Perturbative and colorful lectures on **Strong Interactions**

lecture 1/4

Laurent Favart
IIHE - Université libre de Bruxelles

Belgian Dutch German summer school (BND 2022) - Callantsoog (NL)

September 9, 2022

1. A few words about QCD
   1. running of α_S
   2. asymptotic freedom and confinement
2. quarks and gluons from e^+e^- annihilation
   1. quark electric charges
   2. quark and gluon spins
   3. QCD gauge parameters
3. Structure of hadrons
   1. elastic scattering
   2. deep inelastic scattering
   3. proton structure functions
   4. DGLAP evolution equation
4. hadron - hadron interactions
   1. jet production
   2. Drell-Yan process

A few preliminary remarks

	proton	neutron
mass	938.280 MeV	939.573 MeV
lifetime	stable	898 ± 16 sec
charge	$+1$	0
spin	$1/2$	$1/2$
magnetic moment	$2.793\mu_N$	$-1.913\mu_N$

A few preliminary remarks

	proton	neutron
mass	938.280 MeV	939.573 MeV
lifetime	stable	898 ± 16 sec
charge	$+1$	0
spin	$1/2$	$1/2$
magnetic moment	$2.793\mu_N$	$-1.913\mu_N$

lifetime and masses

- $\tau_n \simeq 15$ min - very long for wi
  $n \rightarrow p\, e^-\, \bar{\nu}_e$
- $\tau_p > 10^{33}$ y - stable (if free state)
- $m_n/m_p = 1.0014$!
  $\Rightarrow$ very similar internal bounding fields

A few preliminary remarks

	proton	neutron
mass	938.280 MeV	939.573 MeV
lifetime	stable	898 ± 16 sec
charge	$+1$	0
spin	1/2	1/2
magnetic moment	$2.793\mu_N$	$-1.913\mu_N$

lifetime and masses

- $\tau_n \simeq 15$ min - very long for wi
  $n \to p\, e^-\, \bar{\nu}_e$

- $\tau_p > 10^{33}$ y - stable (if free state)

- $m_n/m_p = 1.0014$!
  $\Rightarrow$ very similar internal bounding fields

- if $m_n \searrow$: $He/H \nearrow$ - $\nexists$ stars like $\odot$

- if $m_n \nearrow$: all atoms unstable (except H)

A few preliminary remarks

	proton	neutron
mass	938.280 MeV	939.573 MeV
lifetime	stable	898 ± 16 sec
charge	$+1$	0
spin	$1/2$	$1/2$
magnetic moment	$2.793\mu_N$	$-1.913\mu_N$

magnetic moment

Bohr magneton: $\mu_N = e\hbar/2m_N c$

we would naively expect $\mu_p = 1$ and $\mu_n = 0$

$\Rightarrow$ first sign of nucleon charged sub-structure !

- Part 1 -

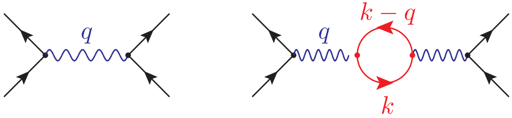
α_S running, asymptotic freedom and confinement

Screening effect in QED

- in QFT the vacuum fluctuates in virtual particle-antiparticle pairs
- in presence of an electric charge, these pairs get polarised
- this leads to an effective charge which depends on the distance to the probe

At LO :

$$(-i)e_0\gamma^\mu \cdot (-i)\frac{g_{\mu\nu}}{q^2} \cdot (-i)e_0\gamma^\nu = ie_0^2\gamma^\mu\frac{g_{\mu\nu}}{q^2}\gamma^\nu$$



with one loop :

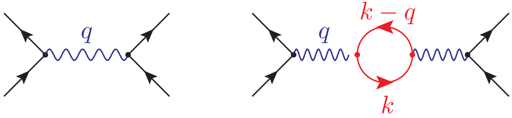
$$(-i)e_0\gamma^\mu(-i)\frac{g_{\mu\rho}}{q^2}(-1)\int\frac{d^4k}{(2\pi)^4}Tr\left[(-i)e_0\gamma^\rho\frac{i(\not{k}+m)}{k^2-m^2}(-i)e_0\gamma^\lambda\frac{i(\not{k}-\not{q}+m)}{(k-q)^2-m^2}\right](-i)\frac{g_{\lambda\nu}}{q^2}(-i)e_0\gamma^\nu$$

Screening effect in QED

- in QFT the vacuum fluctuates in virtual particle-antiparticle pairs
- in presence of an electric charge, these pairs get polarised
- this leads to an effective charge which depends on the distance to the probe

At LO :

$$(-i)e_0\gamma^\mu \cdot (-i)\frac{g_{\mu\nu}}{q^2} \cdot (-i)e_0\gamma^\nu = ie_0^2\gamma^\mu \frac{g_{\mu\nu}}{q^2}\gamma^\nu$$



with one loop :

$$(-i)e_0\gamma^\mu(-i)\frac{g_{\mu\rho}}{q^2}(-1)\int \frac{d^4k}{(2\pi)^4} Tr\left[(-i)e_0\gamma^\rho \frac{i(\not{k}+m)}{k^2-m^2}(-i)e_0\gamma^\lambda \frac{i(\not{k}-\not{q}+m)}{(k-q)^2-m^2}\right](-i)\frac{g_{\lambda\nu}}{q^2}(-i)e_0\gamma^\nu$$

