Simple first hit likelihood

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Proposition 1. *For any PMT, the probability of a first hit occurring at any time, is the same as the probability of at least one hit occurring over all time.*

Given that a first hit occurs, the probability of this first hit occurring at time residual Δt is,

$$h_{1\text{st}}(\Delta t) = \frac{n(\Delta t)}{N_{\text{total}}} e^{-N(\Delta t)},$$

and the probability of at least one hit over all time,

$$P(\text{n.p.e.} > 1) = 1 - e^{-N_{\text{total}}},$$

we apply the condition above to find the normalization constant C by integrating over all time,

$$\frac{C}{N_{\text{total}}} \int_{\infty} n(\Delta t) e^{-N(\Delta t)} = 1 - e^{-N_{\text{total}}}.$$

Since $N(0) = 0$ and $\lim_{x \to \infty} N(x) = N_{\text{total}}$, $\frac{C}{N_{\text{total}}}$ is found to be 1. We now have the probability that a first hit occurs, and that it occurs at time residual Δt,

$$p_{1\text{st}}(\Delta t) = n(\Delta t) e^{-N(\Delta t)}. \quad (1)$$

Using this first hit probability to define the log likelihood, we encapsulate both the hit and hit time information,

$$\sum_{i}^{\text{PMT}} \log(n(\Delta t)) + \sum_{i}^{\text{PMT}} -N(\Delta t).$$