EFFECTIVE FIELD THEORIES

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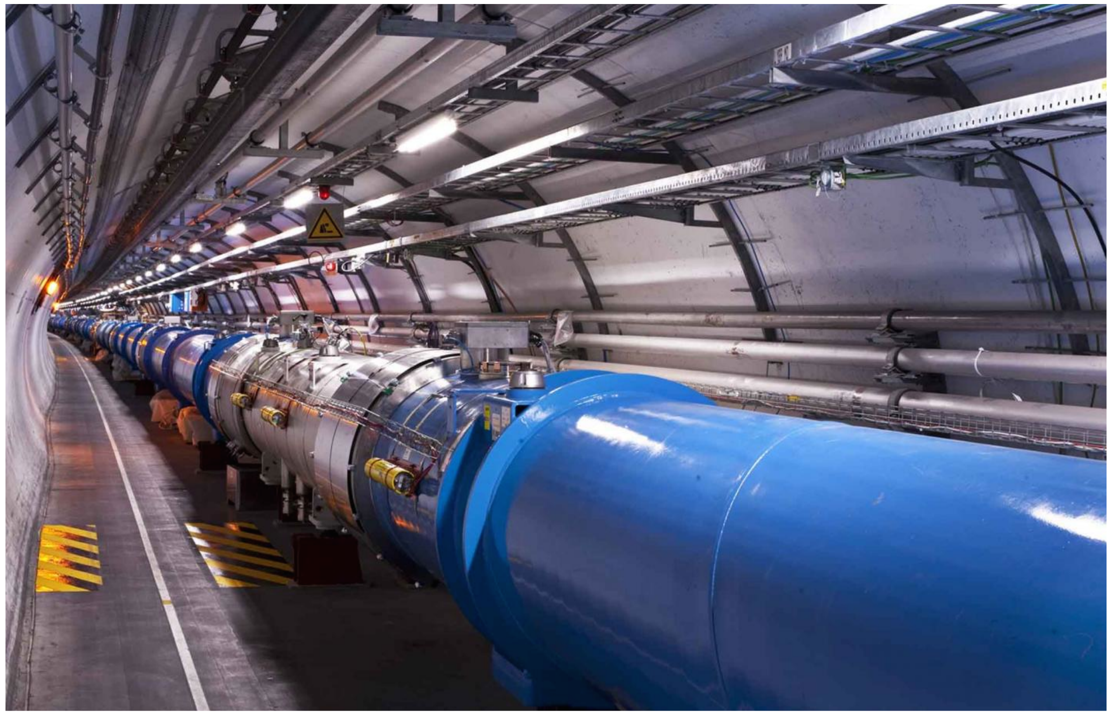
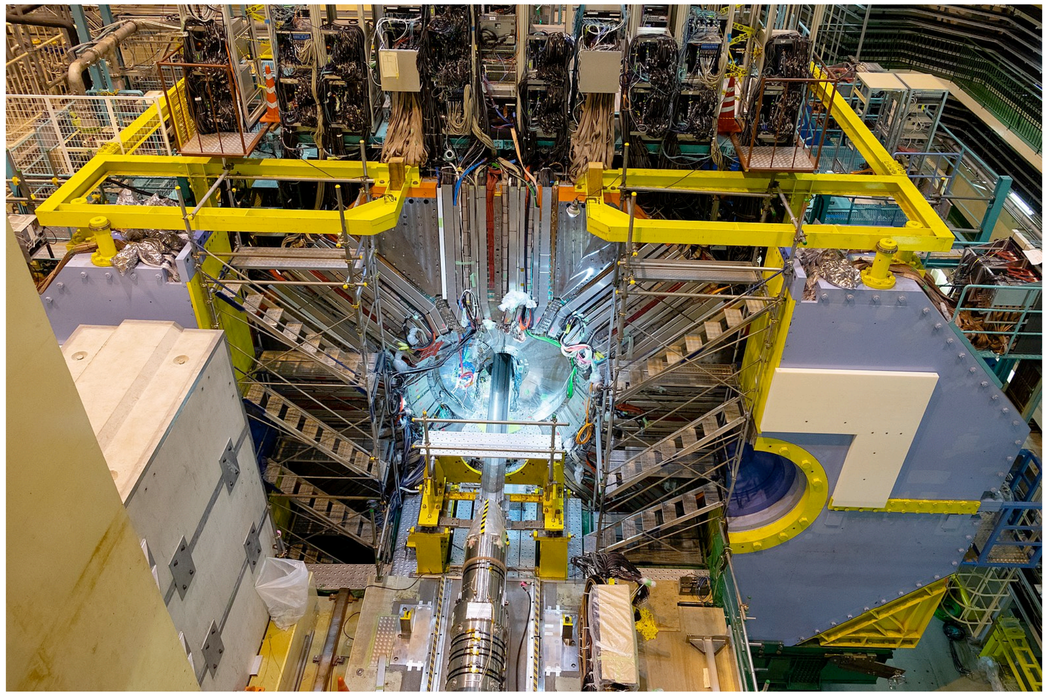
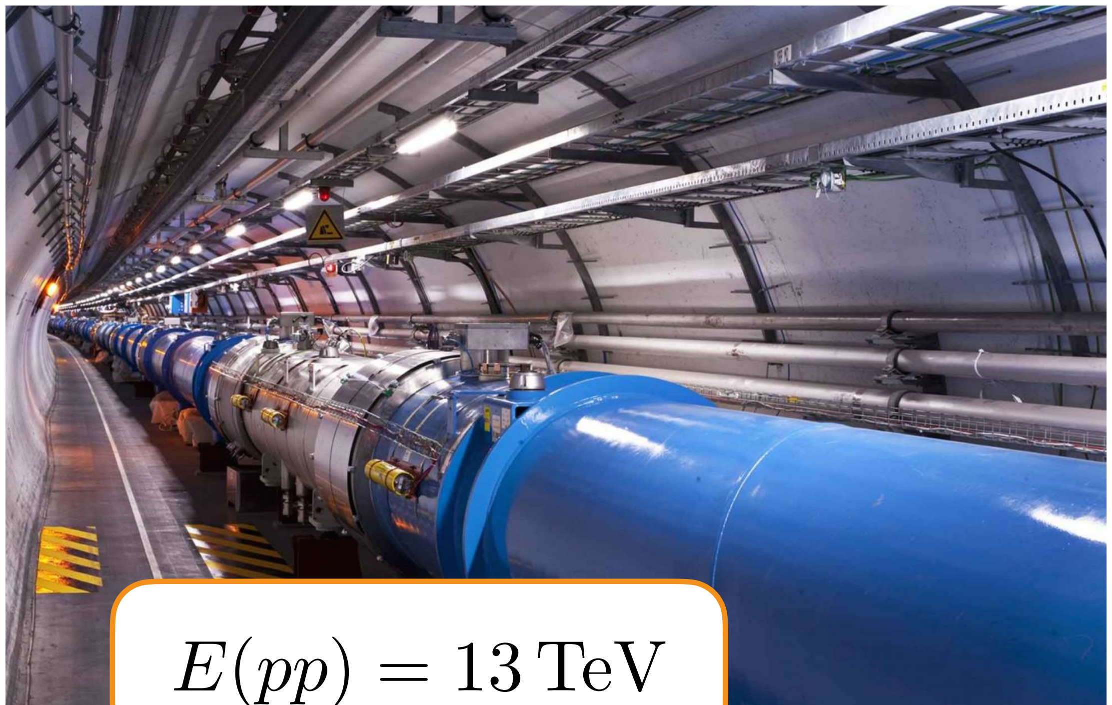
Home of the fundamental physics