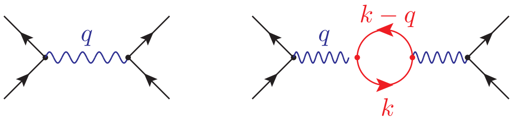
fermion loop

Screening effect in QED

- in QFT the vacuum fluctuates in virtual particle-antiparticle pairs
- in presence of an electric charge, these pairs get polarised
- this leads to an effective charge which depends on the distance to the probe

At LO :

$$(-i)e_0\gamma^\mu \cdot (-i)\frac{g_{\mu\nu}}{q^2} \cdot (-i)e_0\gamma^\nu = ie_0^2\gamma^\mu\frac{g_{\mu\nu}}{q^2}\gamma^\nu$$



with one loop :

$$(-i)e_0\gamma^\mu(-i)\frac{g_{\mu\rho}}{q^2}(-1)\int\frac{d^4k}{(2\pi)^4}Tr\left[(-i)e_0\gamma^\rho\frac{i(\not{k}+m)}{k^2-m^2}(-i)e_0\gamma^\lambda\frac{i(\not{k}-\not{q}+m)}{(k-q)^2-m^2}\right](-i)\frac{g_{\lambda\nu}}{q^2}(-i)e_0\gamma^\nu$$

trace because we sum over fermions spin states does the trace converge ?

$$\int \frac{d^4 k}{(k^2 - m^2)^2} = \int \frac{k^3}{(k^2 - m^2)^2}\, dk\; d\Omega$$

divergent for $k \to \infty$

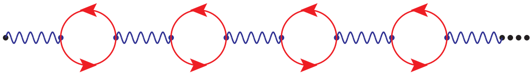
$$\int^{\infty} \frac{dk}{k} \to \int^{\mu_R} \frac{dk}{k} \qquad \to \text{we introduce a cutoff}$$

the integral give a log, the **effective** charge is given by:

$$e_{eff}^2(1loop) = i e_0^2 \gamma^\mu \frac{g_{\mu\nu}}{q^2} \gamma^\nu \left[1 + \frac{e_0^2}{12\pi^2} \ln\left(\frac{m^2 - q^2}{\mu_R^2}\right) + \frac{e_0^2}{12\pi^2} F(q^2) \right]$$

where $F(q^2)$ is a finite function vanishing for $q^2 \to \infty$.

Adding more loops

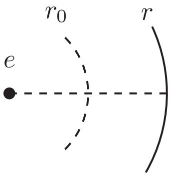


$$e^2_{eff} = e_0^2 \left[1 + \frac{e_0^2}{3\pi} \ln\left(\frac{m^2 - q^2}{\mu^2_{UV}}\right) + \left(\frac{e_0^2}{3\pi} \ln\left(\frac{m^2 - q^2}{\mu^2_{UV}}\right)\right)^2 + \cdots \right]$$

behaves arithmetic sequence, whose sum is (using $Q^2 = -q^2$):

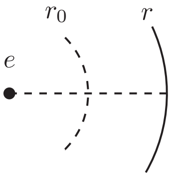
$$e^2_{eff}(Q^2) = \frac{e_0^2}{1 - \frac{e_0^2}{3\pi} \ln \frac{Q^2}{\mu_R^2}}$$

Interpretation : using the distance $r^2 \sim 1/Q^2$ (sign in front of ln changes)



$$e^2(r) = \frac{e^2(r_0)}{1 + \frac{2e^2(r_0)}{3\pi} \ln \frac{r}{r_0}}$$

Interpretation : using the distance $r^2 \sim 1/Q^2$ (sign in front of ln changes)

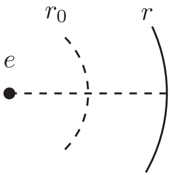


$$e^2(r) = \frac{e^2(r_0)}{1 + \frac{2e^2(r_0)}{3\pi} \ln \frac{r}{r_0}}$$

if $r \gg r_0$ $\nearrow$ $\simeq \frac{e^2(r_0)}{e^2(r_0)\cdots} \rightarrow \alpha = 1/137$

independ. of r_0

Interpretation : using the distance $r^2 \sim 1/Q^2$ (sign in front of ln changes)

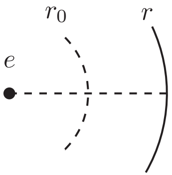


$$e^2(r) = \frac{e^2(r_0)}{1 + \frac{2e^2(r_0)}{3\pi} \ln \frac{r}{r_0}}$$

if $r \gg r_0$ $\nearrow$ $\simeq \frac{e^2(r_0)}{e^2(r_0)\cdots} \rightarrow \alpha = 1/137$ independ. of r_0

if $r_0 \rightarrow 0$ $\searrow$ $\simeq \frac{e^2(r_0)}{\infty}$ only possible if $e^2(r_0 \rightarrow 0) \rightarrow \infty$

Interpretation : using the distance $r^2 \sim 1/Q^2$ (sign in front of ln changes)

