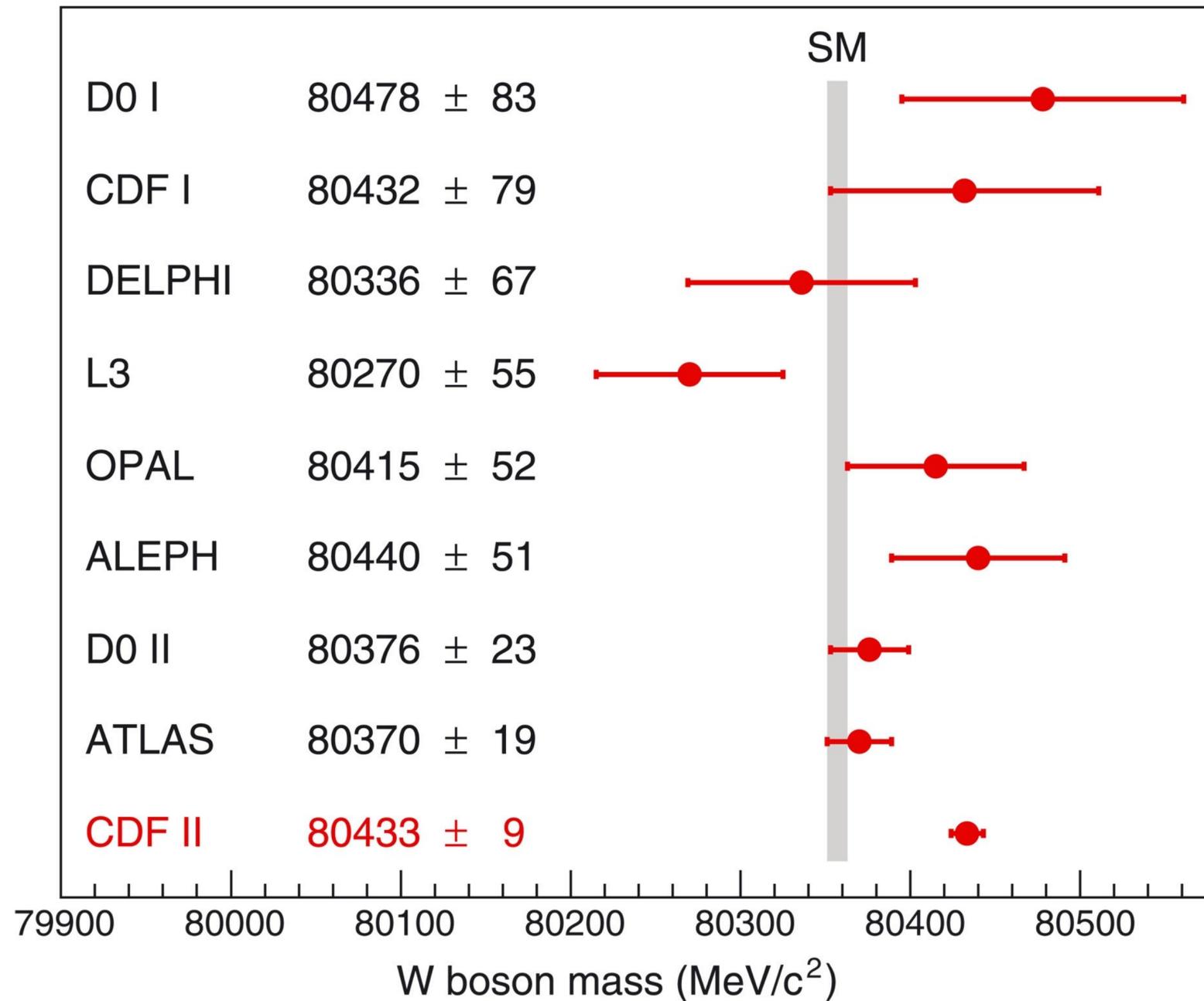


# W-mass and the Standard Model EFT

I adapted (**read: stole**) most of these slides from **Tom Tong**  
(postdoc in my group in U-Mass, now in Siegen)

*Jordy de Vries*  
*Theory Meets Experiment, June 3*

# The CDF W mass measurement



Shots to prevent cancer show early promise p. 126

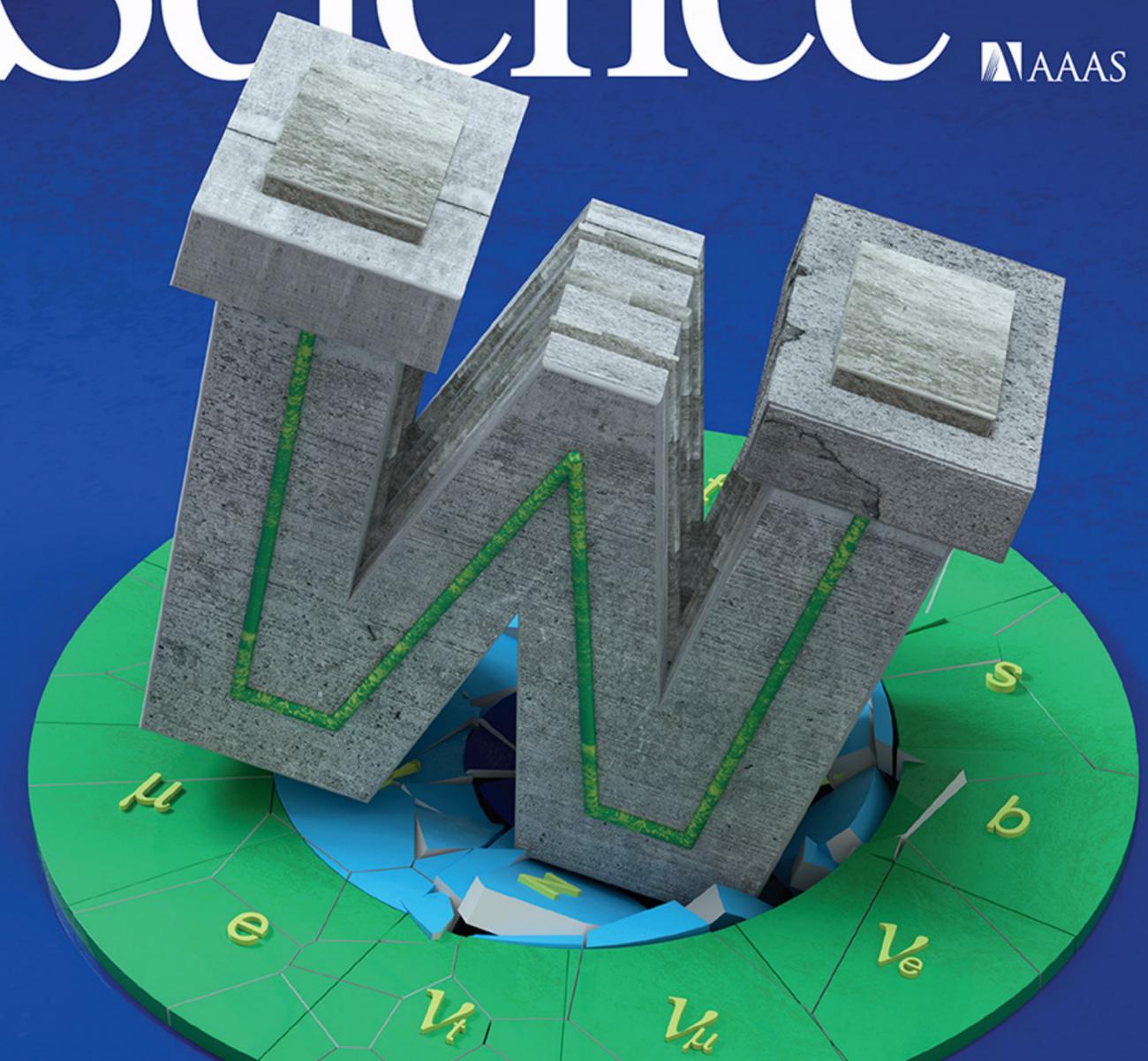
Visualizing a key step in cytokine signaling pp. 139 & 163

