Optimally sensitive observables for global EFT fits

NNV Annual meeting 2021
November 5th

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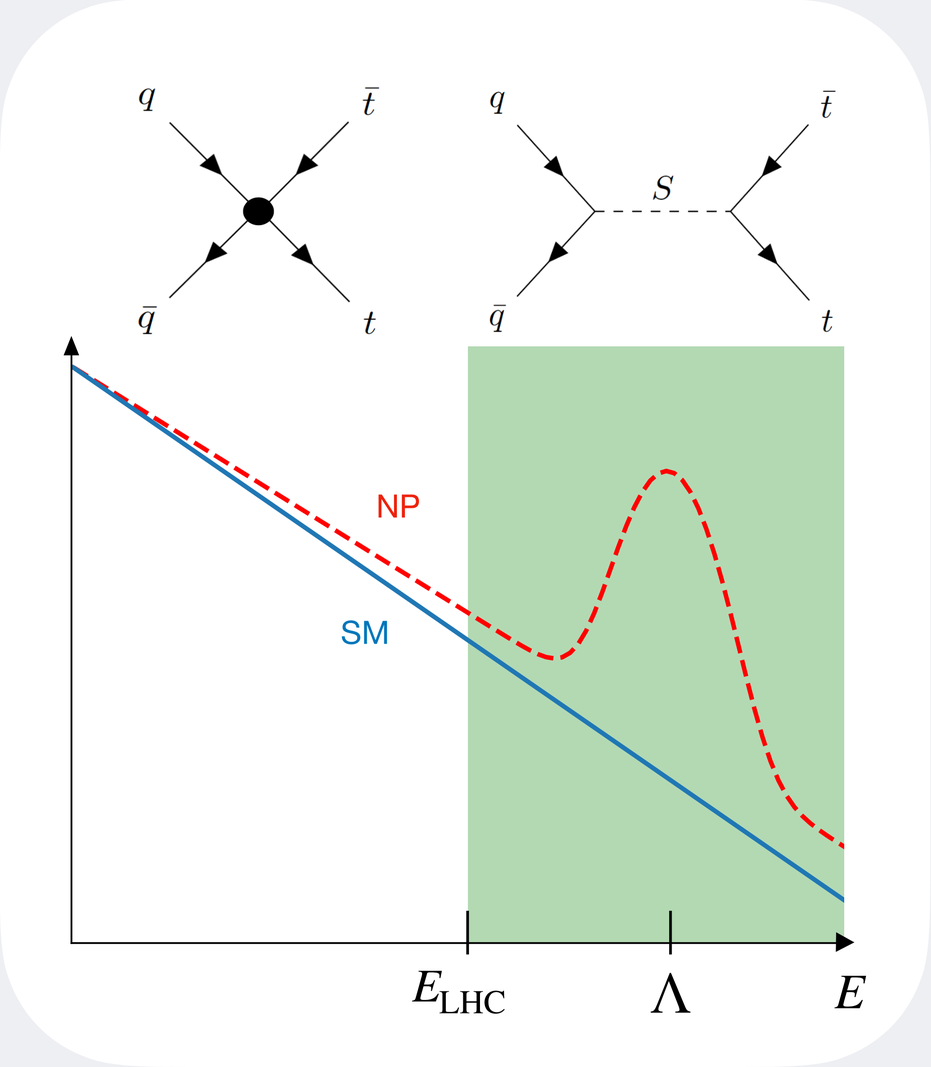
Work done in collaboration with R. Gomez Ambrosio, M. Madigan, J. Rojo, V. Sanz

The Standard Model Effective Field Theory

- Systematic parameterisation of the **theory space** close to the Standard Model
- Study the **fingerprints** of NP at low energies through higher dimensional operators (2499!)
- Assumes the **SM field content** and **gauge symmetries**

$$\mathscr{L}_{\text{SMEFT}} = \mathscr{L}_{\text{SM}} + \sum_i^{N_{d6}} \frac{c_i}{\Lambda^2} \mathcal{O}_i^{(6)} + \sum_i^{N_{d8}} \frac{b_i}{\Lambda^4} \mathcal{O}_i^{(8)} + \ldots$$



The Standard Model Effective Field Theory

- Systematic parameterisation of the **theory space** close to the Standard Model
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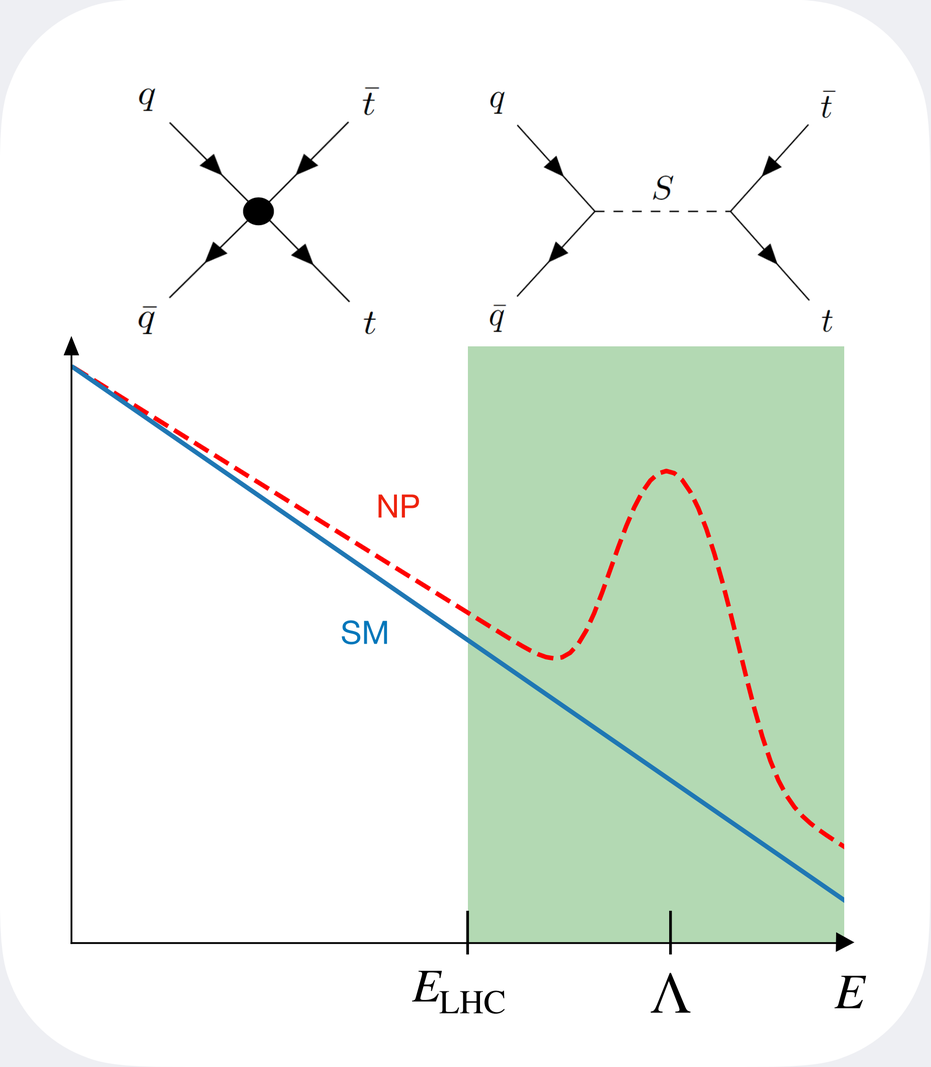
$$\sigma = \sigma_{\mathrm{SM}} + \sum_{i}^{N_{d6}} \frac{c_i}{\Lambda^2}\kappa_i + \sum_{i,j}^{N_{d6}} \frac{c_i c_j}{\Lambda^4}\tilde{\kappa}_{ij} + \ldots$$

