Jacobians

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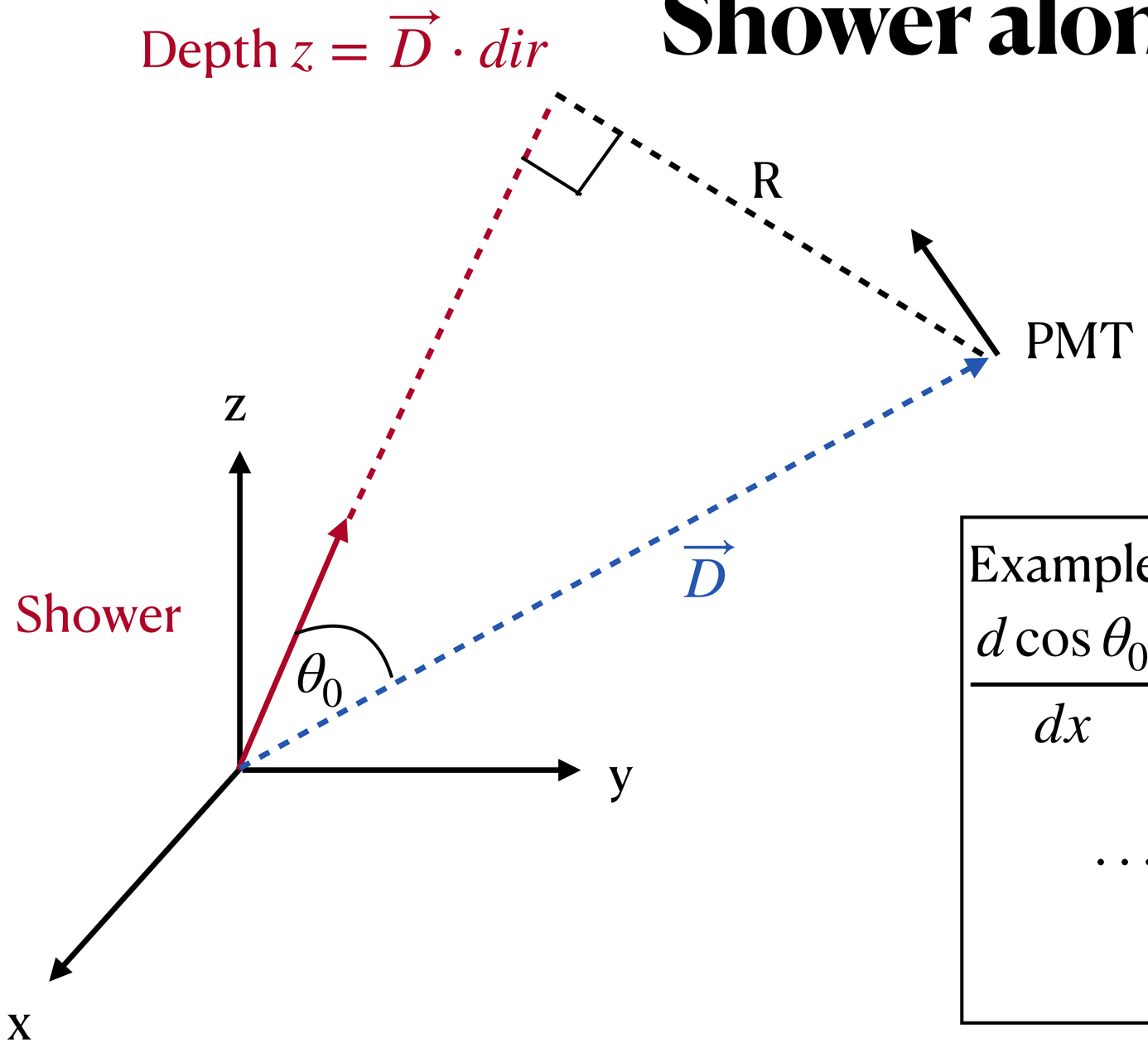
Derivatives of the likelihood