Silk-wrapped food wins BII & Science Prize p. 146

# Science

\$15  
8 APRIL 2022  
science.org

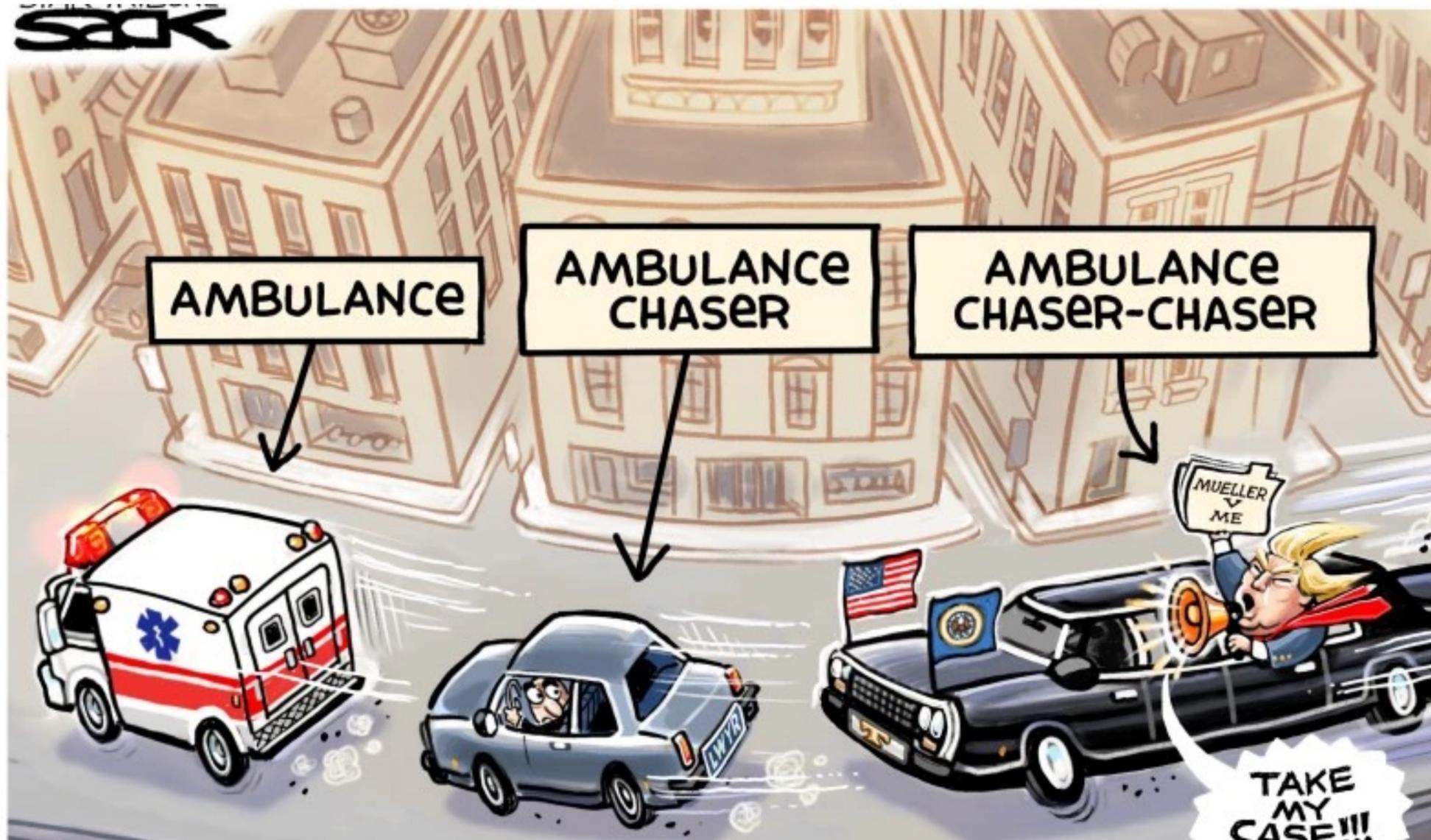
AAAS



## HEAVYWEIGHT

W boson mass measures higher than expected pp. 125, 136, & 170

# Wow! A new ambulance in town



- 131 citations as of this morning
- Although with some controversy, e.g. ResBos1 vs 2
- It's somebody's job (kind of...) to chase it
- What do people usually do?
- Basically in two ways

*A story of chasing the ambulance-chasers*

# Explaining an anomaly

- Step 1: Pick up a model you like, e.g. scalar triplet, 2HDM, yada yada
- Step 2: Calculate relevant observables (the tedious part although a lot is automatized nowadays)
- Step 3: Compare them with the experiments including the new  $W$  mass
- Step 4: Fit parameters and write paper
- Step 5: Collect citations and go to step 1