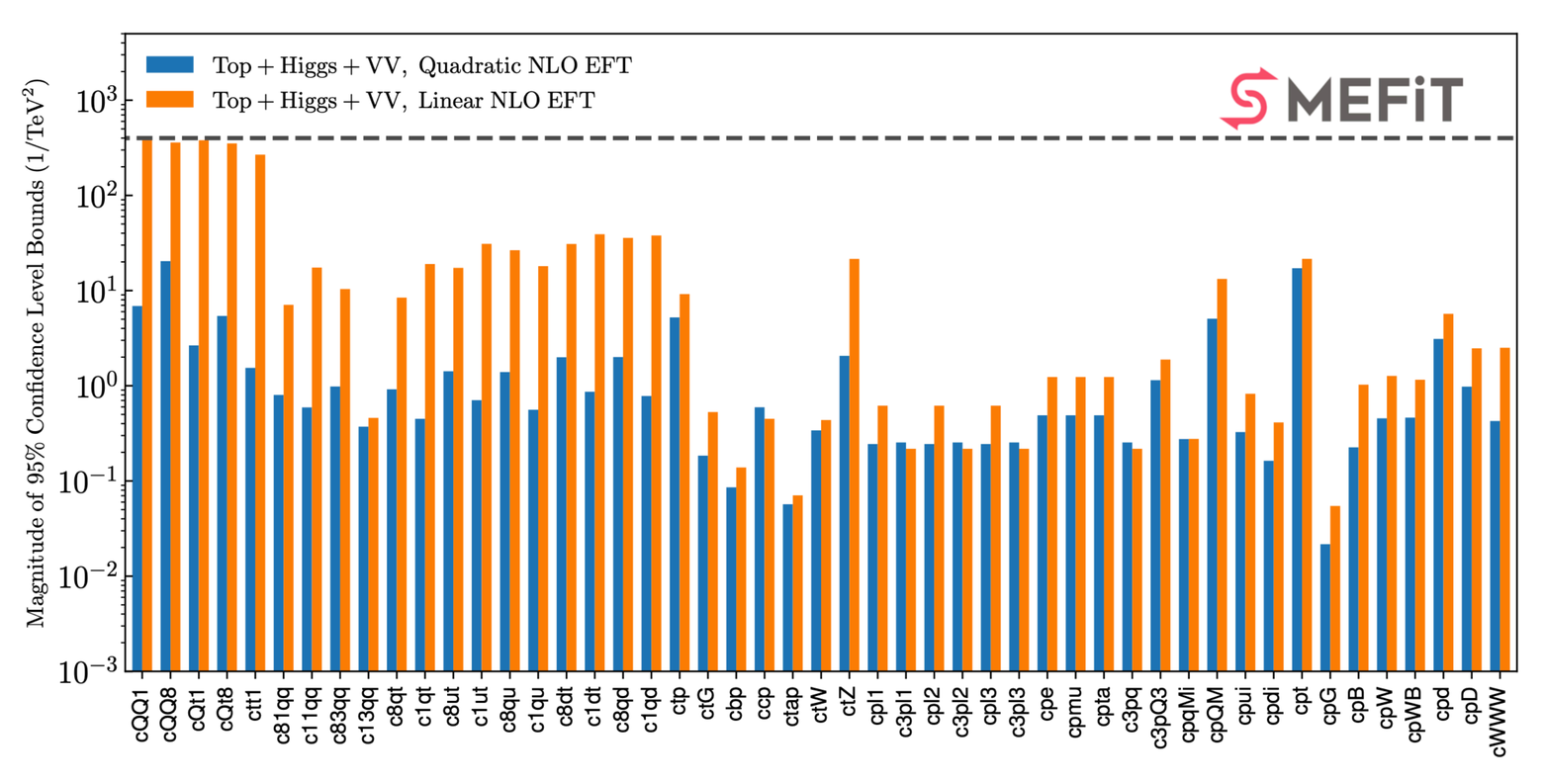
Interference

Quadratic corrections



Global EFT fits

- Status of the global EFT program: **Top + Higgs + diboson** data
- Based on traditional unfolded cross section distributions



J.J. Ethier et al. [2105.00006]

Optimal sensitivity

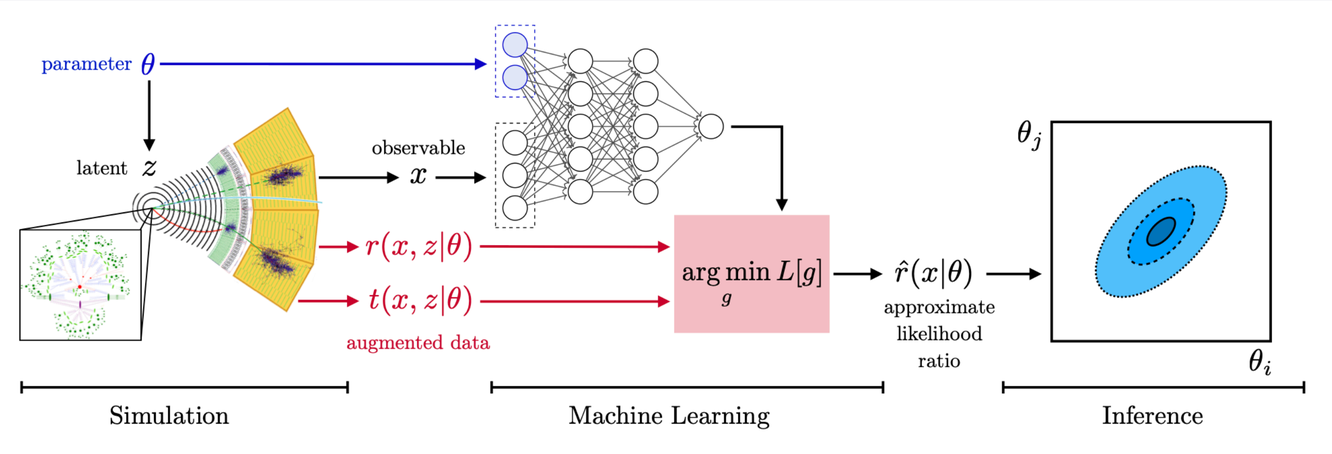
Key question: given a collider process, how can one be optimally sensitive to the EFT coefficients?

- We lose information in the process of binning
- To what degree can binned analyses achieve statistically optimal bounds?
- Even for bins, the precise choice of binning is not clear

Goal: develop statistically optimal observables and integrate them into global EFT fits

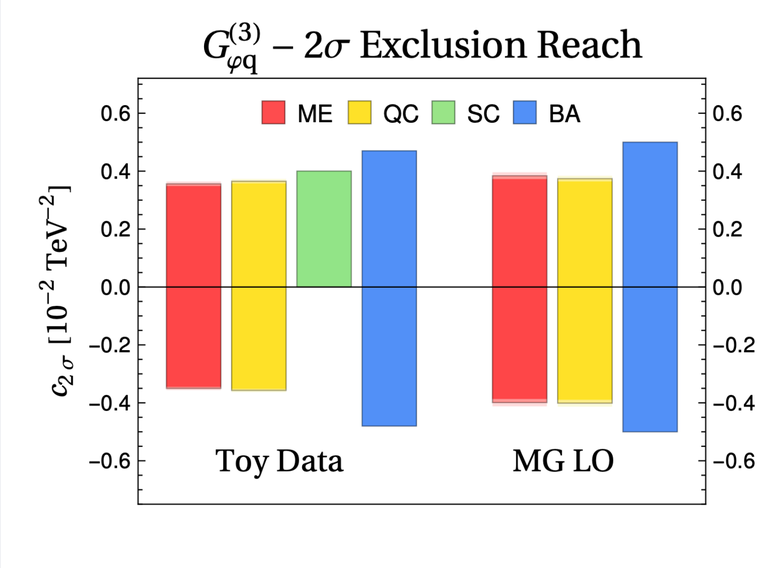
Related work

- The **likelihood** (ratio) as central object
- Matrix Element Method (MEM): transfer functions
- Parameterise the likelihood ratio with **Neural Networks**
- Current studies are limited to a **small number of EFT coefficients**



J. Brehmer, K. Cranmer [2010.06439]

S. Chen, A. Glioti, G. Panico, A. Wulzer [2007.10356]



Finding optimal observables

Neyman-Pearson: the most powerful statistical test at fixed size (significance level) between two *simple* hypotheses H_0 and H_1 is the (log) likelihood ratio:

$$t_c(D) \equiv \log \frac{\mathscr{L}(H_1 \,|\, D)}{\mathscr{L}(H_0 \,|\, D)}$$

- Any other test statistic has less power, i.e. gives suboptimal bounds
- No longer applies in case of systematics: profile likelihood ratio

Finding optimal observables

Key idea: train a NN classifier to learn the *extended* likelihood ratio

SM hypothesis

Number of events

$$t_c \equiv \log \frac{\mathscr{L}(H_1 \,|\, D)}{\mathscr{L}(H_0 \,|\, D)} = \nu^{\text{eft}} - \nu^{\text{sm}} - \sum_{i=1}^{n} \log \frac{d\sigma(x_i, c)}{d\sigma(x_i, 0)}$$

Extended likelihood ratio

EFT hypothesis (null)

Expected number of events under the SM

Cross section ratio

The events x_i can be invariant masses, rapidities, scattering angles, p_T, …

Binary classifier

- Train a **classifier** by minimising the cross entropy (or the quadratic loss) loss functional

$$L[f(x)] = -\int dx \frac{d\sigma_0}{dx} \log(1-f) - \int dx \frac{d\sigma_1}{dx} \log f$$

which gives

The choice of loss functional is not unique!

