Testing General Relativity with gravitational waves higher order modes

Anna Puecher

C. Kalaghatgi, S. Roy, Y. Setyawati, C. van den Broeck

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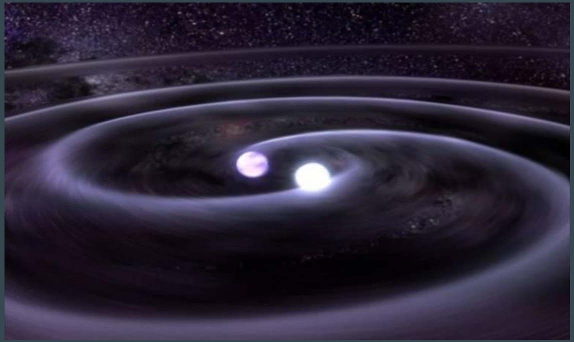
Introduction

- Gravitational waves predicted by General Relativity
- Mass acceleration:

  different sources, main Compact Binaries Coalescence



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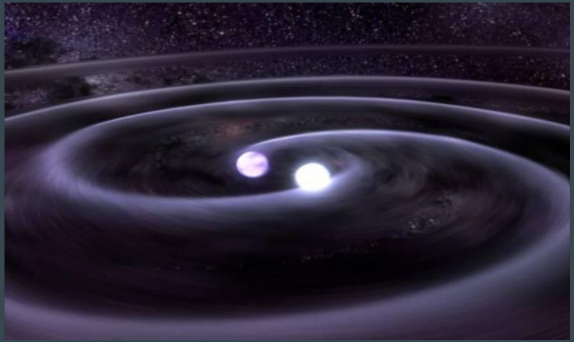

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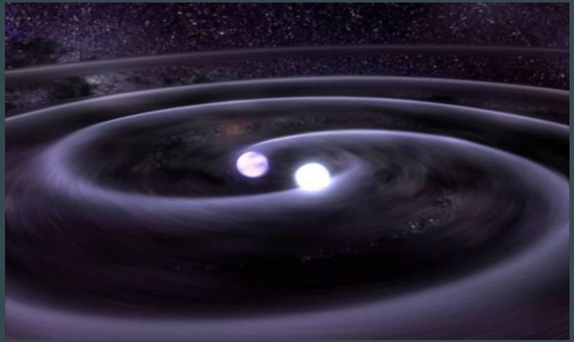

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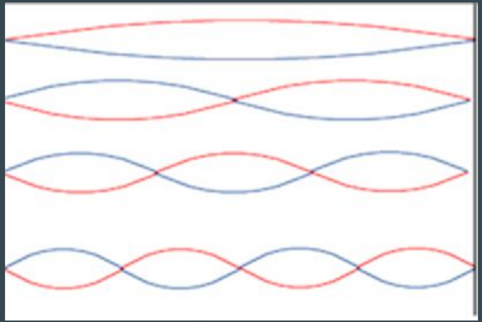
- Dominant quadrupolar mode, but also **higher multipoles**

  $$M^{i_1 i_2 \ldots i_N} \equiv \frac{1}{c^2}\int d^3\boldsymbol{x} T^{00} x^{i_1} x^{i_2} \ldots x^{i_N}$$

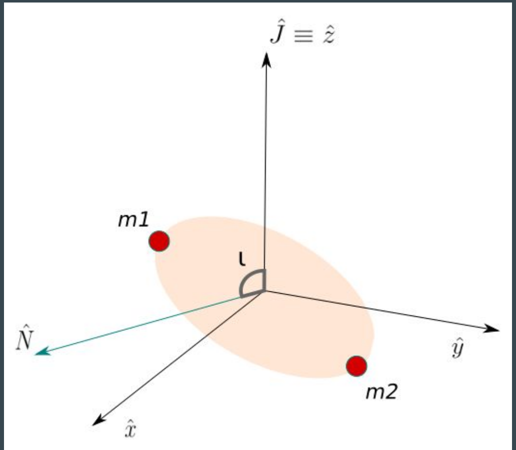


- Complete description

  $$h(t;\iota,\phi_0,\vec{\lambda}) = \sum_{l=2}^{\infty}\sum_{m=-l}^{l} Y_{-2}^{lm}(\iota,\phi_0)h_{lm}(t;\vec{\lambda})$$

Introduction

- Strength of higher harmonics depends on:
  - total mass $M = m_1 + m_2$
  - inclination angle ι
  - relative mass ratio $\Delta = (m_1 - m_2)/M$