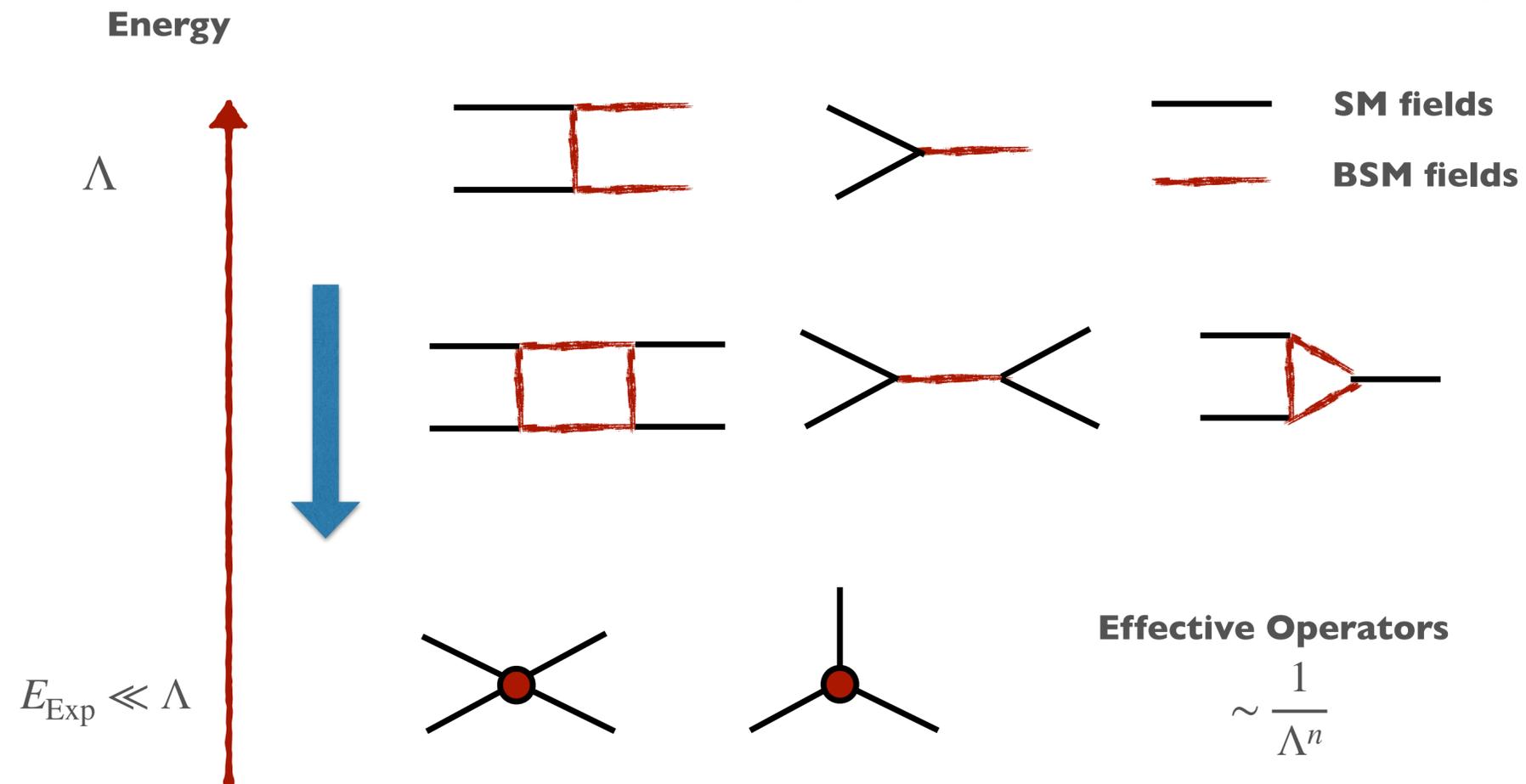


**LEGO Master Model Builder**

# The 'model-independent' way

- Step 1: Use the Standard Model EFT

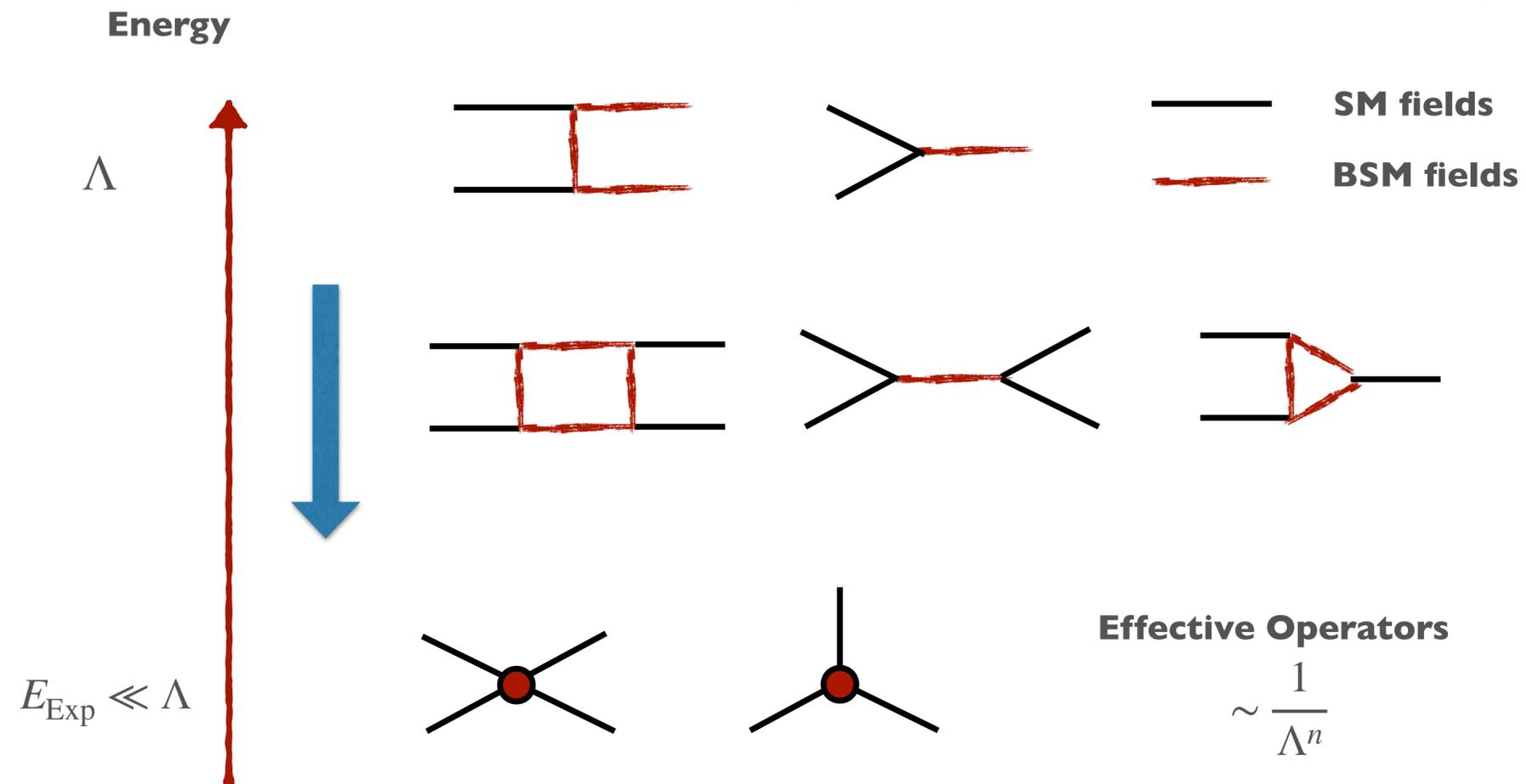
$$\mathcal{L}_{\text{SMEFT}}^{\text{dim-6}} = \mathcal{L}_{\text{SM}} + \sum_i C_i \mathcal{O}_i^{\text{dim-6}}$$



# The 'model-independent' way

- Step 1: Use the Standard Model EFT

$$\mathcal{L}_{\text{SMEFT}}^{\text{dim-6}} = \mathcal{L}_{\text{SM}} + \sum_i^{2499} C_i \mathcal{O}_i^{\text{dim-6}}$$



# The 'model-independent' way

- Step 1: use the Standard Model Effective Field Theory

$$\mathcal{L}_{\text{SMEFT}}^{\text{dim-6}} = \mathcal{L}_{\text{SM}} + \sum_i^{2499} C_i \mathcal{O}_i^{\text{dim-6}}$$

- ~~Step 2: Constrain all the Wilson coefficients with all the observables~~
- Step 2: Make some assumptions to simplify the SMEFT, say oblique, flavor universal, MFV, etc.
- Step 3: Choose **relevant** operators and relevant observables
- Step 4: Global fit (well global within assumptions)!
- But what are **relevant** operators and observables?

# Relevant to the W mass, of course!

|                          |  |
|--------------------------|--|
| $\mathcal{O}_{HWB}$      | $H^\dagger \tau^I H W_{\mu\nu}^I B^{\mu\nu}$                                     |
| $\mathcal{O}_{HD}$       | $ H^\dagger D_\mu H ^2$  |
| $\mathcal{O}_{Hl}^{(3)}$ | $(H^\dagger i \overleftrightarrow{D}_\mu^I H) (\bar{l}_p \tau^I \gamma^\mu l_r)$ |
| $\mathcal{O}_{Hq}^{(3)}$ | $(H^\dagger i \overleftrightarrow{D}_\mu^I H) (\bar{q}_p \tau^I \gamma^\mu q_r)$ |
| $\mathcal{O}_{ll}$       | $(\bar{l}_p \gamma_\mu l_r) (\bar{l}_s \gamma^\mu l_t)$                          |
| $\mathcal{O}_{lq}^{(3)}$ | $(\bar{l}_p \gamma_\mu l_r) (\bar{q}_s \gamma^\mu q_t)$                          |

$$C_i \sim \frac{1}{\Lambda^2}$$

- W mass is one of the EWPO

|   | Measurement           |
|---|-----------------------|
| $M_W$ [GeV]   | $80.413 \pm 0.015$    |
| $\Gamma_W$ [GeV]  | $2.085 \pm 0.042$     |
| $\sin^2 \theta_{\text{eff}}^{\text{lept}} (Q_{\text{FB}}^{\text{had}})$ | $0.2324 \pm 0.0012$   |
| $P_\tau^{\text{pol}} = \mathcal{A}_\ell$                                | $0.1465 \pm 0.0033$   |
| $\Gamma_Z$ [GeV]  | $2.4955 \pm 0.0023$   |
| $\sigma_h^0$ [nb]   | $41.480 \pm 0.033$    |
| $R_\ell^0$  | $20.767 \pm 0.025$    |
| $A_{\text{FB}}^{0,\ell}$  | $0.0171 \pm 0.0010$   |
| $\mathcal{A}_\ell$ (SLD)  | $0.1513 \pm 0.0021$   |
| $R_b^0$   | $0.21629 \pm 0.00066$ |
| $R_c^0$   | $0.1721 \pm 0.0030$   |
| $A_{\text{FB}}^{0,b}$   | $0.0996 \pm 0.0016$   |
| $A_{\text{FB}}^{0,c}$   | $0.0707 \pm 0.0035$   |
| $\mathcal{A}_b$   | $0.923 \pm 0.020$     |
| $\mathcal{A}_c$   | $0.670 \pm 0.027$     |
| $\mathcal{A}_s$   | $0.895 \pm 0.091$     |
| $\text{BR}_{W \rightarrow \ell \bar{\nu}_\ell}$                         | $0.10860 \pm 0.00090$ |
| $\sin^2 \theta_{\text{eff}}^{\text{lept}} (\text{HC})$                  | $0.23143 \pm 0.00025$ |
| $R_{uc}$  | $0.1660 \pm 0.0090$   |

- In SMEFT @ dim-6, W mass is mainly corrected by

$$\frac{\delta m_W^2}{m_W^2} = v^2 \frac{s_w c_w}{s_w^2 - c_w^2} \left[ \underbrace{2 C_{HWB}}_S + \frac{c_w}{2s_w} \underbrace{C_{HD}}_T + \frac{s_w}{c_w} \underbrace{\left( 2 C_{Hl}^{(3)} - C_{ll} \right)}_{GF} \right]$$

Trott et al '15 '16

# Universal/Obllique corrections

Peskin-Takeuchi, PRL 65, 964 (1990)

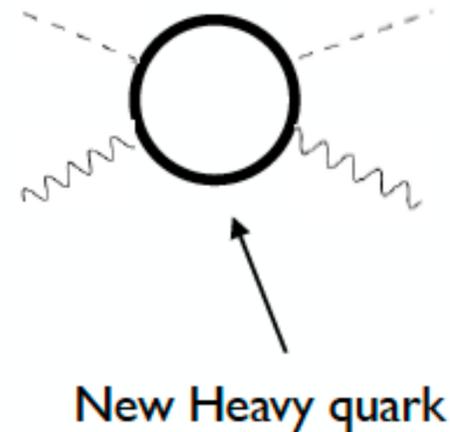
Barbieri-Pomarol-Rattazzi-Strumia hep-ph/0405040

Wells-Zhang, 1510.08462

- Universal new physics

$$\frac{\delta m_W^2}{m_W^2} = v^2 \frac{s_w c_w}{s_w^2 - c_w^2} \left[ 2 C_{HWB} + \frac{c_w}{2s_w} C_{HD} + \dots \right]$$

↓                      ↓  
**S**                      **T**



## “Universal theories”

- New physics couples to SM bosons, and / or to SM fermions through SM currents
- Consistent framework to analyze EW precision tests (oblique corrections, etc)
- Evade flavor constraints (Minimal Flavor Violation is automatic), scale can be low

# Universal/Obllique corrections

Peskin-Takeuchi, PRL 65, 964 (1990)

