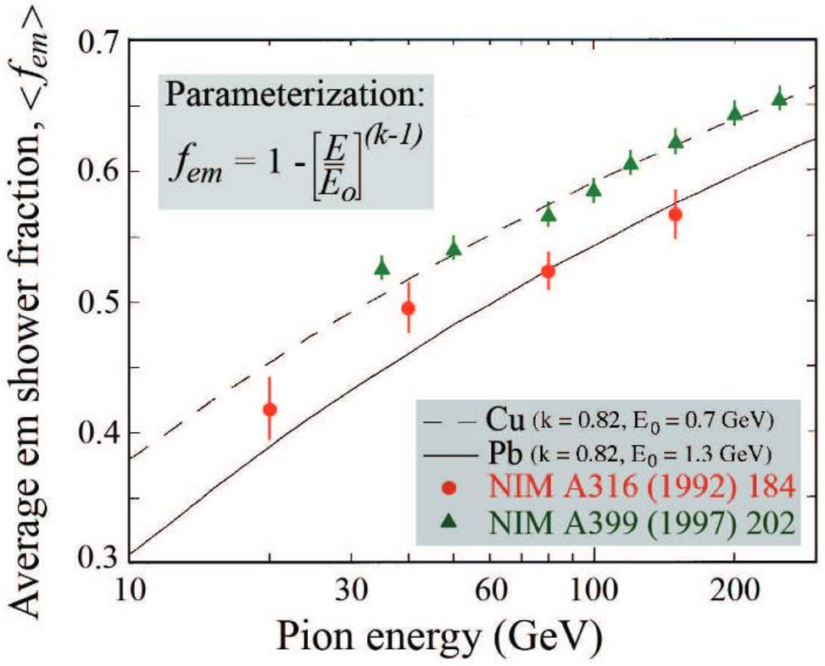
JShowerReconstruction

(from a newcomer's perspective)

Why not one all-purpose shower fitter?

- Low-energy (1-20 GeV) showers are difficult to detect!
  - Few hit coincidences (**low L1 hit probability**)

- Relatively large hadronic components
  - Vast majority (~90%) of hadronic shower particles are pions
  - Neutral pions decay into 2γ's
    --> EM showers
  - At lower shower energies, smaller EM component
    (from ~30% at 10 GeV to ~50% at 100 GeV)
  - The higher the hadronic component,
    the higher the fraction of invisible energy
    (due to nuclear interactions)



R. Wigmans – Lecture series on Calorimetry (2008)

The reconstruction chain

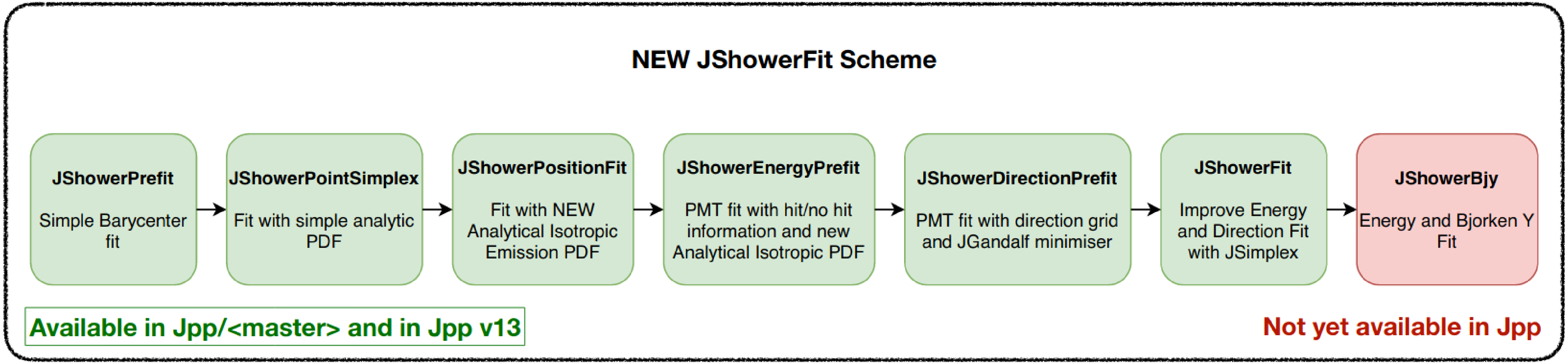
- At present, six parts:

  I. Vertex + direction prefit
  II. 1st vertex likelihood minimization (Powell's method)
  III. 2nd vertex likelihood minimization (Levenberg-Marquardt)

  Vertex

  IV. Energy prefit
  V. 2nd direction prefit
  VI. Energy and direction likelihood minimization (Powell's method)

  Energy + direction



NEW JShowerFit Scheme

JShowerPrefit
Simple Barycenter fit

JShowerPointSimplex
Fit with simple analytic PDF

JShowerPositionFit
Fit with NEW Analytical Isotropic Emission PDF

JShowerEnergyPrefit
PMT fit with hit/no hit information and new Analytical Isotropic PDF

JShowerDirectionPrefit
PMT fit with direction grid and JGandalf minimiser

JShowerFit
Improve Energy and Direction Fit with JSimplex

JShowerBjy
Energy and Bjorken Y Fit

Available in Jpp/<master> and in Jpp v13

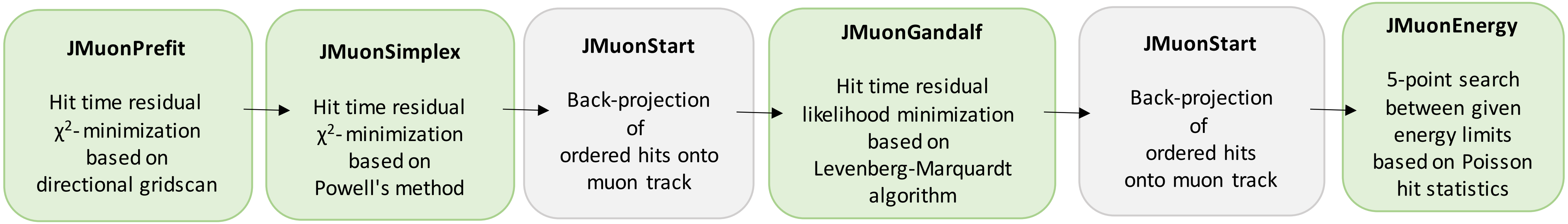
Not yet available in Jpp

From Alba's June 2020 collaboration meeting presentation

The reconstruction chain

- At present, six parts

- C.f. the JMuonReconstruction chain:



JMuonPrefit
Hit time residual χ^2- minimization based on directional gridscan

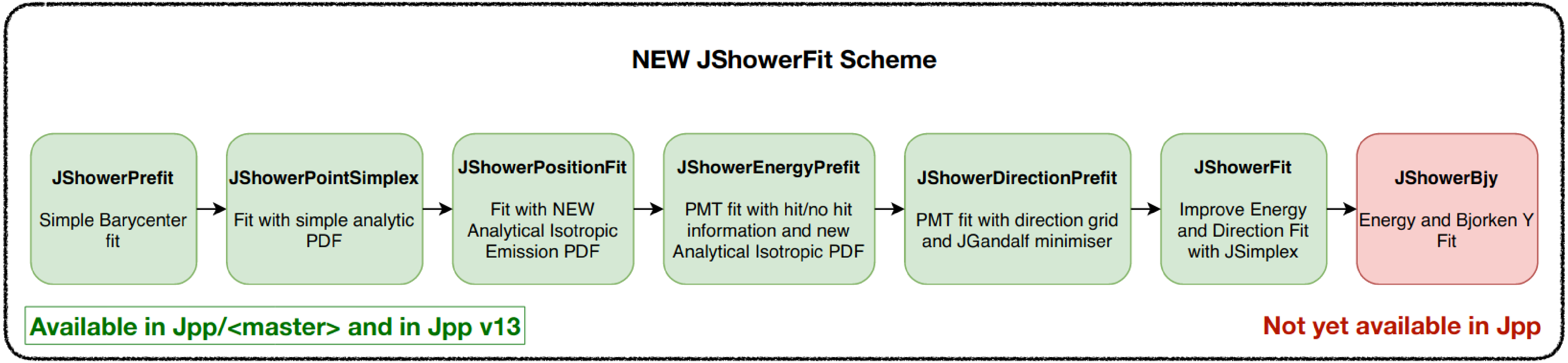
JMuonSimplex
Hit time residual χ^2- minimization based on Powell's method

JMuonStart
Back-projection of ordered hits onto muon track

JMuonGandalf
Hit time residual likelihood minimization based on Levenberg-Marquardt algorithm

JMuonStart
Back-projection of ordered hits onto muon track

JMuonEnergy
5-point search between given energy limits based on Poisson hit statistics



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From Alba's June 2020 collaboration meeting presentation

Vertex prefit

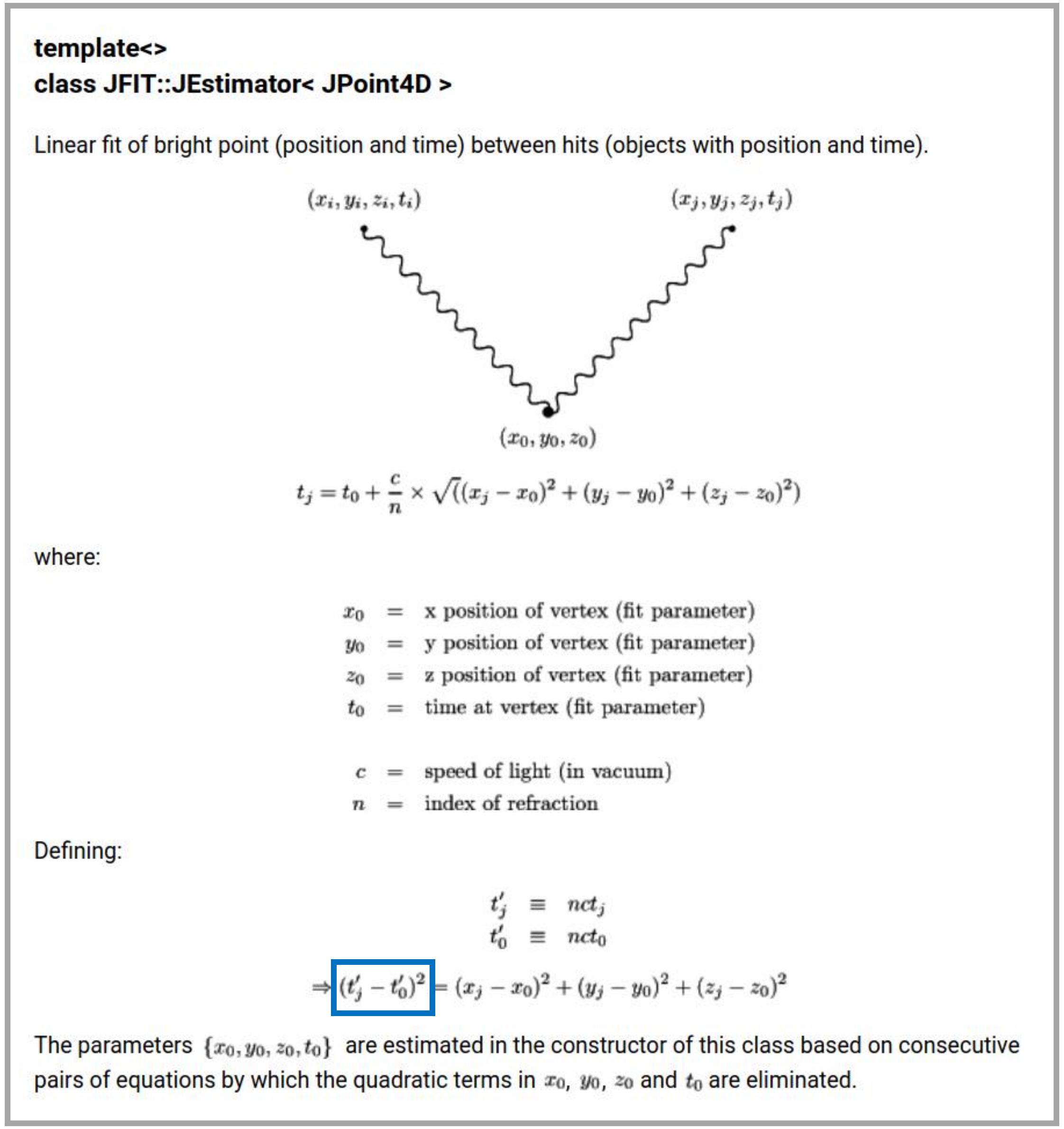
- The vertex prefit consists of roughly 2 steps:

  I. Hit selection

    - Find a cluster of causally related hits originating from a single emission point (vertex)
    - Minimize background

  II. Least squares vertex (barycenter) fit based on **time of flight**

    - Find the neutrino vertex which minimizes squared hit time residuals



template<>
class JFIT::JEstimator< JPoint4D >

Linear fit of bright point (position and time) between hits (objects with position and time).

$$t_j = t_0 + \frac{c}{n} \times \sqrt{((x_j - x_0)^2 + (y_j - y_0)^2 + (z_j - z_0)^2)}$$

where:

x_0 = x position of vertex (fit parameter)
y_0 = y position of vertex (fit parameter)
z_0 = z position of vertex (fit parameter)
t_0 = time at vertex (fit parameter)

c = speed of light (in vacuum)
n = index of refraction

Defining:

$$t'_j \equiv nct_j$$
$$t'_0 \equiv nct_0$$

$$\Rightarrow \boxed{(t'_j - t'_0)^2} = (x_j - x_0)^2 + (y_j - y_0)^2 + (z_j - z_0)^2$$

The parameters $\{x_0, y_0, z_0, t_0\}$ are estimated in the constructor of this class based on consecutive pairs of equations by which the quadratic terms in x_0, y_0, z_0 and t_0 are eliminated.