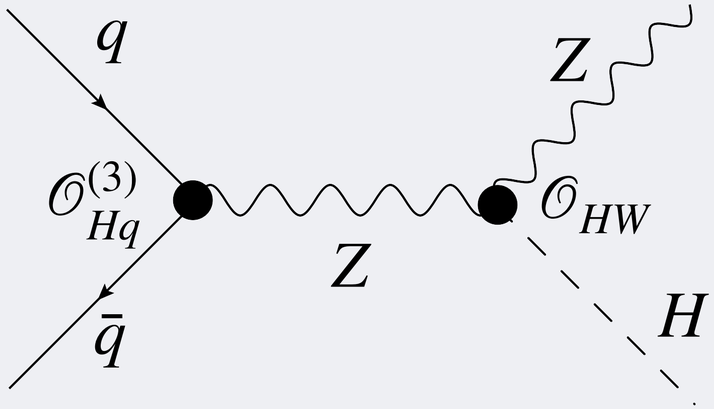
$$\frac{\delta L}{\delta f(x')} = \frac{d\sigma_0}{1-f} - \frac{d\sigma_1}{f} = 0 \implies \hat{f} = \frac{1}{1 + d\sigma_0/d\sigma_1}$$

- This is a **one-to-one estimator** of the likelihood ratio!

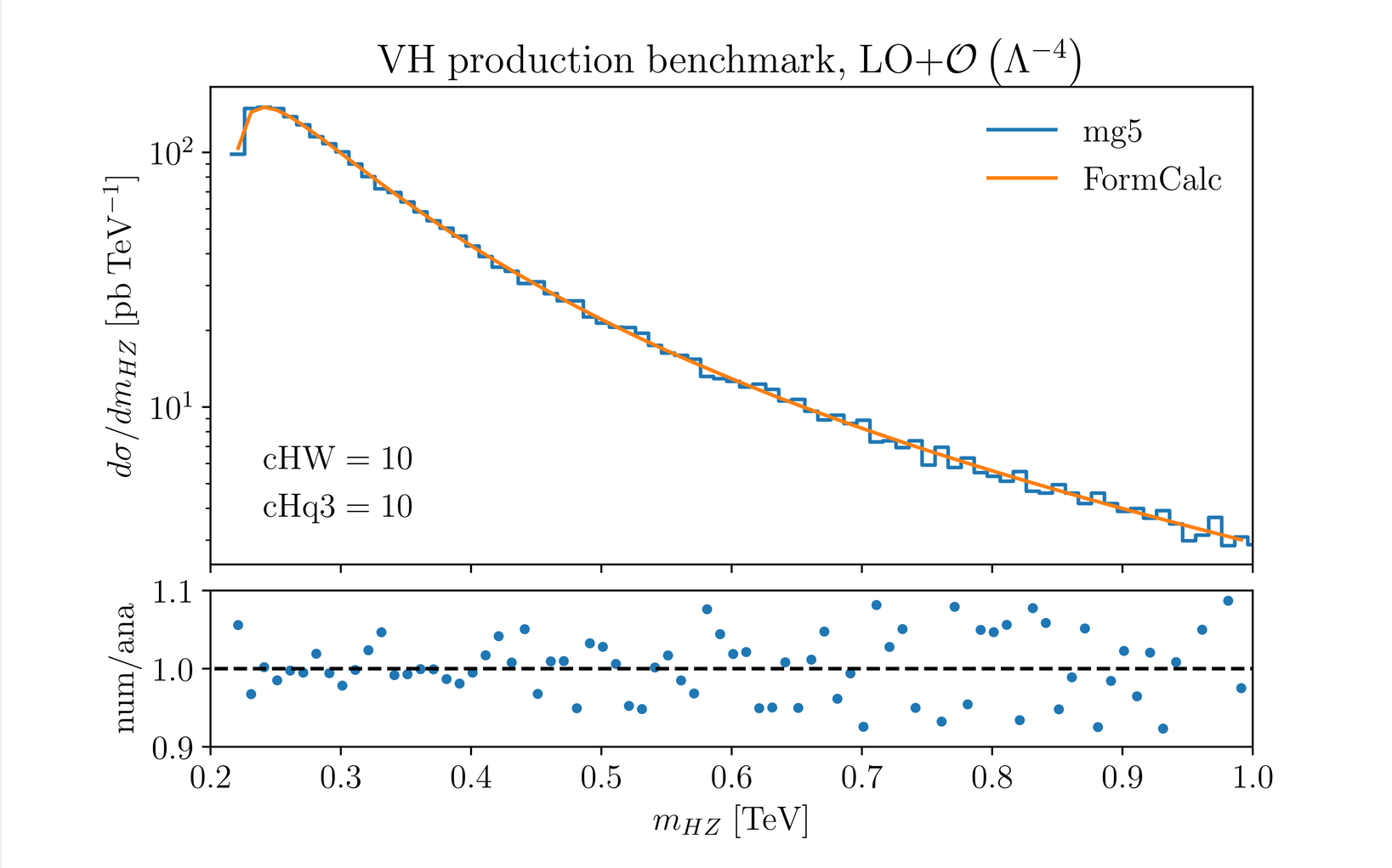
VH production

- Need access to underlying truth to assess the NN accuracy
- Efficient **pipeline** using **FeynRules, SMEFTsim, FeynArts / FormCalc** to obtain analytical predictions
- LO parton level, but the method is applicable to any final state
- Study $\mathcal{O}_{HW}$, $\mathcal{O}_{HWB}$, $\mathcal{O}_{HB}$, $\mathcal{O}_{HD}$ and $\mathcal{O}_{Hq}^{(3)}$ up to $\mathcal{O}\left(\Lambda^{-4}\right)$ differential in the rapidity and invariant mass m_{VH}

$$\frac{d\sigma}{dm_{VH}dY} = \frac{2m_{VH}}{s}\left[\sum_f f_f\left(x_1, Q\right) f_{\bar{f}}\left(x_2, Q\right) \hat{\sigma}_{q\bar{q}\to VH}\right]$$



VH production: benchmark



Training the likelihood ratio

We separate the learning problem by exploiting the structure inherent to the EFT parameter space:

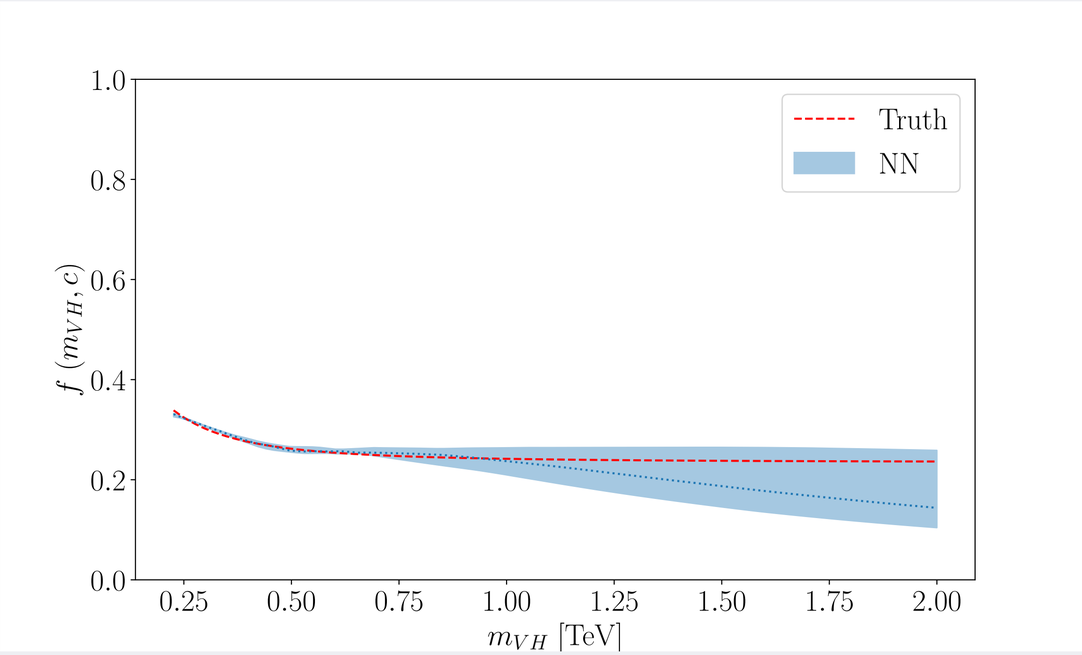
$$r(x, \mathbf{c}) = 1 + c_1\alpha_1(x) + c_2\alpha_2(x) + c_1^2\beta_{11}(x) + c_1c_2\beta_{12}(x) + \beta_{22}c_2^2$$

1. Train the linear coefficient functions in parallel
2. Switch on quadratic corrections and train the quadratic coefficients
3. The cross terms can finally be extracted