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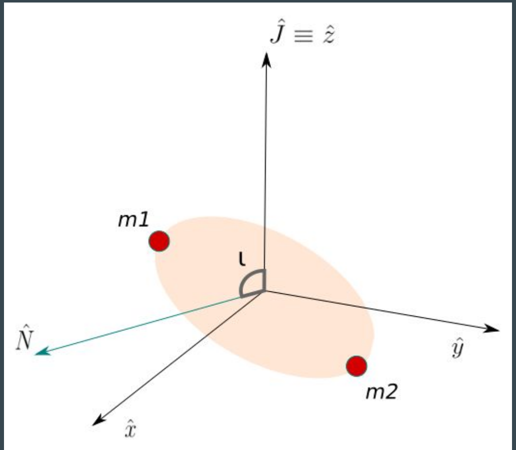

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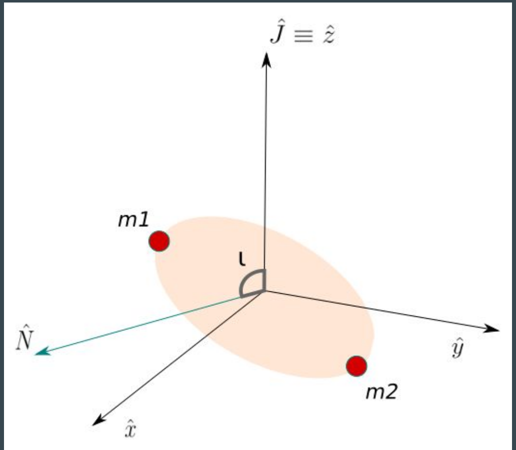

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- Already detected!

GW190412

$m_1 \sim 30 M_\odot \quad m_2 \sim 8 M_\odot$

GW190814

$m_1 \sim 23 M_\odot \quad m_2 \sim 2.6 M_\odot$

Testing GR with Higher Order Modes

$$h(t;\iota,\phi_0,\vec{\lambda}) = \sum_{l=2}^{\infty}\sum_{m=-l}^{l} Y_{-2}^{lm}(\iota,\phi_0)h_{lm}(t;\vec{\lambda})$$

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$$\Downarrow$$

$$h(t;\iota,\phi_0,\vec{\lambda}) = \sum_{\pm 2} Y_{-2}^{2m}(\iota,\phi_0) h_{2m}(t;\vec{\lambda})$$

$$+ \sum_{HOM} \sum_{m=-l}^{m=l} (1 + c_{lm}) Y_{-2}^{lm}(\iota,\phi_0) h_{lm}(t;\vec{\lambda})$$

GR: $c_{lm} = 0$

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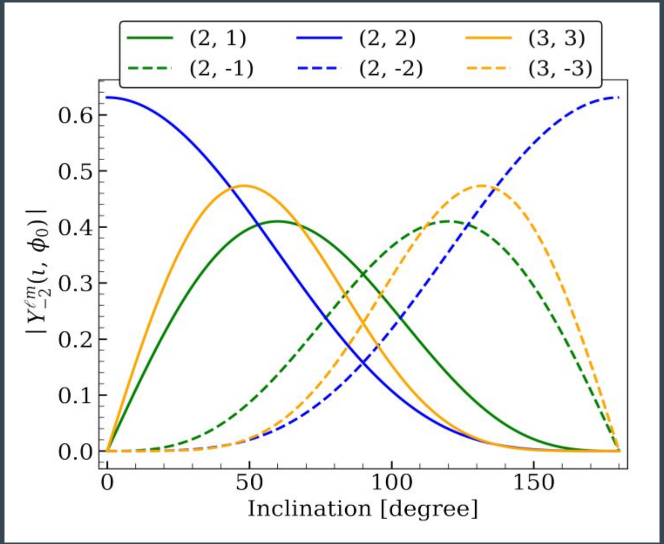
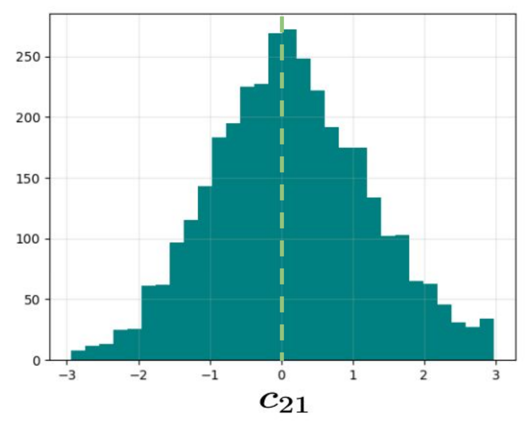


