\Delta^+_{ds}\right)\right)}{\Delta^-_{ct}\left(\hat{I}_{10} + y_u^2\,\hat{I}_9 + y_b^2\left(\hat{I}_7 + y_u^2\left(\hat{I}_5 - 2\Delta^+_{ct}\Delta^+_{ds}\right)\right) - (y_u^4 + y_c^2 y_t^2)\,(y_b^4 + y_d^2 y_s^2)\right)}\right]^{1/2}$$

$$s_{12} = \left[\frac{\Delta^-_{db}\left(\hat{I}_{10} + y_s^2\left(\hat{I}_7 - y_c^2 y_t^2 \Delta^+_{db}\right)\right) + y_u^2\,\Delta^-_{bd}\left(-\hat{I}_9 - y_s^2\,\hat{I}_5 + \Delta^+_{sb}\Delta^+_{ct}\Delta^+_{ds}\right) + y_u^4 y_s^2\,(y_b^4 - y_d^4)}{\Delta^-_{ds}\left(\hat{I}_{10} + y_u^2\,\hat{I}_9 + y_b^2\left(\hat{I}_7 + y_u^2\left(\hat{I}_5 - 2\Delta^+_{ct}\Delta^+_{ds}\right)\right) - (y_b^4 + y_d^2 y_s^2)\,(y_u^4 + y_c^2 y_t^2)\right)}\right]^{1/2}$$

- When combined with the CP-odd 11th invariant, one also can derive the Dirac CP-violating phase (and its sign!)

$$s_\delta = \frac{4}{3}\, I^-_{11}\,\left[\Delta^-_{tc}\Delta^-_{tu}\Delta^-_{cu}\Delta^-_{bs}\Delta^-_{bd}\Delta^-_{sd}\, s_{12}s_{13}s_{23}\,\left(1 - s_{23}^2\right)^{1/2}\left(1 - s_{12}^2\right)^{1/2}\left(1 - s_{13}^2\right)\right]^{-1}$$

Here one notices the proportionality to the Jarlskog as well!

Numerical Checks

[2107.03951]

- To test the validity of our formulae, we wrote a script to compare values of Dirac parameters predicted from our formulae vs. those extracted with numerical techniques. It did so by…

  (a) computing the eigenvectors of $\left[\mathcal{Y}^{(u,d)}\mathcal{Y}^{(u,d),\dagger}\right]$. These are normalized to unit vectors $\mathbb{v}_i$ and then the numerical matrices are defined by $U_{(u,d)} \equiv \left(\mathbb{v}_1^{(u,d),T}, \mathbb{v}_2^{(u,d),T}, \mathbb{v}_3^{(u,d),T}\right)$.

  (b) computing the CKM matrix as $V_{CKM} = U_u^\dagger \cdot U_d$.

  (c) uniquely extracting the s_{13} mixing angle from V_{13}, $s_{13} = |V_{13}|$.

  (d) uniquely extracting the s_{23} mixing angle from V_{23}, $s_{23} = |V_{23}|/\sqrt{1-s_{13}^2}$.

  (e) uniquely extracting the s_{12} mixing angle from V_{12}, $s_{12} = |V_{12}|/\sqrt{1-s_{13}^2}$.

  (f) uniquely extracting the s_δ phase in a phase-convention-independent manner from the Jarlskog invariant J.

- Computations done in arbitrary flavor/weak bases (i.e. with full but arbitrary 3D structure in matrices)

mass dimension up to n = 10

new physics between 2-10 TeV

- **Conclusion**: complete agreement up to numerical tolerance of 10^{10} !! ✅

- Note that numerical checks in environments with $\{\mathbb{Y}'_u, \mathbb{Y}'_d\} = \{U_\chi^{u\,\dagger}\,\mathbb{Y}_u\,U_\chi^u, U_\chi^{d\,\dagger}\,\mathbb{Y}_d\,U_\chi^d\}$ also confirm LACK of ability to predict CKM angles with our formulae in this instance, as expected!

These formulae…

- are **exact**, and **analytically** relate the fundamental Lagrangian parameters to the `physical' masses, mixings, and phase (*for the first time, to my knowledge*).

- **complete** the list of all-orders Lagrangian parameters in the Dirac flavor sector of the geoSMEFT

- are **basis independent** (as long as the information required is present in the basis in question)

- are applicable to explicit **(B)SM models and EFTs**, when global U(3)_Q flavor rotations control flavor parameters.

Powerful tools in the description of (B)SM flavor physics!

Applicability

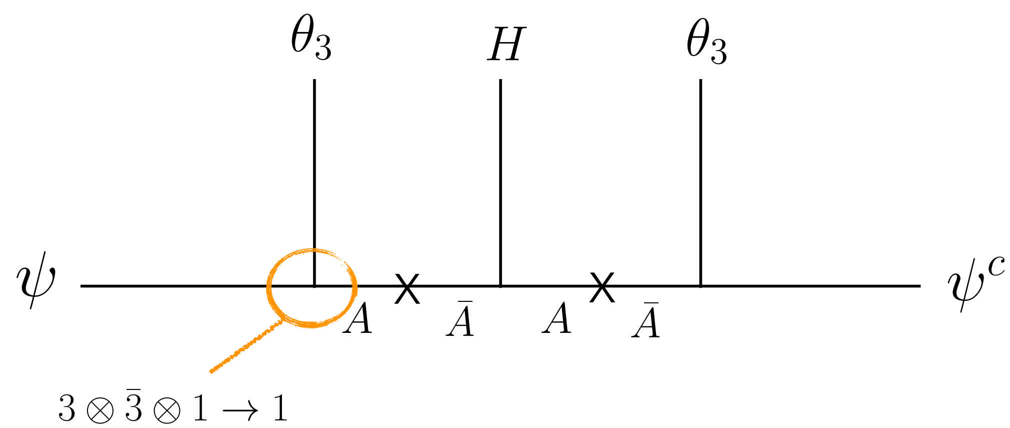
Applications: UV-completing flavor

[2107.03951]

The Universal Texture Zero Model

Fields	$\psi_{q,e,\nu}$	$\psi^c_{q,e,\nu}$	H_5	Σ	S	θ_3	θ_{23}	θ_{123}	θ	θ_X
$\Delta(27)$	3	3	1_{00}	1_{00}	1_{00}	$\bar{3}$	$\bar{3}$	$\bar{3}$	$\bar{3}$	3
Z_N	0	0	0	2	-1	0	-1	2	0	x