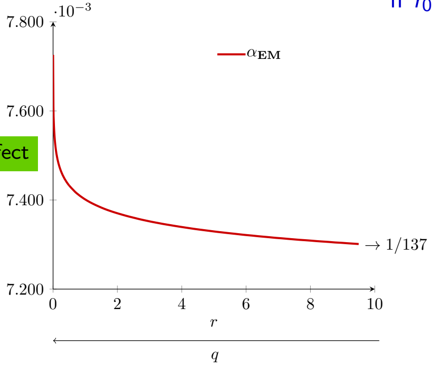


$$e^2(r) = \frac{e^2(r_0)}{1 + \frac{2e^2(r_0)}{3\pi} \ln \frac{r}{r_0}}$$

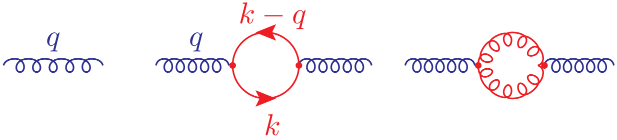
if $r \gg r_0$ $\simeq \frac{e^2(r_0)}{e^2(r_0)\cdots} \rightarrow \alpha = 1/137$ independ. of r_0

if $r_0 \rightarrow 0$ $\simeq \frac{e^2(r_0)}{\infty}$ only possible if $e^2(r_0 \rightarrow 0) \rightarrow \infty$

screening effect



in QCD: situation partially similar, on additional gluon loop

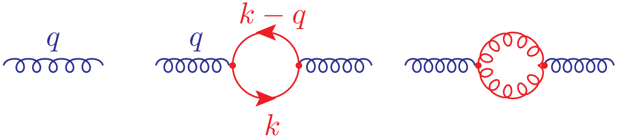


- the fermion loop (N_f) is similar to the QED case
- the additional gluon loop comes with a (-1)
- sum over N_c colors

$$\alpha_s(Q^2) = \frac{\alpha_s(\mu_R^2)}{1 - \frac{2N_f - 11N_c}{6\pi}\alpha_s(\mu_R^2)\ln\frac{Q^2}{\mu_R^2}}$$

- competition between N_f and N_c
- as $b = 2N_f - 11N_c < 0 \Rightarrow$ anti-screening effect

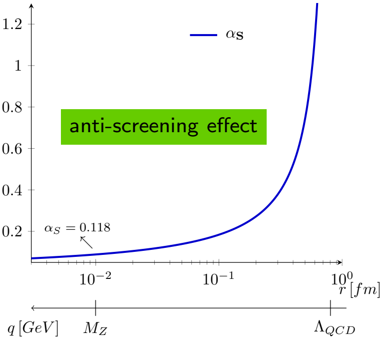
in QCD: situation partially similar, on additional gluon loop



- the fermion loop (N_f) is similar to the QED case
- the additional gluon loop comes with a (-1)
- sum over N_c colors

$$\alpha_s(Q^2) = \frac{\alpha_s(\mu_R^2)}{1 - \frac{2N_f - 11N_c}{6\pi}\alpha_s(\mu_R^2)\ln\frac{Q^2}{\mu_R^2}}$$

- competition between N_f and N_c
- as $b = 2N_f - 11N_c < 0 \Rightarrow$ anti-screening effect



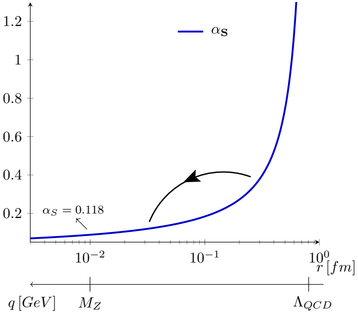
- QFT gives us the evolution but not the values
- in one direction or the other

$$\alpha_s(Q^2) = \frac{\alpha_s(\mu_R^2)}{1 - \frac{2N_f - 11N_c}{6\pi}\alpha_s(\mu_R^2)\ln\frac{Q^2}{\mu_R^2}}$$

- Λ_{QCD} is defined as the divergent scale value:

$$\alpha_s(\Lambda_{QCD}^2) = \frac{\alpha_s(\mu_R^2)}{0}$$

- $\Lambda_{QCD} \simeq m_\pi \simeq 100$ MeV



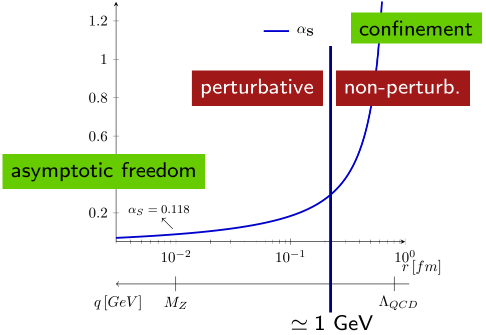
- QFT gives us the evolution but not the values
- in one direction or the other

$$\alpha_s(Q^2) = \frac{\alpha_s(\mu_R^2)}{1 - \frac{2N_f - 11N_c}{6\pi}\alpha_s(\mu_R^2)\ln\frac{Q^2}{\mu_R^2}}$$