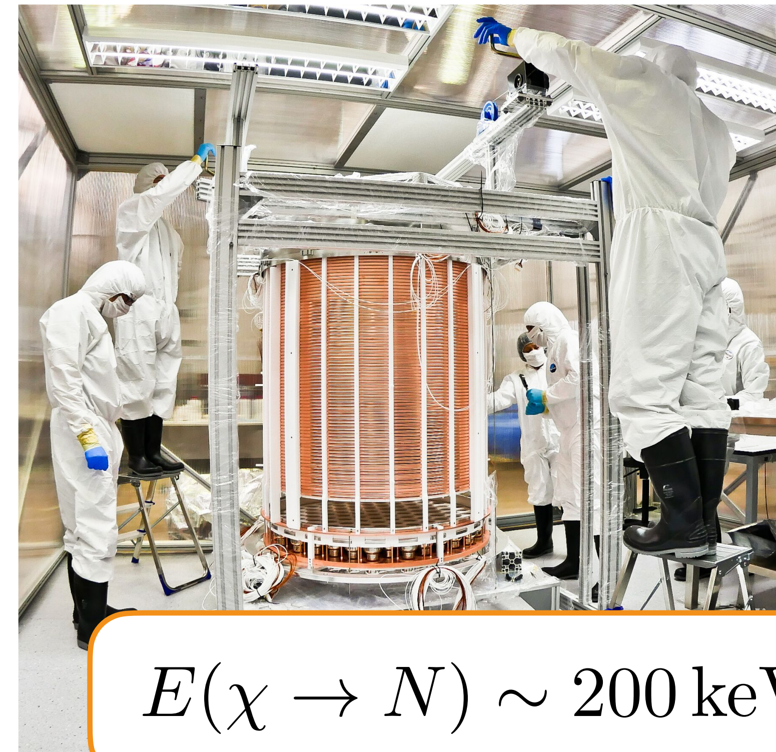




Physics at different scales

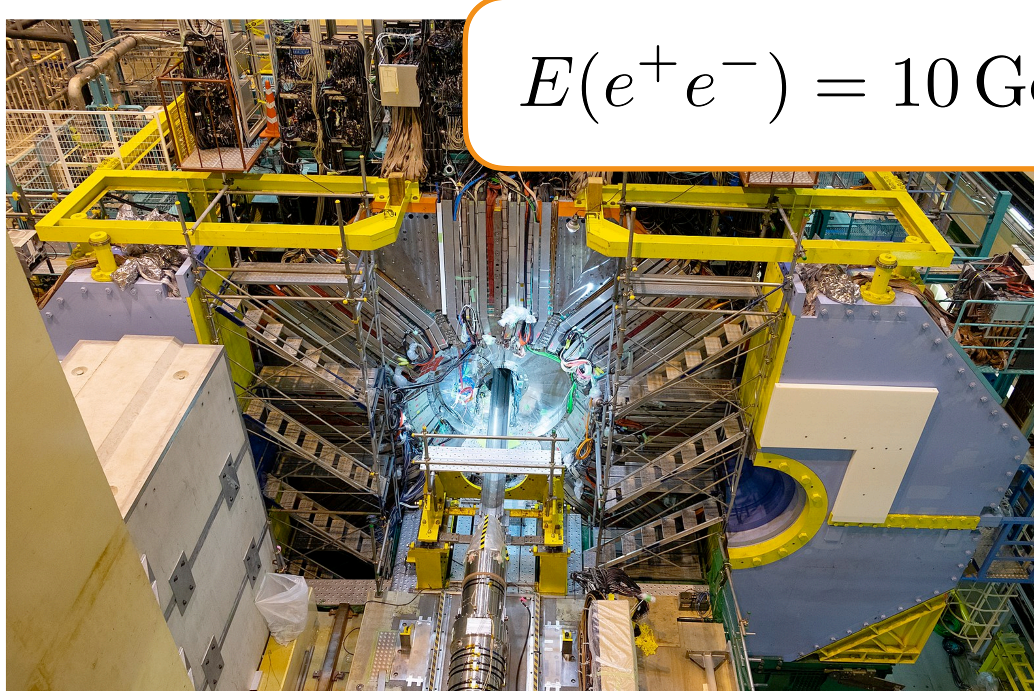


$E(pp) = 13\,\mathrm{TeV}$



$E(\chi \to N) \sim 200\,\mathrm{keV}$

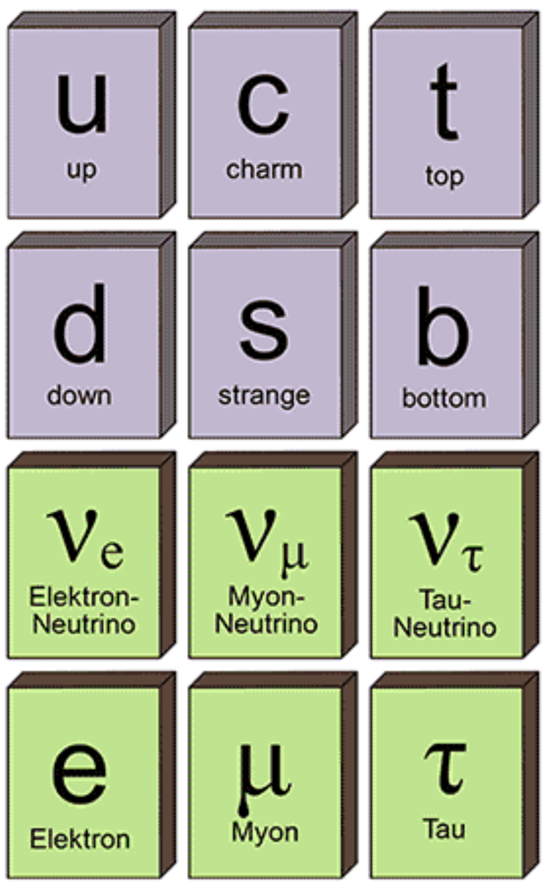
$E(e^+e^-) = 10\,\mathrm{GeV}$

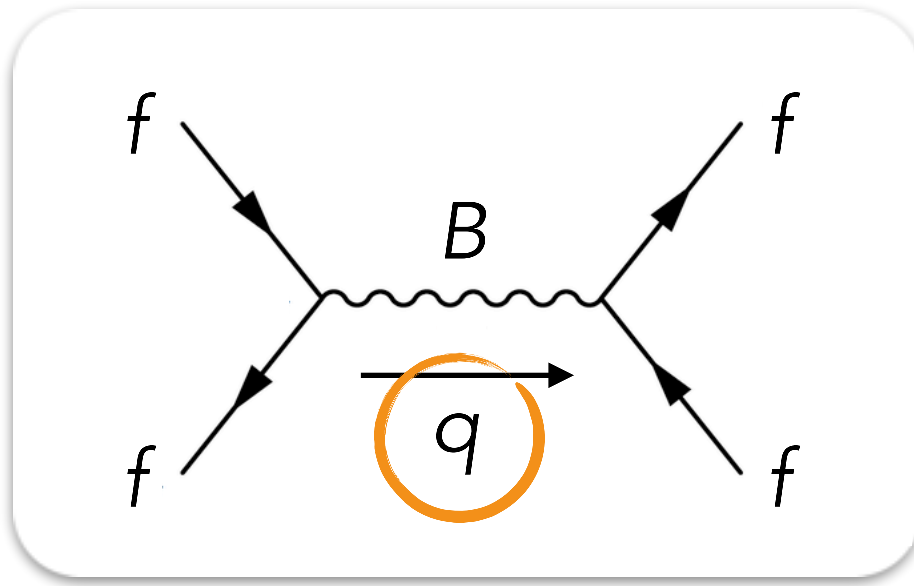


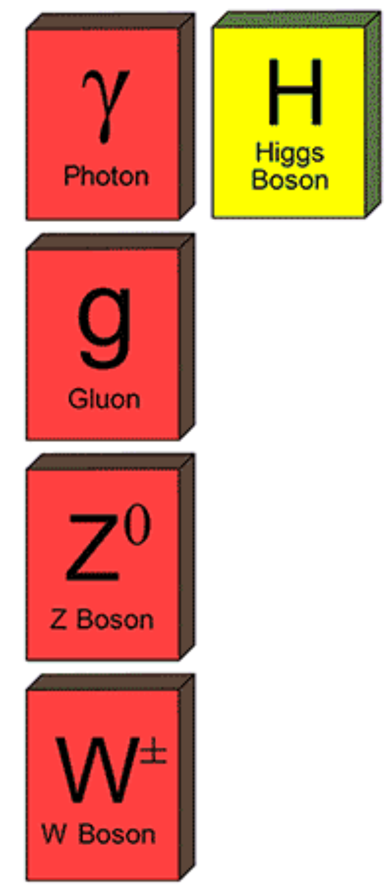
Standard Model interactions

matter particles

force carriers







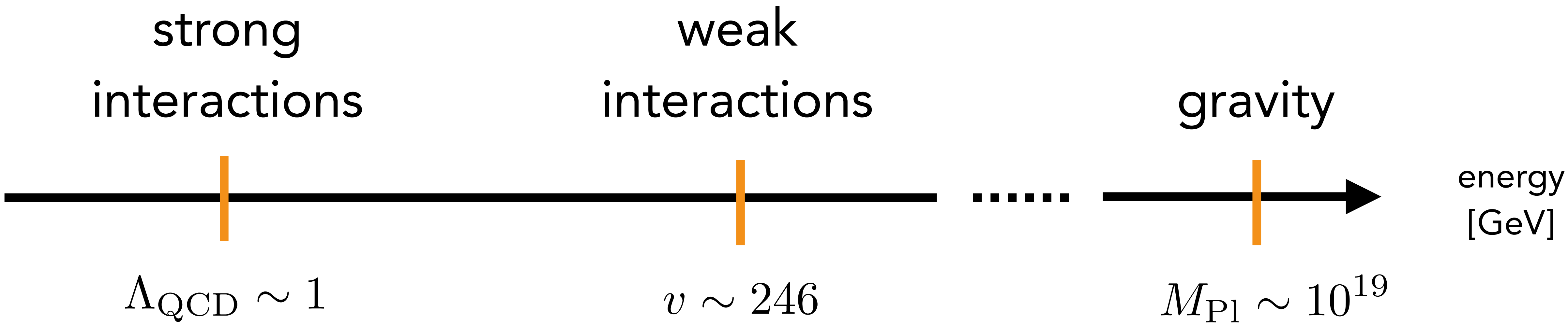
fermions

gauge symmetry

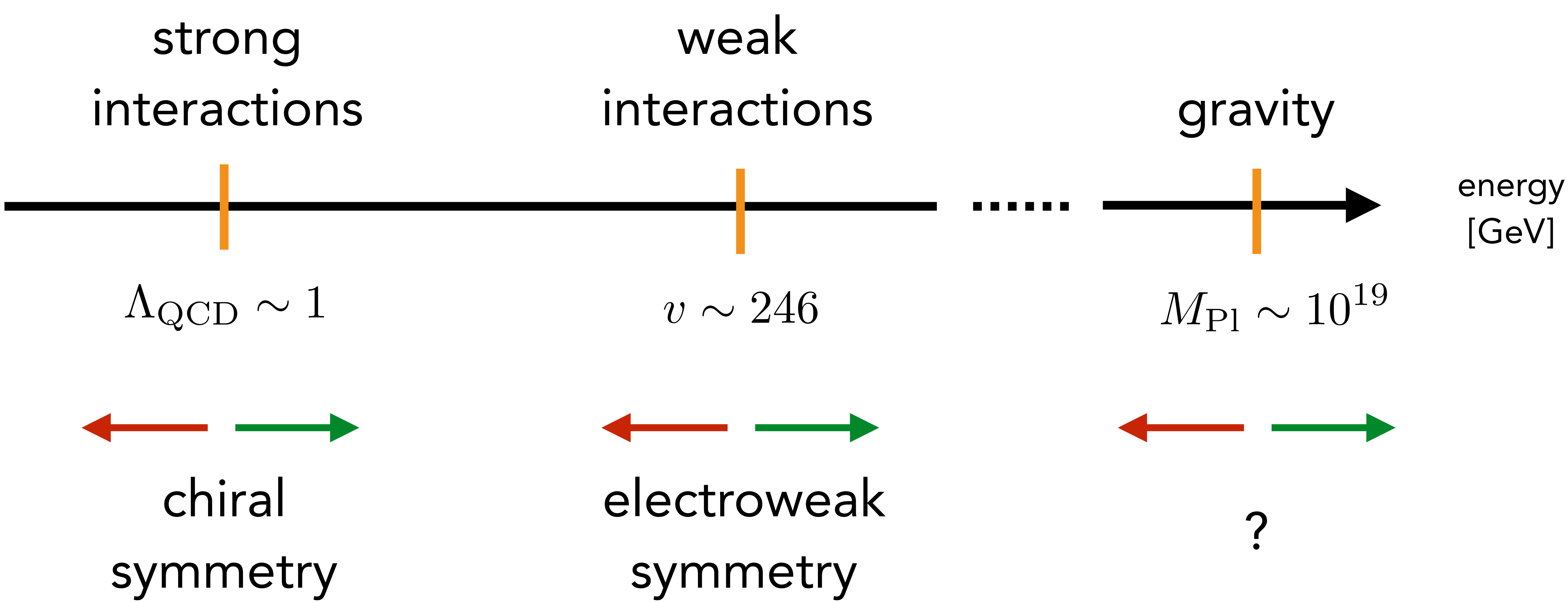
bosons

$$SU(3)_C \times SU(2)_L \times U(1)_Y$$

Interactions across the scales



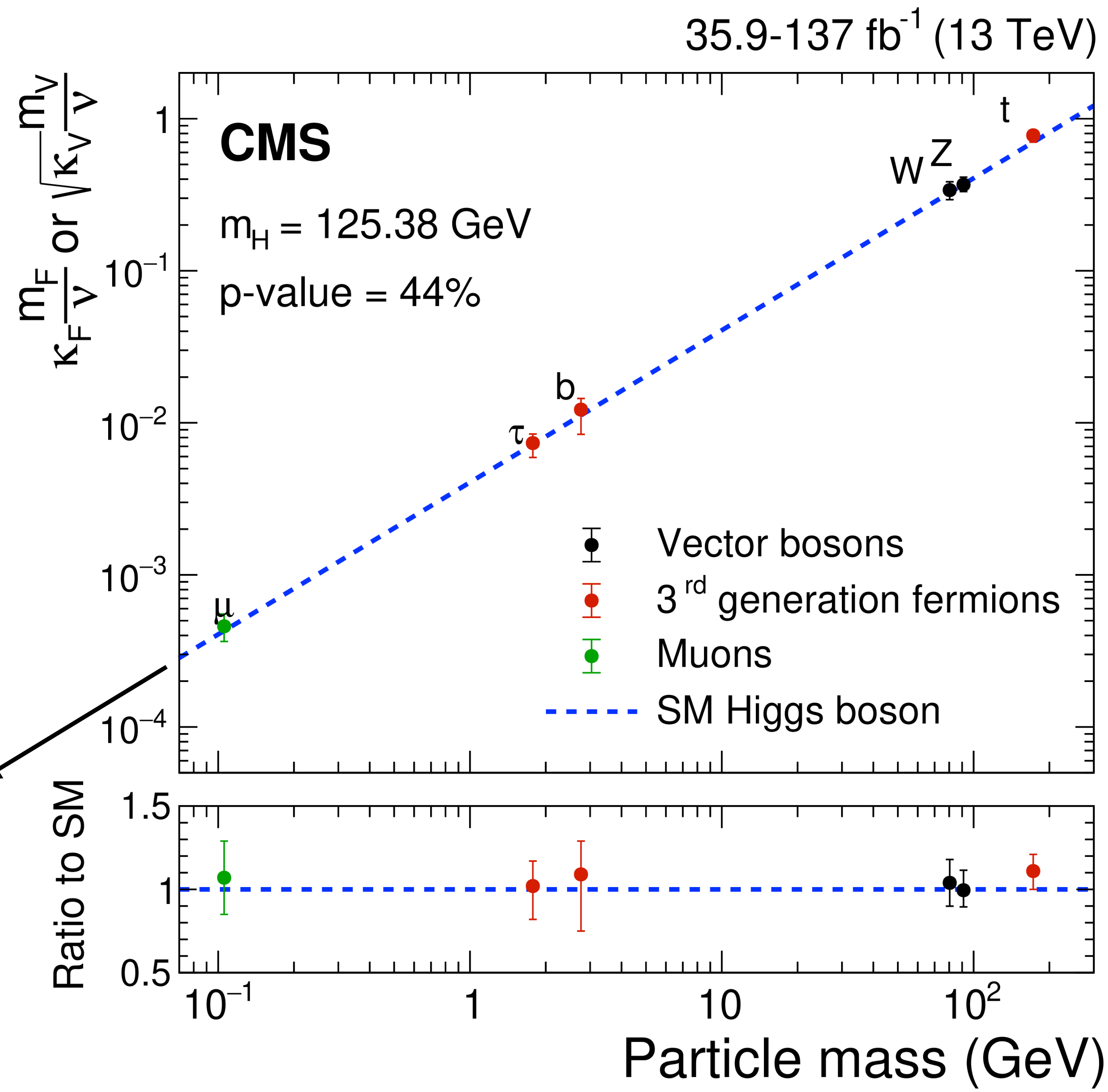
Interactions across the scales



symmetry breaking affects physics at low energies

electrodynamics: unbroken symmetry $\longrightarrow$ no scale