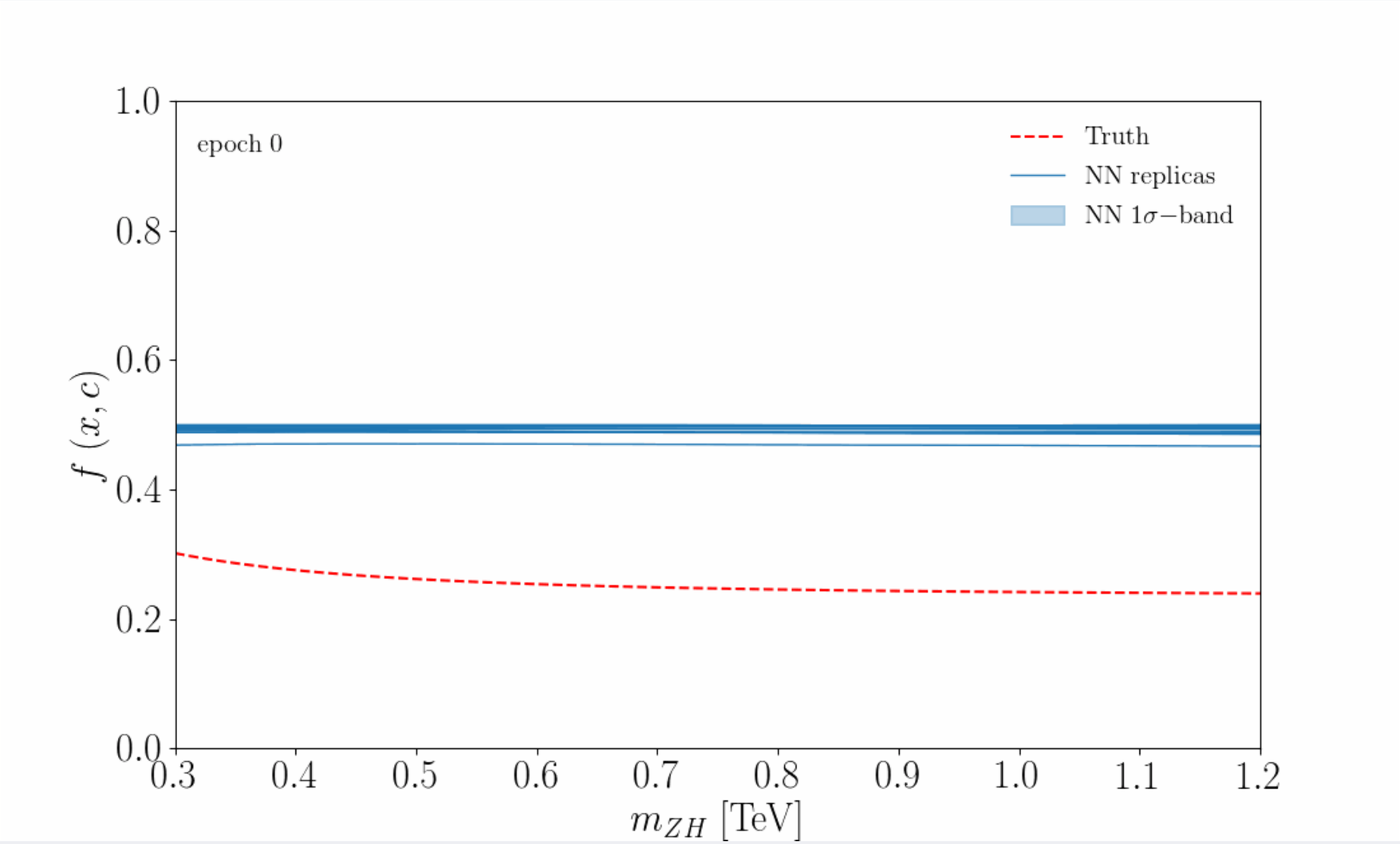
New: this allows for efficient scaling (quadratically) and parallel training for n EFT parameters

NN model uncertainties

- We systematically assess the **model uncertainties** associated to the NN parameterisation of the likelihood ratio
- **Replica**: an independent MC training set to propagate the error to the space of models
- Train 30 independent replicas **in parallel**
- Translate to the error on the Wilson coefficients

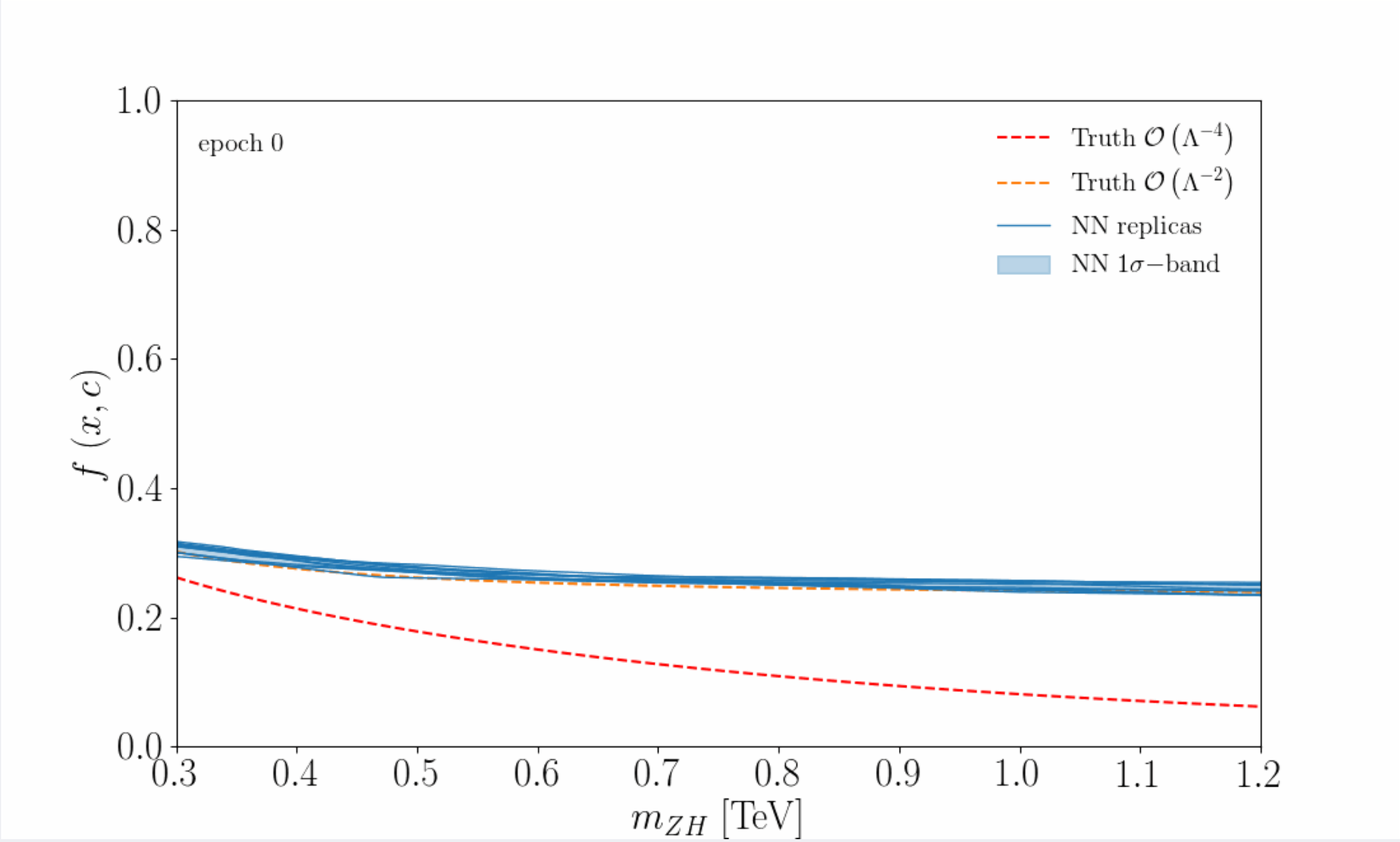


Seeing the training at work



- Trained on 30 replicas
- 100K events in SM and EFT
- Cross validation
- Architecture: {2, 5x30, 1} with ReLU activation functions
- Standardised training data to zero mean and unit variance