- Λ_{QCD} is defined as the divergent scale value:

$$\alpha_s(\Lambda^2_{QCD}) = \frac{\alpha_s(\mu_R^2)}{0}$$

- $\Lambda_{QCD} \simeq m_\pi \simeq 100$ MeV
- to apply perturbative methods: $\alpha_S \ll 1$
i.e. $Q \gg \Lambda_{QCD}$
- if non-pert. (no Fey. diag, etc) but $\mathcal{L}_{QCD}$ is valid



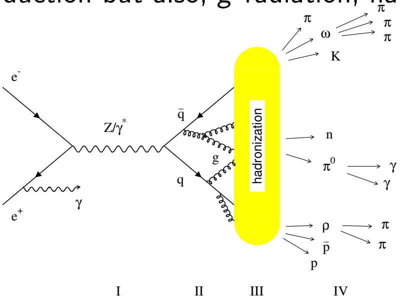
- Part 2 -

e^+e^- annihilation and QCD

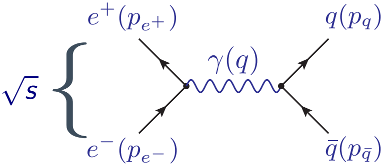
quark production from e^-e^+ annihilation

- the cleanest way to produce quarks is from e^-e^+ annihilations:
- no color charge in the initial state
- pure electroweak process (at LO)
- inital state momentum fully transfered to the $q\bar{q}$

$\rightarrow$ allows us to study q production but also, g radiation, hadronisation,...



- at LO, for γ exchange only:

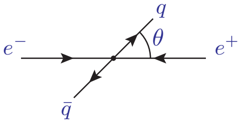


- negl. mass terms :

$$\overline{|\mathcal{M}|^2} = 8(4\pi)^2 \frac{\alpha^2}{q^4} N_c\, Q_i^2\, \{(p_{e^+} \cdot p_q)(p_{e^-} \cdot p_{\bar{q}}) + (p_{e^+} \cdot p_{\bar{q}})(p_{e^-} \cdot p_q)\}$$

- only 1 degree of freedom
- in the c.m.s.:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}cos\theta} = \frac{\pi\alpha^2}{2s} N_c\, Q_i^2\, (1 + \cos^2\theta)$$



acces to quark electric charges

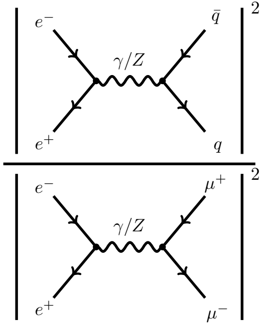
- as all hadrons come from quarks and that hadrons only come from quarks :

$$\sigma(e^+e^- \to X(\text{any hadron})) = \sum_{i=1}^{N_f} \sigma(e^+e^- \to q\bar{q}) = \frac{4\pi\alpha^2}{3s} N_c \sum_{i=1}^{N_f} Q_i^2$$

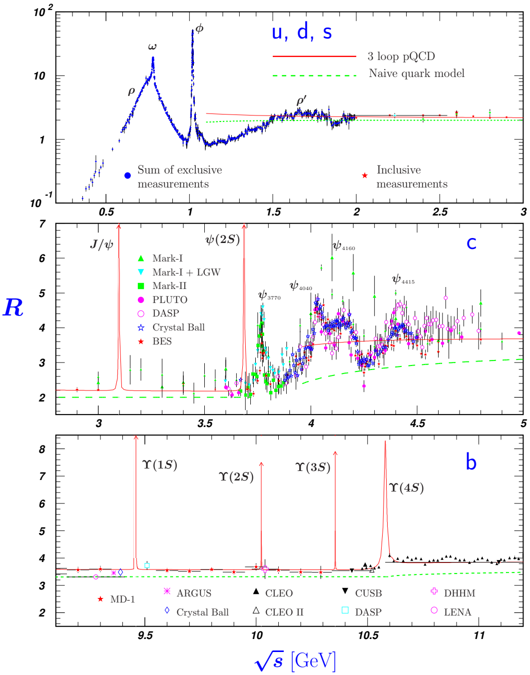
- we can study the observable:

$$R = \frac{\sigma(e^+e^- \to X)}{\sigma(e^+e^- \to \mu^+\mu^-)} = \frac{\frac{4\pi\alpha^2}{3s} N_c \sum_{i=1}^{N_f} Q_i^2}{\frac{4\pi\alpha^2}{3s}} = N_c \sum_{i=1}^{N_f} Q_i^2$$

$R \sim$



- is this very simple model realistic ?

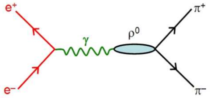


$$
\begin{aligned}
R &= \frac{\sigma(e^+e^- \to X)}{\sigma(e^+e^- \to \mu^+\mu^-)} = N_c \sum_{i=1}^{N_f} Q_i^2 \\
&= 3\left[\left(\frac{2}{3}\right)^2 + \left(\frac{-1}{3}\right)^2 + \left(\frac{-1}{3}\right)^2 + \ldots\right] \\
&= 2 \qquad \text{for } 2m_s \ll \sqrt{s} \ll 2m_c \\
&= 10/3 \quad \text{for } 2m_c \ll \sqrt{s} \ll 2m_b \\
&= 11/3 \quad \text{for } 2m_b \ll \sqrt{s} \ll m_Z
\end{aligned}
$$