Particles across the scales



e, ν

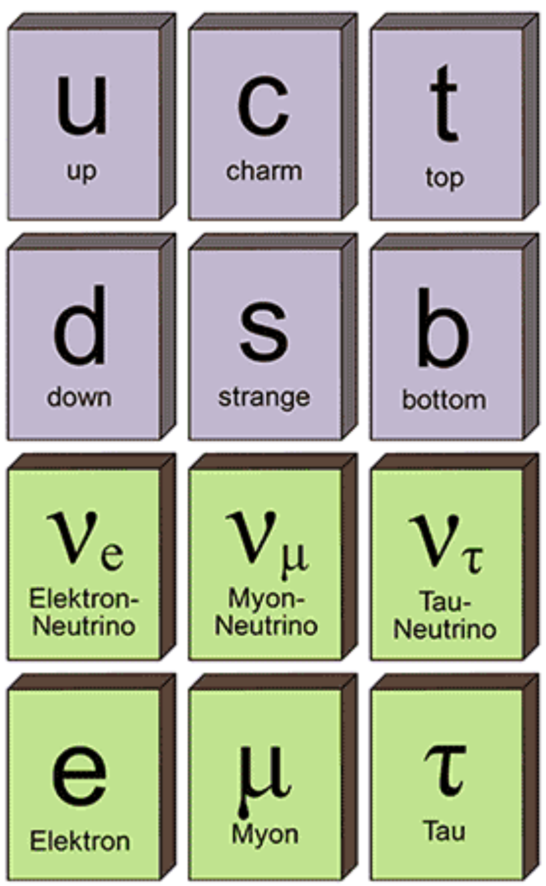
u, d, c, s

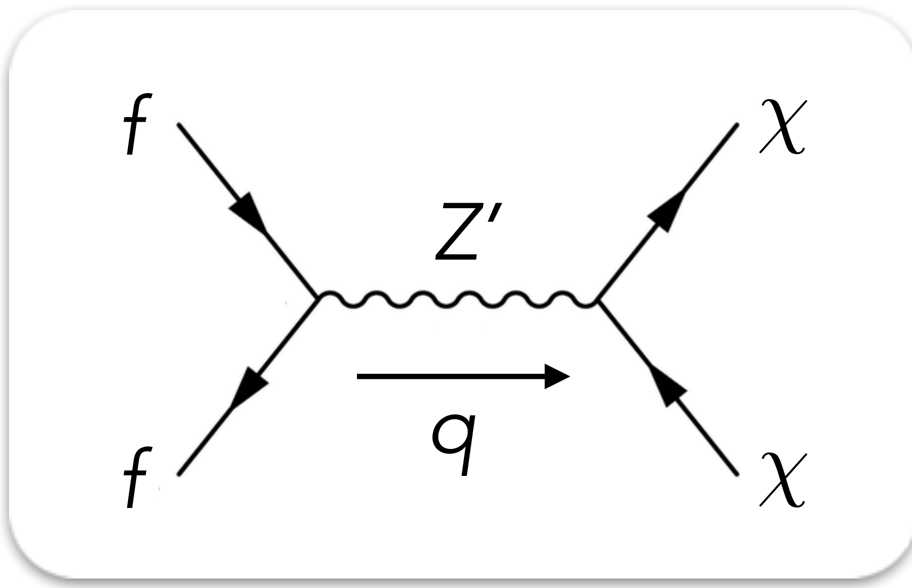
$$m_f = y_f \frac{v}{\sqrt{2}}$$

multiple mass scales - origin unknown

New interactions - a dark matter example

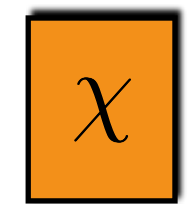
visible matter



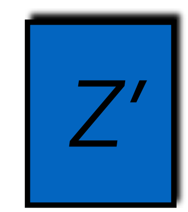


$U(1)', \mathbb{Z}_2$

dark matter



mediator

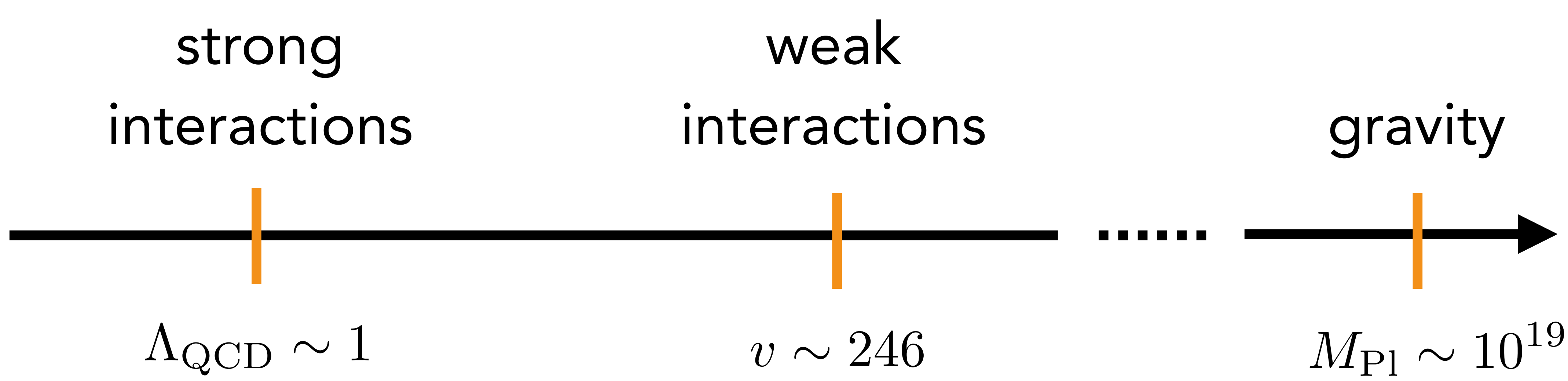


new symmetries? new scales? origin?

Effective Field Theories

in particle physics

- describe particle interactions involving multiple scales
- virtual effects of heavy particles at low energies



Effective Field Theories

beyond the particle zoo

- superconductivity: BCS theory
- gravity: radiation emission from binary stars
- condensed matter
- …