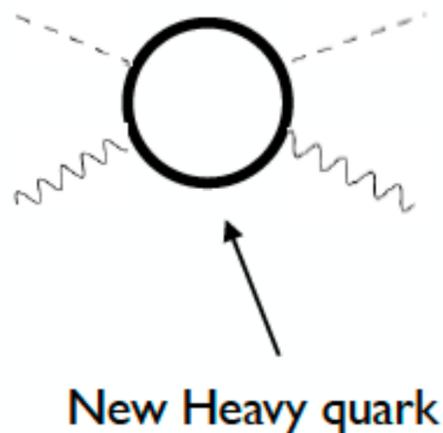
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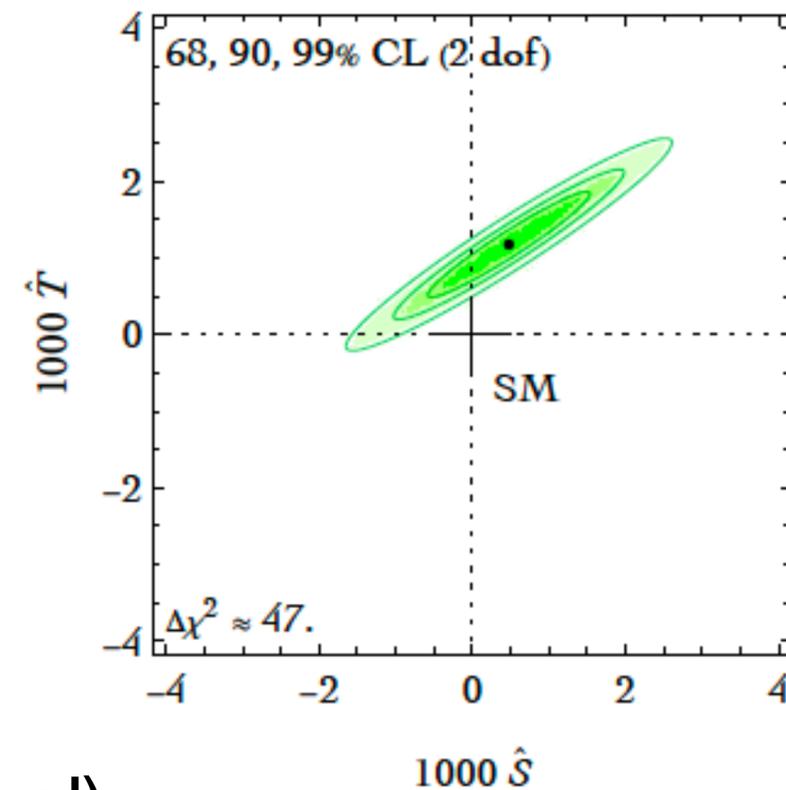
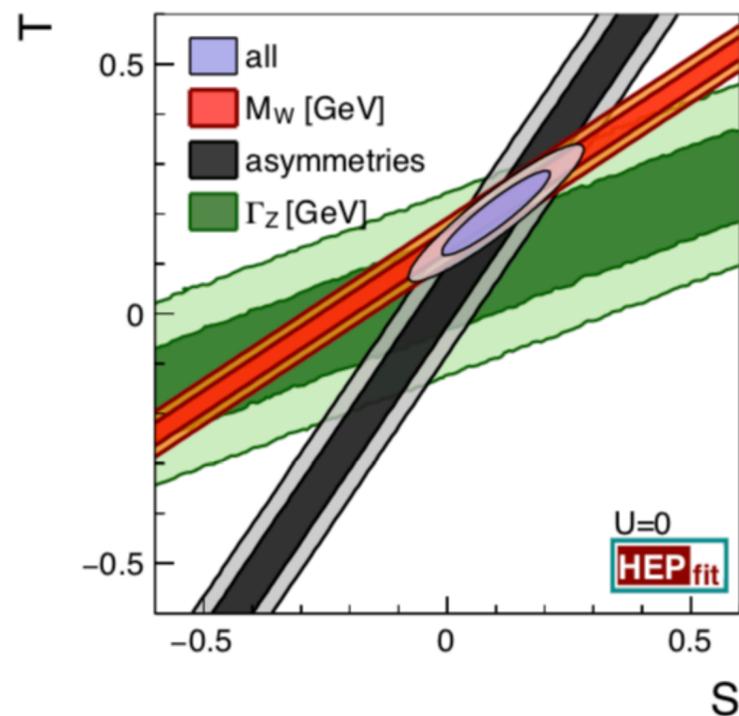
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“Universal theories”

- New physics couples to SM bosons, and / or to SM fermions through SM currents
- Consistent framework to analyze EW precision tests (oblique corrections, etc)
- Evade flavor constraints (Minimal Flavor Violation is automatic), scale can be low

| Model                   | Pred. $M_W$ [GeV]    | Pull         |
|-------------------------|----------------------|--------------|
| <i>standard average</i> |                      |              |
| SM                      | $80.3499 \pm 0.0056$ | $6.5 \sigma$ |
| ST                      | $80.366 \pm 0.029$   | $1.6 \sigma$ |



- Quite a few papers do this (results from Strumia and de Blas et al)

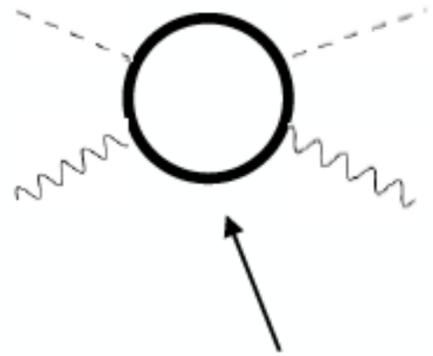
# Universal/Obllique corrections

Peskin-Takeuchi, PRL 65, 964 (1990)

Barbieri-Pomarol-Rattazzi-Strumia hep-ph/0405040

Wells-Zhang, 1510.08462

- Universal new physics



New Heavy quark

## “Universal theories”

- New physics couples to SM bosons, and / or to SM fermions through SM currents
- Consistent framework to analyze EW precision tests (oblique corrections, etc)
- Evade flavor constraints (Minimal Flavor Violation is automatic), scale can be low

- Does not fully explain the discrepancy (still  $\sim 2$  sigma left)

- The scale of new physics is at the level of a 5-7 TeV

$$C_i \sim \frac{1}{\Lambda^2}$$

- Strumia: this means it must be **tree-level new physics (many models on arxiv e.g. Z', little Higgs, etc)**

- Otherwise the new particles would be O(300 GeV) and thus should have been seen at the LHC (your mileage may vary in ‘tuned’ models)

$$C'_i \sim \frac{1}{\Lambda^2} \frac{\alpha_w}{4\pi}$$

# Flavor universal assumption

- Under Flavor ‘universality’ or MFV

$$U(3)_q \times U(3)_u \times U(3)_d \times U(3)_l \times U(3)_e$$

- There are 8 combinations of SMEFT operators relevant to the EWPO
- This includes the ‘S’ and ‘T’ operators and operators that couple to fermions

Impact of the recent measurements of the top-quark and W-boson masses on electroweak precision fits #1  
J. de Blas (CAFPE, Granada and Granada U.), M. Pierini (CERN), L. Reina (Florida State U.), L. Silvestrini (INFN, Rome) (Apr 8, 2022)  
e-Print: [2204.04204](#) [hep-ph]  
[pdf](#) [cite](#) ↻ 71 citations

$$\hat{C}_{\varphi f}^{(1)} = C_{\varphi f}^{(1)} - \frac{Y_f}{2} C_{\varphi D}, \quad f = l, q, e, u, d,$$
$$\hat{C}_{\varphi f}^{(3)} = C_{\varphi f}^{(3)} + \frac{c_w^2}{4s_w^2} C_{\varphi D} + \frac{c_w}{s_w} C_{\varphi WB}, \quad f = l, q,$$

# Flavor universal assumption

|                             | Result             | Correlation Matrix                                  |       |       |       |       |       |       |      |  |
|-----------------------------|--------------------|---|-------|-------|-------|-------|-------|-------|------|--|
|                             |                    | (IC <sub>SMEFT</sub> /IC <sub>SM</sub> = 31.8/80.2) |       |       |       |       |       |       |      |  |
| $\hat{C}_{\varphi l}^{(1)}$ | $-0.007 \pm 0.011$ | 1.00  |       |       |       |       |       |       |      |  |
| $\hat{C}_{\varphi l}^{(3)}$ | $-0.042 \pm 0.015$ | -0.68   | 1.00  |       |       |       |       |       |      |  |
| $\hat{C}_{\varphi e}$       | $-0.017 \pm 0.009$ | 0.48  | 0.04  | 1.00  |       |       |       |       |      |  |
| $\hat{C}_{\varphi q}^{(1)}$ | $-0.018 \pm 0.044$ | -0.02   | -0.06 | -0.13 | 1.00  |       |       |       |      |  |
| $\hat{C}_{\varphi q}^{(3)}$ | $-0.113 \pm 0.043$ | -0.03   | 0.04  | -0.16 | -0.37 | 1.00  |       |       |      |  |
| $\hat{C}_{\varphi u}$       | $0.090 \pm 0.150$  | 0.06  | -0.04 | 0.04  | 0.61  | -0.77 | 1.00  |       |      |  |
| $\hat{C}_{\varphi d}$       | $-0.630 \pm 0.250$ | -0.13   | -0.05 | -0.30 | 0.40  | 0.58  | -0.04 | 1.00  |      |  |
| $\hat{C}_{ll}$              | $-0.022 \pm 0.028$ | -0.80   | 0.95  | -0.10 | -0.06 | -0.01 | -0.04 | -0.05 | 1.00 |  |