[de Medeiros Varzielas, Ross, **Talbert**: 1710.01741]



$$\mathcal{L}_{\text{UTZ}} \supset \psi_p \left(\frac{1}{M^2_{3,f}} \theta^p_3 \theta^r_3 + \frac{1}{M^3_{23,f}} \theta^p_{23} \theta^r_{23} \Sigma + \frac{1}{M^3_{123,f}} (\theta^p_{123} \theta^r_{23} + \theta^p_{23} \theta^r_{123}) S \right) \psi^c_r H + \mathcal{O}(1/M^4) + \dots$$

- After flavor- and EW-symmetry breaking, the EFT/model shapes Yukawa/mass matrices of the form

$$\mathcal{M}^D_f = \begin{pmatrix} 0 & a\,e^{i\gamma} & a\,e^{i\gamma} \\ a\,e^{i\gamma} & \left(b\,e^{-i\gamma} + 2a\,e^{-i\delta}\right) e^{i(\gamma+\delta)} & b\,e^{i\delta} \\ a\,e^{i\gamma} & b\,e^{i\delta} & 1 - 2a\,e^{i\gamma} + b\,e^{i\delta} \end{pmatrix}_f$$

- Proof-in-principle fits to global flavor data yield post-dictions for mass (ratios) and CKM mixing angles:

$$\frac{m_u}{m_t} = 7.16 \cdot 10^{-6}\,, \qquad \frac{m_c}{m_t} = 0.0027\,, \qquad \frac{m_d}{m_b} = 0.00090\,, \qquad \frac{m_s}{m_b} = 0.020$$

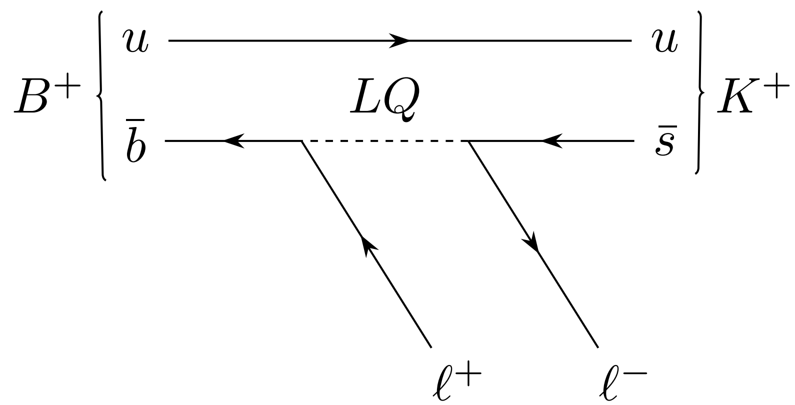
$$s_{12} = 0.226, \quad s_{23} = 0.0191, \quad s_{13} = 0.0042, \quad s_\delta = 0.5609$$

✅ currently working on an MCMC fit to the UTZ!

Applications: BSM States in the IR

[2107.03951]

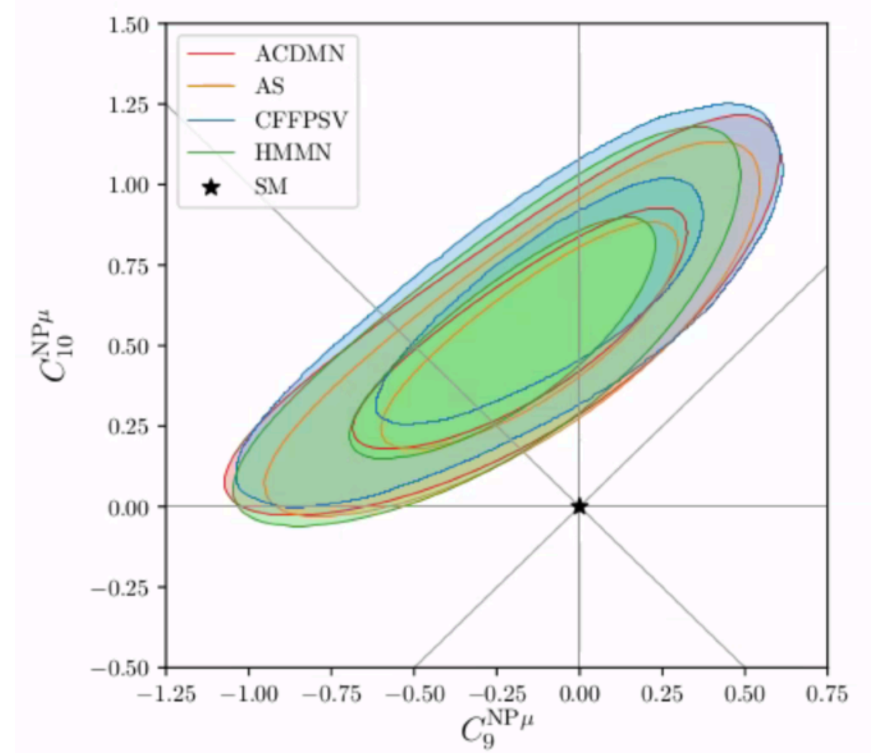
$$R_H \equiv \frac{\int_{q^2_{\min}}^{q^2_{\max}} \frac{d\mathcal{B}(B \to H\mu^+\mu^-)}{dq^2} dq^2}{\int_{q^2_{\min}}^{q^2_{\max}} \frac{d\mathcal{B}(B \to He^+e^-)}{dq^2} dq^2}$$



$$\Delta_3 \sim (\bar{\mathbf{3}}, \mathbf{3}, 1/3)$$

$$\mathcal{G}_{SM} \equiv SU(3)_C \times SU(2)_L \times U(1)_Y$$

$$\mathcal{L} \supset y^{LL}_{3,ij} \bar{Q}^{C\,i,a}_L \epsilon^{ab} (\tau^k \Delta^k_3)^{bc} L^{j,c}_L + z^{LL}_{3,ij} \bar{Q}^{C\,i,a}_L \epsilon^{ab} ((\tau^k \Delta^k_3)^\dagger)^{bc} Q^{j,c}_L + \text{h.c.}$$



fit to LFU observables + $B_s \to \mu\mu$

[Capdevila et al, 2021 Flavor Anomaly Workshop]

- Regardless of the introduction of new IR flavor violation, Dirac mass and mixing still predictable!