Vertex prefit (hit selection)

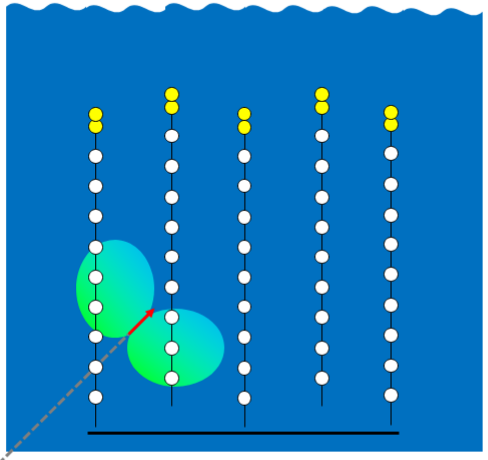
- Based on the MX (mix) trigger
  - Uses a mix of L0 and L1 hits

Hit rates:

- RL0 ~ 115 x 18 x 31 x 5 kHz = 320 MHz
- RL1 ~ 115 x 18 x 1 kHz = 2 MHz

In typical time window of ~ 100 ns, expect ~ **O(1) L1 hit** and ~ **O(10-100) L0 hits**

- Which L0's are causally related to a L1 hit?
  - Low energy showers are well-contained --> **use distance** as selection criterion



size of shower much smaller than size of detector

⇩

limit distance between L1/L0 hits

See [Maarten's presentation on JTriggerMXShower]

Vertex prefit (hit selection)

- Start with a cluster of causally connected L2s ($\Delta t < 20$ ns and $\alpha < 135$ deg) that satisfy:

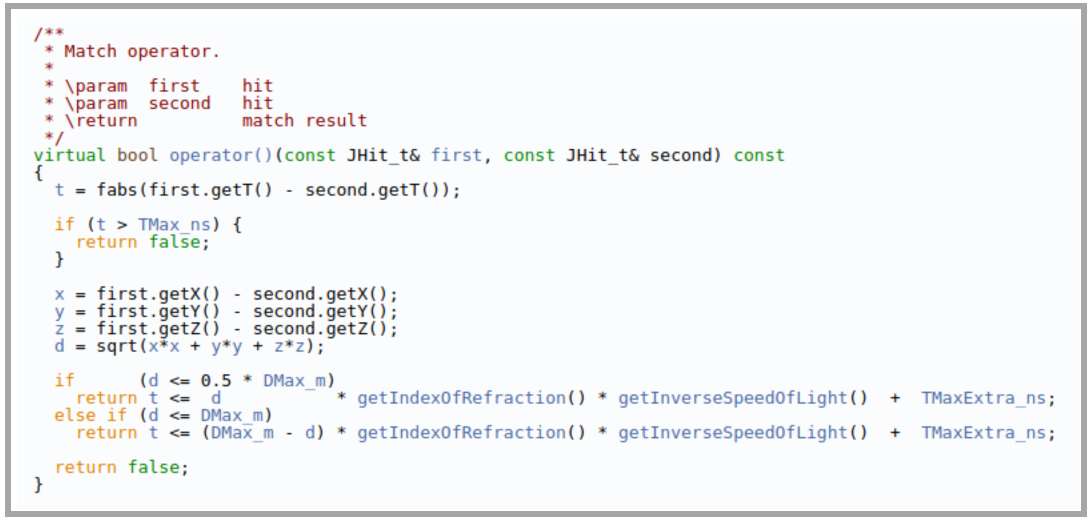
$$|t_i - t_j| \leq \boxed{|\vec{x}_i - \vec{x}_j|/c_{water}} + \boxed{T_{extra}} \quad \text{with } |\vec{x}_i - \vec{x}_j| = 50 \text{ m and } T_{extra} = 20 \text{ ns.}$$

i.e.: hit time difference smaller than **TOF between the two hit DOMs**
+ some extra time (to account for scattering and hit time resolution)

c_{water} ~ 0.29979 / 1.34 ~ 0.22 m/ns $\longrightarrow$ $|t_i - t_j| \leq$ 220 ns

- Subsequently check if any surrounding L0s are causally connected with either of the L2 hits (using same condition) and add them to the time-sorted L2 cluster

- Matching operator defined in **JMatch3G**



```
/**
 * Match operator.
 *
 * \param  first    hit
 * \param  second   hit
 * \return          match result
 */
virtual bool operator()(const JHit_t& first, const JHit_t& second) const
{
  t = fabs(first.getT() - second.getT());

  if (t > TMax_ns) {
    return false;
  }

  x = first.getX() - second.getX();
  y = first.getY() - second.getY();
  z = first.getZ() - second.getZ();
  d = sqrt(x*x + y*y + z*z);

  if      (d <= 0.5 * DMax_m)
    return t <=  d          * getIndexOfRefraction() * getInverseSpeedOfLight()  +  TMaxExtra_ns;
  else if (d <= DMax_m)
    return t <= (DMax_m - d) * getIndexOfRefraction() * getInverseSpeedOfLight()  +  TMaxExtra_ns;

  return false;
}
```

Vertex prefit (fitting algorithm)

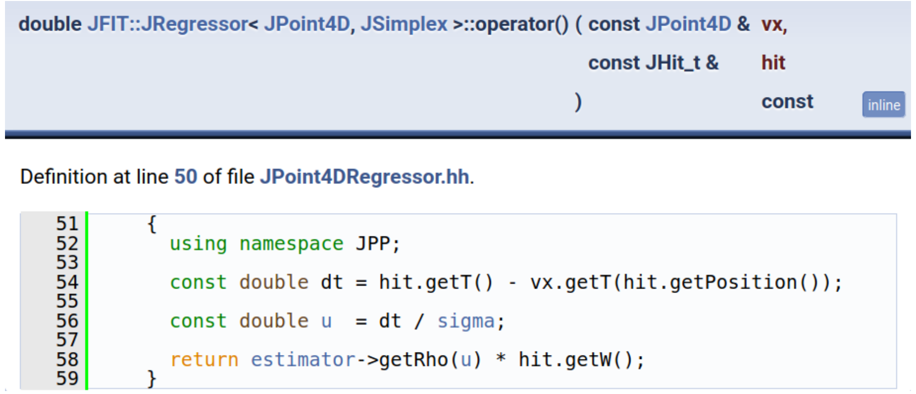
- The least squares fit for the arrival time residuals is largely analogous to what is done in JMuonPrefit
    - See also the appendix of Brían's track reconstruction document
- But it involves additional iterative steps, meant to enhance the precision (whilst limiting CPU time!)