GR: $c_{lm} = 0$

Fundamental mode (l,m) = (2,2)

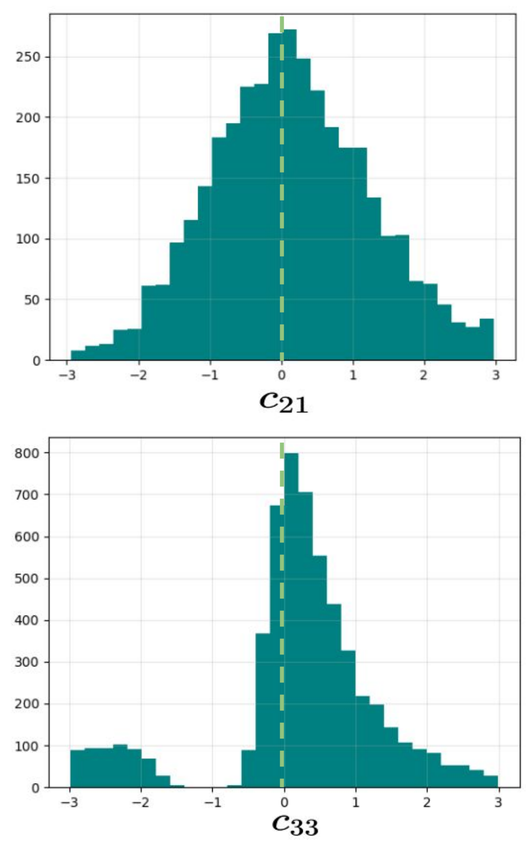
For this test (l,m) = (2,1) and (l,m) = (3,3) modes used

Example of posteriors



Measure c_{21} and c_{33} together with the other parameters

Example of posteriors



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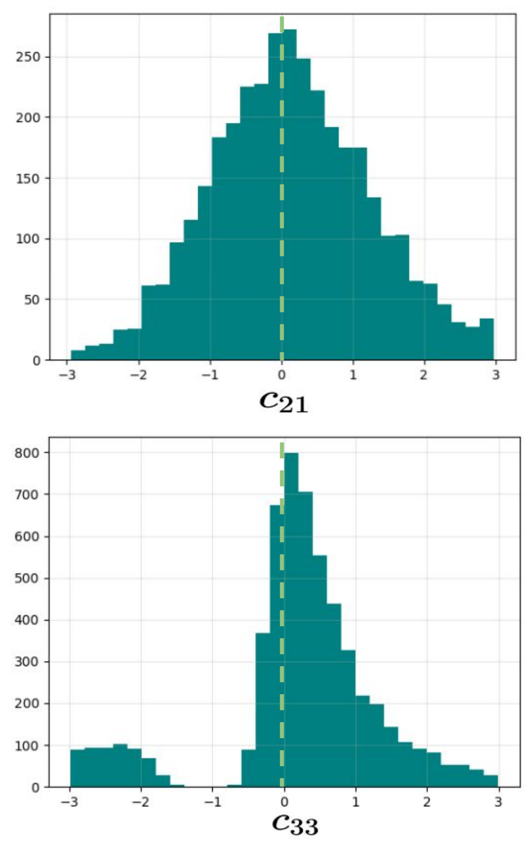

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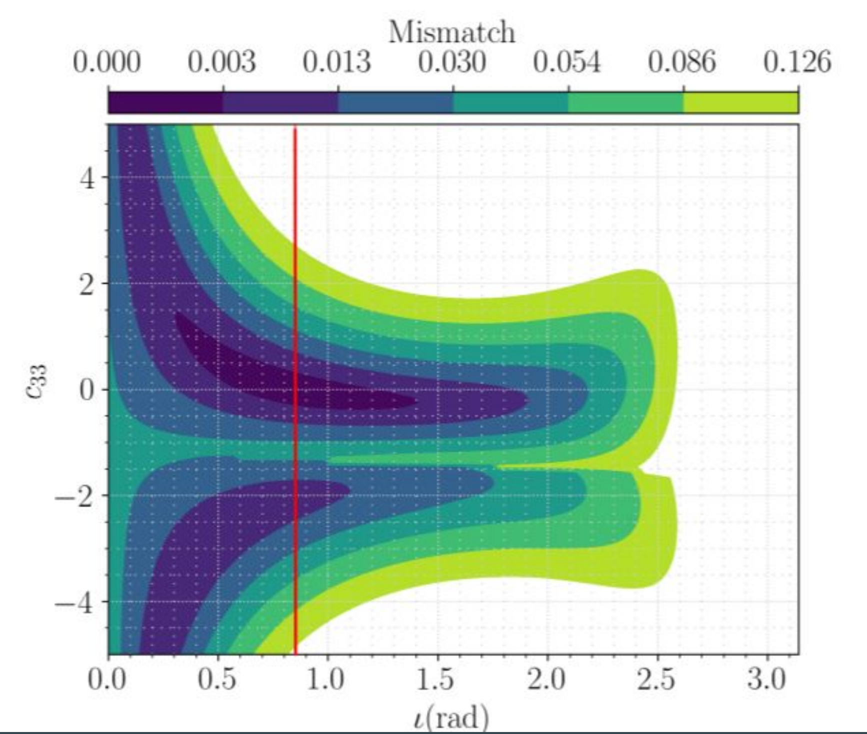