EFT for CKM + PMNS + Leptoquarks

	Q''^1	Q''^{23}	u''^1_R	u''^2_R	u''^3_R	d''^1_R	d''^2_R	d''^3_R	ϕ_u	ϕ_d
D_{15}	$\mathbf{1}_-$	$\mathbf{2_1}$	$\mathbf{1}_-$	$\mathbf{1}$	$\mathbf{1}_-$	$\mathbf{1}_-$	$\mathbf{1}$	$\mathbf{1}_-$	$\mathbf{2_1}$	$\mathbf{2_1}$

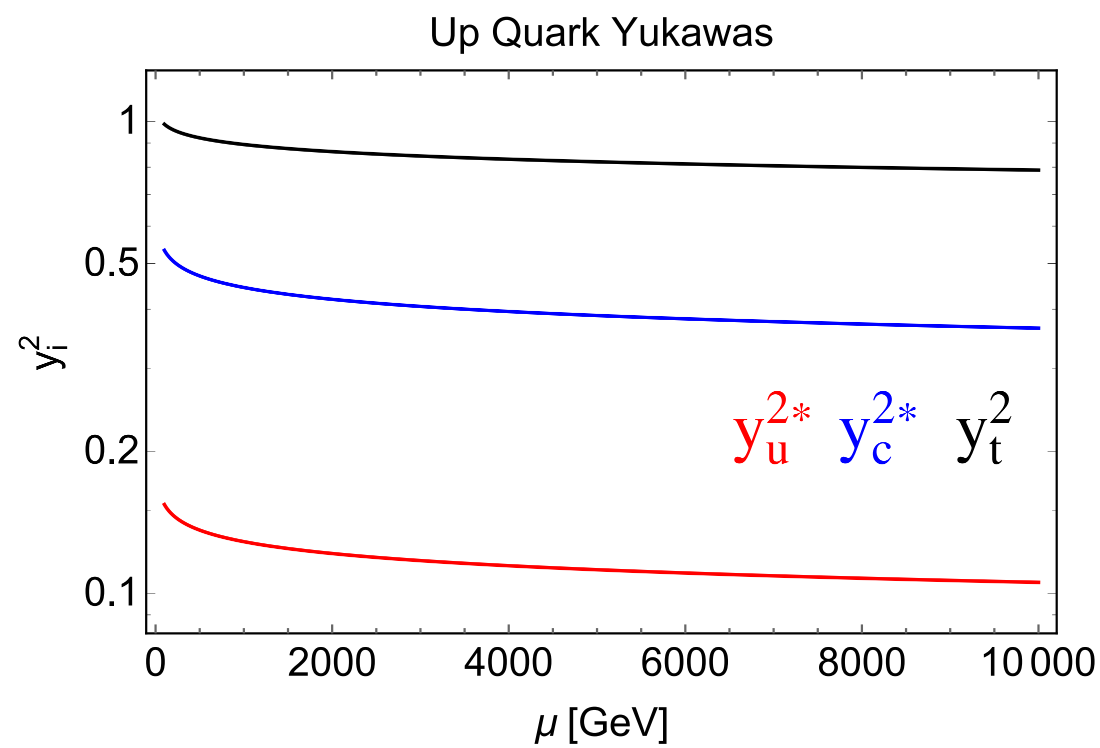
[Bernigaud, de Medeiros Varzielas, **Talbert**: 2005.12293]

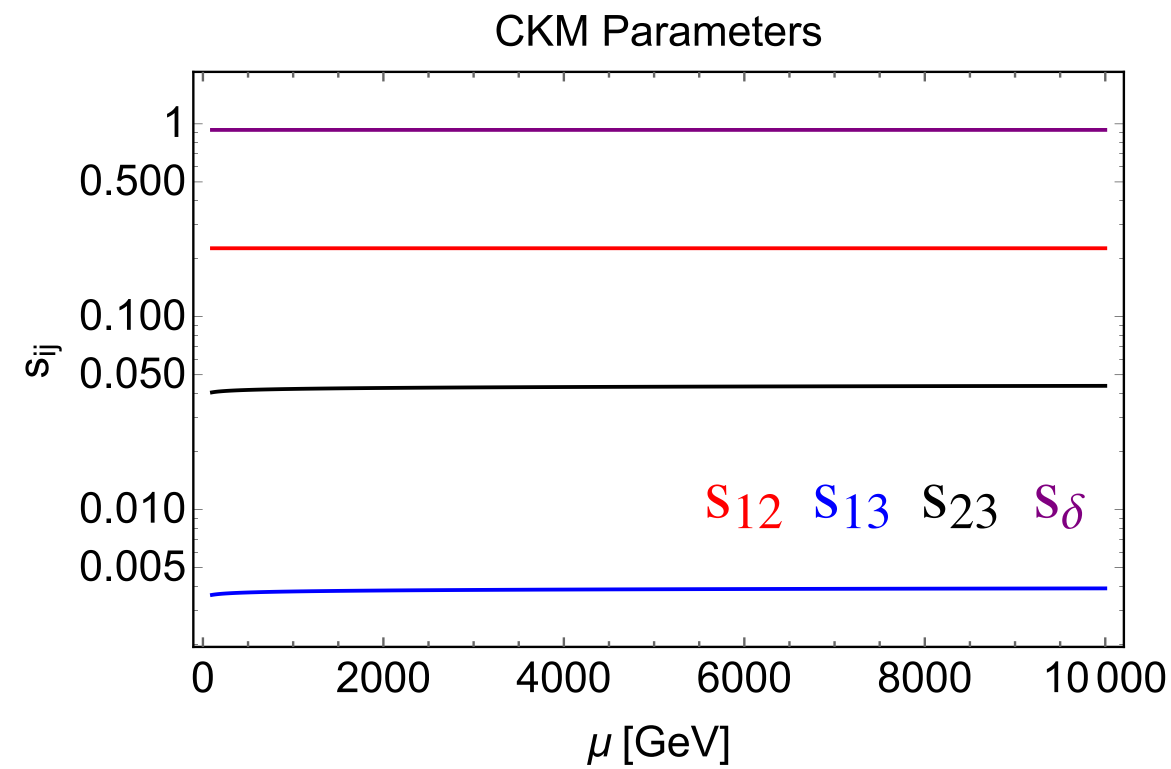
$$\mathcal{L}_Y \supset a_u \bar{Q}''^1_L u''^1_R + b_u \left[\bar{Q}''^{23}_L \phi_u\right]_{\mathbf{1}} u''^2_R + c_u \left[\bar{Q}''^{23}_L \phi_u\right]_{\mathbf{1}_-} u''^3_R$$
$$+ a_d \bar{Q}''^1_L d''^1_R + b_d \left[\bar{Q}''^{23}_L \phi_d\right]_{\mathbf{1}} d''^2_R + c_d \left[\bar{Q}''^{23}_L \phi_d\right]_{\mathbf{1}_-} d''^3_R,$$

$$Y''_u = P^\dagger \Lambda_d V^\dagger_{CKM} \cdot \begin{pmatrix} y_u & 0 & 0 \\ 0 & y_c & 0 \\ 0 & 0 & y_t \end{pmatrix} \cdot \Lambda^\dagger_U P, \qquad Y''_d = P^\dagger \Lambda_d \cdot \begin{pmatrix} y_d & 0 & 0 \\ 0 & y_s & 0 \\ 0 & 0 & y_b \end{pmatrix} \cdot \Lambda^\dagger_D P$$

(even in an absurd model basis!)