- Used to improve/speed up maximisation
- Needed: $\frac{d(\text{Log Lik})}{d(\text{fit parameter})}$
  - Fit parameters (x,y,z,t,θ, ϕ,E)
- From the PDFs you get $\frac{d(\text{Log Lik})}{d(\text{Jpp parameters})}$
  - Jpp Parameters (R, D,cos θ_0, θ_{pmt}, ϕ_{pmt})

Missing element: Jacobians $\frac{d(\text{Jpp parameters})}{d(\text{fit parameter})}$

Shower along z



$$|\vec{D}| = \sqrt{x^2 + y^2 + z^2}$$

$$R = |\vec{D}|^2 - z^2$$

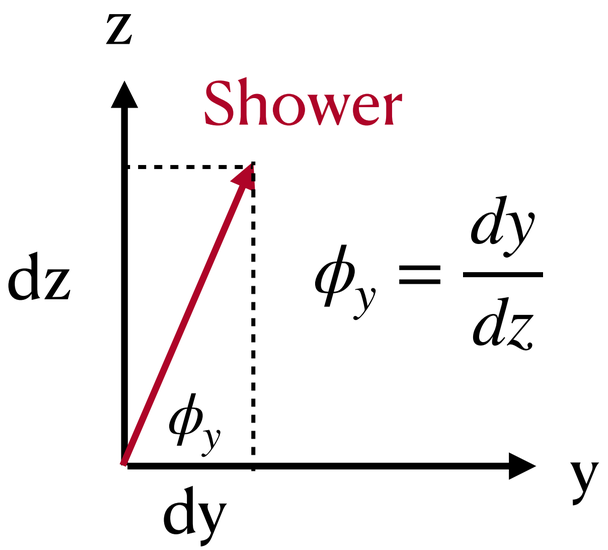
$$\cos\theta_0 = \frac{z}{|\vec{D}|}$$

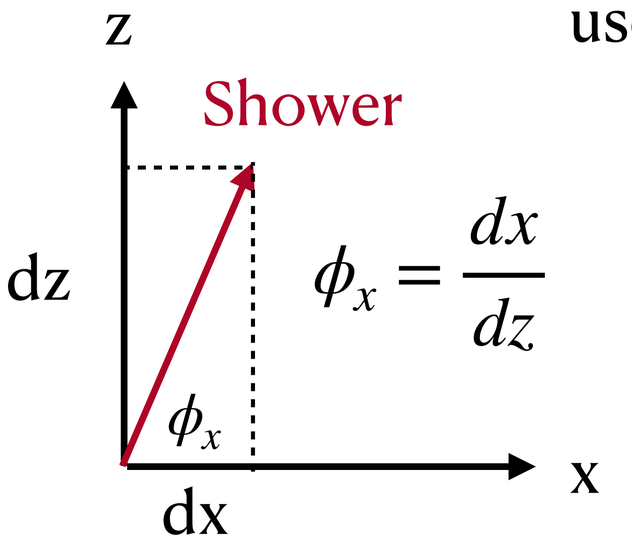
Example derivative

$$\frac{d\cos\theta_0}{dx} = \frac{d}{dx}\frac{z}{|\vec{D}|} = \frac{d}{dx}\left(\frac{z}{\sqrt{x^2 + y^2 + z^2}}\right)$$

$$\cdots = -\frac{1}{2}\frac{2zx}{(x^2 + y^2 + z^2)^{\frac{3}{2}}} = -\frac{zx}{|\vec{D}|^3}$$