1. An extra distinction based on L2 cluster size:
    - If total L2+L0 hits, $N_{tot} < 41$:
        1. Repeat the chi2-minimization $N_{tot} - 4$ times, removing 1 L0 hit in each iteration
        2. Choose the fit which yields the least chi2
    - If total L2+L0 hits, Ntot > 40:
        1. Perform the chi2-minimization just once, using all hits within the cluster

JSimplex

- JSimplex performs a first, simple likelihood minimization, using **Powell's method:**
    - Iteratively finds a set of N mutually **conjugate directions**, with which to minimize the likelihood landscape
    - Does not require any gradients!
- For each step, the likelihood is re-evaluated:

$\prod$ hits

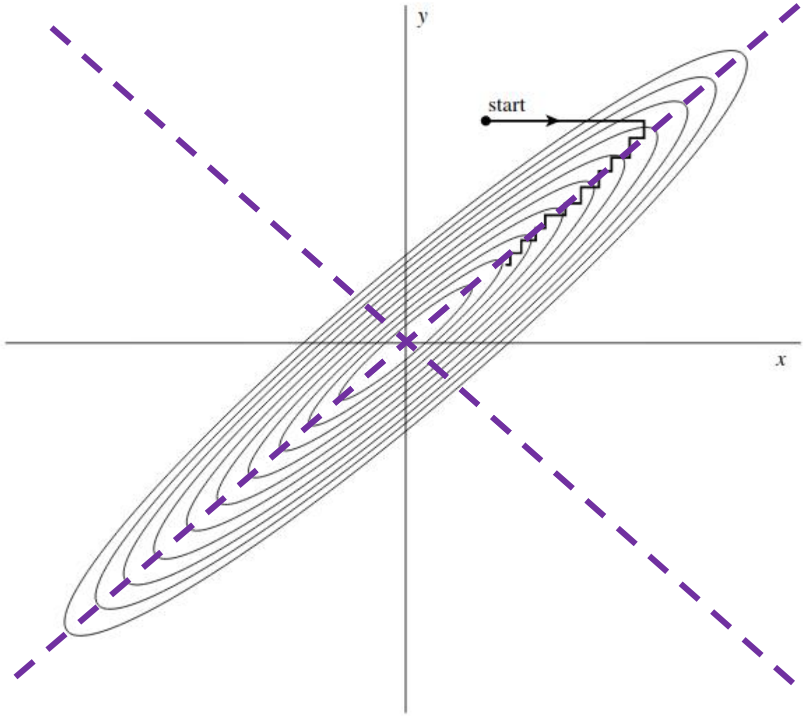


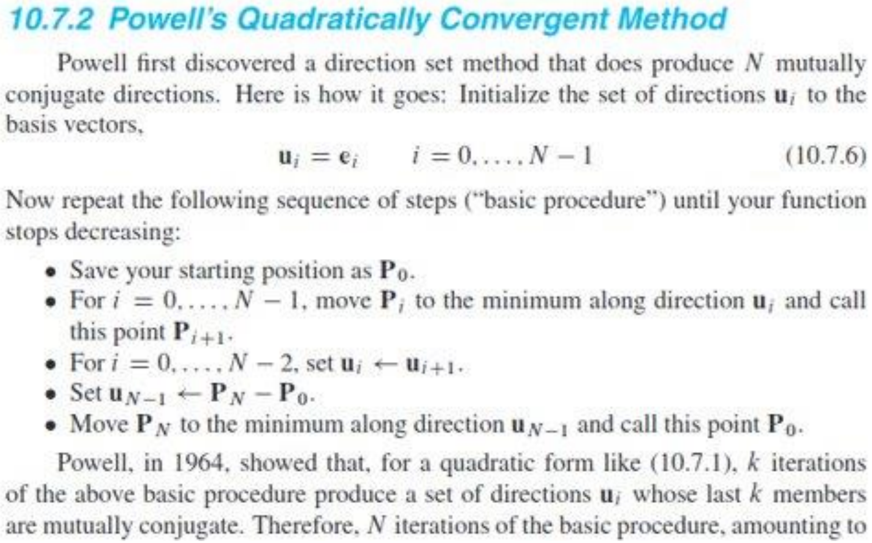
double JFIT::JRegressor< JPoint4D, JSimplex >::operator() (const JPoint4D & vx,
const JHit_t & hit
) const inline

Definition at line 50 of file JPoint4DRegressor.hh.

```
{
  using namespace JPP;
  const double dt = hit.getT() - vx.getT(hit.getPosition());
  const double u  = dt / sigma;
  return estimator->getRho(u) * hit.getW();
}
```

- Terminate when fractional improvement becomes negligible (default 1e-4)





10.7.2 Powell's Quadratically Convergent Method

Powell first discovered a direction set method that does produce N mutually conjugate directions. Here is how it goes: Initialize the set of directions $\mathbf{u}_i$ to the basis vectors,

$$\mathbf{u}_i = \mathbf{e}_i \qquad i = 0, \ldots, N-1 \tag{10.7.6}$$

Now repeat the following sequence of steps ("basic procedure") until your function stops decreasing:

- Save your starting position as $\mathbf{P}_0$.
- For $i = 0, \ldots, N-1$, move $\mathbf{P}_i$ to the minimum along direction $\mathbf{u}_i$ and call this point $\mathbf{P}_{i+1}$.
- For $i = 0, \ldots, N-2$, set $\mathbf{u}_i \leftarrow \mathbf{u}_{i+1}$.
- Set $\mathbf{u}_{N-1} \leftarrow \mathbf{P}_N - \mathbf{P}_0$.
- Move $\mathbf{P}_N$ to the minimum along direction $\mathbf{u}_{N-1}$ and call this point $\mathbf{P}_0$.

Powell, in 1964, showed that, for a quadratic form like (10.7.1), k iterations of the above basic procedure produce a set of directions $\mathbf{u}_i$ whose last k members are mutually conjugate. Therefore, N iterations of the basic procedure, amounting to

Press, WH et al. (2007); Numerical Recipes: The Art of Scientific Computing (3rd ed.)