Applications: Renormalization Group Flow





[2107.03951]

$$\dot{s}_\delta = s_\delta \left[\frac{\dot{I}_{11}^-}{I_{11}^-} - \sum_{(ij)\,\in\,\mathbb{s}_2} \frac{\dot{\Delta}_{ij}^-}{\Delta_{ij}^-} - \dot{s}_{12}\frac{(1-2\,s_{12}^2)}{s_{12}c_{12}^2} - \dot{s}_{23}\frac{(1-2\,s_{23}^2)}{s_{23}c_{23}^2} - \dot{s}_{13}\frac{(1-3\,s_{13}^2)}{s_{13}c_{13}^2} \right]$$

(e.g)

$$\mu\frac{dI_{11}^-}{d\mu} \simeq (6a_0 + 6b_0 + 2a_1 I_1 + 2b_1 I_2)\, I_{11}^-$$

$$a_0 = \frac{3}{8\pi^2}\left(I_1 + I_2 + \frac{I_1 - I_2}{2n_g}\right) - 2\frac{\alpha_s}{\pi}, \qquad a_1 = \frac{3}{16\pi^2}$$

$$b_0 = \frac{3}{8\pi^2}\left(I_1 + I_2 + \frac{I_2 - I_1}{2n_g}\right) - 2\frac{\alpha_s}{\pi}, \qquad b_1 = \frac{3}{16\pi^2}$$

[1507.00328]

- Note however that latter formulae only hold for MFV theories (numerics done for SM limit)!
- Would be interesting to pursue more generic RGE studies in SMEFT (e.g. 2005.12283).

Applications: CKM Fits?

Wolfenstein Parameterization

$$W_j \equiv \{\lambda, A, \bar\rho, \bar\eta\}$$

$$\begin{pmatrix} 1-\frac{1}{2}\lambda^2-\frac{1}{8}\lambda^4 & \lambda & A\lambda^3(1+\frac{1}{2}\lambda^2)(\bar\rho-i\bar\eta) \\ -\lambda+A^2\lambda^5(\frac{1}{2}-\bar\rho-i\bar\eta) & 1-\frac{1}{2}\lambda^2-\frac{1}{8}\lambda^4(1+4A^2) & A\lambda^2 \\ A\lambda^3(1-\bar\rho-i\bar\eta) & -A\lambda^2+A\lambda^4(\frac{1}{2}-\bar\rho-i\bar\eta) & 1-\frac{1}{2}A^2\lambda^4 \end{pmatrix}$$

2020 PDG Global Fit

12. CKM Quark-Mixing Matrix

Revised March 2020 by A. Ceccucci (CERN), Z. Ligeti (LBNL) and Y. Sakai (KEK).

$$\begin{pmatrix} 0.97401 \pm 0.00011 & 0.22650 \pm 0.00048 & 0.00361^{+0.00011}_{-0.00009} \\ 0.22636 \pm 0.00048 & 0.97320 \pm 0.00011 & 0.04053^{+0.00083}_{-0.00061} \\ 0.00854^{+0.00023}_{-0.00016} & 0.03978^{+0.00082}_{-0.00060} & 0.999172^{+0.000024}_{-0.000035} \end{pmatrix}$$

- CKM parameter fits big business in flavor physics — critical tests of the SM.
- However, as we have seen, BSM physics encoded in Wilson coefficients impacts the definition of the CKM matrix. A consistent treatment of such effects critical for interpretation of NP bounds.

$$O_i^{\text{input}} = O_{i,\text{SM}}^{\text{input}}(W_j)\big[(1+f(L_k)\big] = O_{i,\text{SM}}^{\text{input}}(W_j)\big[1+g(C_k)\big]$$

LEFT *SMEFT*

$$O_i^{\text{input}} = O_{i,\text{SM}}^{\text{input}}(\widetilde{W}_j)$$

$$\widetilde{W}_j = W_j\left(1+\frac{\delta W_j}{W_j}\right)$$

[1812.08163]

$$O_\alpha = O_{\alpha,\text{SM}}(W_j) + \delta O_{\alpha,\text{NP}}^{\text{direct}} = O_{\alpha,\text{SM}}(\widetilde{W}_j) + \delta O_{\alpha,\text{NP}}^{\text{indirect}} + \delta O_{\alpha,\text{NP}}^{\text{direct}}$$

$$\delta O_{\alpha,\text{NP}}^{\text{indirect}} = -\frac{\partial O_{\alpha,\text{SM}}}{\partial W_i}\delta W_i + \mathcal{O}(\Lambda^{-4})$$

'indirect' and 'direct' NP effects contribute at the same order in v/Λ!

Applications: CKM Fits?