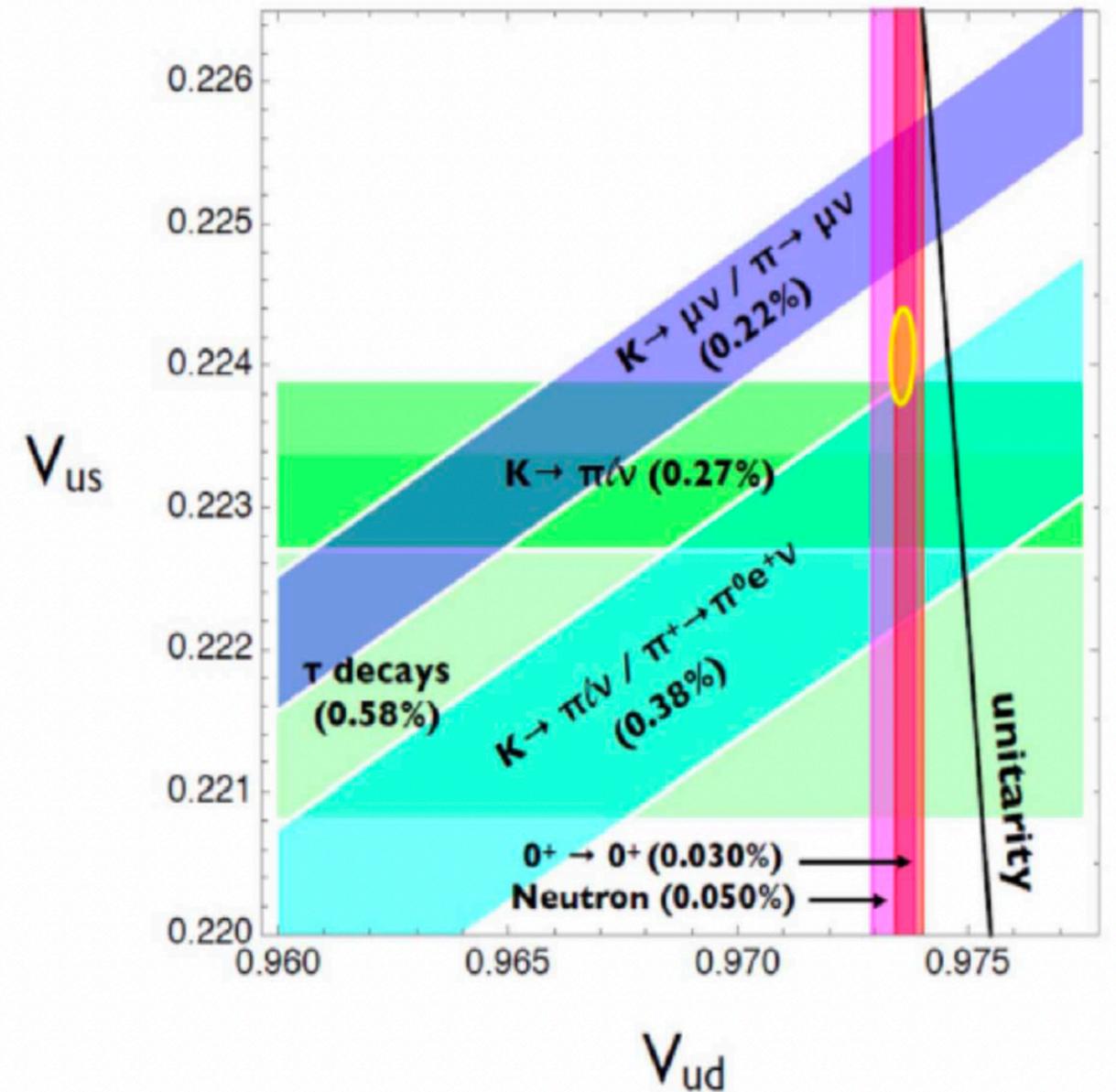
- The preferred ‘solution’ is rather different than just S and T
- This would be the guide for model building: try to build models consistent with these values (beyond ‘universal models’ link to other anomalies?)
- **But can one treat the EWPO in isolation ?**

# First-row CKM unitarity

$$\Delta_{\text{CKM}} \equiv |V_{ud}|^2 + |V_{us}|^2 - 1$$

- $V_{ud}$  and  $V_{us}$  are obtained from nuclear beta decay and Kaon decays.
- Requires detailed understanding of radiative corrections (very interesting, you should really read my papers, but not for today)
- Very precise determinations are in tension with CKM unitarity

$$\Delta_{\text{CKM}}^{\text{PDG}} \approx - (0.15 \pm 0.06) \%$$



# First-row CKM in SMEFT (with MFV)

Beta-decay implications for the  $W$ -boson mass anomaly

---

Vincenzo Cirigliano,<sup>a</sup> Wouter Dekens,<sup>a</sup> Jordy de Vries,<sup>b,c</sup> Emanuele Mereghetti,<sup>d</sup> Tom Tong<sup>e</sup>

<sup>a</sup>*Institute for Nuclear Theory, University of Washington, Seattle WA 91195-1550, USA*

<sup>b</sup>*Institute for Theoretical Physics Amsterdam and Delta Institute for Theoretical Physics, University of Amsterdam, Science Park 904, 1098 XH Amsterdam, The Netherlands*

<sup>c</sup>*Nikhef, Theory Group, Science Park 105, 1098 XG, Amsterdam, The Netherlands*

<sup>d</sup>*Theoretical Division, Los Alamos National Laboratory, Los Alamos, NM 87545, USA*

<sup>e</sup>*Center for Particle Physics Siegen, University of Siegen, 57068 Siegen, Germany*

2204.08440

$$\Delta_{\text{CKM}} \equiv |V_{ud}|^2 + |V_{us}|^2 - 1$$

$$= 2 \frac{v^2}{\Lambda^2} \left[ C_{Hq}^{(3)} - C_{Hl}^{(3)} + C_{ll} - C_{lq}^{(3)} \right]$$

$C_{\Delta}$

where  $C_{lq}^{(3)}$  is irrelevant to the EWPO and does not play a role in the EWPO fit

- We combine the relevant Wilson coefficients into  $C_{\Delta}$
- Same couplings enter CKM and  $W$ -mass !

# Oops!

- From the re-fit, we obtain a large, %-level, deviation from the first-row CKM unitarity

$$\Delta_{CKM}^{fit} \approx - (1 \pm 0.4) \%$$

- Based on up-to-date predictions of  $0^+ \rightarrow 0^+$  nuclear beta-decays and Kaon decays, the PDG average indicates that

$$\Delta_{CKM}^{PDG} \approx - (0.15 \pm 0.06) \%$$

- Refitting while including CKM shifts the values of the LECs  $\rightarrow$  Would point to other models !

|                             | Result              | Result with CKM    |
|-----------------------------|---------------------|--------------------|
| $\hat{C}_{\varphi l}^{(1)}$ | $-0.007 \pm 0.011$  | $-0.013 \pm 0.009$ |
| $\hat{C}_{\varphi l}^{(3)}$ | $-0.042 \pm 0.015$  | $-0.034 \pm 0.014$ |
| $\hat{C}_{\varphi e}$       | $-0.017 \pm 0.009$  | $-0.021 \pm 0.009$ |
| $\hat{C}_{\varphi q}^{(1)}$ | $-0.0181 \pm 0.044$ | $-0.048 \pm 0.04$  |
| $\hat{C}_{\varphi q}^{(3)}$ | $-0.114 \pm 0.043$  | $-0.041 \pm 0.015$ |
| $\hat{C}_{\varphi u}$       | $0.086 \pm 0.154$   | $-0.12 \pm 0.11$   |
| $\hat{C}_{\varphi d}$       | $-0.626 \pm 0.248$  | $-0.38 \pm 0.22$   |
| $C_{\Delta}$                | $-0.19 \pm 0.09$    | $-0.027 \pm 0.011$ |

# Let's include more high energy data: 2204.05260

## SMEFT Analysis of $m_W$

Emanuele Bagnaschi,<sup>a</sup> John Ellis,<sup>b,a,c</sup> Maeve Madigan,<sup>d</sup> Ken Mimasu,<sup>b</sup>  
Veronica Sanz<sup>e,f</sup> and Tevong You<sup>b,d,g</sup>

<sup>a</sup>Theoretical Physics Department, CERN, CH-1211 Geneva 23, Switzerland

<sup>b</sup>Theoretical Particle Physics and Cosmology Group, Department of Physics,  
King's College London, London WC2R 2LS, UK

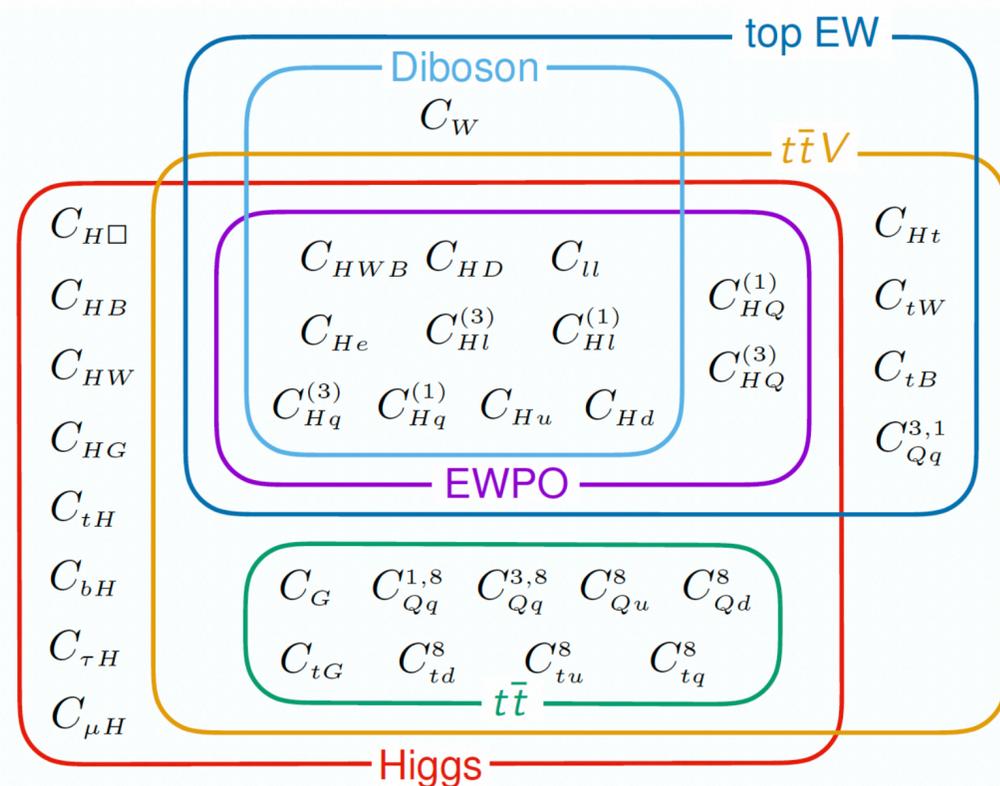
<sup>c</sup>National Institute of Chemical Physics & Biophysics, R vala 10, 10143 Tallinn, Estonia