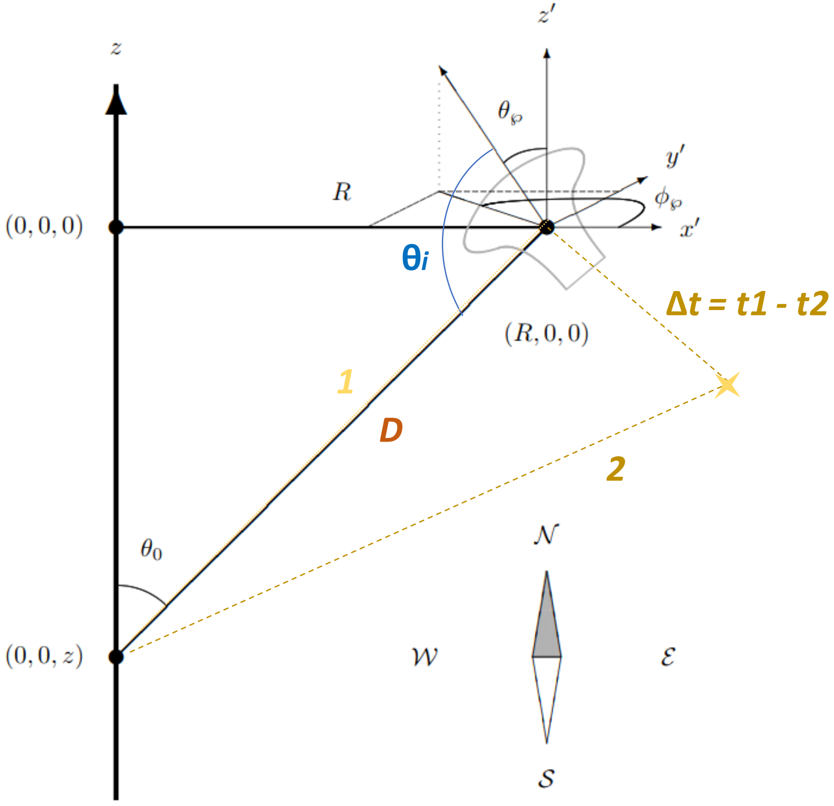
JShowerPositionFit

- The fits from JPrefit and JSimplex should now be sufficiently close to the true optimum, to allow for a final, full MLE, including non-linear effects such as scattering

- JShowerPositionFit constitutes this final minimization step
  - Exploits the analytical arrival time PDFs, dependent on:
    i. **Δt**: **the arrival time residual**
    ii. D: the **(nu-vertex, PMT)-distance**
    iii. Cos(**θi**): **cosine angle of incidence**

- The fit probability is defined as:
  - P = E * PDF(D, cos(**θi**), **Δ**t)

- Minimization is based on the **Levenberg-Marquard Algorithm**
  - Requires analytical calculation of the Hessian matrix



JPDF documentation – Maarten de Jong (see doxygen)

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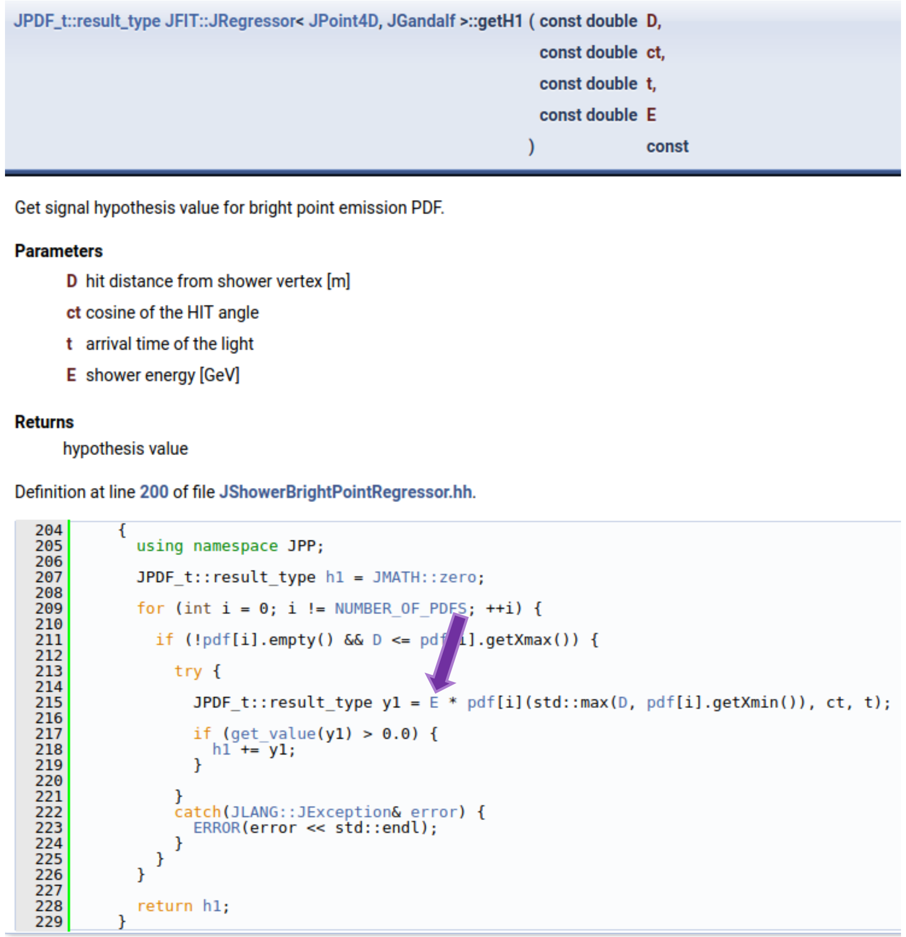


JPDF_t::result_type JFIT::JRegressor< JPoint4D, JGandalf >::getH1 (const double D,
const double ct,
const double t,
const double E
) const

Get signal hypothesis value for bright point emission PDF.

Parameters

D	hit distance from shower vertex [m]
ct	cosine of the HIT angle
t	arrival time of the light
E	shower energy [GeV]