[1812.08163]

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The CKM parameters in the SMEFT

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$\Gamma(K \to \mu\nu_\mu)/\Gamma(\pi \to \mu\nu_\mu), \quad \Gamma(B \to \tau\nu_\tau), \quad \Delta M_d, \quad \Delta M_s$

CKMfitter (SM) [14]	UTfit (SM) [15]	This work (SMEFT)
$\lambda = 0.224747^{+0.000254}_{-0.000059}$	$\lambda = 0.2250 \pm 0.0005$	$\tilde{\lambda} = 0.22537 \pm 0.00046$
$A = 0.8403^{+0.0056}_{-0.0201}$	$A = 0.826 \pm 0.012$	$\tilde{A} = 0.828 \pm 0.021$
$\bar{\rho} = 0.1577^{+0.0096}_{-0.0074}$	$\bar{\rho} = 0.148 \pm 0.013$	$\tilde{\rho} = 0.194 \pm 0.024$
$\bar{\eta} = 0.3493^{+0.0095}_{-0.0071}$	$\bar{\eta} = 0.348 \pm 0.010$	$\tilde{\eta} = 0.391 \pm 0.048$

- As expected, reabsorption of BSM effects into 'SM' parameters leads to non-trivial bounds on NP when calculating other flavored processes:

$$\Gamma(\pi \to \mu\nu) = \left|1 - \frac{\tilde{\lambda}^2}{2} - \frac{\tilde{\lambda}^4}{8}\right|^2 \frac{f_{\pi^\pm}^2 m_{\pi^\pm} m_\mu^2}{16\pi\tilde{v}^4}\left(1 - \frac{m_\mu^2}{m_{\pi^\pm}^2}\right)^2 (1+\delta_{\pi\mu})\left[1 + \widetilde{\Delta}_{\pi\mu 2}\right] \quad \textit{(e.g)}$$

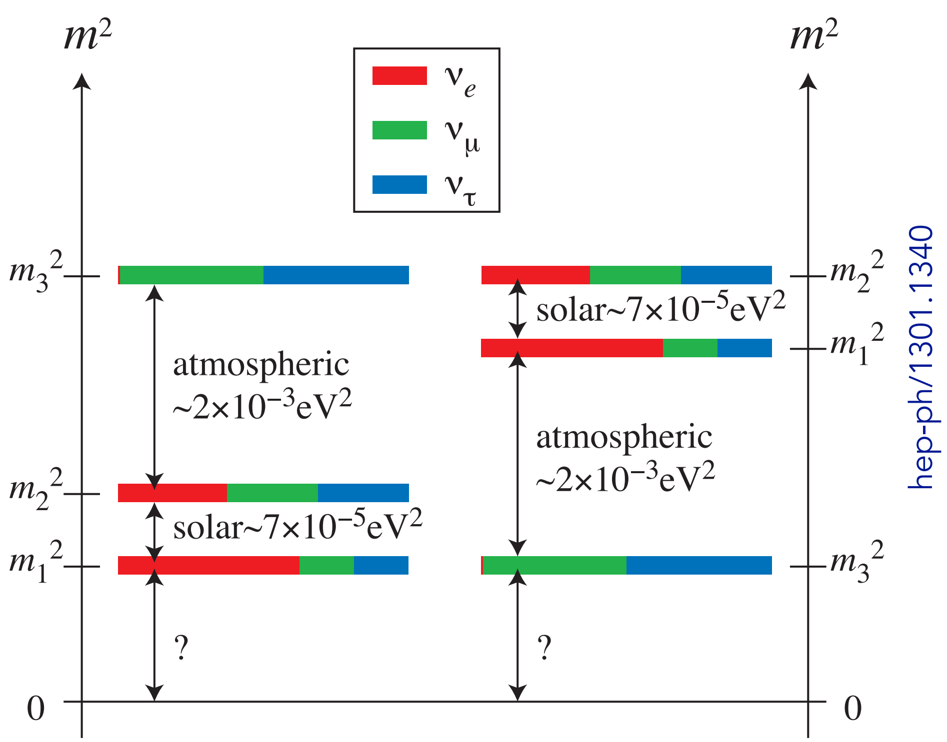
$$\mathcal{B}(\pi \to \mu\nu) = 0.9998770(4) \quad + \quad \tau_\pi = 2.6033(5)\cdot 10^{-8}s \quad \Rightarrow \quad \widetilde{\Delta}_{\pi\mu 2} = 0.004 \pm 0.013$$

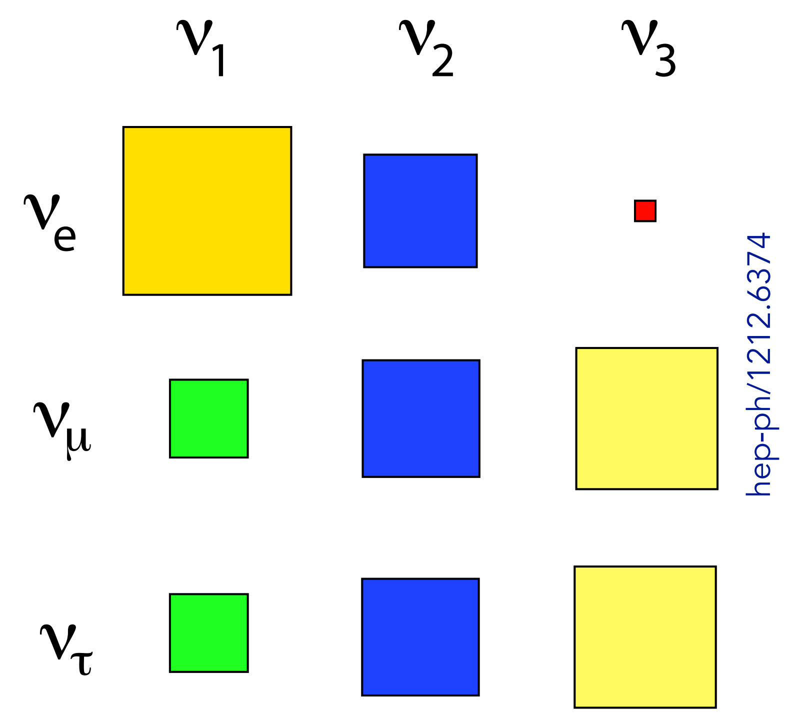
$$\widetilde{\Delta}_{\pi\mu 2} = 2\,\mathrm{Re}(\epsilon_A^{\mu ud}) - \frac{2m_{\pi^\pm}^2}{(m_u+m_d)m_\mu}\mathrm{Re}(\epsilon_P^{\mu ud}) + 4\frac{\delta v}{v} + 2\tilde{\lambda}(1+\tilde{\lambda}^2)\delta\lambda + \mathcal{O}(\Lambda^{-4}, \tilde{\lambda}^6)$$

- Formalism with flavored geoSMEFT can potentially push fits to higher order in v/Λ.
- Of relevance to potential *Cabibbo Angle Anomaly* — see e.g. 2109.06065.