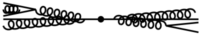
- amazing success if away from resonnances

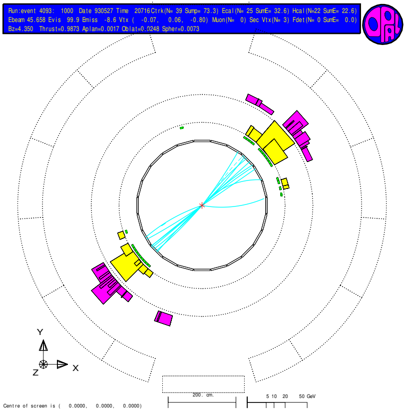
quark charges are as expected

- if close to a resonnance, the important npQCD corrections only in the numerator

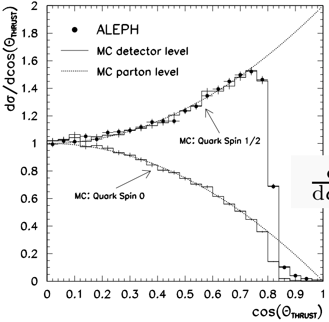


jets - during the had. process, hadrons are forming around the initial quark direction

jet {  } jet



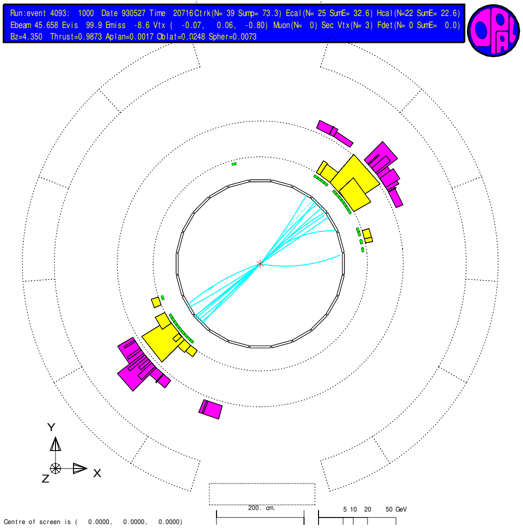
- two particles below to different jets only if $p_i \cdot p_j \gg \Lambda^2_{QCD}$
- angular measurement:

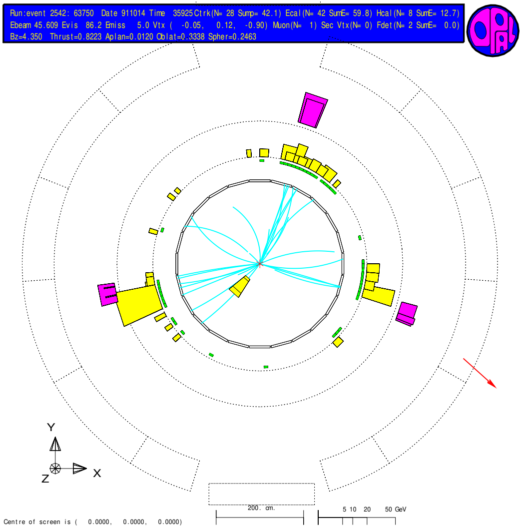


$$\frac{d\sigma}{d\cos\theta} \sim (1 + \cos^2\theta)$$

the quark are spin 1/2 !

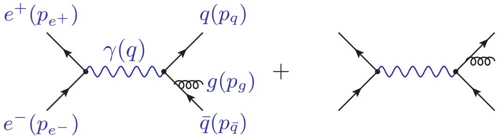
- excellent agreement between jet and quark directions


Run:event 4093: 1000 Date 930527 Time 20716 Ctrk(N= 39 Sump= 73.3) Ecal(N= 25 SumE= 32.6) Hcal(N=22 SumE= 22.6)
Ebeam 45.658 Evis 99.9 Emiss -8.6 Vtx (-0.07, 0.06, -0.80) Muon(N= 0) Sec Vtx(N= 3) Fdet(N= 0 SumE= 0.0)
Bz=4.350 Thrust=0.9873 Aplan=0.0017 Oblat=0.0248 Spher=0.0073
Y
X
Z
200. cm.
5 10 20 50 GeV
Centre of screen is (0.0000, 0.0000, 0.0000)


Run:event 2542: 63750 Date 911014 Time 35925 Ctrk(N= 28 Sump= 42.1) Ecal(N= 42 SumE= 59.8) Hcal(N= 8 SumE= 12.7)
Ebeam 45.609 Evis 86.2 Emiss 5.0 Vtx (-0.05, 0.12, -0.90) Muon(N= 1) Sec Vtx(N= 0) Fdet(N= 2 SumE= 0.0)
Bz=4.350 Thrust=0.8223 Aplan=0.0120 Oblat=0.3338 Spher=0.2463
Y
X
Z
200. cm.
5 10 20 50 GeV
Centre of screen is (0.0000, 0.0000, 0.0000)

First pQCD correction

- up to now, what we saw was driven by EW ME + hadronisation
- additional jets are due to pQCD effects: gluon radiation from the quark lines