Injections study

How well can we recover deviations if present?

Simulations with parameters like GW190814 and GW190412

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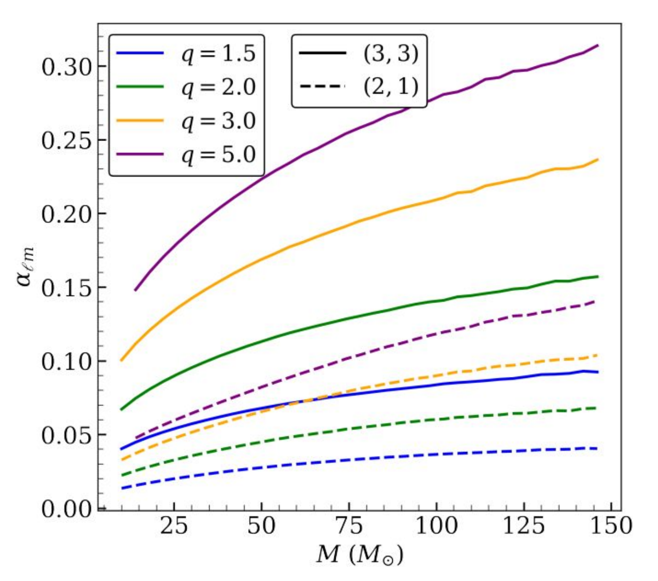
Simulations with parameters like GW190814 and GW190412

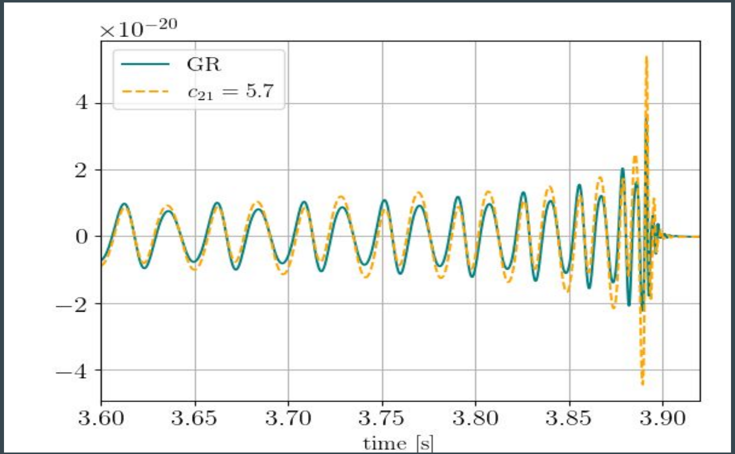
Simulate gravitational wave signal from different binaries

