Likelihoods & Derivatives

Full likelihood fit of 1 shower

- Shower hypothesis with variables $[x, y, z, t, \text{zenith}, \text{azimuth}, E]$ fitted using the full likelihood
- Hypothesis is converted to Jpp coordinates (relative to a PMT) $[E, d, cd, \theta, \phi, r]$

Maximise the full likelihood!

The full likelihood

$$\text{Log L} = \quad + \sum_{\text{hit pmts}} \text{Log}\left(1 - \exp^{-N_{tot}(\hat{x})}\right) \quad ...$$

$$+ \sum_{\text{empty pmts}} \text{Log}\left(\exp^{-N_{tot}(\hat{x})}\right) \quad ...$$

$$+ \sum_{\text{1st hits}} \text{Log}\left(\frac{1}{e^{-N(\hat{x},T_{start})} - e^{-N(\hat{x},T_{end})}} n(\hat{x}, t) e^{-N(\hat{x},t)}\right)$$

Where $\hat{x} = E, d, cd, \theta, \phi,$ and $N_{tot}(\hat{x}), N(\hat{x}, t), n(\hat{x}, t)$ are PDF calls

Derivatives of the likelihood

Maximisation of Log L improves when including $\dfrac{d(\text{Log L})}{d(x, y, z, t, zen, azi, E)}$

- Jpp PDFs return derivative with respect to residual time for free!
- Calculate Jacobean to fitted parameters yourself:
- $\dfrac{d[n(\hat{x}, t)]}{dr} \dfrac{dr}{dt}$
- $\dfrac{d[n(\hat{x}, t)]}{dr} \dfrac{dr}{d(x, y, z)}$
- $\dfrac{d[n(\hat{x}, t)]}{dr} \dfrac{dr}{d(azi, zen)}$
- $\dfrac{d[n(\hat{x}, t)]}{dr} \dfrac{dr}{dE}$

Becomes clear when choosing z as supposed shower direction and using dz/dx, dz/dy as fitted direction parameters

No dependence energy-time

Energy

- PDF of the shower depends linearly on the energy

Typically: fit the energy separate when other parameters are fixed using the amplitude information:

- $\sum_{\text{hit pmts}} + \sum_{\text{empty pmts}}$

Questions

What if you want to fit the direction, but you want to use the amplitude AND timing information?

- Terms of N_{tot} in the likelihood that do not depend on time —> no derivatives
- You still need to assume an energy (and two in the case of a double bang): estimation/prefit/something else?