Effective Field Theories

in particle physics

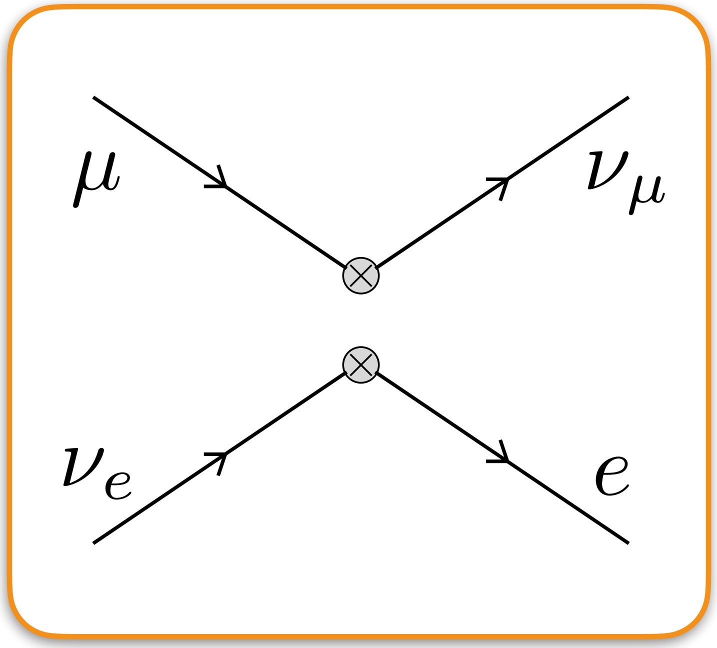
Part I: Basics

Part II: Standard Model Effective Theories

Part III: Dark Matter Effective Field Theory

Part I

Basics of Particle EFT



Literature

M. Neubert, 2005: Effective Field Theory and Heavy Quark Physics

D. Kaplan, 2005: Five Lectures on Effective Field Theory

A. Manohar 2018: Introduction to Effective Field Theories

H. Georgi, 1993: Effective Field Theory

Quantum Field Theory

classical mechanics

x, p

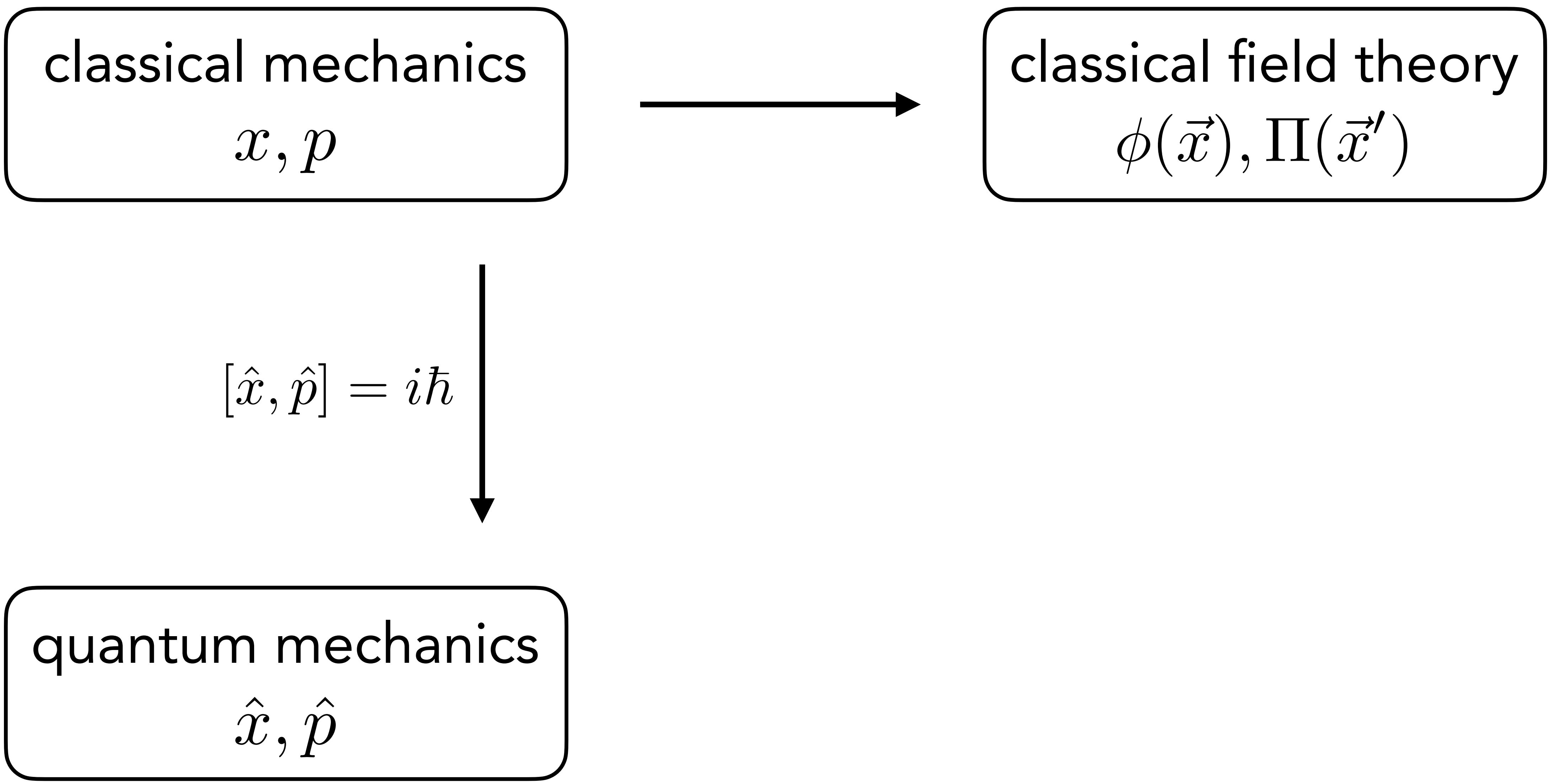
$[\hat{x}, \hat{p}] = i\hbar$

↓

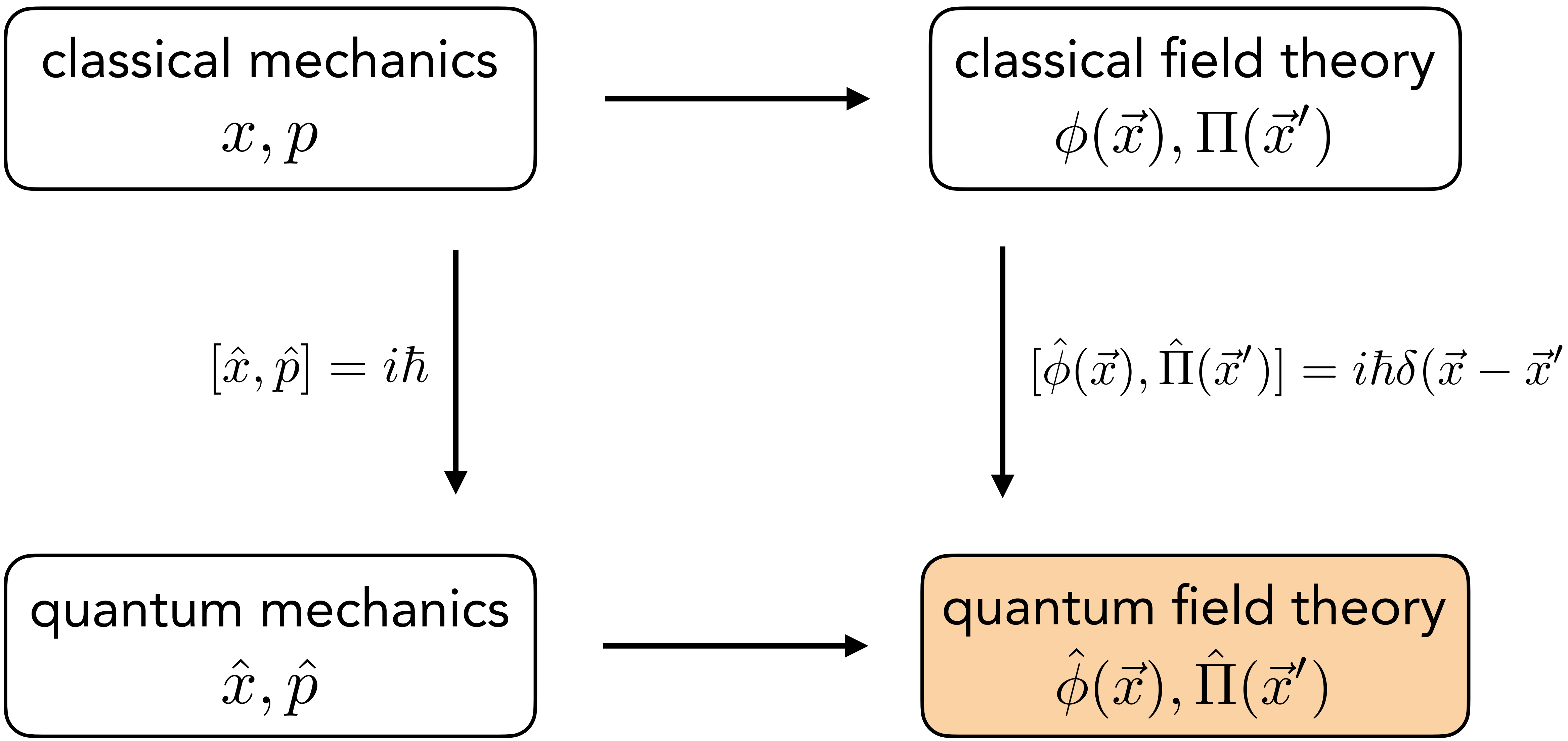
quantum mechanics

$\hat{x}, \hat{p}$

Quantum Field Theory



Quantum Field Theory



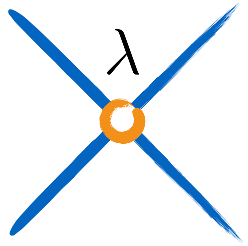
Goal: describe particle interactions - scattering, decay.

Particle interactions in a nutshell

- action $$\mathcal{S}(\phi) = \int d^4x\, \mathcal{L}(\phi(x))$$

fields: $\{\phi\}$

Lagrangian: $\mathcal{L}(\phi) = \mathcal{L}_0 + \mathcal{L}_{\text{int}} = -\frac{m^2}{2}\phi^2 - \frac{\lambda}{4!}\phi^4 + \dots$



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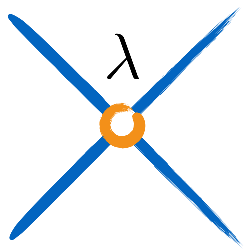

- scattering matrix $S = \mathbf{1} + i\mathbf{T}$

transition amplitude: $(\mathbf{T})_{fi} = \langle f | T \left[\int d^4x \, \mathcal{L}_{\text{int}}(\phi(x)) \right] | i \rangle$

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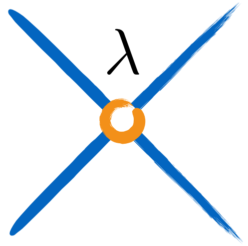