Adding quadratic corrections



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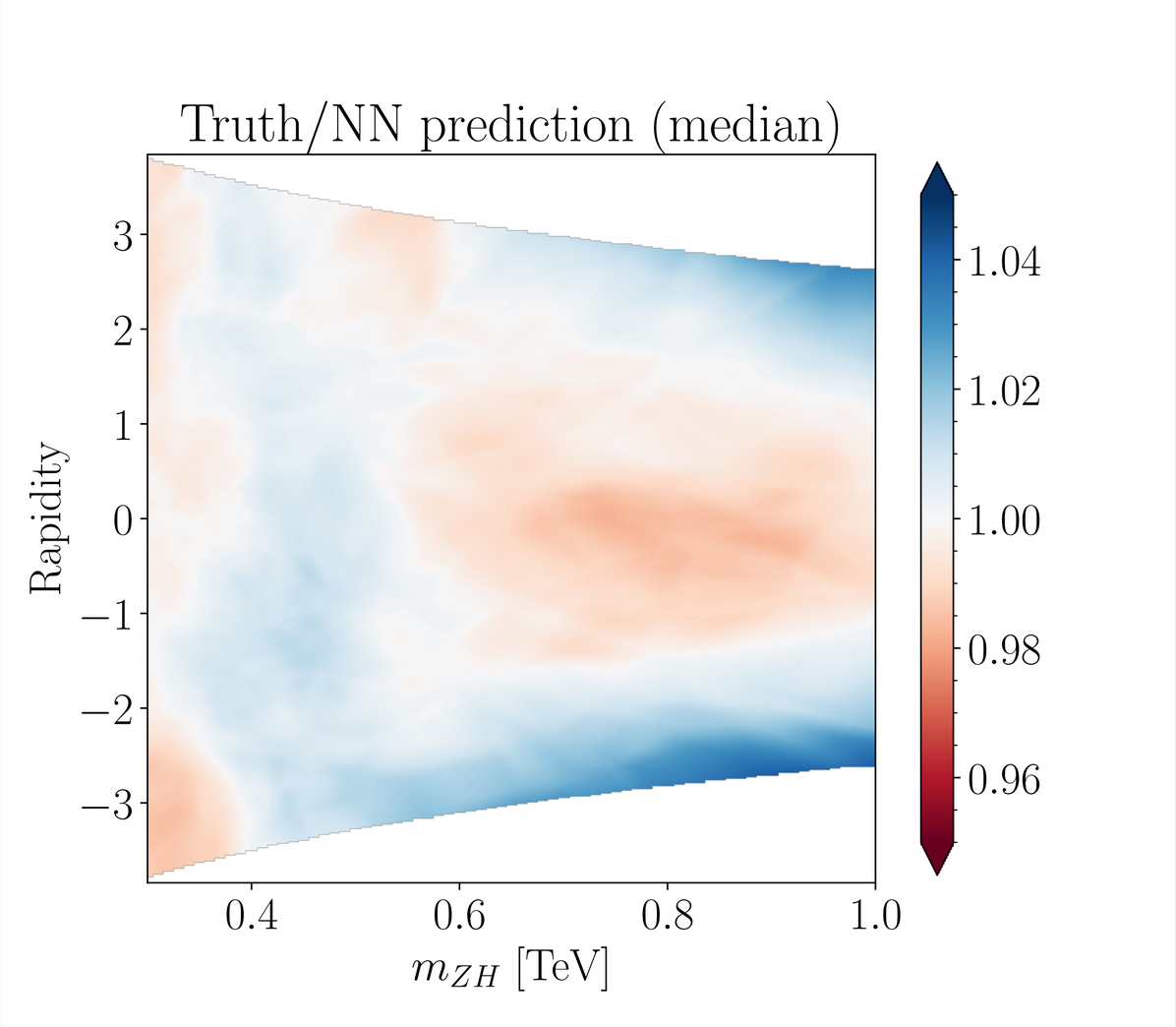
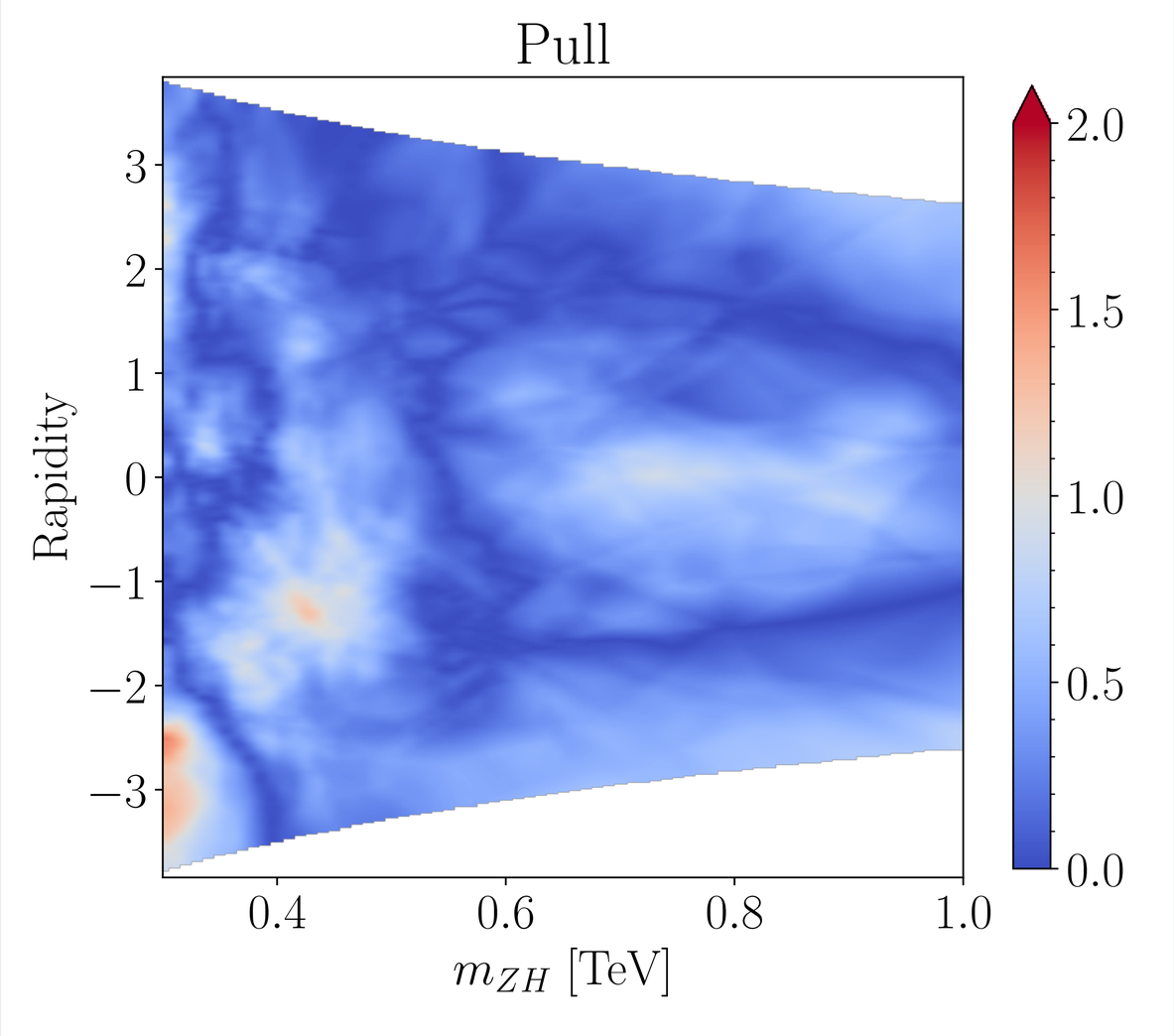
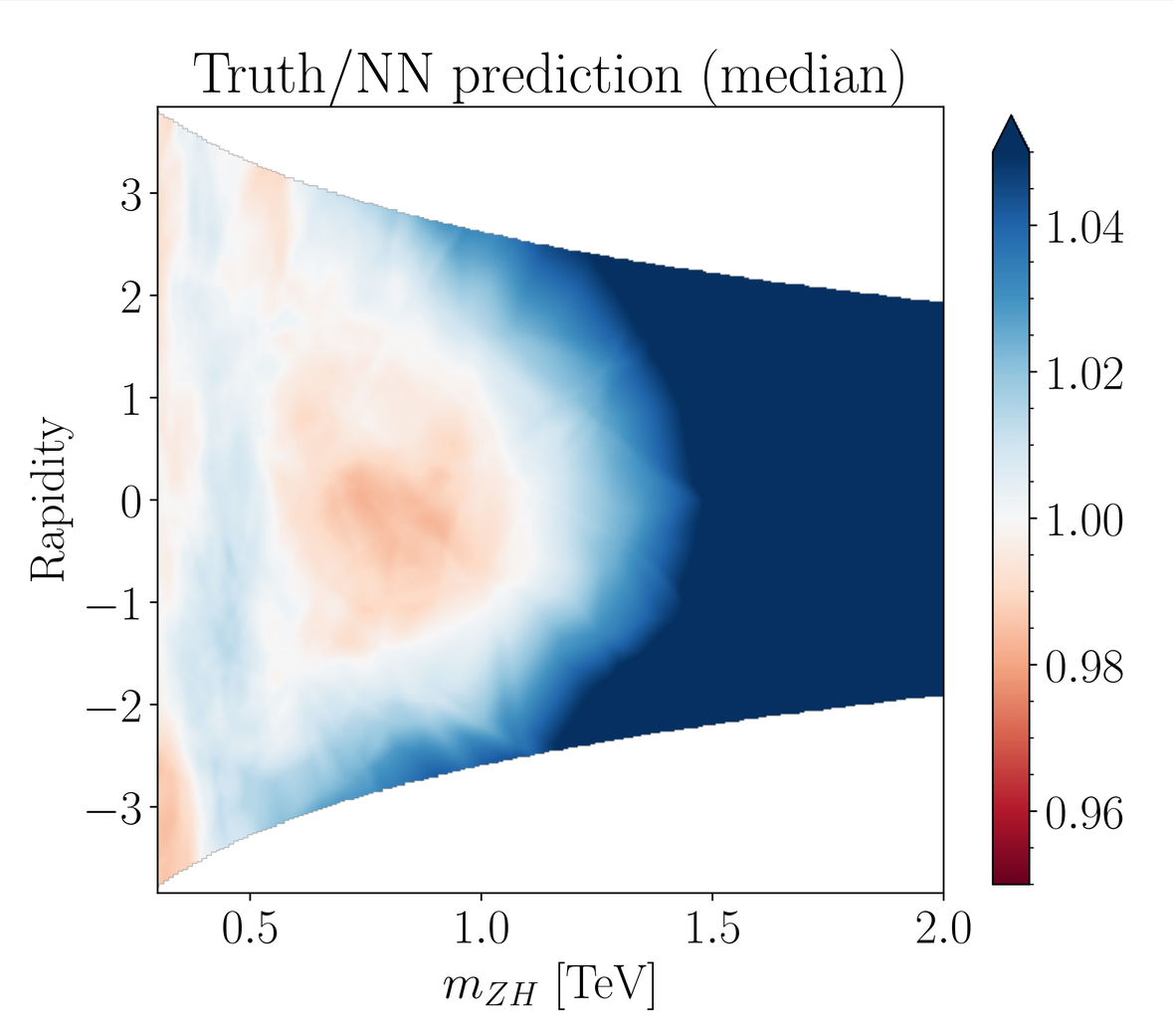
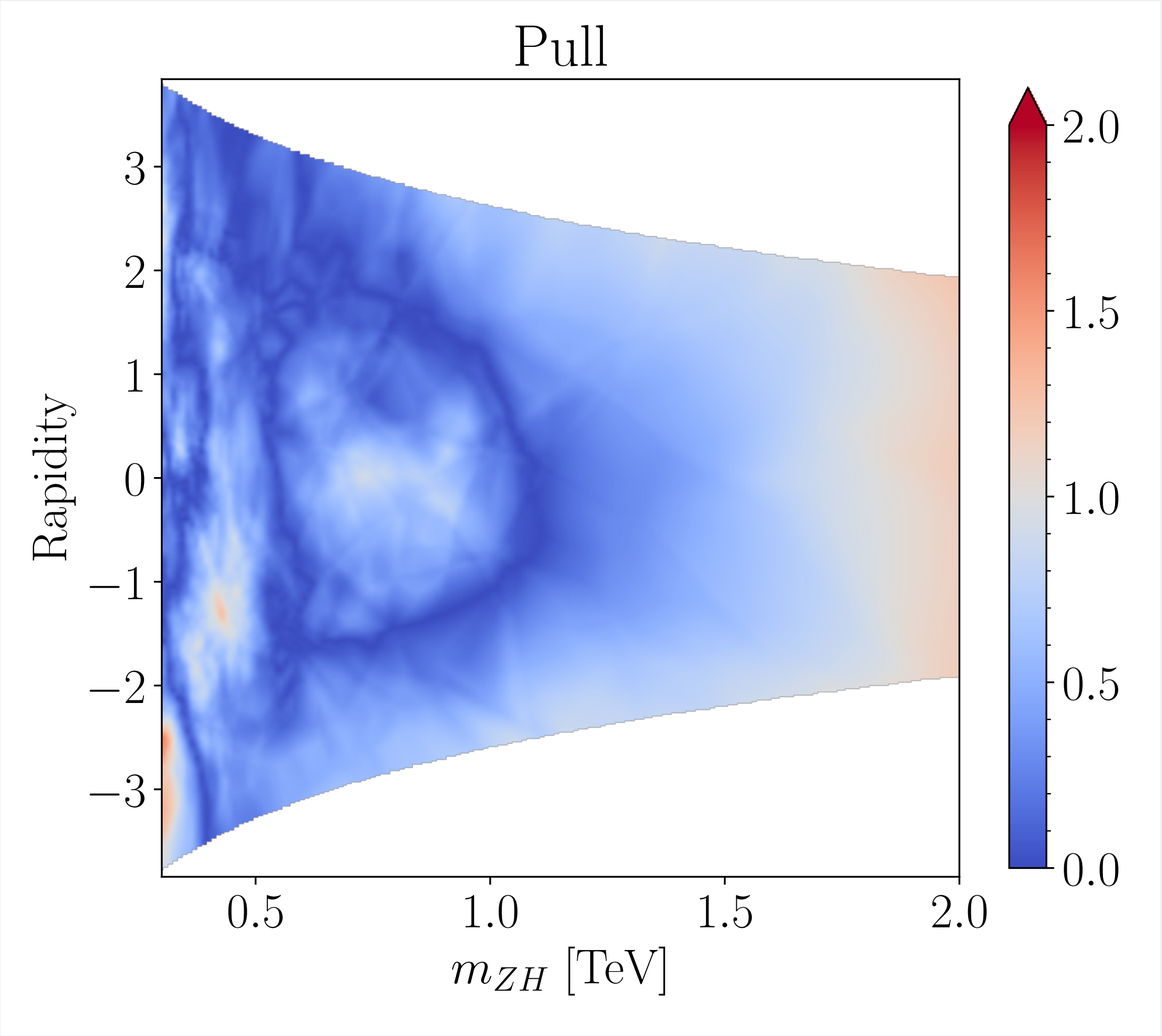
Training performances