<sup>d</sup>DAMTP, University of Cambridge, Wilberforce Road, Cambridge CB3 0WA, UK

<sup>e</sup>Instituto de F sica Corpuscular (IFIC), Universidad de Valencia-CSIC, E-46980 Valencia, Spain

<sup>f</sup>Department of Physics and Astronomy, University of Sussex, Brighton BN1 9QH, UK

<sup>g</sup>Cavendish Laboratory, University of Cambridge, J.J. Thomson Avenue, Cambridge CB3 0HE,  
UK



- EWPO + Diboson + Top + Higgs
- More observables, more relevant operators
- Global-fit with 20 operators (flavor universal)
- Well, the same percent-level CKM unitarity violation
- Also if one includes other flavor assumptions (Zupan et al)

# Conclusion (not really...)

- A SMEFT global-fit including only the high energy data will cause %-level damage to the first-row CKM unitarity.
- One has to include low-energy beta decay data into the fits .

*Unitarity*

*Damaged!*

# Is it really $W$ mass the perpetrator?

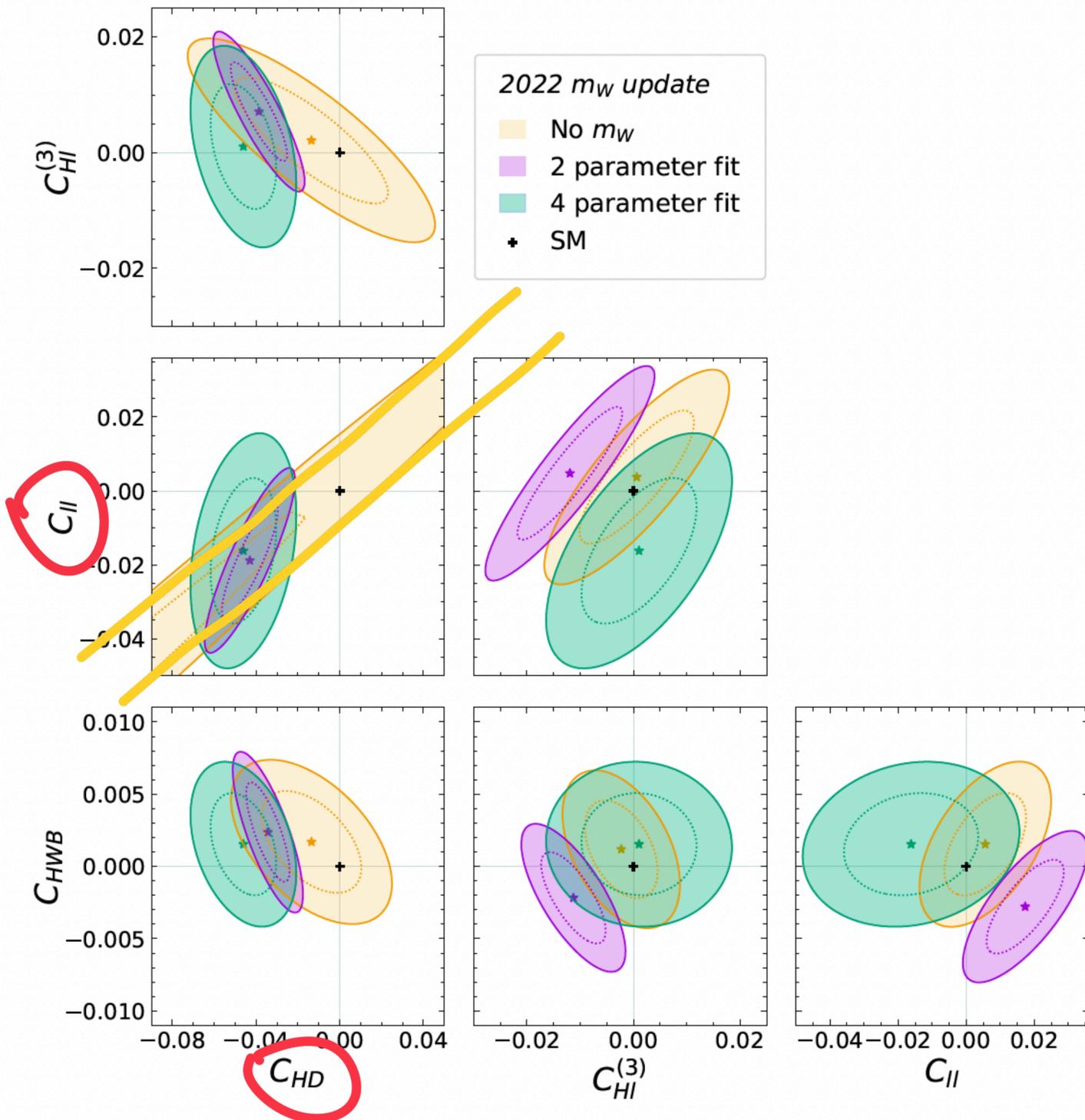
- If not, then the global-fit should be in bad tension with CKM even before the new CDF results
- We redid the old EWPT fits
- It was only  $-(0.4 \pm 0.4) \%$  in 0908.1754
- And a similar value indicated by 2012.02779, which is the old version of the 20-parameter fit
- It seems that roughly about half of the deviation was already there, and the CDF  $W$  mass has doubled that.

THE  
PERPETRATORS



$$\Delta_{CKM}^{fit} \approx -(1 \pm 0.4) \%$$

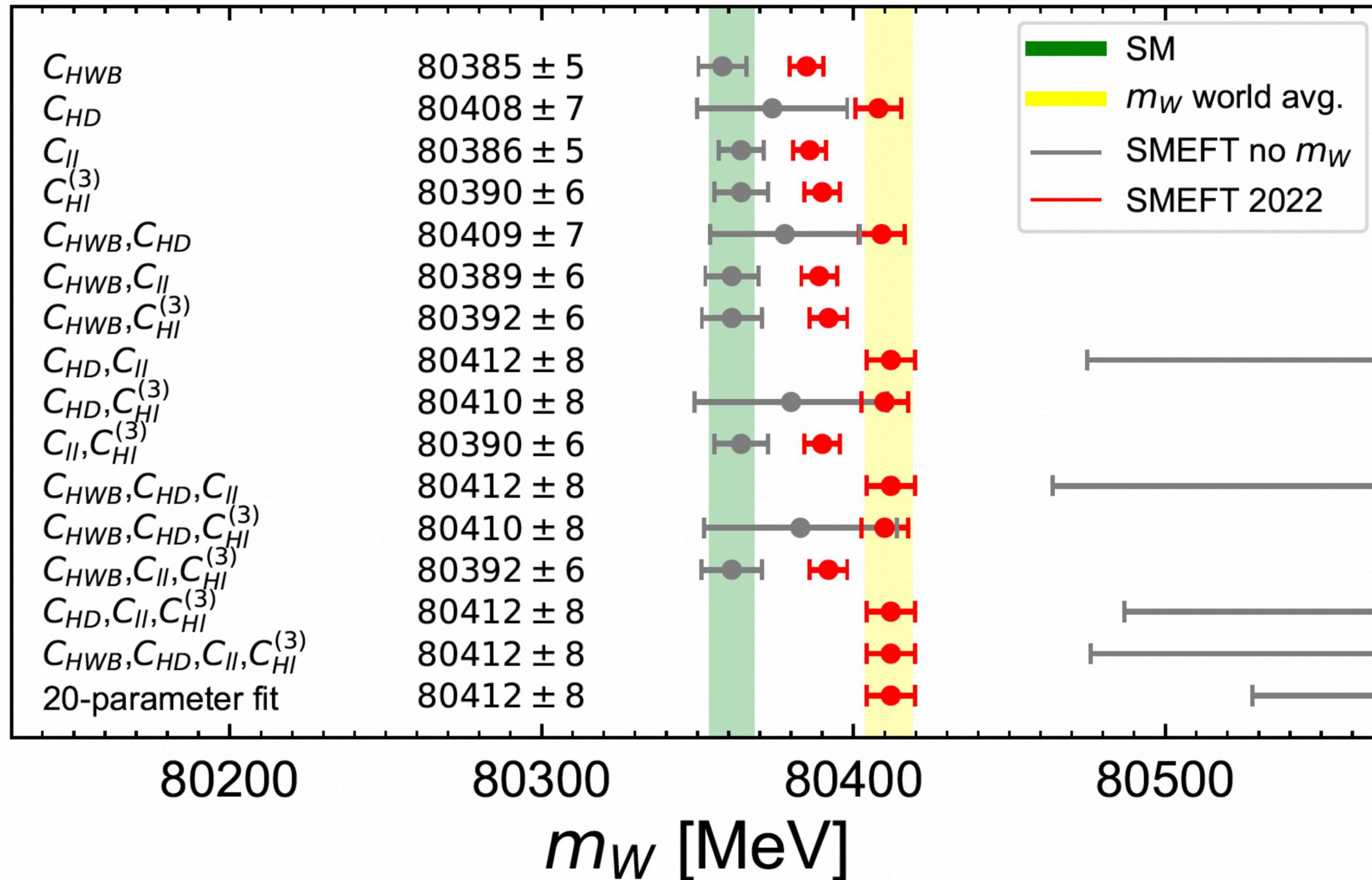
# Flat directions



$$\frac{\delta m_W^2}{m_W^2} = v^2 \frac{s_w c_w}{s_w^2 - c_w^2} \left[ 2 C_{HWB} + \frac{c_w}{2s_w} C_{HD} + \frac{s_w}{c_w} \left( 2 C_{HI}^{(3)} - C_{II} \right) \right]$$