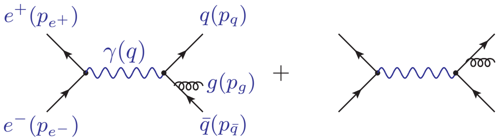
$e^+e^- \to q\bar{q}$: $\overline{|\mathcal{M}|}^2 = 8(4\pi)^2\alpha^2 N_c\, Q_i^2 \dfrac{(p_{e^+}\cdot p_q)(p_{e^-}\cdot p_{\bar{q}}) + (p_{e^+}\cdot p_{\bar{q}})(p_{e^-}\cdot p_q)}{q^4}$

$e^+e^- \to q\bar{q}g$: $\overline{|\mathcal{M}|}^2 = 8(4\pi)^3\alpha^2\alpha_S C_F N_c\, Q_i^2 \dfrac{(p_{e^+}\cdot p_q)(p_{e^-}\cdot p_{\bar{q}}) + (p_{e^+}\cdot p_{\bar{q}})(p_{e^-}\cdot p_q)}{(p_{e^+}\cdot p_{e^-})(p_q\cdot p_g)(p_{\bar{q}}\cdot p_g)}$

1) couplings: $\alpha^2 \to \alpha^2\alpha_S$ - first QCD correction

First pQCD correction

- up to now, what we saw was driven by EW ME + hadronisation
- additional jets are due to pQCD effects: gluon radiation from the quark lines



$e^+e^- \to q\bar{q}$: $$\overline{|\mathcal{M}|}^2 = 8(4\pi)^2\alpha^2 N_c\, Q_i^2\, \frac{(p_{e^+}\cdot p_q)(p_{e^-}\cdot p_{\bar{q}}) + (p_{e^+}\cdot p_{\bar{q}})(p_{e^-}\cdot p_q)}{q^4}$$

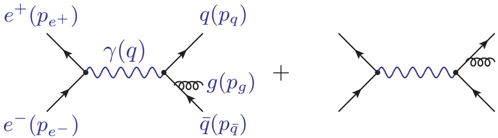
$e^+e^- \to q\bar{q}g$: $$\overline{|\mathcal{M}|}^2 = 8(4\pi)^3\alpha^2\alpha_S C_F N_c\, Q_i^2\, \frac{(p_{e^+}\cdot p_q)(p_{e^-}\cdot p_{\bar{q}}) + (p_{e^+}\cdot p_{\bar{q}})(p_{e^-}\cdot p_q)}{(p_{e^+}\cdot p_{e^-})(p_q\cdot p_g)(p_{\bar{q}}\cdot p_g)}$$

2) factors: $N_c \to C_F N_c$

C_F is the color combinatory for one gluon radiation from a quark with a given color, $C_F = 4/3$ (coming from the λ^a)

First pQCD correction

- up to now, what we saw was driven by EW ME + hadronisation
- additional jets are due to pQCD effects: gluon radiation from the quark lines



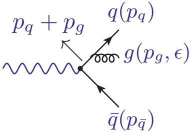
$e^+e^- \to q\bar{q}$: $\overline{|\mathcal{M}|^2} = 8(4\pi)^2\alpha^2 N_c\, Q_i^2\, \frac{(p_{e^+}\cdot p_q)(p_{e^-}\cdot p_{\bar{q}}) + (p_{e^+}\cdot p_{\bar{q}})(p_{e^-}\cdot p_q)}{q^4}$

$e^+e^- \to q\bar{q}g$: $\overline{|\mathcal{M}|^2} = 8(4\pi)^3\alpha^2\alpha_S C_F N_c\, Q_i^2\, \frac{(p_{e^+}\cdot p_q)(p_{e^-}\cdot p_{\bar{q}}) + (p_{e^+}\cdot p_{\bar{q}})(p_{e^-}\cdot p_q)}{(p_{e^+}\cdot p_{e^-})(p_q\cdot p_g)(p_{\bar{q}}\cdot p_g)}$

3) kinematics:

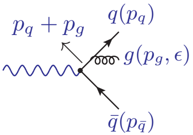
- note that: $q^4 = 4(p_{e^+}\cdot p_{e^-})(p_q\cdot p_{\bar{q}})$
- so, the kin effect is: $(p_q\cdot p_g) \to (p_q\cdot p_g)(p_{\bar{q}}\cdot p_g)$ - where does it come from ?

- adding a quark propagator in the ME calculation
- neglecting masses and in soft gluon approximation:

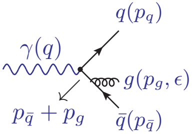


$$\sim \frac{(p_q + p_g) + m_q}{(p_q + p_g)^2 - m_q^2} \not{\epsilon} \overset{m_q=0}{\simeq} \frac{p_q + p_g}{2(p_q \cdot p_g)} \not{\epsilon} \overset{|p_g| \ll |p_q|}{\simeq} \frac{1}{2} \frac{\epsilon \cdot p_q}{p_q \cdot p_g}$$

- adding a quark propagator in the ME calculation
- neglecting masses and in soft gluon approximation:

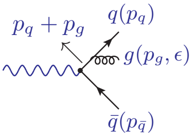


$$\sim \frac{(p_q + p_g) + m_q}{(p_q + p_g)^2 - m_q^2} \not\epsilon \overset{m_q=0}{\simeq} \frac{p_q + p_g}{2(p_q \cdot p_g)} \not\epsilon \overset{|p_g| \ll |p_q|}{\simeq} \frac{1}{2} \frac{\epsilon \cdot p_q}{p_q \cdot p_g}$$