Returns

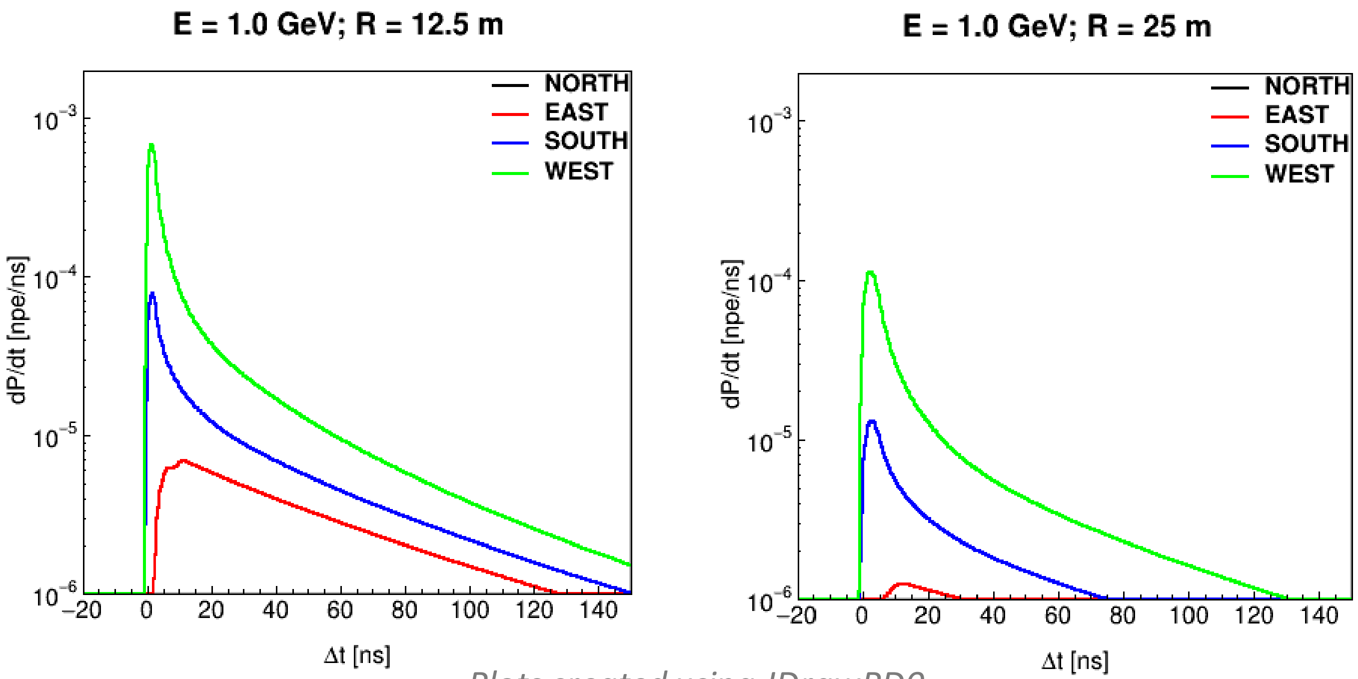
hypothesis value

Definition at line 200 of file JShowerBrightPointRegressor.hh.

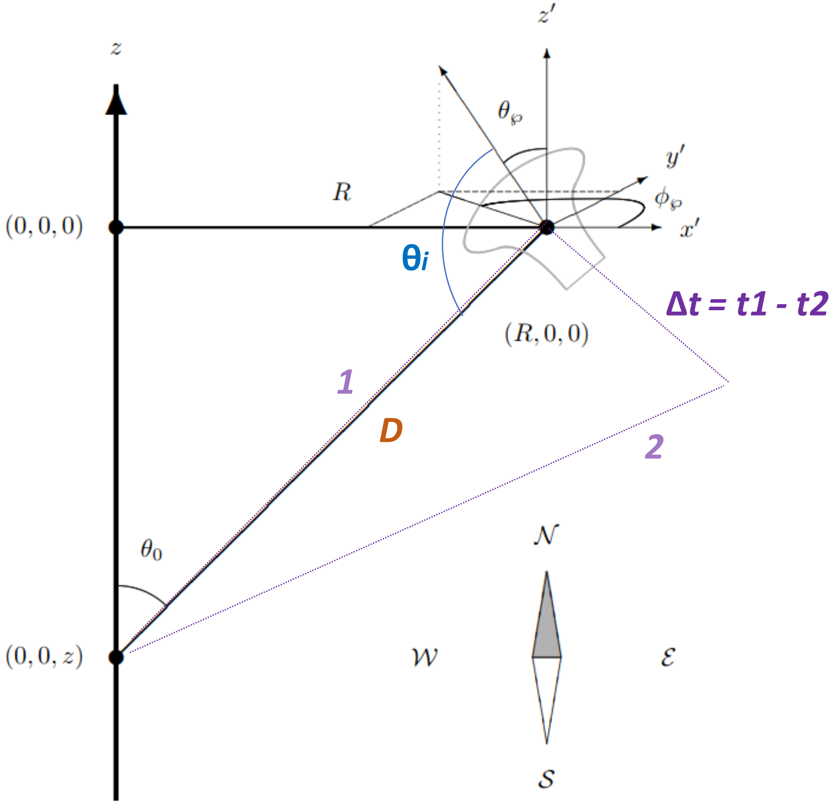
```
{
  using namespace JPP;

  JPDF_t::result_type h1 = JMATH::zero;

  for (int i = 0; i != NUMBER_OF_PDFS; ++i) {

    if (!pdf[i].empty() && D <= pdf[i].getXmax()) {

      try {

        JPDF_t::result_type y1 = E * pdf[i](std::max(D, pdf[i].getXmin()), ct, t);

        if (get_value(y1) > 0.0) {
          h1 += y1;
        }

      }
      catch(JLANG::JException& error) {
        ERROR(error << std::endl);
      }
    }
  }

  return h1;
}
```

JShowerPositionFit

- The fits from JPrefit and JSimplex should now be sufficiently close to the true optimum, to allow for a final, full MLE, including non-linear effects such as scattering
- JShowerPositionFit constitutes this final minimization step



Plots created using JDrawPD0



JPDF documentation – Maarten de Jong (see doxygen)

JShowerEnergyPrefit

- Similar in working to JMuonEnergy
  - Energy estimation based on a binomial hit/no-hit likelihood estimation

- Uses the Jpp isotropic emission PDFs to calculate the hit probabilities, given a PMT at distance D from the vertex, with photon incidence angle θi, originating from a shower with total energy E

- An iterative five-point search is performed between a given energy range
  - In each iteration, pick the point with maximal hit/no-hit likelihood and refine the five points

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    - Energy estimation based on a binomial hit/no-hit likeliho

- Uses the Jpp isotropic emission PDFs to calculate the hit proba
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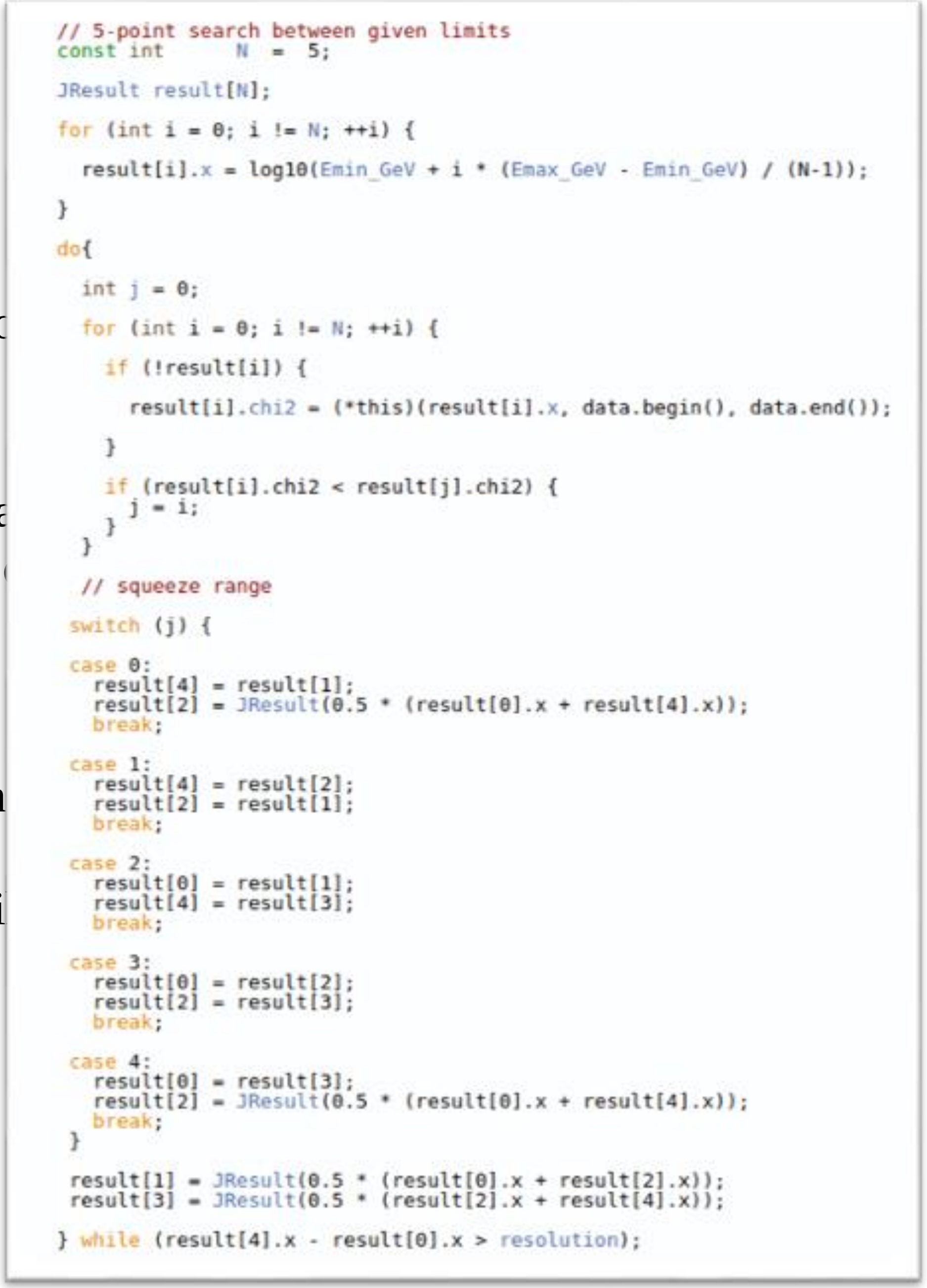