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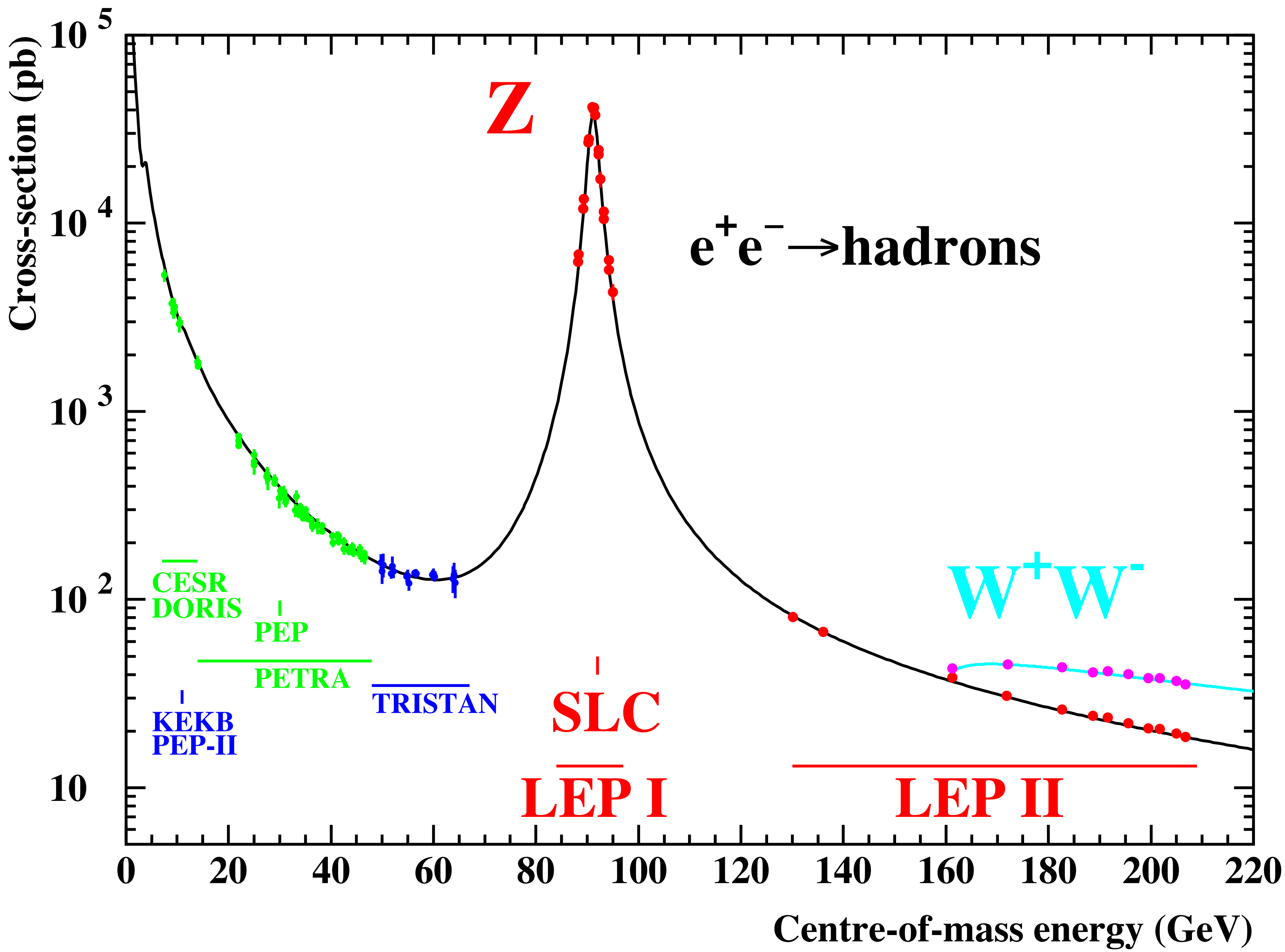
transition amplitude: $(\mathbf{T})_{fi} = \langle f | T \left[\int d^4x\, \mathcal{L}_{\rm int}(\phi(x)) \right] | i \rangle$

$= \mathcal{M}_{fi}\, \delta^{(4)}(\{k_i\} - \{k_f\})$ ‚matrix element'

- cross section $$\sigma_{ij\to f} = \frac{1}{2E_iE_j|v_i - v_j|} \int d\Pi_n |\mathcal{M}(k_i, k_j \to \{k_f\})|^2$$

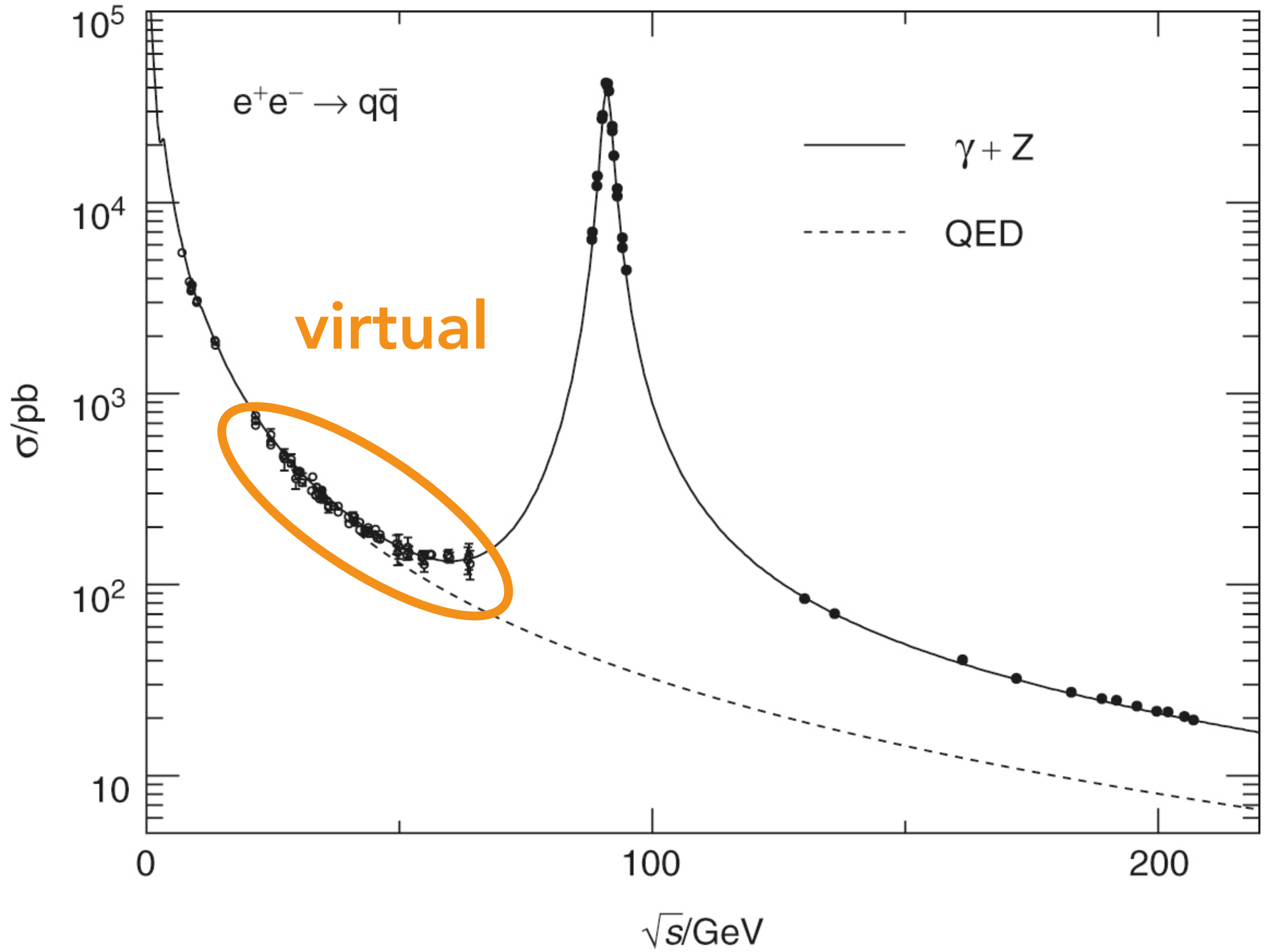
Why care about *effective* field theory?