- Total mass $m_1 + m_2$: 65, 120 $M_\odot$
- $q = m_1/m_2$: 3,5,7,9
- SNR = 25



Injections study - mismatch

Mismatch = 'how different' a waveform is with respect to GR case



Injections study - mismatch

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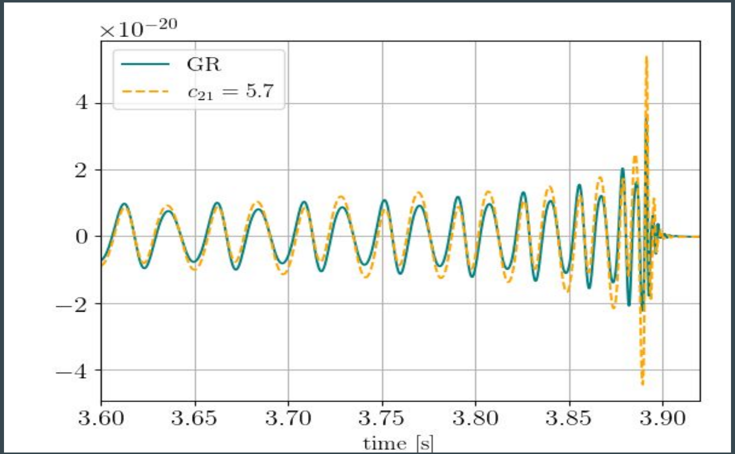
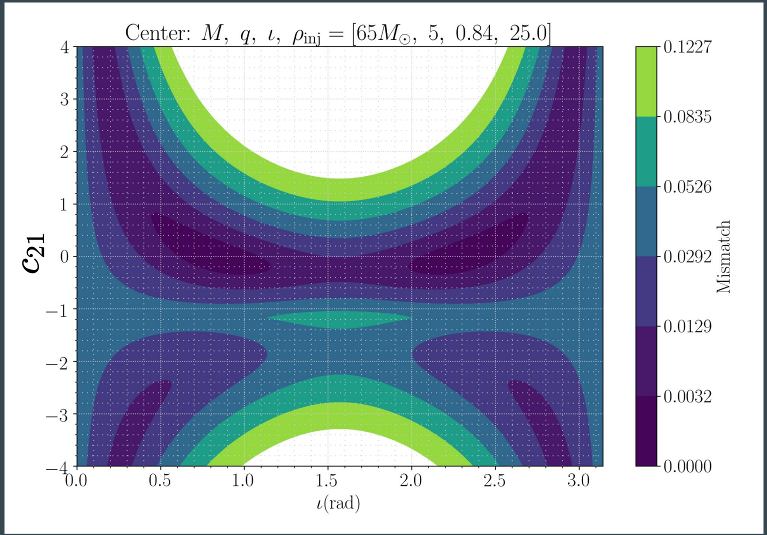
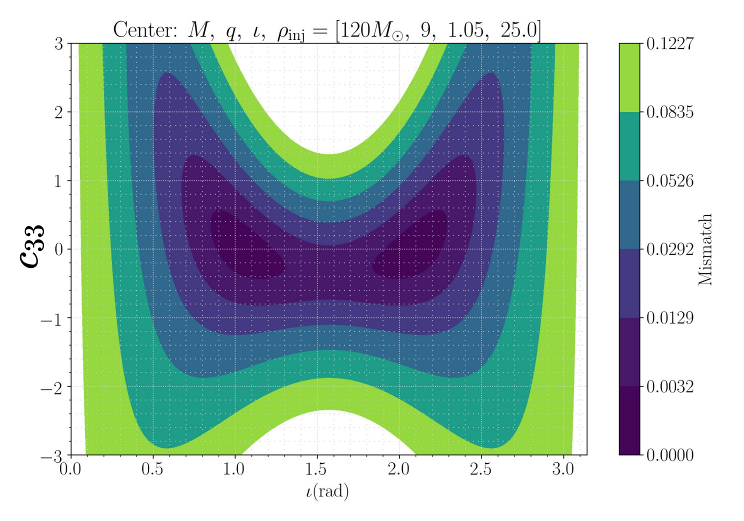
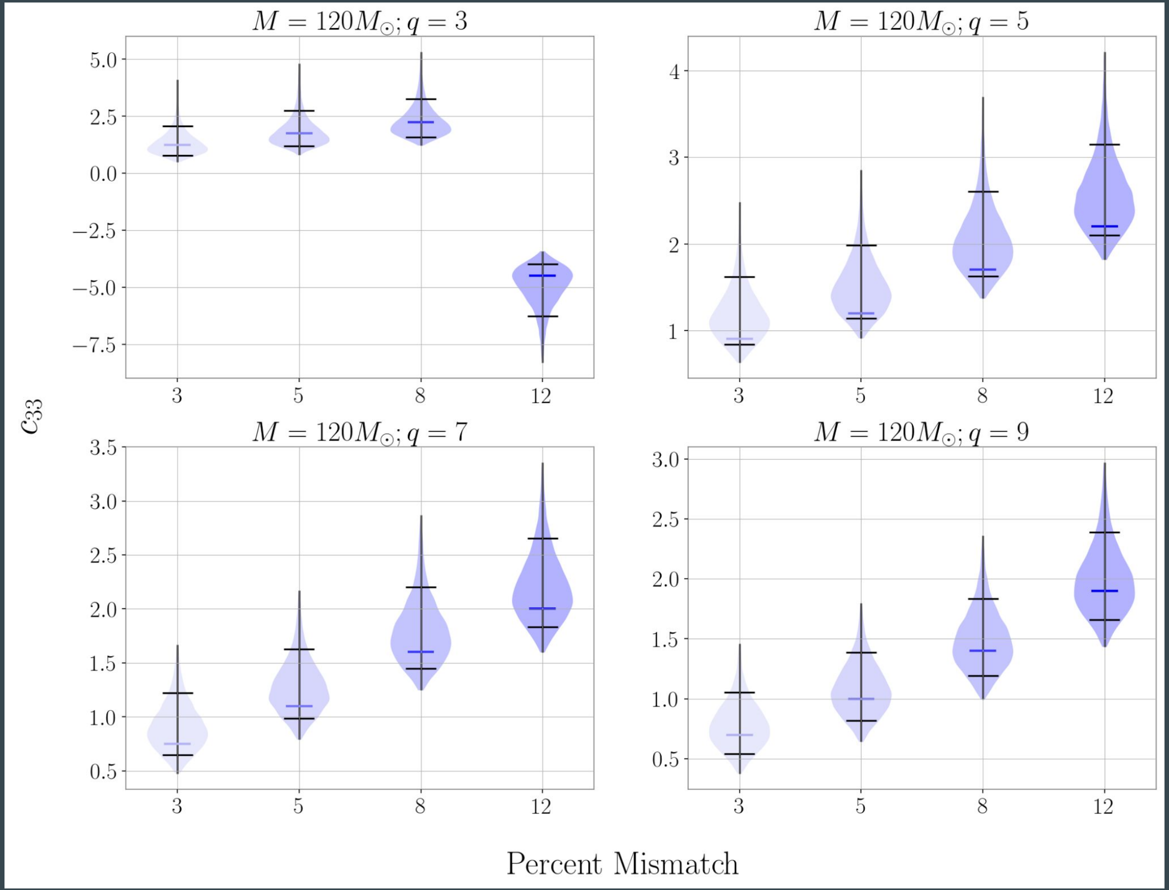
Choose ι and c_{lm} values to give mismatch of 3%, 5%, 8% and 12%