Shower direction

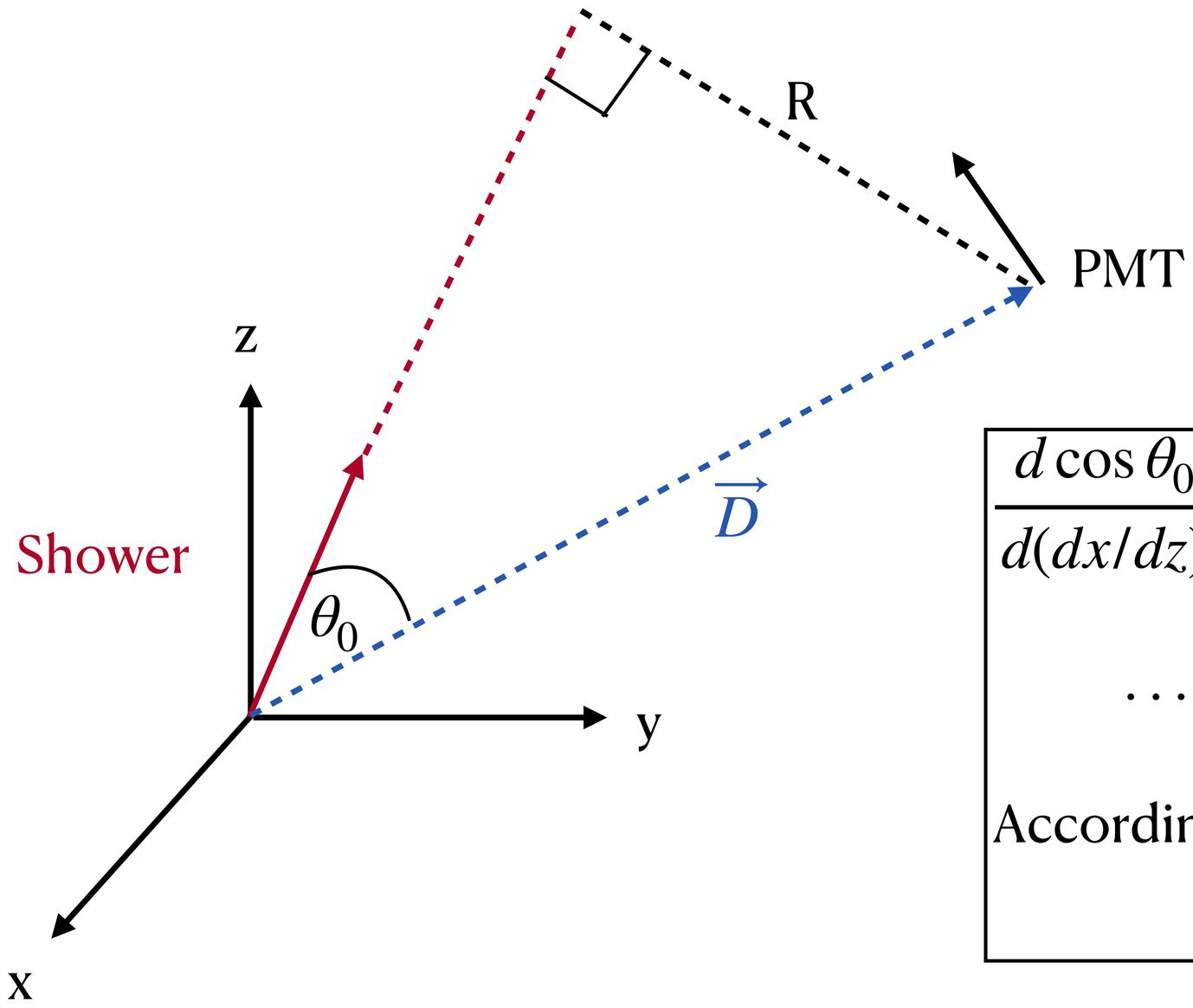




Question: if you construct shower direction along z-axis, how can you use $\frac{dx}{dz}, \frac{dy}{dz}$ as direction?