Towards Neutrinos

Neutrino Masses and Mixings





Hierarchy Problem

Neutrino Masses

CP Violation

Dark Matter

Flavor Problem

…

$$V_{PMNS} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}\,e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}\,e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}\,e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}\,e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}\,e^{i\delta} & c_{23}c_{13} \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha_1} & 0 \\ 0 & 0 & e^{i\alpha_2} \end{pmatrix}$$

- Neutrino mass and mixing is an **experimental fact**, and represents a clear departure from the naive SM. Massive experimental effort underway to pin down neutrino properties…
- Known: there is a gigantic hierarchy between neutrino mass scales and (e.g.) the top mass, and the mixing in the neutrino sector is large and non-hierarchical.

neutrinos key to understanding critical BSM physics

Neutrinos in the (geo)SMEFT

- Neutrino masses described by dim-5 Weinberg operator at leading-order in SMEFT:

$$\mathcal{L}^{(5)} = \frac{c_{ij}^{(5)}}{2}\left(\ell_i^T \tilde{H}^\star\right) C \left(\tilde{H}^\dagger \ell_j\right) + \text{h.c.}$$

EWSB ⇒

$$\mathcal{L} \supset -\frac{m_{\nu,k}}{2}\overline{\nu_L^{c,k}}\nu_L^k + \text{h.c.}$$

$$m_{\nu,k} = -\frac{v^2}{2}(U_\nu^T)_{ki}\, c_{ij}^{(5)}\, U_{\nu,jk}$$

- A field-space connection that describes this mass generation at all-orders should be found…

$$\mathcal{L} \supset \eta(\phi)_{\alpha\beta}\,\ell^\alpha \ell^\beta$$

	Mass Dimension				
Field space connection	5	7	9	11	13
$\eta(\phi)_{\alpha\beta}\,\ell^\alpha\ell^\beta + \text{h.c.}$	$2\cdot 2N_f$	?	?	?	?

✅

for N_f = 3…

- Furthermore, Hilbert series and associated basis of invariants known for N_f = 3!

$$H(q) = \frac{1+q^6+2q^8+4q^{10}+8q^{12}+7q^{14}+9q^{16}+10q^{18}+9q^{20}+7q^{22}+8q^{24}+4q^{26}+2q^{28}+q^{30}+q^{36}}{(1-q^2)^2\,(1-q^4)^3\,(1-q^6)^4\,(1-q^8)^2\,(1-q^{10})}$$

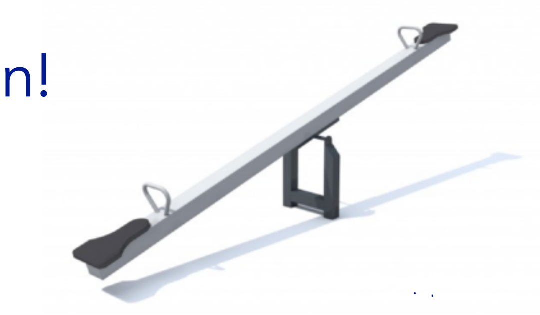
All-Orders flavor formalism analogous to quark sector within (quick) reach!

✅ [0907.4763] [2107.06274]

5261596067
#of mass-dimension 14
operators in Nf = 3 nuSMEFT
— Thanks to ECO!!
[2005.09521]

Neutrinos in the (geo)υSMEFT

- Introducing a light sterile neutrino N changes the EFT under consideration!



$$\mathcal{L}_N = \frac{1}{2}(\bar{N}_p i\partial\!\!\!/ N_p - \bar{N}_p M_{pr} N_r) - [\bar{N}_p \omega_{p\beta} \tilde{H}^\dagger l_\beta + \text{H.c.}]$$

It's all about scales!

- A geometric υSMEFT can (and will) be developed. Obvious field-space connections are:

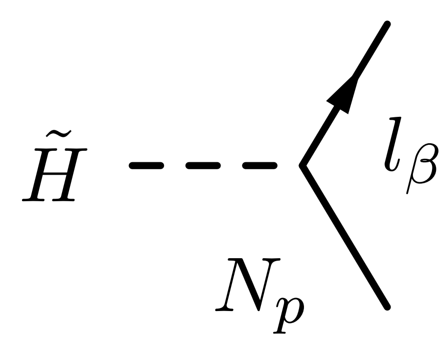
$$\mathcal{L} \supset Y^{\nu}_{pr}(\phi)\,\overline{L}\,N + M_{pr}(\phi)\,\overline{N}\,N$$

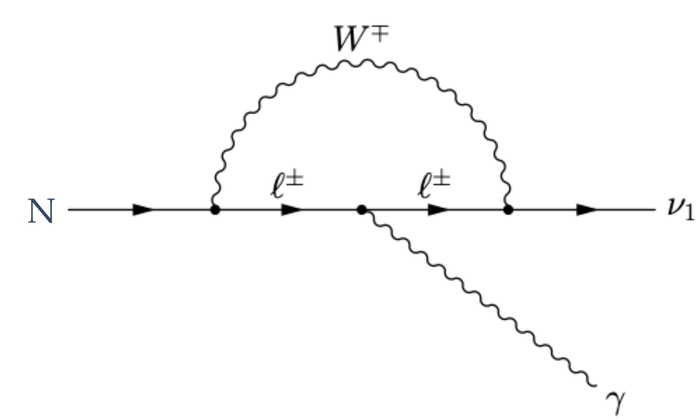
	Mass Dimension				
Field space connection	6	8	10	12	14
$Y^{\nu}_{pr}(\phi)\,\overline{L}\,N + \text{h.c.}$	$2\,N_f^2$	?	?	?	?
$M_{pr}(\phi)\,\overline{N}\,N + \text{h.c.}$	$2 \cdot 2N_f$	?	?	?	?

✅

for N_f = 3…

All-orders amplitudes in such a theory could be a big boon to precision neutrino phenomenology!!





- The Hilbert Series for the complete three-generation Lagrangian was found in **1010.3161**.

Summary and outlook

- One can construct basis-independent flavor formalisms using invariant theory.
- These formalisms depend exclusively on flavor symmetry and free parameters.
- As a result, they hold at all-orders in effective field theories, e.g. the (geo)SMEFT.
- We have presented analytic formulae for the Dirac masses and mixings present in the (geo)SM(EFT). They are useful in any number of (B)SM contexts.
- Phenomenological applications are obvious, including fits to mass and mixing.
- The extension of the formalism to neutrino physics is ongoing, and rich in application.
- Flavor & neutrino physics offer prime opportunities for low- and high-energy complementarity!

THANK YOU!