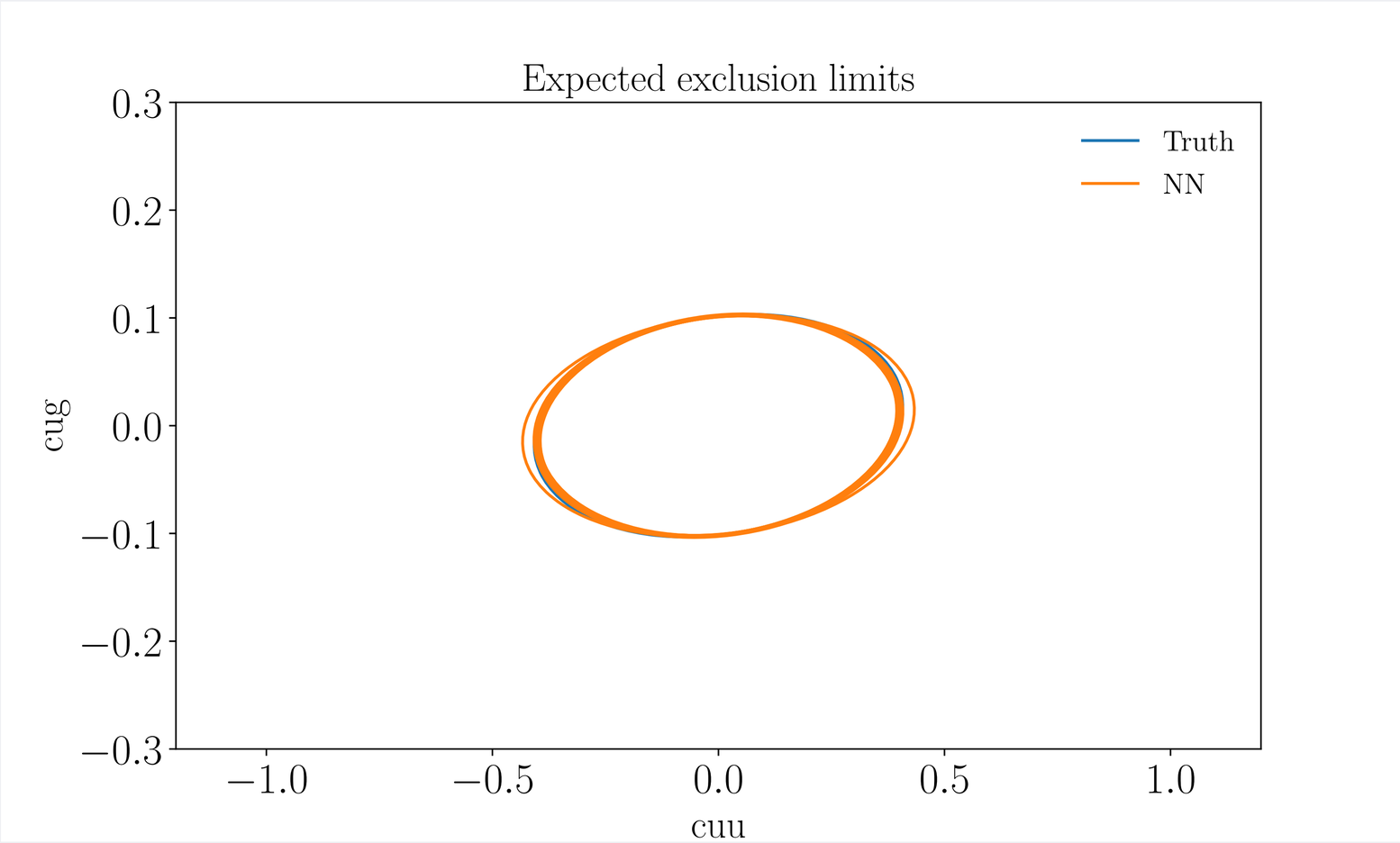


Training performances





Limits: NN versus Truth



$$\mathcal{Q}_{uG} = (\bar{q}_p \sigma^{\mu\nu} T^a u_r)\tilde{H} G^a_{\mu\nu}$$

$$\mathcal{Q}_{uu} = (\bar{u}_p \gamma_\mu u_r)(\bar{u}_s \gamma^\mu u_t)$$

- $t\bar{t}$ production with two EFT coefficients
- One ellipse per replica
- Good agreement between truth and NN proxy!