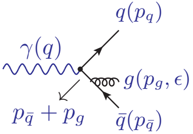


$$\sim \frac{1}{2} \frac{\epsilon \cdot p_{\bar{q}}}{p_{\bar{q}} \cdot p_g}$$

- adding a quark propagator in the ME calculation
- neglecting masses and in soft gluon approximation:



$$\sim \frac{(p_q + p_g) + m_q}{(p_q + p_g)^2 - m_q^2} \not{\epsilon} \overset{m_q=0}{\simeq} \frac{p_q + p_g}{2(p_q \cdot p_g)} \not{\epsilon} \overset{|p_g| \ll |p_q|}{\simeq} \frac{1}{2} \frac{\epsilon \cdot p_q}{p_q \cdot p_g}$$



$$\sim \frac{1}{2} \frac{\epsilon \cdot p_{\bar{q}}}{p_{\bar{q}} \cdot p_g}$$

- putting them together and after sum over the gluon polarisation states (only 2 tranverse states):

$$\mathrm{d}\sigma_{q\bar{q}g} = \frac{\alpha_S}{2\pi} C_F \left[\frac{p_{\bar{q}}}{p_{\bar{q}} \cdot p_g} - \frac{p_q}{p_q \cdot p_g} \right]^2 \frac{\mathrm{d}^3 p_g}{(2\pi)^3 p_g^0} \, \mathrm{d}\sigma_{qq}$$

- note : the sign difference comes from the orientation of $\vec{\epsilon}$

$$\cdots \left[\frac{p_{\bar{q}}}{p_{\bar{q}} \cdot p_g} - \frac{p_q}{p_q \cdot p_g} \right]^2 \mathrm{d}\sigma_{qq} \simeq \cdots 2 \frac{p_{\bar{q}} \cdot p_q}{(p_{\bar{q}} \cdot p_g)(p_q \cdot p_g)} \mathrm{d}\sigma_{qq}$$

- with the term from $\mathrm{d}\sigma_{qq}$:

$$2 \frac{p_{\bar{q}} \cdot p_q}{(p_{\bar{q}} \cdot p_g)(p_q \cdot p_g)} \frac{\overset{\nearrow 1/q^4}{1}}{4(p_{e^+} \cdot p_{e^-})(p_q \cdot p_{\bar{q}})} = \frac{1}{2(p_{e^+} \cdot p_{e^-})(p_{\bar{q}} \cdot p_g)(p_q \cdot p_g)}$$

$\Rightarrow$ we find back:

$e^+ e^- \to q\bar{q}g$: $\overline{|\mathcal{M}|}^2 = 8(4\pi)^3 \alpha^2 \alpha_S C_F N_c \, Q_i^2 \, \dfrac{(p_{e^+} \cdot p_q)(p_{e^-} \cdot p_{\bar{q}}) + (p_{e^+} \cdot p_{\bar{q}})(p_{e^-} \cdot p_q)}{(p_{e^+} \cdot p_{e^-})(p_q \cdot p_g)(p_{\bar{q}} \cdot p_g)}$

$$\cdots \left[\frac{p_{\bar{q}}}{p_{\bar{q}} \cdot p_g} - \frac{p_q}{p_q \cdot p_g} \right]^2 \mathrm{d}\sigma_{qq} \simeq \cdots 2 \frac{p_{\bar{q}} \cdot p_q}{(p_{\bar{q}} \cdot p_g)(p_q \cdot p_g)} \mathrm{d}\sigma_{qq}$$

- with the term from $\mathrm{d}\sigma_{qq}$:

$1/q^4$

$$2 \frac{p_{\bar{q}} \cdot p_q}{(p_{\bar{q}} \cdot p_g)(p_q \cdot p_g)} \; \frac{1}{4(p_{e^+} \cdot p_{e^-})(p_q \cdot p_{\bar{q}})} = \frac{1}{2(p_{e^+} \cdot p_{e^-})(p_{\bar{q}} \cdot p_g)(p_q \cdot p_g)}$$

$\Rightarrow$ we find back:

$\boxed{e^+ e^- \to q\bar{q}g :}$ $$\overline{|\mathcal{M}|}^2 = 8(4\pi)^3 \alpha^2 \alpha_S C_F N_c \, Q_i^2 \, \frac{(p_{e^+} \cdot p_q)(p_{e^-} \cdot p_{\bar{q}}) + (p_{e^+} \cdot p_{\bar{q}})(p_{e^-} \cdot p_q)}{(p_{e^+} \cdot p_{e^-})(p_q \cdot p_g)(p_{\bar{q}} \cdot p_g)}$$

⚠ it presents singularities

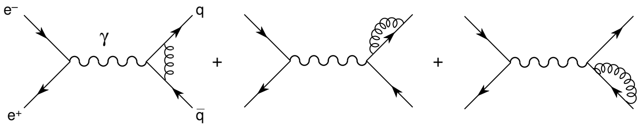
if $p_q \cdot p_g \to 0$ and/or $p_{\bar{q}} \cdot p_g \to 0$

two singularities

$$
\begin{aligned}
p_q \cdot p_g &= E_q E_g - \vec{p}_q \cdot \vec{p}_g = E_q E_g - |\vec{p}_q||\vec{p}_g| \cos\theta_{qg} \\
&\simeq E_q E_g (1 - \cos\theta_{qg})
\end{aligned}
$$