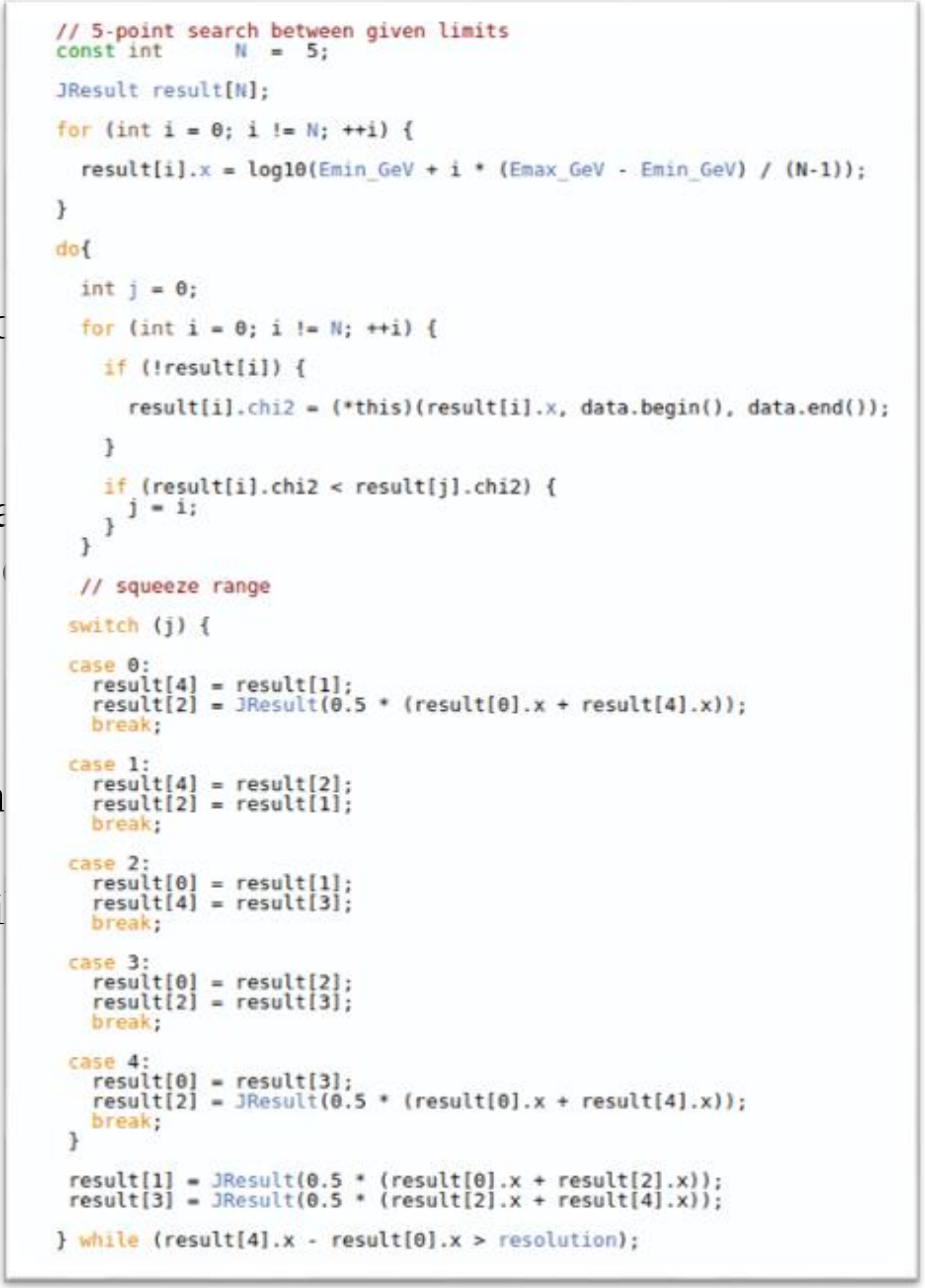
```
// 5-point search between given limits
const int      N  =  5;

JResult result[N];

for (int i = 0; i != N; ++i) {

  result[i].x = log10(Emin_GeV + i * (Emax_GeV - Emin_GeV) / (N-1));

}

do{

  int j = 0;

  for (int i = 0; i != N; ++i) {

    if (!result[i]) {

      result[i].chi2 = (*this)(result[i].x, data.begin(), data.end());

    }

    if (result[i].chi2 < result[j].chi2) {
      j = i;
    }
  }

  // squeeze range

 switch (j) {

 case 0:
   result[4] = result[1];
   result[2] = JResult(0.5 * (result[0].x + result[4].x));
   break;

 case 1:
   result[4] = result[2];
   result[2] = result[1];
   break;

 case 2:
   result[0] = result[1];
   result[4] = result[3];
   break;

 case 3:
   result[0] = result[2];
   result[2] = result[3];
   break;

 case 4:
   result[0] = result[3];
   result[2] = JResult(0.5 * (result[0].x + result[4].x));
   break;
 }

 result[1] = JResult(0.5 * (result[0].x + result[2].x));
 result[3] = JResult(0.5 * (result[2].x + result[4].x));

} while (result[4].x - result[0].x > resolution);
```