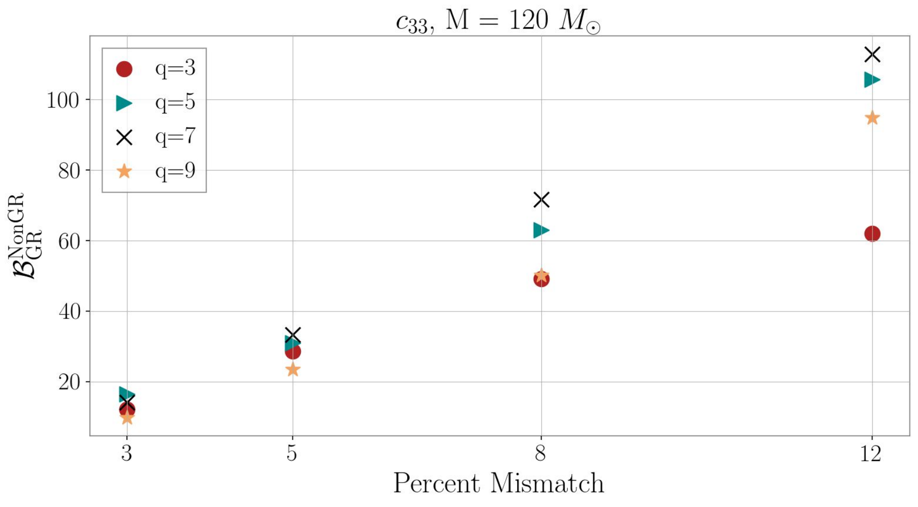
Injections study - parameter recovery



Injection study - model selection

Recovery both with GR and non-GR model: compare Bayes factors

$$\mathcal{B}^{NonGR}_{GR} = \frac{prob(nonGR|data)}{prob(GR|data)}$$