$\Rightarrow$ singularities for $p_q \cdot p_g \to 0$ (same for $\bar{q}g$):
- $E_g \to 0$ (soft gluon limit)
- $\theta_{qg} \to 0$ (collinear limit)

- they correspond to a double pole (when both limits occur at the same time) and a single pole.
- these IR poles are exactly cancelled by the virtual correction UV poles



comparison to data

$$\mathrm{d}\sigma_{q\bar{q}g} = \frac{\alpha_S}{2\pi} C_F \left[\frac{p_{\bar{q}}}{p_{\bar{q}} \cdot p_g} - \frac{p_q}{p_q \cdot p_g} \right]^2 \frac{\mathrm{d}^3 p_g}{(2\pi)^3 p_g^0} \mathrm{d}\sigma_{qq}$$

- the cross section has 2 physical degrees of freedom :

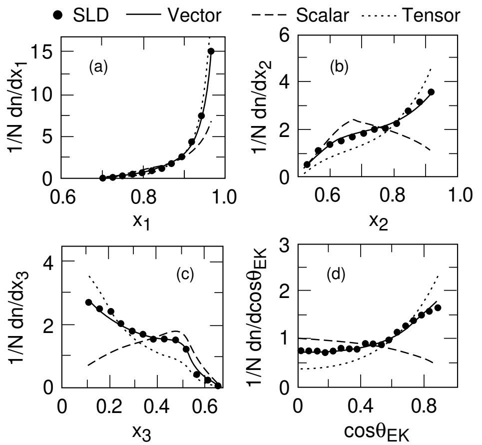
  $+9$ free variables $(\mathrm{d^3p_q, d^3p_{\bar{q}}, d^3p_g})$
  -4 relations ($E, \vec{P}$ conservation)
  -3 independent (non relevant here - unpolaratised case) Euler angles
  $= 2$

- we choose : $x_q = 2E_q/\sqrt{s}$ $\quad x_{\bar{q}} = 2E_{\bar{q}}/\sqrt{s}$ $\quad x_g = 2E_g/\sqrt{s}$
  related by $x_q + x_{\bar{q}} + x_g = 2$ and $1 - x_i = \frac{1}{2} x_j x_k (1 - \cos\theta_{jk})$

$$\frac{d^2\sigma_{q\bar{q}g}}{dx_q dx_{\bar{q}}} = \frac{\alpha_s}{2\pi} C_F \frac{x_q^2 + x_{\bar{q}}^2}{(1 - x_q)(1 - x_{\bar{q}})} \sigma_{q\bar{q}}$$

- 2 sigularities correspond to $x_q \to 1$ and $x_{\bar{q}} \to 1$

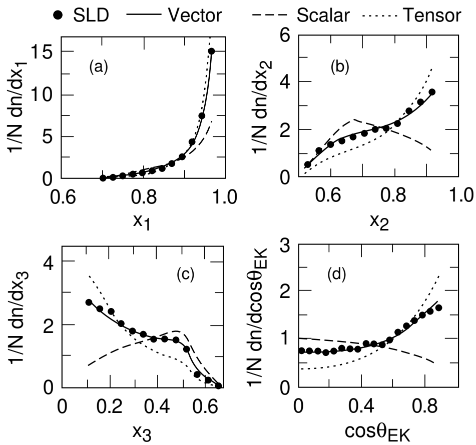
- in data we don't know which is q jet, ...
- sort the 3 jets by decreasing energy :
  - $E_1 > E_2 > E_3$
  - define: $x_i \;=\; 2E_i/\sqrt{s}$



- jet 1: $q/\bar{q}$ jet almost unaffected ($x_1 \to 1$ pole)
- jet 2: $q/\bar{q}$ jet with significant energy loss $+$ g jet such that $E_{q/\bar{q}} > E_g > E_{q/\bar{q}}$
- jet 3: mainy the gluon jet (falling distribution) $+$ $q/\bar{q}$ jet of the above case

- $\cos\theta_{EK} = \frac{\sin\theta_2 - \sin\theta_3}{\sin\theta_1}$

- in data we don't know which is q jet, ...
- sort the 3 jets by decreasing energy :
  - $E_1 > E_2 > E_3$
  - define: $x_i \ = \ 2E_i/\sqrt{s}$



- jet 1: $q/\bar{q}$ jet almost unaffected ($x_1 \to 1$ pole)
- jet 2: $q/\bar{q}$ jet with significant energy loss $+$ g jet such that $E_{q/\bar{q}} > E_g > E_{q/\bar{q}}$
- jet 3: mainy the gluon jet (falling distribution) $+$ $q/\bar{q}$ jet of the above case

- $\cos\theta_{EK} = \frac{\sin\theta_2 - \sin\theta_3}{\sin\theta_1}$

gluons are spin 1 !

pure vectorial current (γ^μ)

That's all for today