- Fitting to the all EWPO but the W-mass, there exists an almost flat direction involving  $C_{HD}$  and  $C_{II}$
- It can only be lifted by the W mass
- The value of W mass largely dominates the constraints on  $C_{HD}$  and  $C_{II}$  along this flat direction

# Flat directions 2



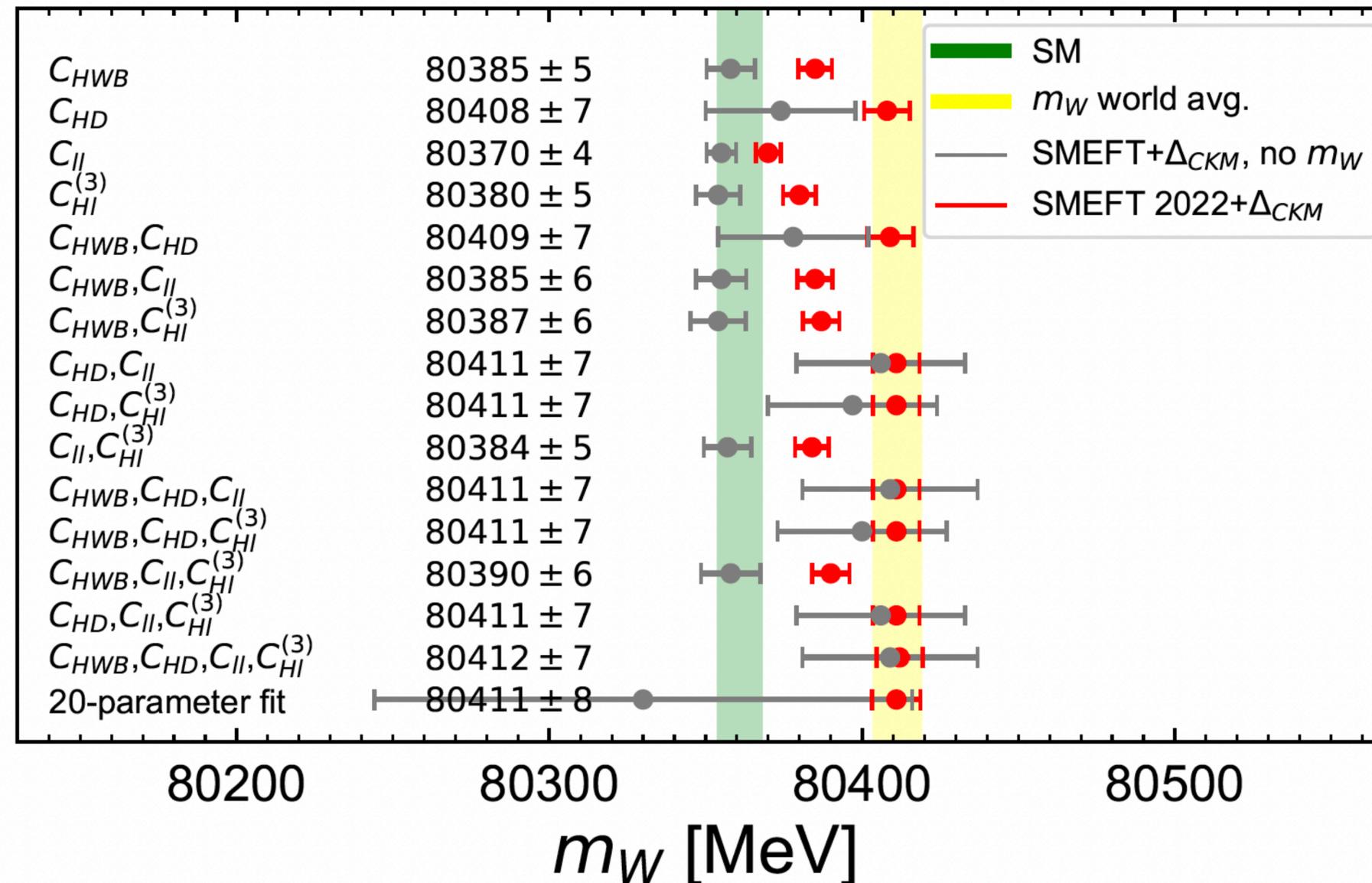
- Grey bars: Fitting results to the high energy data but *without* W mass



- Not even compatible with the real W mass at all, if both  $C_{HD}$  and  $C_{II}$  are present

# Flat directions III

- The fits are now updated in v2 of 2204.05260



“

 $C_{lq}^{(3)}$ 

•

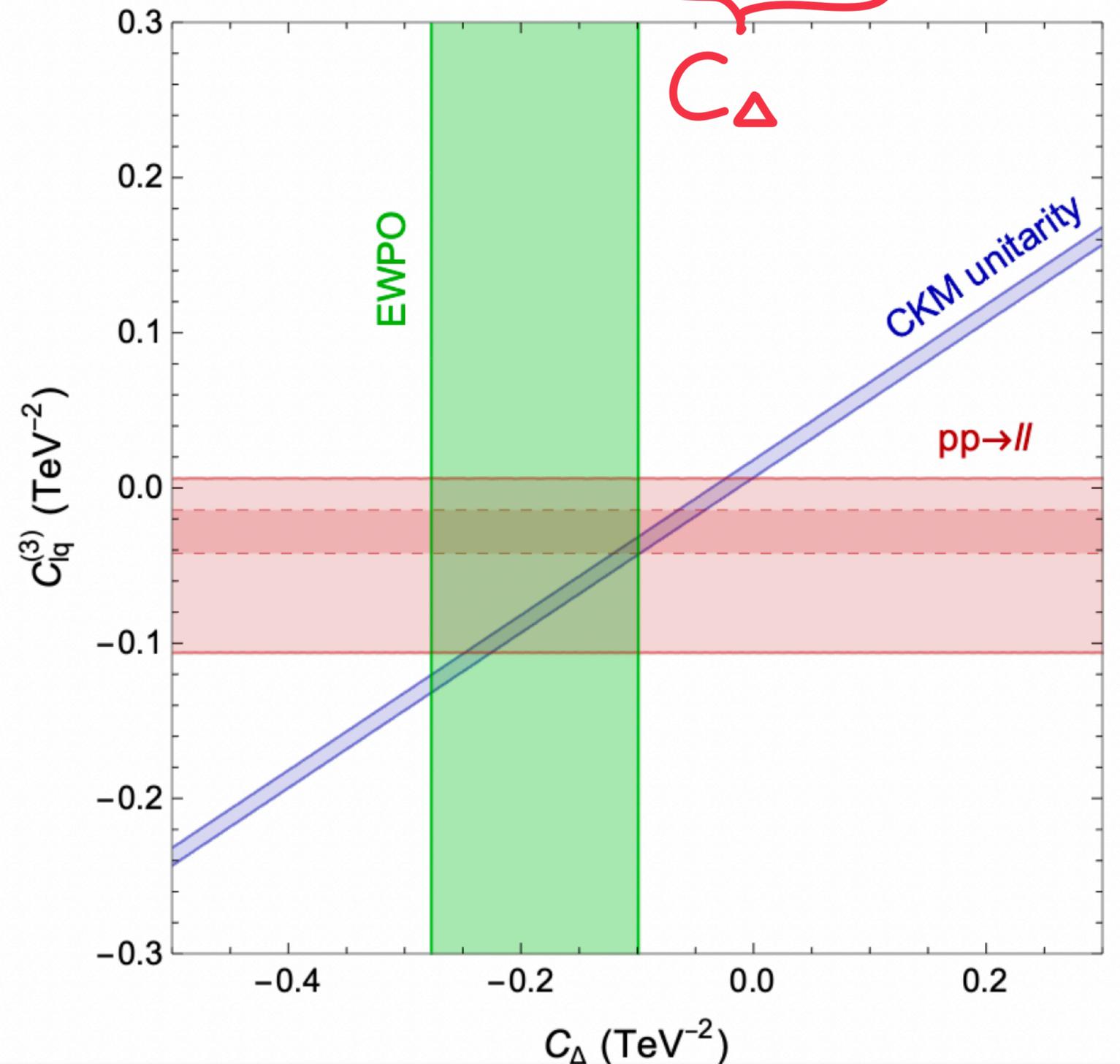


”

$$\Delta_{\text{CKM}} \equiv |V_{ud}|^2 + |V_{us}|^2 - 1$$

$$= 2 \frac{v^2}{\Lambda^2} \left[ C_{Hq}^{(3)} - C_{Hl}^{(3)} + C_{ll} - C_{lq}^{(3)} \right]$$