Summary

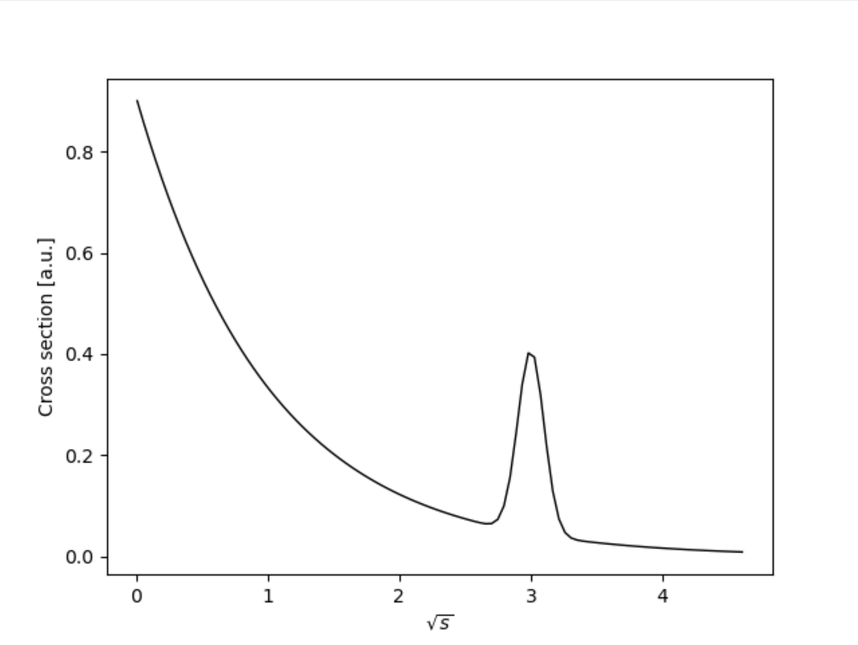
Key question: given a collider process, how can one be optimally sensitive to the EFT coefficients?

- Train a surrogate of the **likelihood ratio**
- **Efficient scaling** properties to n EFT parameters necessary for global EFT fits
- Good reconstruction **performances** throughout phase space
- Outlook: Include **systematics** with the profile likelihood ratio

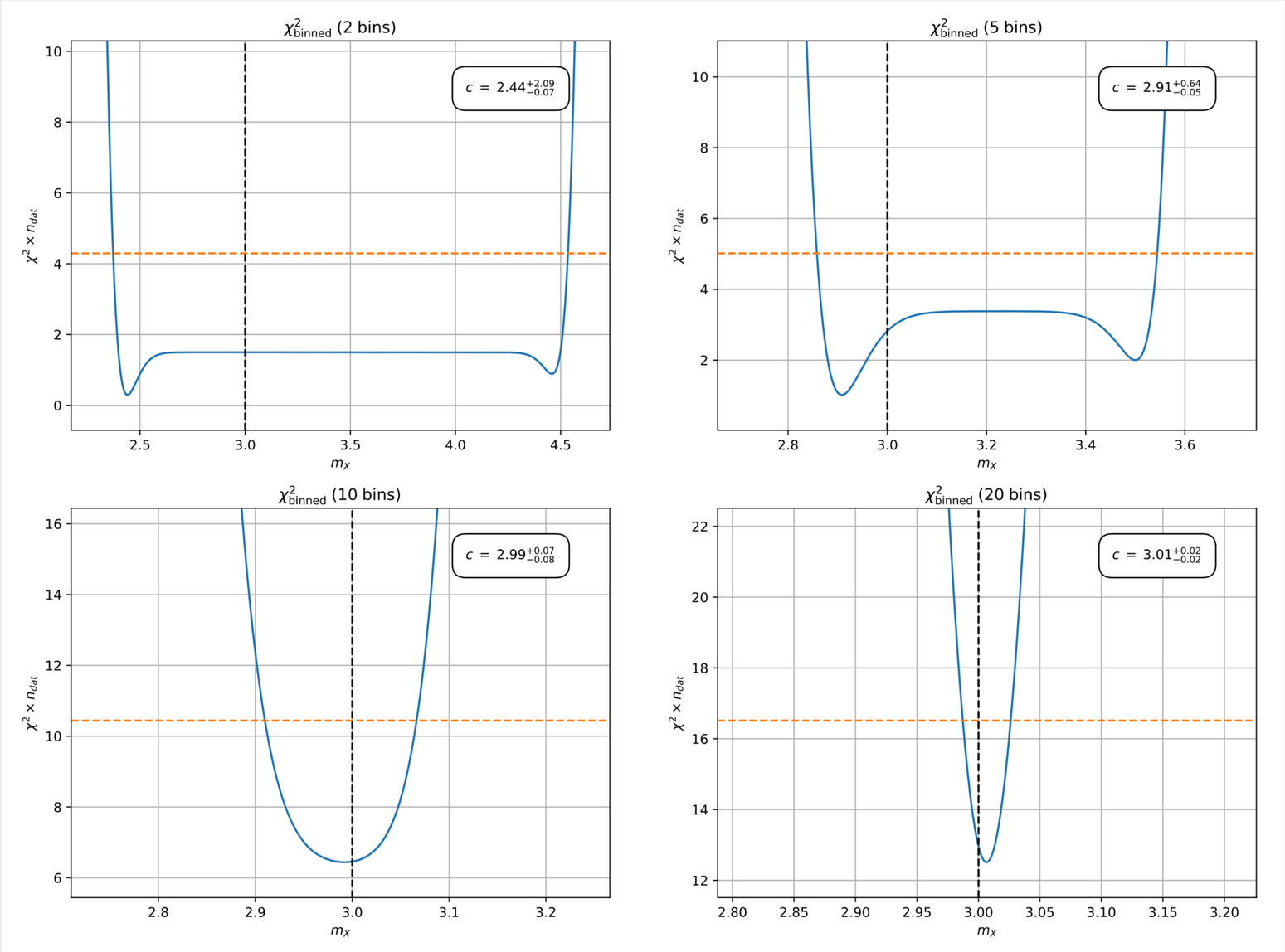
Thank you!
Questions?

Backup

Binned vs unbinned



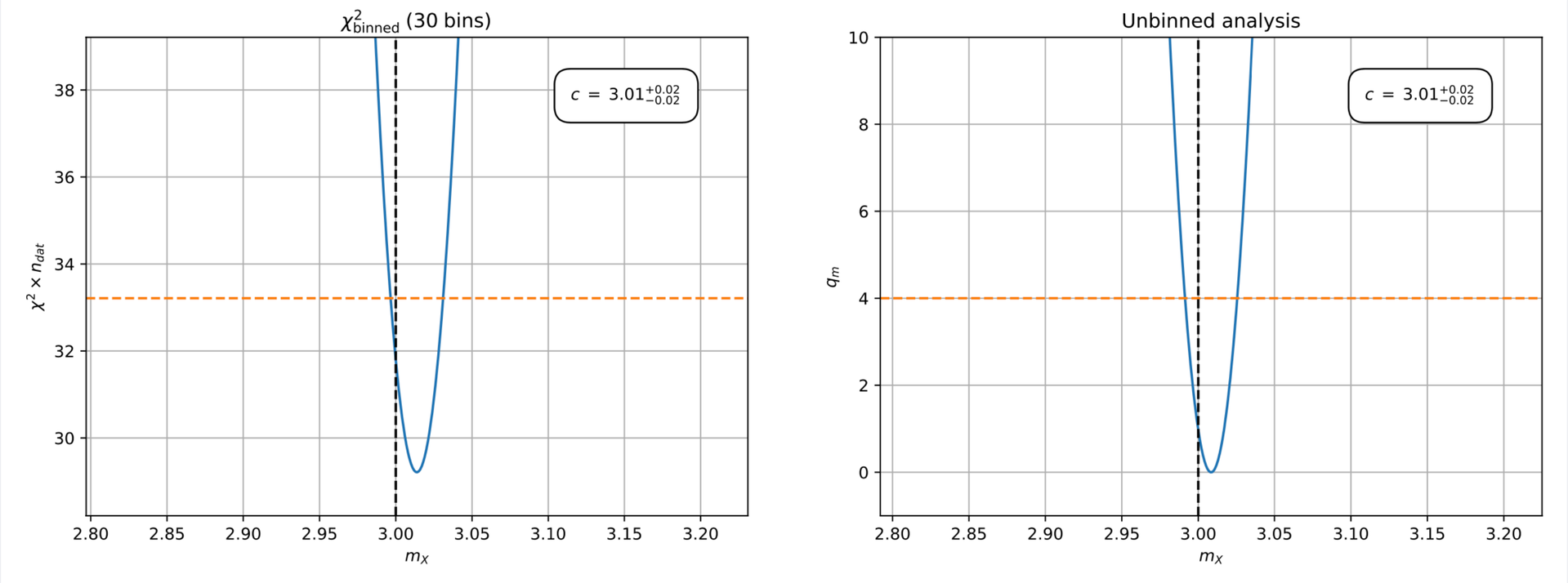
$$\chi^2_{\text{binned}} = \sum_{i=1}^{n} \frac{(n_i - \nu_i)^2}{n_i}$$