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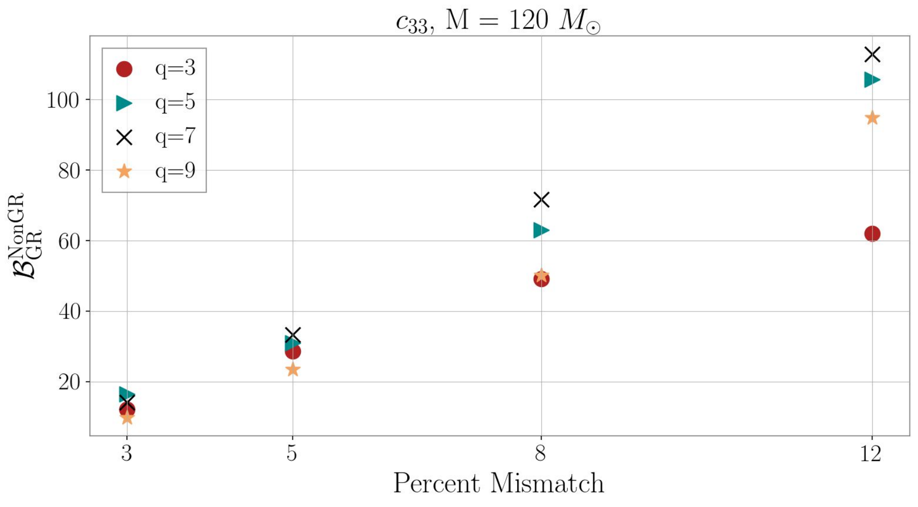
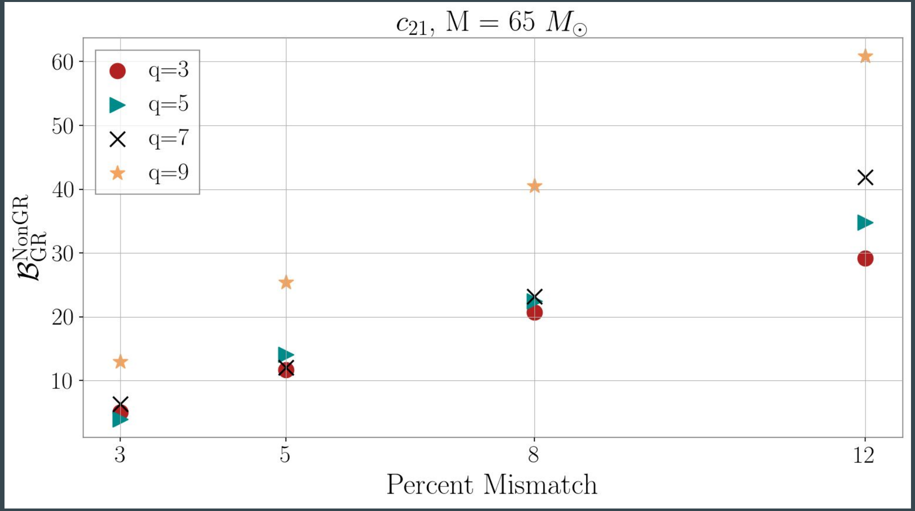
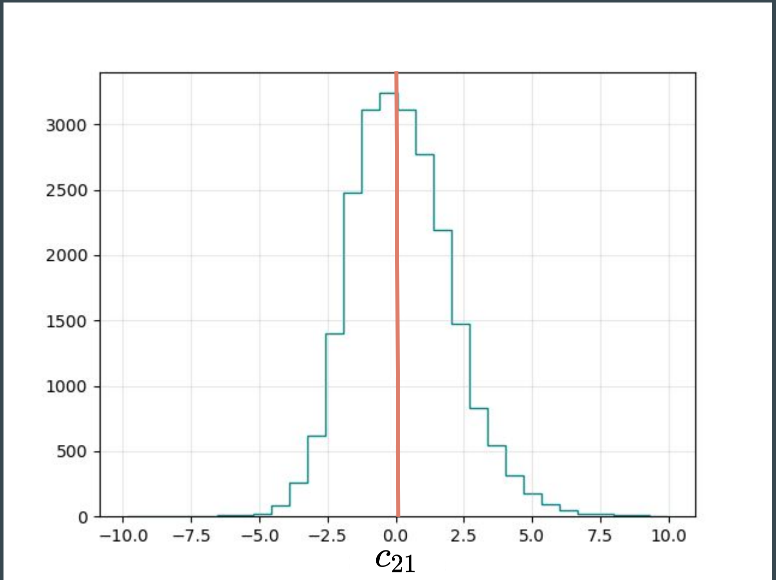
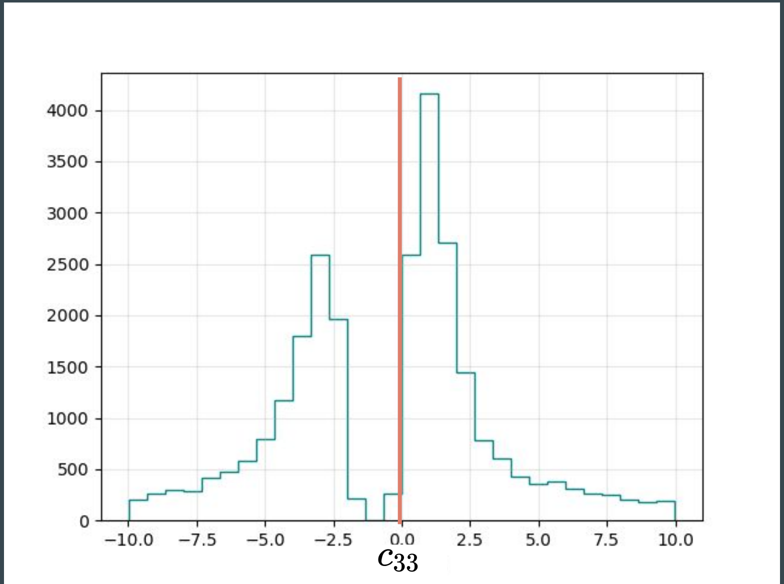