Backup Slides

Applications: Flavor Violation Pheno [2005.12283]

LL RGE evolution for Yukawa and Wilson Coefficients known:

$$Y_d(\mu_{\rm EW}) = Y_d(\Lambda) - \delta Y_d \frac{3y_t^2}{32\pi^2} \ln\left(\frac{\mu_{\rm EW}}{\Lambda}\right) + \ldots$$

$$\left[\widetilde{\mathcal{C}}_a(\mu_{\rm EW})\right]_{ij} = \left[\mathcal{C}_a(\Lambda)\right]_{ij} + \frac{(\beta_{ab})^{ijkl}}{16\pi^2} \ln\left(\frac{\mu_{\rm EW}}{\Lambda}\right) \left[\mathcal{C}_b(\Lambda)\right]_{kl}$$

At EW scale, Yukawa (and Wilson Coefficients) must be re-rotated to (physical) fermion mass-eigenstates!

$$\left[\mathcal{C}_a(\mu_{\rm EW})\right]_{ij} = U^\dagger_{ik} \left[\widetilde{\mathcal{C}}_a(\mu_{\rm EW})\right]_{kl} U_{lj}$$

$$U_{d_L} = \begin{pmatrix} -0.93 + 0.37i & 1.6\cdot 10^{-5} + 2.5\cdot 10^{-7}\, i & -3.8\cdot 10^{-4} \\ -1.2\cdot 10^{-5} + 1.1\cdot 10^{-5}\, i & -0.93 + 0.37i & 1.6\cdot 10^{-3} - 6.7\cdot 10^{-4}\, i \\ 2.7\cdot 10^{-4} - 2.6\cdot 10^{-4}\, i & -1.6\cdot 10^{-3} + 6.1\cdot 10^{-4}\, i & -0.93 + 0.37\, i \end{pmatrix}$$

compare to $\kappa^{ij}_{RGE} = \frac{\lambda_t^{ij}}{16\pi^2} \ln\left(\frac{\mu_{\rm EW}}{\Lambda}\right) \approx 9\cdot 10^{-4} - 2\cdot 10^{-5} i.$

Flavour Violating Effects of Yukawa Running in SMEFT

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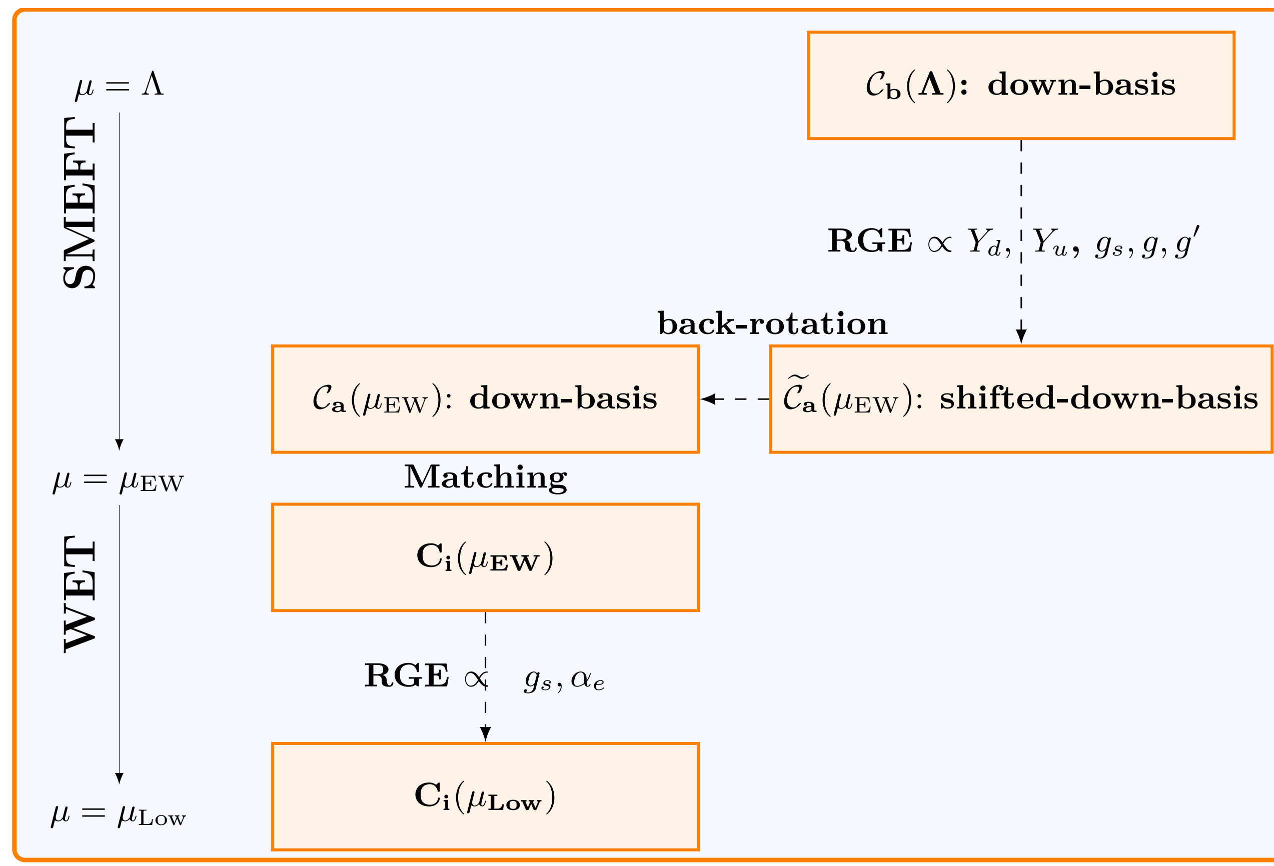

- The resulting NP bounds derived from (e.g.) ΔF=2 or b->sll processes are very important!
- *Q1: what is the correspondence between RGE of flavor invariants and (known) non-MFV relations?*
- *Q2: what is the phenomenological impact of higher-order RGE of physical parameters?*

Partial Sq. vs. Full Dim-8: Fermionic Z Decay

- Consider all-order geoSMEFT width for Z-boson decay to fermions:

$$\bar{\Gamma}_{Z\to\bar{\psi}\psi} = \sum_{\psi} \frac{N_c^{\psi}}{24\pi}\sqrt{\bar{m}_Z^2}|g_{\rm eff}^{Z,\psi}|^2\left(1-\frac{4\bar{M}_\psi^2}{\bar{m}_Z^2}\right)^{3/2} \qquad g_{\rm eff}^{Z,\psi} = \frac{\bar{g}_Z}{2}\left[(2s_{\theta_Z}^2\, Q_\psi - \sigma_3)\delta_{pr} + \bar{v}_T\langle L_{3,4}^{\psi,pr}\rangle + \sigma_3\bar{v}_T\langle L_{3,3}^{\psi,pr}\rangle\right]$$

defined at all orders in v/Λ !!