Shower direction

Depth $z = \vec{D} \cdot dir$



$$|\vec{D}| = \sqrt{x^2 + y^2 + z^2}$$

$$R = |\vec{D}|^2 - z^2$$

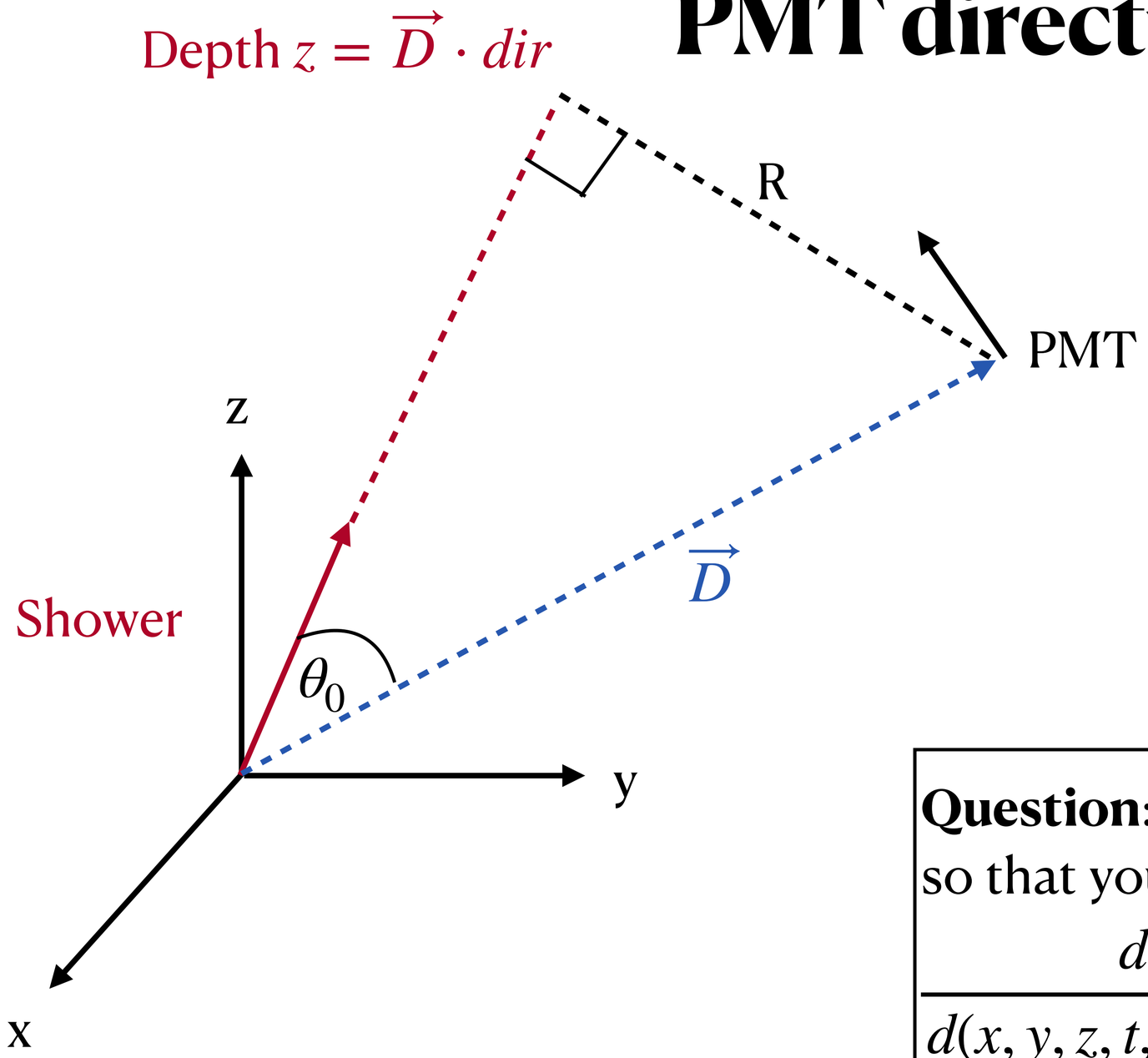
$$\cos\theta_0 = \frac{z}{|\vec{D}|}$$

$$\frac{d\cos\theta_0}{d(dx/dz)} = \frac{d}{d(dx/dz)}\frac{z}{|\vec{D}|} = \ldots ?$$

$$\ldots = -\frac{xz^2}{|\vec{D}|R} \ldots ?$$

According to Jpp JShower3EZRegressor

PMT direction



Rotate PMT to x-z plane

$$x = D_x - z\frac{dx}{dz}$$

$$y = D_y - z\frac{dy}{dz}$$

Rotation over $-\text{atan}(y, x)$

Gives θ, ϕ

Question: how do you express θ, ϕ so that you can calculate

$$\frac{d(\theta, \phi)}{d(x, y, z, t, dx/dz, dy/dz)}$$