non-GR model preferred ✓

Real events results-GW190412

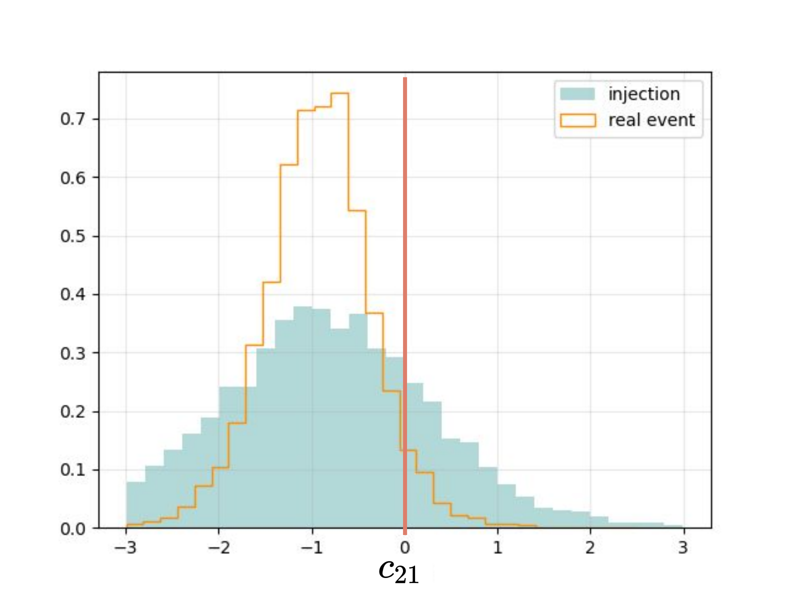


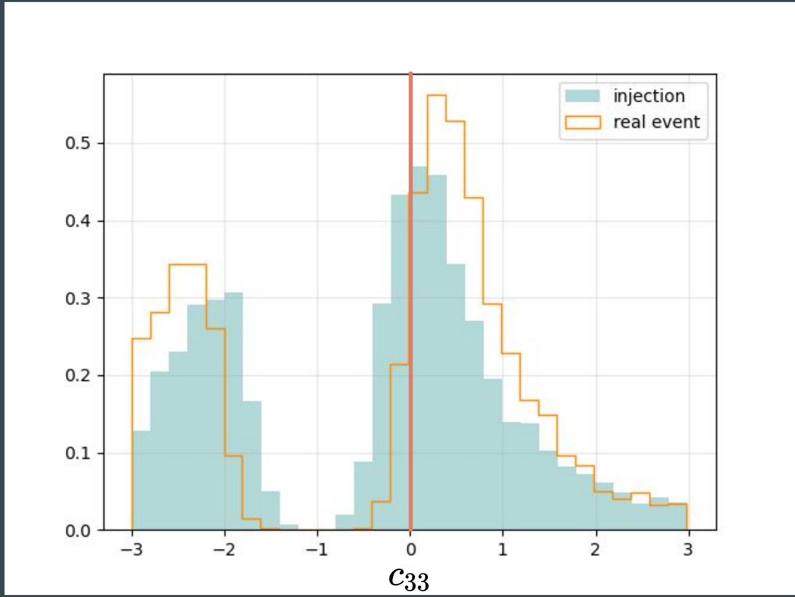


GR ✓

Posterior shape $\Longleftrightarrow$ correlation with iota

Real events results-GW190814





Comparison with a GR injection with GW190814-like parameters: GR ✓

Conclusions

- New method to test GR with Higher Order Modes
- Strong correlation with some binary parameters, like inclination
- Injections study showed that if GR violation, our model would
  - i) recover deviation value quite well
  - ii) have higher Bayes Factor than GR model
- Apply to real events (GW190412, GW190814): no GR violations found
- Hopefully more unequal-mass detection in the future

Thank you for your attention