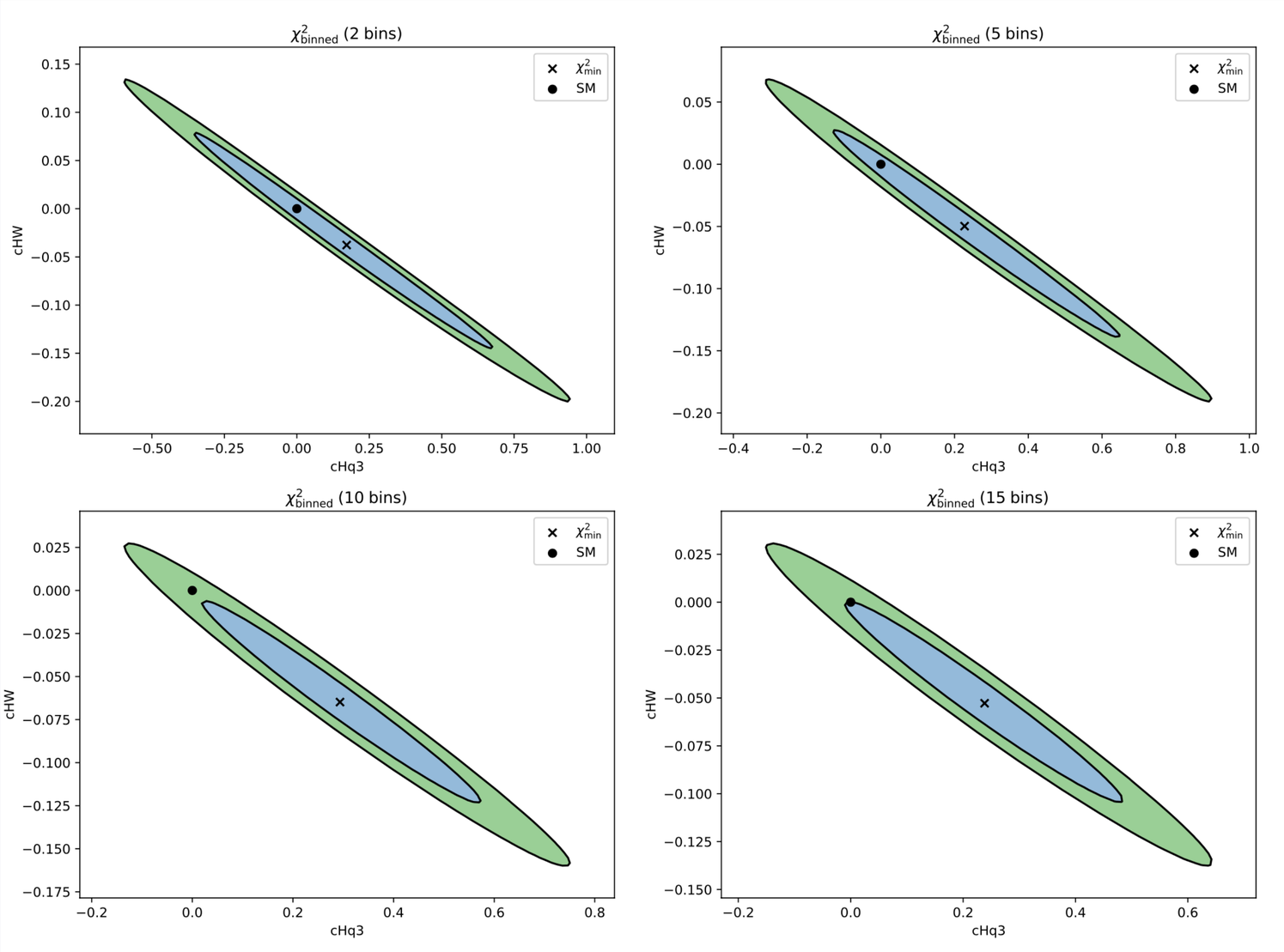
Binned versus unbinned

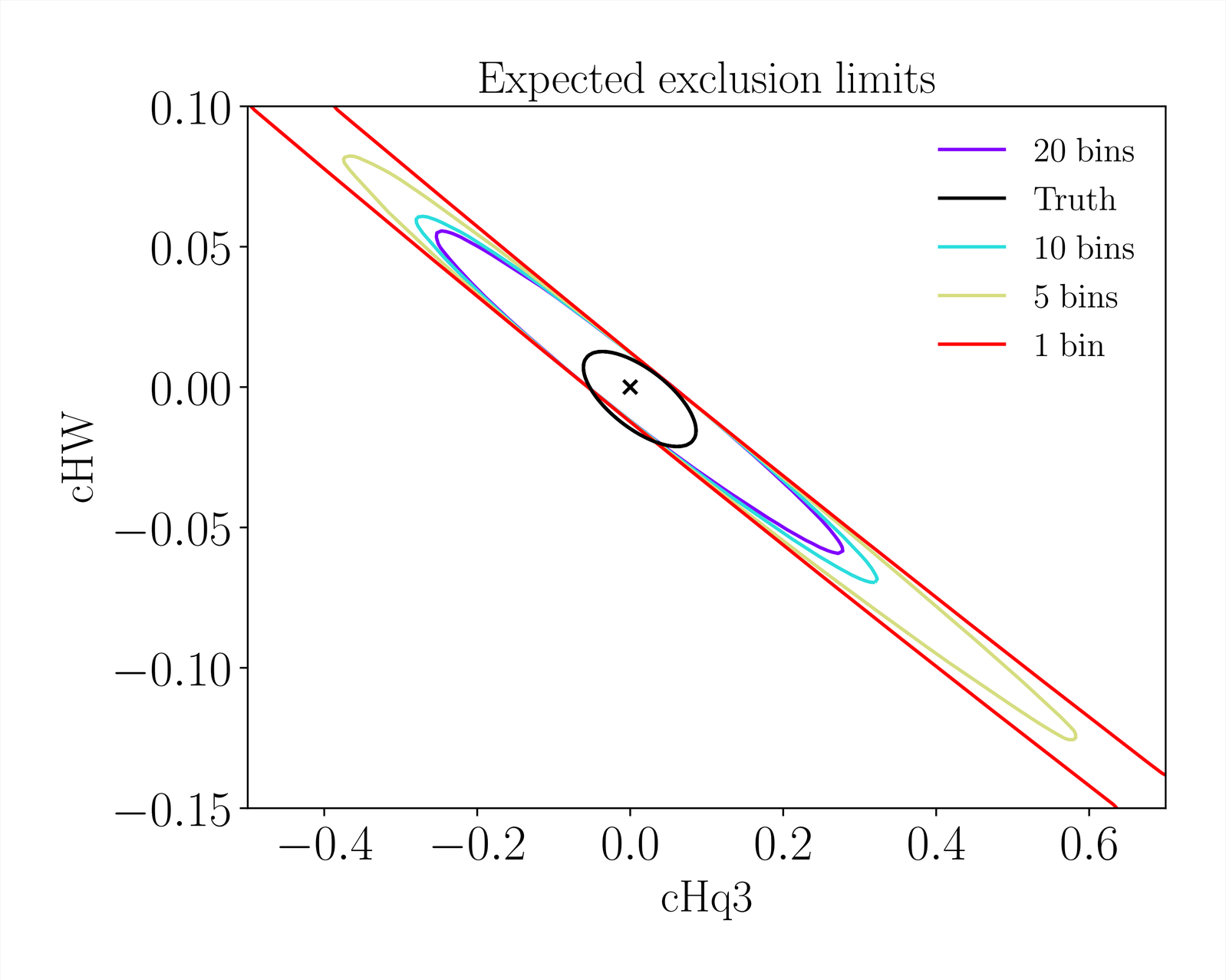
$$\chi^2_{\text{binned}} = \sum_{i=1}^{n} \frac{(n_i - \nu_i)^2}{n_i}$$

$$q_m = -2 \log \frac{p(x \mid m)}{p(x \mid \hat{m})}$$



VH production




Expected exclusion limits
20 bins
Truth
10 bins
5 bins
1 bin
0.10
0.05
0.00
−0.05
−0.10
−0.15
cHW
−0.4
−0.2
0.0
0.2
0.4
0.6
cHq3