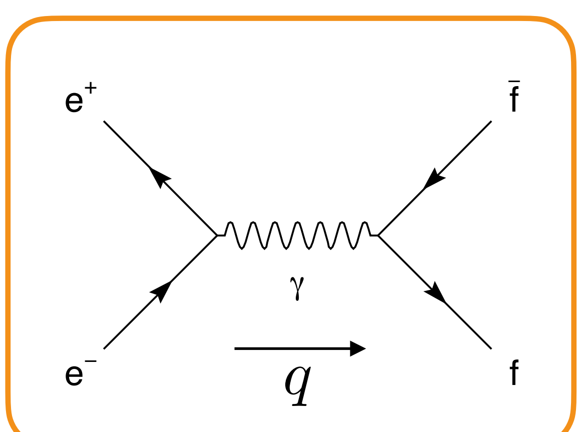
Electroweak production of hadrons



LEP collaboration 2005

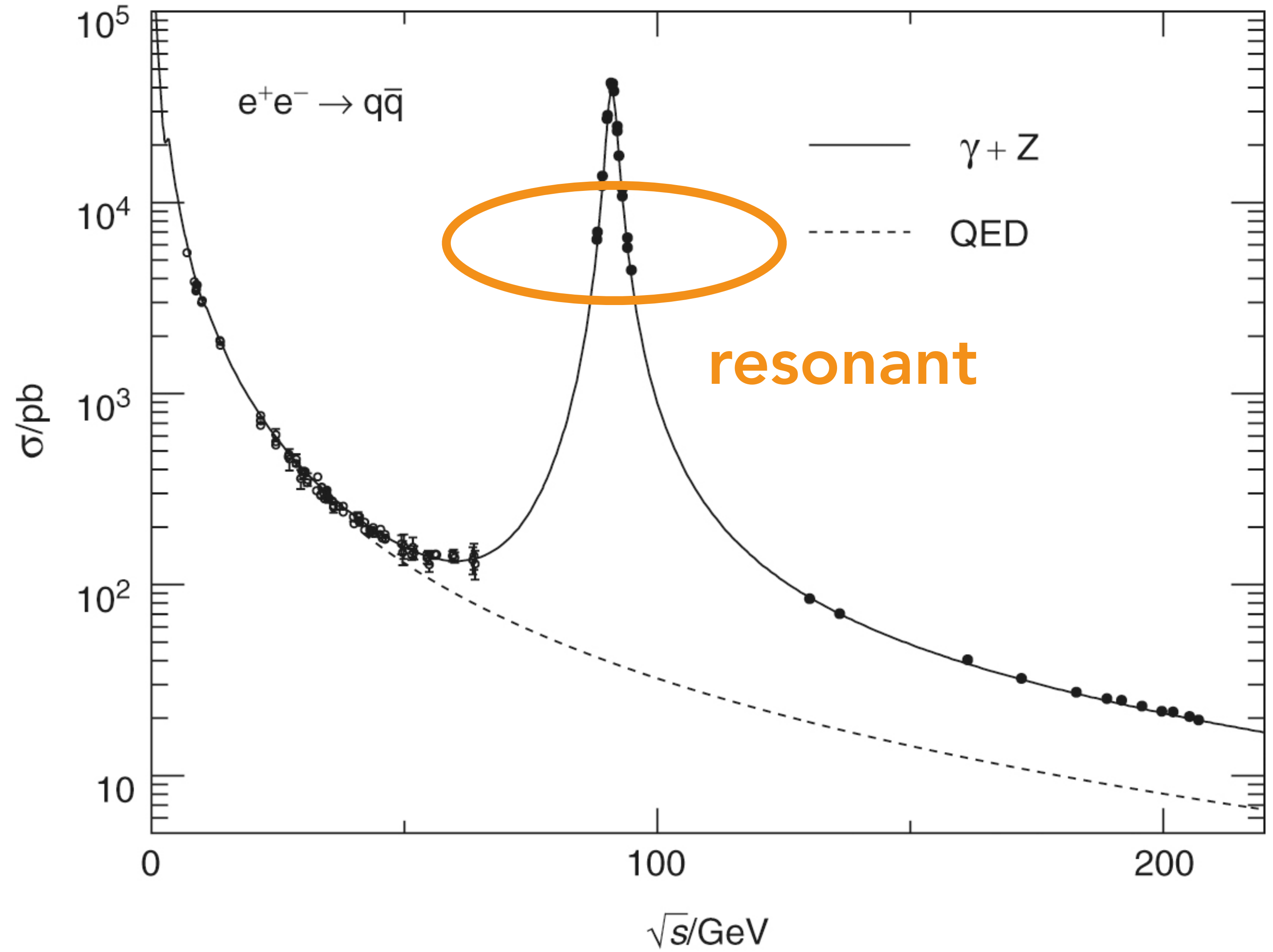
Electrodynamics

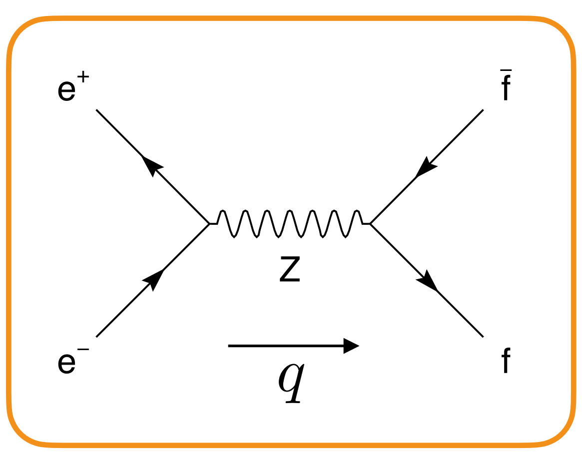




$$\mathcal{M} \sim \frac{1}{q^2}$$

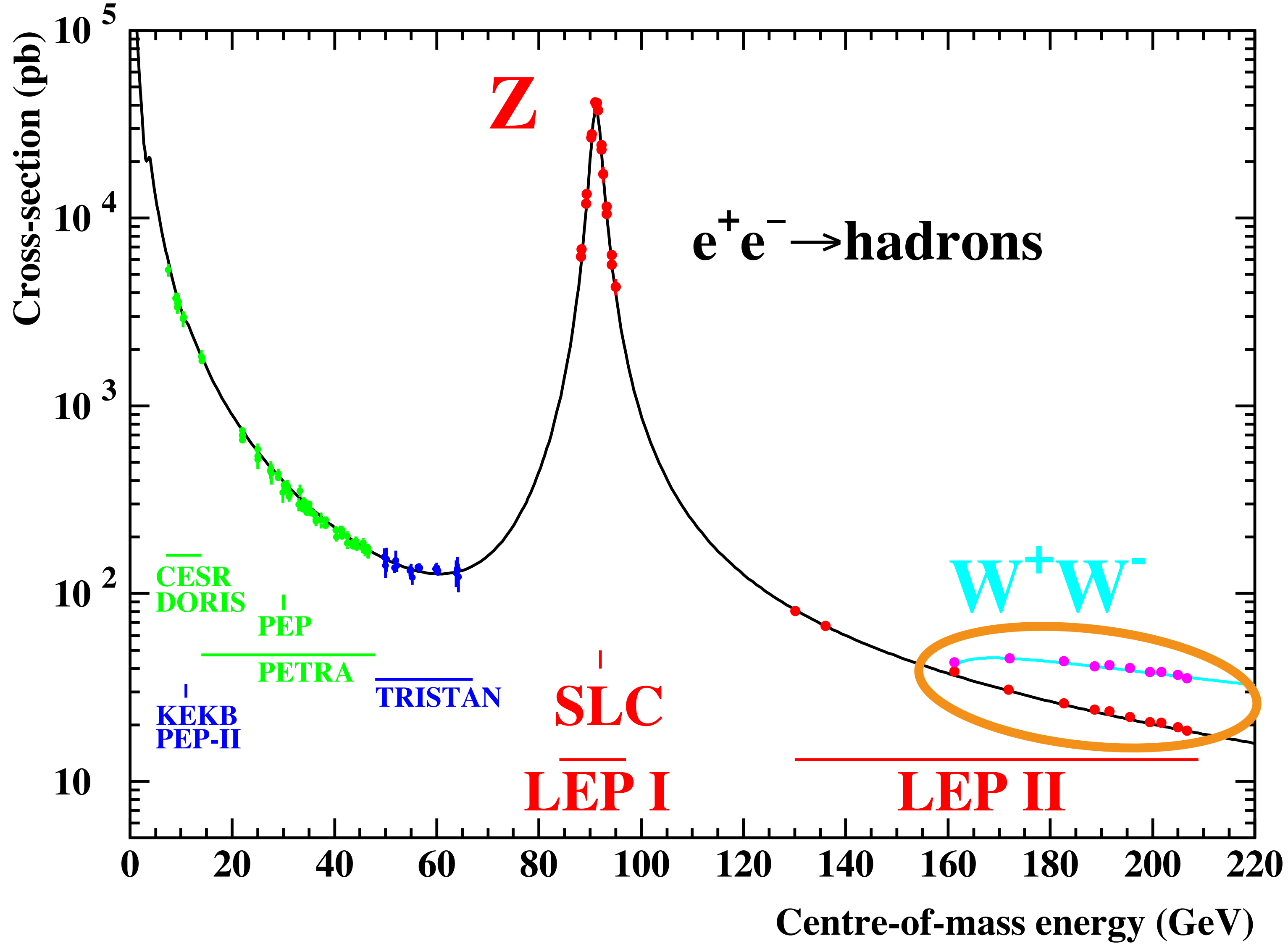
Weak interactions



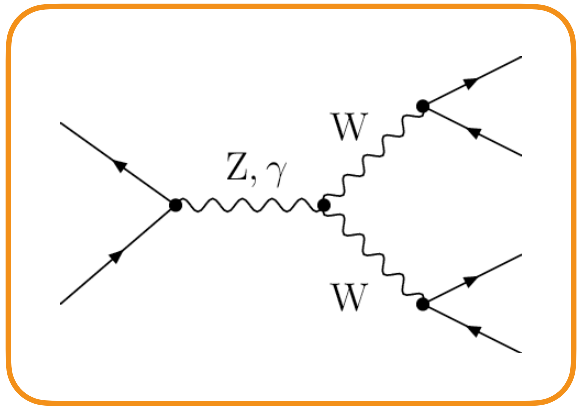


$$\mathcal{M} \sim \frac{1}{q^2 - m_Z^2 + i m_Z \Gamma_Z}$$

W pair production

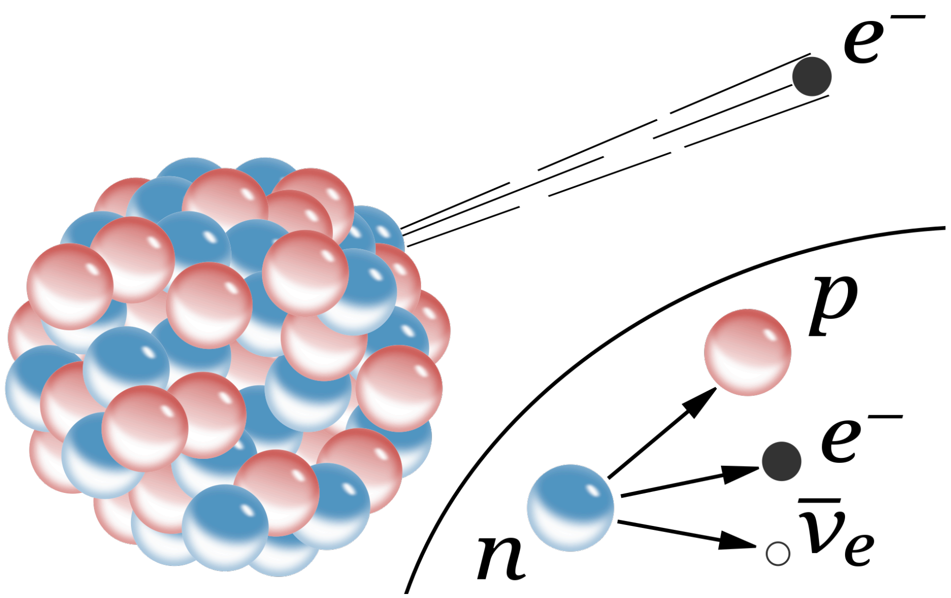


$$q^2 > (2m_W)^2 :$$



Fermi's theory of weak interactions

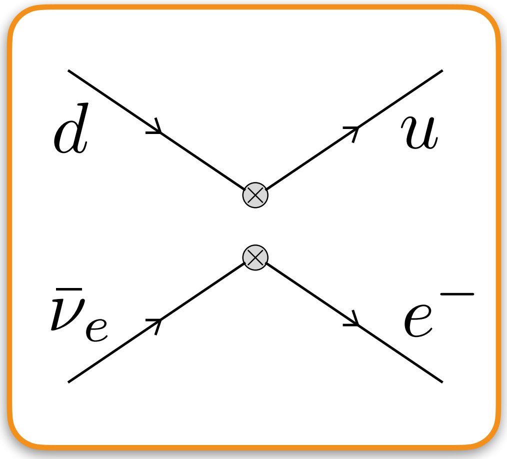
Beta decay:



Enrico Fermi

Fermi constant:

$$G_F = 1.166 \times 10^{-5}\,\mathrm{GeV}^{-2}$$

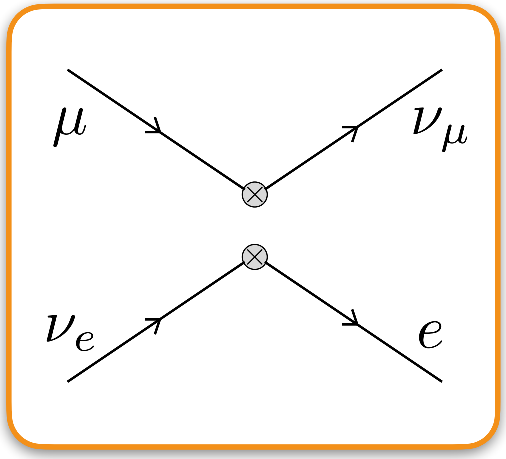


Muon Decay

$$\mu^- \to \nu_\mu e^- \bar{\nu}_e$$

Four-fermion coupling:

$$\mathcal{M}_{\text{eff}} = -i\frac{4G_F}{\sqrt{2}}\left(\bar{e}_L\gamma^\rho\nu_L\right)\left(\bar{\nu}_L\gamma_\rho\mu_L\right)$$