- We may effectively decouple the CKM from EWPO by a non-zero  $C_{lq}^{(3)}$
- $C_{lq}^{(3)}$  is constrained by 8 TeV  $pp \rightarrow ll$  data at the LHC
- Could be tested by 13 TeV data
- And also at the HL-LHC



# Conclusion (for real)

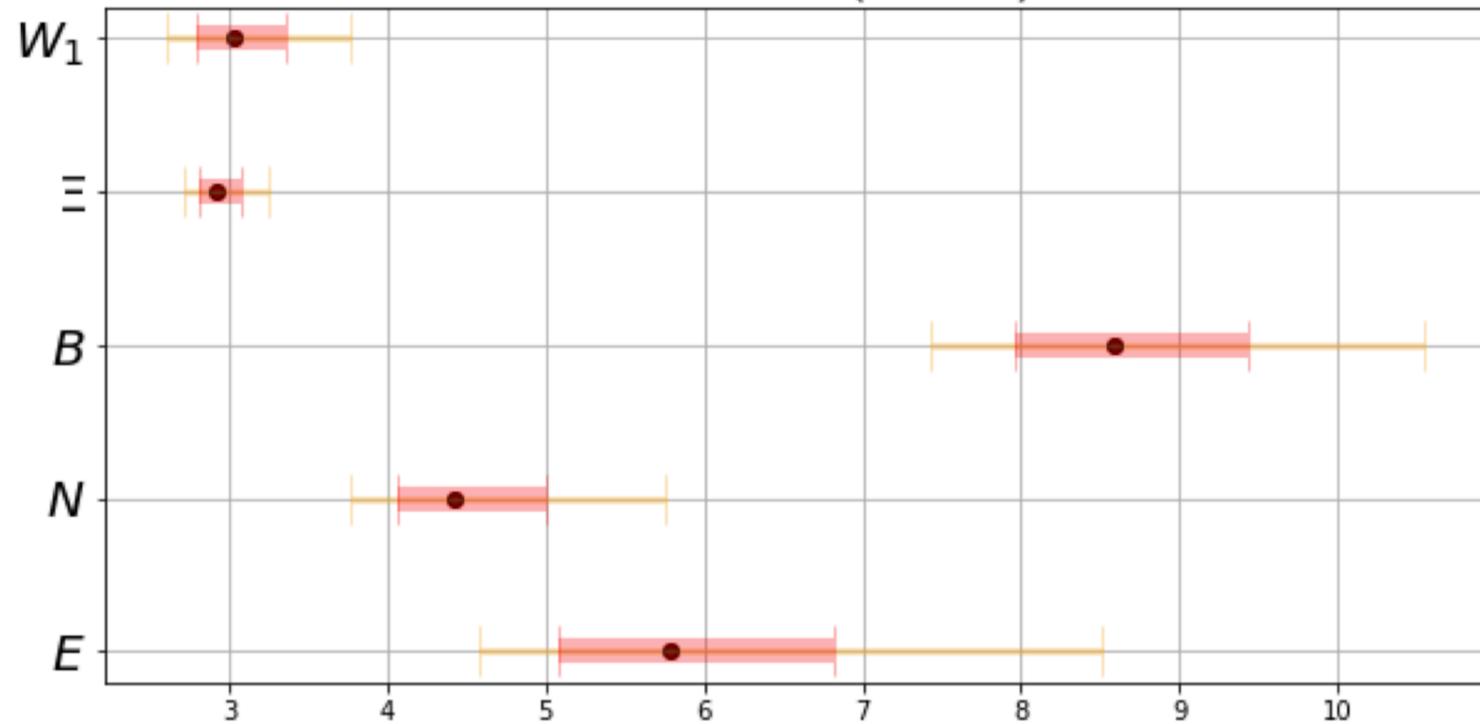
- SMEFT global-fits including only **high energy** data seems to damage **CKM** unitarity
- **Low energy** data is important because they can help lift some of the **flat directions**
- But adding more observables requires more SM-EFT operators and thus more observables and thus more SM-EFT operators and thus more.....
- Would like to make a flavorful global analysis (working in progress...) Also including  $g-2$ , flavor anomalies, etc.



| Model      | Spin          | SU(3) | SU(2) | U(1) | Parameters                           |
|------------|---------------|-------|-------|------|--------------------------------------|
| $S_1$      | 0             | 1     | 1     | 1    | $(M_S, \kappa_S)$                    |
| $\Sigma$   | $\frac{1}{2}$ | 1     | 3     | 0    | $(M_\Sigma, \lambda_\Sigma)$         |
| $\Sigma_1$ | $\frac{1}{2}$ | 1     | 3     | -1   | $(M_{\Sigma_1}, \lambda_{\Sigma_1})$ |
| $N$        | $\frac{1}{2}$ | 1     | 1     | 0    | $(M_N, \lambda_N)$                   |
| $E$        | $\frac{1}{2}$ | 1     | 1     | -1   | $(M_E, \lambda_E)$                   |
| $B$        | 1             | 1     | 1     | 0    | $(M_B, \hat{g}_H^B)$                 |
| $B_1$      | 1             | 1     | 1     | 1    | $(M_{B_1}, \lambda_{B_1})$           |
| $\Xi$      | 0             | 1     | 3     | 0    | $(M_\Xi, \kappa_\Xi)$                |
| $W_1$      | 1             | 1     | 3     | 1    | $(M_{W_1}, \hat{g}_{W_1}^\varphi)$   |
| $W$        | 1             | 1     | 3     | 0    | $(M_W, \hat{g}_W^H)$                 |

| Model      | $C_{HD}$                            | $C_{U}$ | $C_{Hl}^{(3)}$ | $C_{Hl}^{(1)}$  | $C_{He}$ | $C_{H\Box}$                                  | $C_{\tau H}$                            | $C_{tH}$                             | $C_{bH}$                             |
|------------|-------------------------------------|---------|----------------|-----------------|----------|--|---|--------------------------------------|--------------------------------------|
| $S_1$      |                                     | -1      |                |                 |          |  |   |                                      |                                      |
| $\Sigma$   |                                     |         | $\frac{1}{16}$ | $\frac{3}{16}$  |          |  | $\frac{y_\tau}{4}$                      |                                      |                                      |
| $\Sigma_1$ |                                     |         | $\frac{1}{16}$ | $-\frac{3}{16}$ |          |  | $\frac{y_\tau}{8}$                      |                                      |                                      |
| $N$        |                                     |         | $-\frac{1}{4}$ | $\frac{1}{4}$   |          |  |   |                                      |                                      |
| $E$        |                                     |         | $-\frac{1}{4}$ | $-\frac{1}{4}$  |          |  | $\frac{y_\tau}{2}$                      |                                      |                                      |
| $B_1$      | 1                                   |         |                |                 |          | $-\frac{1}{2}$                               | $-\frac{y_\tau}{2}$                     | $-\frac{y_t}{2}$                     | $-\frac{y_b}{2}$                     |
| $B$        | -2                                  |         |                |                 |          |  | $-y_\tau$                               | $-y_t$                               | $-y_b$                               |
| $\Xi$      | $-2 \left(\frac{1}{M_\Xi}\right)^2$ |         |                |                 |          | $\frac{1}{2} \left(\frac{1}{M_\Xi}\right)^2$ | $y_\tau \left(\frac{1}{M_\Xi}\right)^2$ | $y_t \left(\frac{1}{M_\Xi}\right)^2$ | $y_b \left(\frac{1}{M_\Xi}\right)^2$ |
| $W_1$      | $-\frac{1}{4}$                      |         |                |                 |          | $-\frac{1}{8}$                               | $-\frac{y_\tau}{8}$                     | $-\frac{y_t}{8}$                     | $-\frac{y_b}{8}$                     |
| $W$        | $\frac{1}{2}$                       |         |                |                 |          | $-\frac{1}{2}$                               | $-y_\tau$                               | $-y_t$                               | $-y_b$                               |

Mass limits (in TeV)



| Model | Pull | Best-fit mass (TeV) | 1- $\sigma$ mass range (TeV) | 2- $\sigma$ mass range (TeV) | 1- $\sigma$ coupling <sup>2</sup> range |
|-------|------|---------------------|------------------------------|------------------------------|---|
| $W_1$ | 6.4  | 3.0                 | [2.8, 3.6]                   | [2.6, 3.8]                   | [0.09, 0.13]                            |
| $B$   | 6.4  | 8.6                 | [8.0, 9.4]                   | [7.4, 10.6]                  | [0.011, 0.016]                          |
| $\Xi$ | 6.4  | 2.9                 | [2.8, 3.1]                   | [2.7, 3.2]                   | [0.011, 0.016]                          |
| $N$   | 5.1  | 4.4                 | [4.1, 5.0]                   | [3.8, 5.8]                   | [0.040, 0.060]                          |
| $E$   | 3.5  | 5.8                 | [5.1, 6.8]                   | [4.6, 8.5]                   | [0.022, 0.039]                          |

- These models induce too large CKM unitarity violation