- Expand complete dependence at dim-6, dim-8:

$$\langle g_{\rm eff,pr}^{\mathcal{Z},\psi}\rangle_{\rm SM} = \bar{g}_Z^{\rm SM}\left[(s_\theta^{\rm SM})^2\, Q_\psi - \frac{\sigma_3}{2}\right]\delta_{pr}$$

SM

$$\langle g_{\rm eff,pr}^{\mathcal{Z},\psi}\rangle_{\mathcal{O}(v^2/\Lambda^2)} = \frac{\langle\bar{g}_Z\rangle_{\mathcal{O}(v^2/\Lambda^2)}}{\bar{g}_Z^{\rm SM}}\langle g_{\rm eff,pr}^{\mathcal{Z},\psi}\rangle_{\rm SM}\,\delta_{pr} + \bar{g}_Z^{\rm SM}\, Q_\psi\,\langle s_{\theta_Z}^2\rangle_{\mathcal{O}(v^2/\Lambda^2)}\,\delta_{pr} + \frac{\bar{g}_Z^{\rm SM}}{2}\left[\tilde{C}_{\substack{H\psi\\pr}}^{1,(6)} - \sigma_3\,\tilde{C}_{\substack{H\psi\\pr}}^{3,(6)}\right]$$

dim-6

$$\begin{aligned}\langle g_{\rm eff,pr}^{\mathcal{Z},\psi}\rangle_{\mathcal{O}(v^4/\Lambda^4)} = &\frac{\langle\bar{g}_Z\rangle_{\mathcal{O}(v^4/\Lambda^4)}}{\bar{g}_Z^{\rm SM}}\langle g_{\rm eff,pr}^{\mathcal{Z},\psi}\rangle_{\rm SM}\,\delta_{pr} + \bar{g}_Z^{\rm SM}\, Q_\psi\,\langle s_{\theta_Z}^2\rangle_{\mathcal{O}(v^4/\Lambda^4)}\,\delta_{pr} + \langle\bar{g}_Z\rangle_{\mathcal{O}(v^2/\Lambda^2)}\,\langle s_{\theta_Z}^2\rangle_{\mathcal{O}(v^2/\Lambda^2)}\, Q_\psi\delta_{pr}\\ &+ \frac{\langle\bar{g}_Z\rangle_{\mathcal{O}(v^2/\Lambda^2)}}{2}\left[\tilde{C}_{\substack{H\psi\\pr}}^{1,(6)} - \sigma_3\,\tilde{C}_{\substack{H\psi\\pr}}^{3,(6)}\right] + \frac{g_Z^{\rm SM}}{4}\left[\tilde{C}_{\substack{H\psi\\pr}}^{1,(8)} - \sigma_3\,\tilde{C}_{\substack{H\psi\\pr}}^{2,(8)} - \sigma_3\,\tilde{C}_{\substack{H\psi\\pr}}^{3,(8)}\right]\end{aligned}$$

dim-8

- Compare (e.g.) dependence on $(C^{(6)}_{HWB})^2$ using partial square vs. full dim-8 analysis:

Partial Square

$$|g_{\rm eff,pr}^{\mathcal{Z},\psi}|^2_{\rm partial\,square} \supset \frac{g_1^2\, g_2^2\,(\tilde{C}_{HWB}^{(6)})^2}{(g_Z^{\rm SM})^6}\delta_{pr}\left[g_Z^{\rm SM}\,\langle g_{\rm eff,pr}^{\mathcal{Z},\psi}\rangle_{\rm SM} + (g_2^2 - g_1^2)\, Q_\psi)\right]^2$$

Complete Analysis

$$|g_{\rm eff,pr}^{\mathcal{Z},\psi}|^2_{\mathcal{O}(v^4/\Lambda^4)} \supset \frac{g_1^2\, g_2^2\,(\tilde{C}_{HWB}^{(6)})^2\,(g_2^2-g_1^2)^2\, Q_\psi^2}{(g_Z^{\rm SM})^6}\delta_{pr} + (\tilde{C}_{HWB}^{(6)})^2\langle g_{\rm eff,pr}^{\mathcal{Z},\psi}\rangle_{\rm SM}^2\delta_{pr}$$

*There are even *cancellations* such that term ~Q doesn't exist in full expansion…*

Towards PMNS Fits in the (geo)(ν)SMEFT

www.nu-fit.org

NuFIT 5.1 (2021)

	Normal Ordering (best fit)		Inverted Ordering ($\Delta\chi^2 = 2.6$)	
	bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
$\sin^2\theta_{12}$	$0.304^{+0.013}_{-0.012}$	$0.269 \to 0.343$	$0.304^{+0.012}_{-0.012}$	$0.269 \to 0.343$
$\theta_{12}/^\circ$	$33.44^{+0.77}_{-0.74}$	$31.27 \to 35.86$	$33.45^{+0.77}_{-0.74}$	$31.27 \to 35.87$
$\sin^2\theta_{23}$	$0.573^{+0.018}_{-0.023}$	$0.405 \to 0.620$	$0.578^{+0.017}_{-0.021}$	$0.410 \to 0.623$
$\theta_{23}/^\circ$	$49.2^{+1.0}_{-1.3}$	$39.5 \to 52.0$	$49.5^{+1.0}_{-1.2}$	$39.8 \to 52.1$
$\sin^2\theta_{13}$	$0.02220^{+0.00068}_{-0.00062}$	$0.02034 \to 0.02430$	$0.02238^{+0.00064}_{-0.00062}$	$0.02053 \to 0.02434$
$\theta_{13}/^\circ$	$8.57^{+0.13}_{-0.12}$	$8.20 \to 8.97$	$8.60^{+0.12}_{-0.12}$	$8.24 \to 8.98$
$\delta_{CP}/^\circ$	194^{+52}_{-25}	$105 \to 405$	287^{+27}_{-32}	$192 \to 361$

- Given complete flavor formalism(s) in the (geo)(ν)SMEFT, the natural project would be to do a precision fit to mass and mixing, as in CKM case.
- Re-absorption of BSM effects likely important in interpretation of neutrino NSI and associated bounds on new physics…
- Knowledge of matching and RGE to relevant neutrino processes required!