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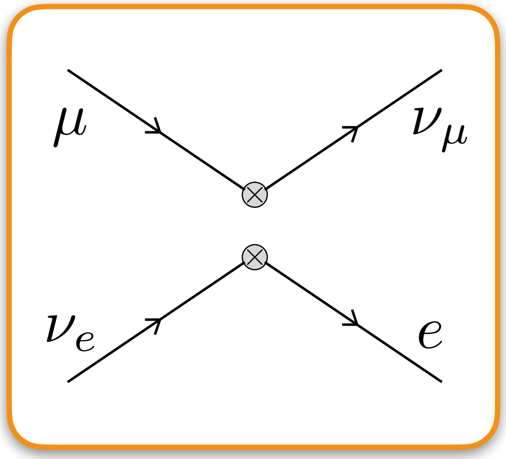
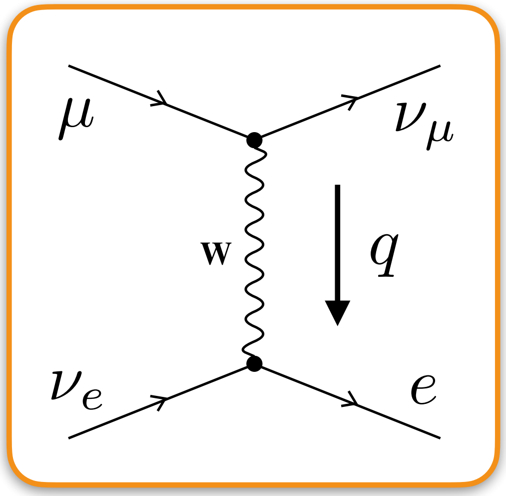


- $q^2 \ll m_W^2$: $\quad \dfrac{1}{q^2 - m_W^2} \approx -\dfrac{1}{m_W^2}\left[1 + \dfrac{q^2}{m_W^2} + \dots\right]$

Weak interaction:

$$\mathcal{M} = i\,\frac{g^2}{2}\left(\bar{e}_L\gamma^\rho\nu_L\right)\frac{g_{\rho\sigma}}{q^2 - m_W^2}\left(\bar{\nu}_L\gamma^\sigma\mu_L\right)$$



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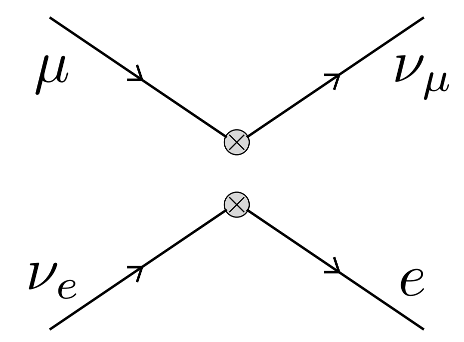
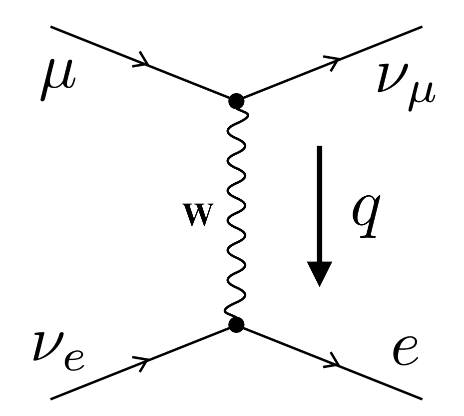


‚matching‘:

$$G_F = \frac{\sqrt{2}g^2}{8m_W^2}$$

Weak interaction:

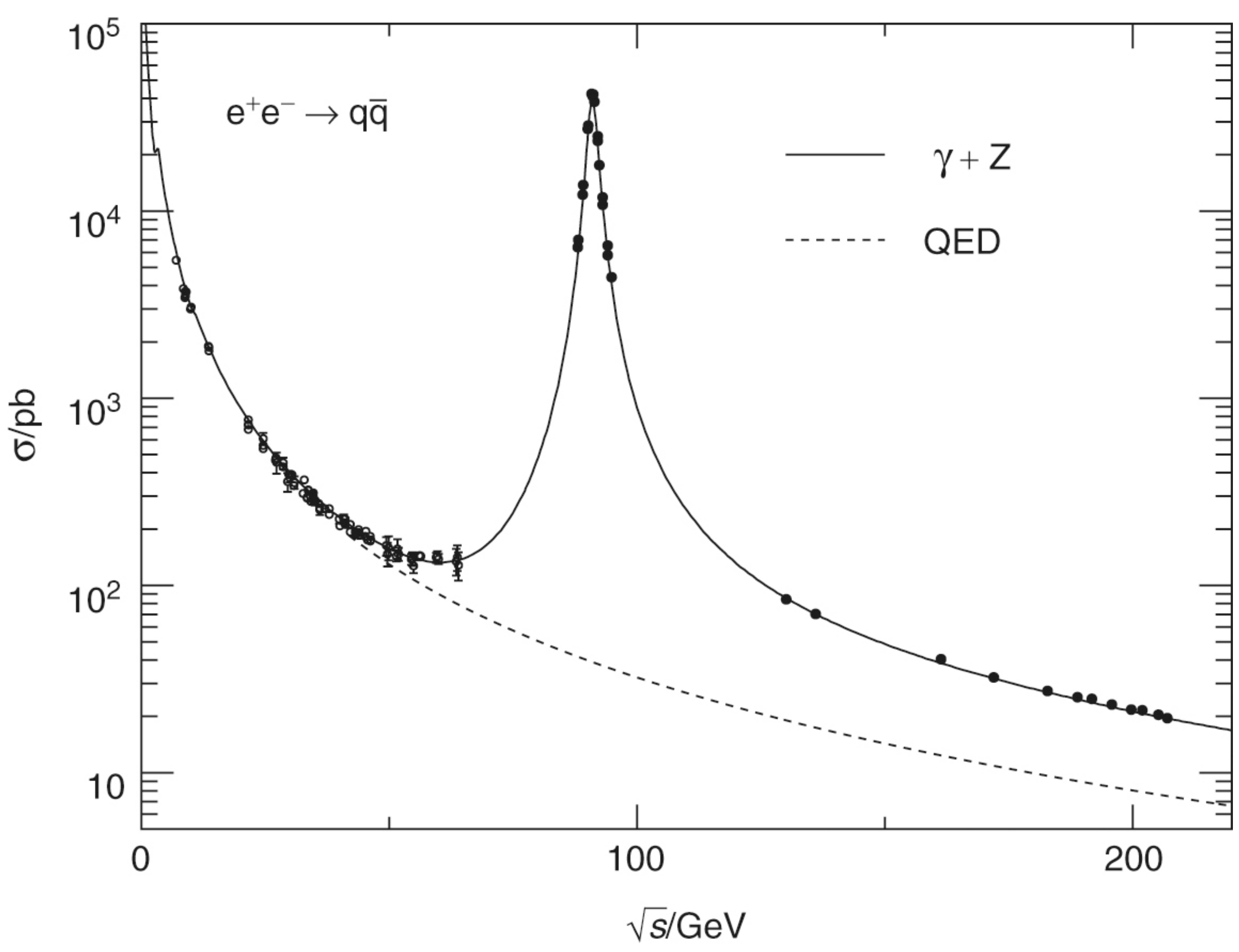
$$\mathcal{M} = i\,\frac{g^2}{2}\left(\bar{e}_L\gamma^\rho\nu_L\right)\frac{g_{\rho\sigma}}{q^2 - m_W^2}\left(\bar{\nu}_L\gamma^\sigma\mu_L\right)$$

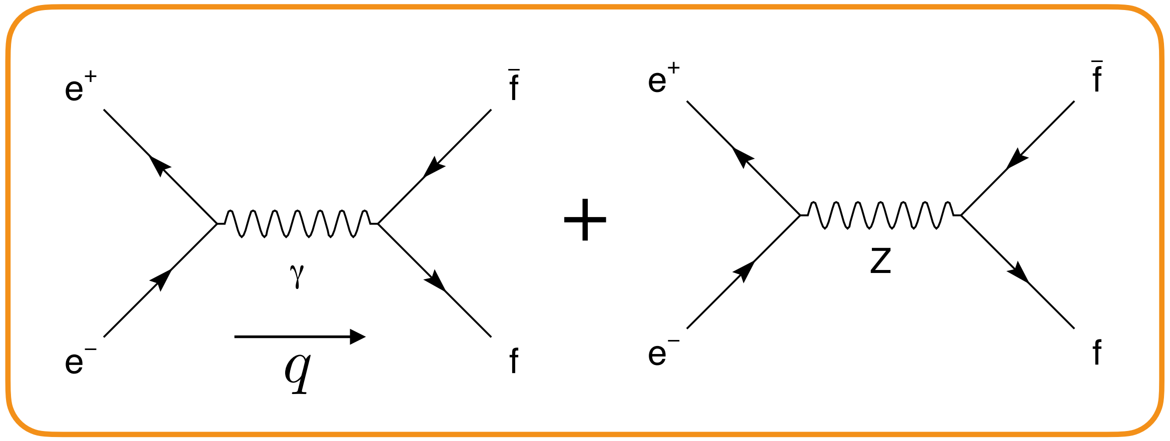


Your turn: Be Fermi

What is Fermi's theory for $e^+e^- \to q\bar{q}$

at energies $q^2 \ll m_Z^2$?

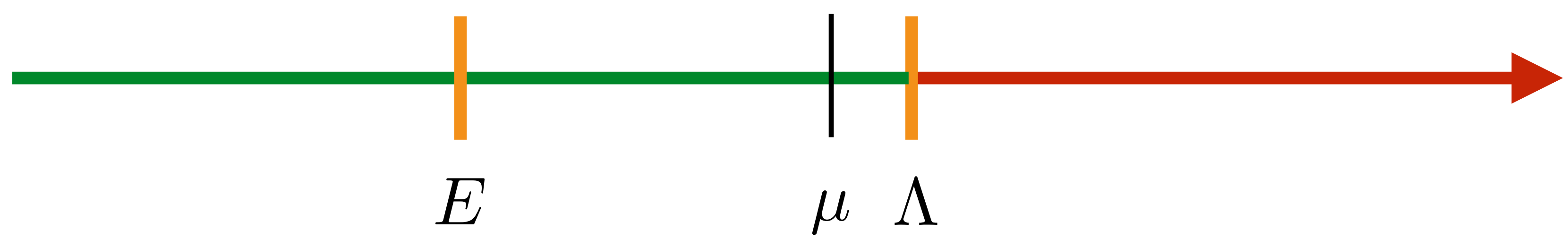




Effective Field Theory

approximation of full theory at experimentally relevant scales

- valid up to **cutoff scale** $\mu < \Lambda$



action in full theory: $\mathcal{S}(\phi) = \mathcal{S}(\phi_{E<\mu}, \phi_{E>\mu})$

intro: Neubert 2005

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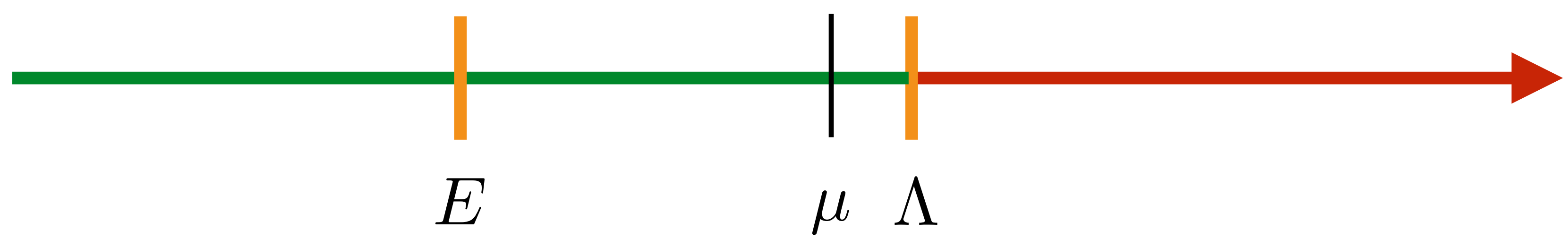

action in full theory: $$\mathcal{S}(\phi) = \mathcal{S}(\phi_{E<\mu}, \phi_{E>\mu})$$

remove high ‚frequencies‘: $$\mathcal{S}_\mu(\phi_{E<\mu}) = \int d^4x\, \mathcal{L}_{\text{eff}}(x)$$

