Quantum Algorithms - State of the Art

(for high-energy physics and/or gravitational waves?)

Ronald de Wolf

CWI

QuSoft

Universiteit van Amsterdam

Quantum computers

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2. Set up initial superposition and manipulate it with gates, so that computational paths leading to correct output interfere constructively, others interfere destructively (computation is efficient if it uses few gates)
3. Measurement of final state then gives classical output

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2. Searching through large spaces
3. Faster optimization
4. Simulating quantum systems

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I will go over these 4 items at a fairly high level, hoping that something triggers a click with the computational needs of high-energy physics and/or gravitational-wave research

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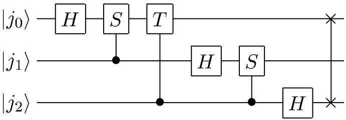

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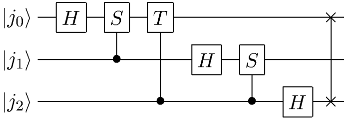

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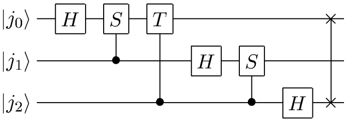

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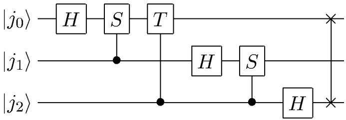

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- Key component in many quantum algorithms. Difficulty is how to "load" your periodic sequence as amplitudes of a state

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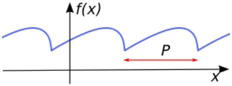

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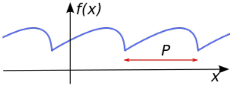

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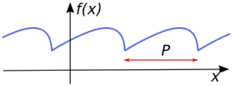
This is a periodic function 

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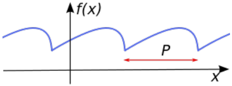

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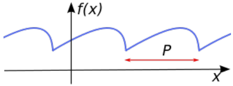
   This is a periodic function 

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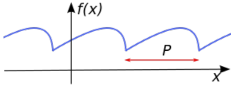

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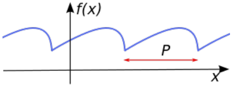

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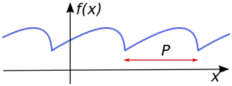
   This is a periodic function 

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   2.4 Do quantum Fourier transform and measure:
   the result gives information about the period P

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- Many applications: basically anything where search appears as a subproblem, and also many estimation problems (e.g., Monte Carlo)

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- Montanaro'14: exponential speed-up on average in higher dimensions (Grover + algorithm for finding "hidden shifts")

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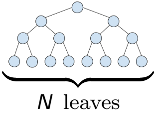

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   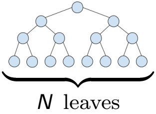

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   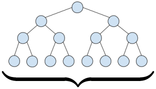

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   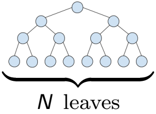

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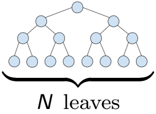

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Grover-based speedups are probably not for the near term

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Quantum speed-ups for **continuous** optimization

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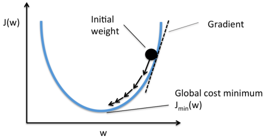

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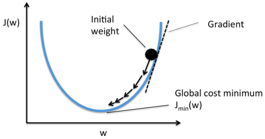

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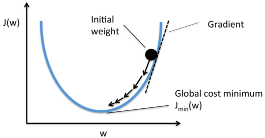

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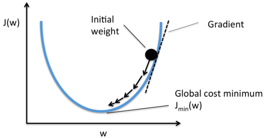

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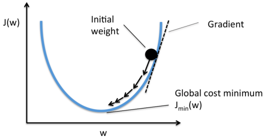

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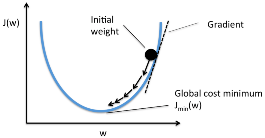

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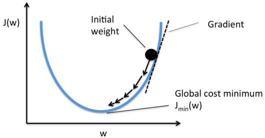

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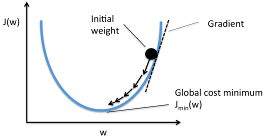

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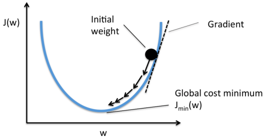

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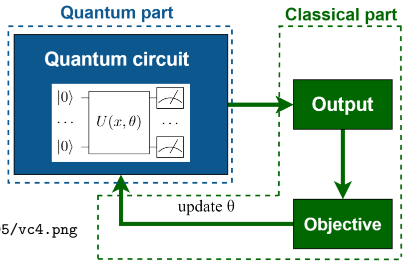

https://dkopczyk.quantee.co.uk/wp-content/uploads/2019/05/vc4.png

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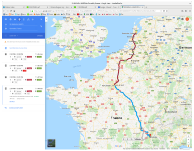

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  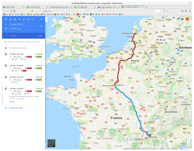

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  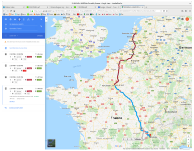

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  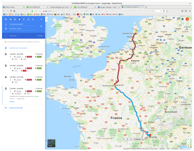

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- NB: Finding ground state energy of H is *much* harder problem

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- Conjecture: quantum computers can't efficiently solve NP-hard problems

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- This is weaker (and more subtle) than classical parallelism, but much faster than classical computers for some computational problems
- I tried to survey the state of the art in quantum algorithms that might be useful for problems in high-energy physics and/or gravitational waves:
  1. Extracting periodicity using the quantum Fourier transform
  2. Searching through large spaces
  3. Faster optimization
  4. Simulating quantum systems