- **effective Lagrangian** $$\mathcal{L}_{\text{eff}}(x) = \sum_i \frac{C_i}{\Lambda^{\gamma_i}} O_i(\phi_{E<\mu}(x))$$

intro: Neubert 2005

Operator expansion

$$\mathcal{L}_{\text{eff}}(x) = \sum_i \frac{C_i}{\Lambda^{\gamma_i}} O_i(\phi_{E<\mu}(x))$$

O_i : **local operators** with mass dimension $\gamma_i + 4$

C_i : **Wilson coefficients**

Operator expansion

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C_i : **Wilson coefficients**

- $\gamma_i > 0 : \; [O_i] > 4 \longrightarrow \mathcal{L}_{\text{eff}} = \sum_i \frac{C_i}{\Lambda} O_i^{(5)} + \sum_j \frac{C_j}{\Lambda^2} O_j^{(6)} + \dots$

power counting:

$$[\mathcal{L}] \overset{!}{=} 4 \longrightarrow [F] = \frac{3}{2},\; [B] = 1,\; [V_{\mu\nu}] = 2,\; [\partial_\mu] = 1$$

examples: $[(\bar{q}\gamma_\mu q)(\bar{\ell}\gamma^\mu \ell)] = 6 \qquad [(\overline{Q} H \sigma_{\mu\nu} q) F^{\mu\nu}] = 6$

Operator expansion

$$\mathcal{L}_{\text{eff}}(x) = \sum_i \frac{C_i}{\Lambda^{\gamma_i}} O_i(\phi_{E<\mu}(x))$$

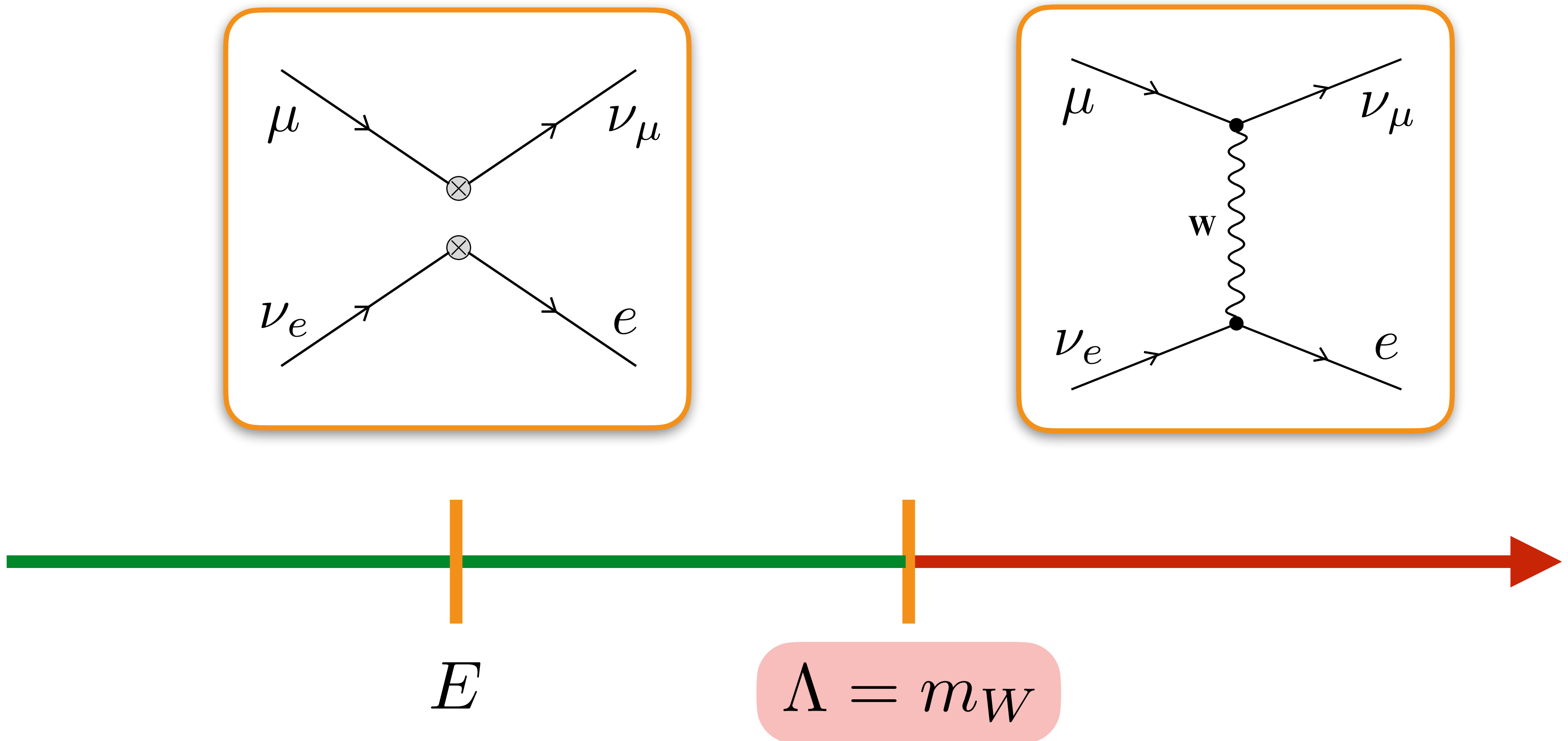
O_i : **local operators** with mass dimension $\gamma_i + 4$

C_i : **Wilson coefficients**

- $\gamma_i > 0: \; [O_i] > 4 \quad \longrightarrow \quad \mathcal{L}_{\text{eff}} = \sum_i \frac{C_i}{\Lambda} O_i^{(5)} + \sum_j \frac{C_j}{\Lambda^2} O_j^{(6)} + \ldots$
- in observables: $\sigma \sim C_i \left(\frac{E}{\Lambda}\right)^{\gamma_i} + \mathcal{O}(C_i^2)$

Operator effects are suppressed for $E \ll \Lambda$.

Interpretation: Fermi's theory



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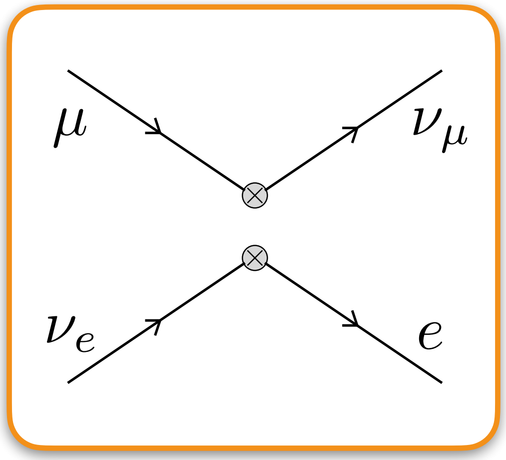

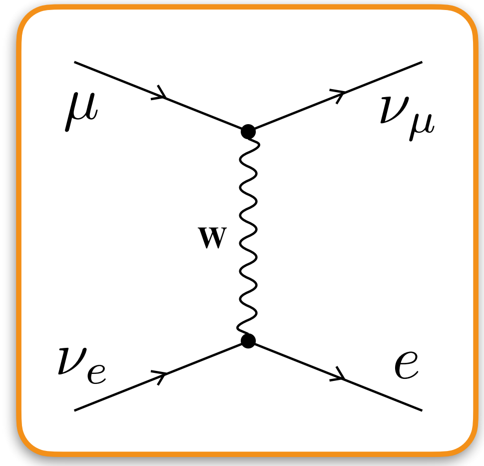

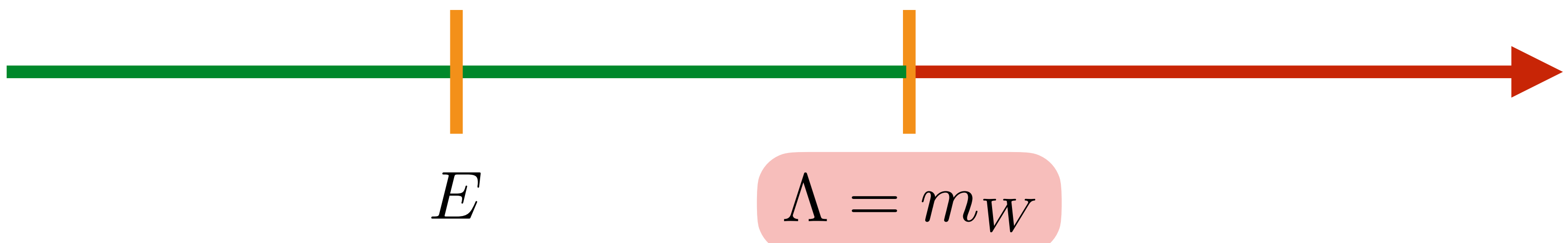

- effective Lagrangian

$$\mathcal{L}_{\text{eff}} = \sum_i \frac{C_i}{\Lambda^2} O_i^{(6)} + \ldots \; = -\frac{g^2}{2m_W^2} \left(\bar{e}_L \gamma^\rho \nu_L\right) \left(\bar{\nu}_L \gamma_\rho \mu_L\right) + \ldots$$

- weak *effective* interactions at low energies $E = m_\mu$:

$$\Gamma(\mu \to \nu_\mu e \bar{\nu}_e) \sim \left(\frac{m_\mu^2}{m_W^2}\right)^2 \longrightarrow \; G_F \cdot \text{GeV}^2 \ll 1$$

Summary Part I

Main features of an effective theory:

- good **approximation** of full theory at energies $E \ll \Lambda$
- **effective Lagrangian** describes particle interactions

$$\mathcal{L}_{\text{eff}} = \sum_i \frac{C_i}{\Lambda^{\gamma_i}} O_i$$

- series of **local operators** O_i with expansion parameter E/Λ
- operators respect **symmetries** of the full theory

Any effective theory is only valid up to cut-off.