JShowerDirectionPrefit

- Similar in working to JMuonEnergy
  - Energy estimation based on a binomial hit/no-hit likeliho

- Uses the Jpp isotropic emission PDFs to calculate the hit proba
  at distance D from the vertex, with photon incidence angle θi,

- An iterative five-point search is performed between a given en
  - In each iteration, pick the point with maximal hit/no-hit li
    and refine the five points

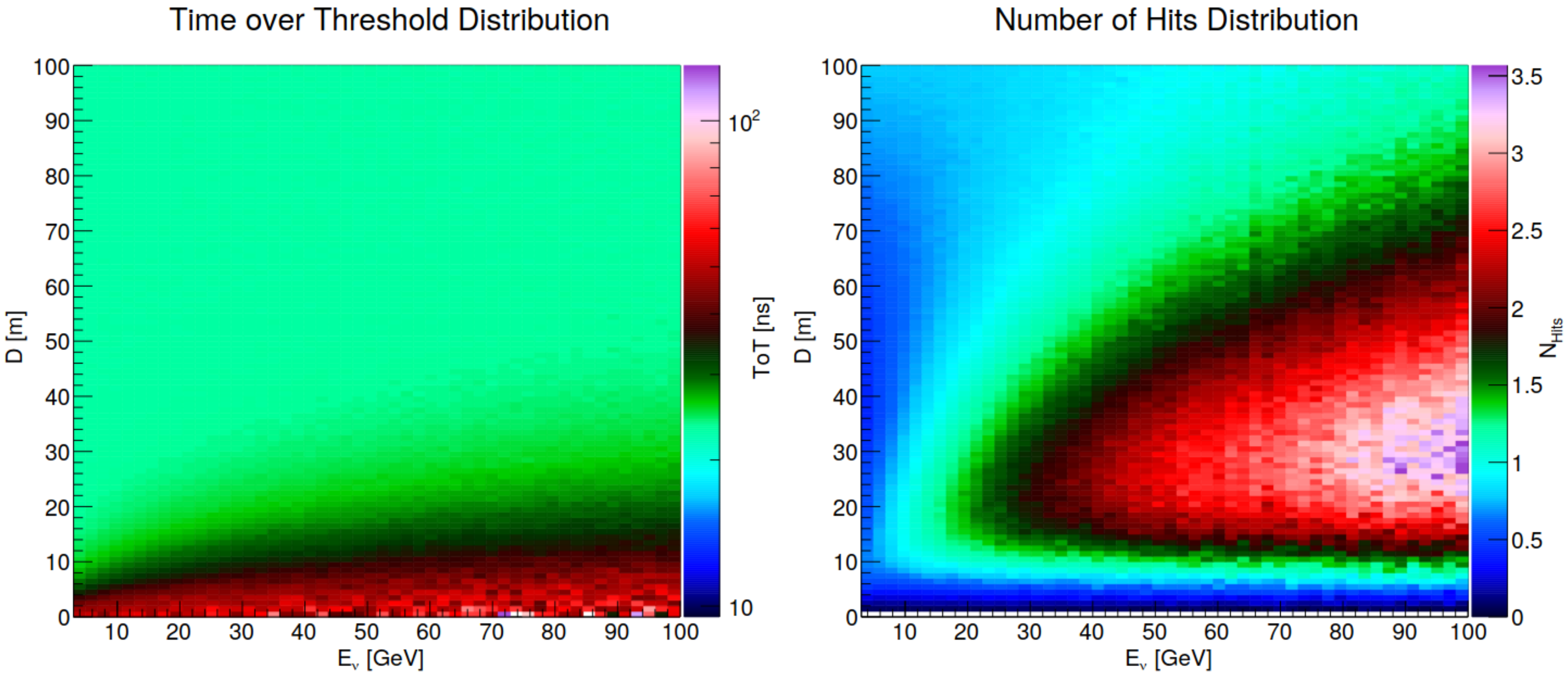


```
// 5-point search between given limits
const int      N  =  5;

JResult result[N];

for (int i = 0; i != N; ++i) {

  result[i].x = log10(Emin_GeV + i * (Emax_GeV - Emin_GeV) / (N-1));

}

do{

  int j = 0;

  for (int i = 0; i != N; ++i) {

    if (!result[i]) {

      result[i].chi2 = (*this)(result[i].x, data.begin(), data.end());

    }

    if (result[i].chi2 < result[j].chi2) {
      j = i;
    }
  }

  // squeeze range

 switch (j) {

 case 0:
   result[4] = result[1];
   result[2] = JResult(0.5 * (result[0].x + result[4].x));
   break;

 case 1:
   result[4] = result[2];
   result[2] = result[1];
   break;

 case 2:
   result[0] = result[1];
   result[4] = result[3];
   break;

 case 3:
   result[0] = result[2];
   result[2] = result[3];
   break;

 case 4:
   result[0] = result[3];
   result[2] = JResult(0.5 * (result[0].x + result[4].x));
   break;
 }

 result[1] = JResult(0.5 * (result[0].x + result[2].x));
 result[3] = JResult(0.5 * (result[2].x + result[4].x));

} while (result[4].x - result[0].x > resolution);
```

Some slides by Brían on direction fit

Bonus

ToT vs number of hits



(a) (b)

Figure 5.5: (a) ToT and (b) number of hits distributions in function of neutrino energy and distance from the neutrino interaction vertex.

Alba Domini – Ph.D. Thesis (2019)

JPrefit – Mathematical foundation

- Define the difference between two hit arrival times:

$$\begin{aligned} t_j'^2 - t_i'^2 - 2(t_j' - t_j')t_0' &= x_j^2 - x_i^2 - 2(x_j - x_j)x_0 \\ &+ y_j^2 - y_i^2 - 2(y_j - y_j)y_0 \,. \end{aligned}$$

- Casting this into matrix form, $H\,\Theta = Y$:

$$H = \begin{pmatrix} 2(x_2 - x_1) & 2(y_2 - y_1) & -2(t_2' - t_1') \\ 2(x_3 - x_2) & 2(y_3 - y_2) & -2(t_3' - t_2') \\ . & . & . \\ . & . & . \\ 2(x_1 - x_n) & 2(y_1 - y_n) & -2(t_1' - t_n') \end{pmatrix}, \qquad \Theta = \begin{pmatrix} x_0 \\ y_0 \\ t_0' \end{pmatrix} \qquad \longrightarrow \qquad Y = \begin{pmatrix} x_2^2 - x_1^2 + y_2^2 - y_1^2 - t_2'^2 + t_1'^2 \\ x_3^2 - x_2^2 + y_3^2 - y_2^2 - t_3'^2 + t_2'^2 \\ . \\ . \\ x_1^2 - x_n^2 + y_1^2 - y_n^2 - t_1'^2 + t_n'^2 \end{pmatrix},$$

- To which the least squares solution is:

$$\boxed{\Theta = \left(H^T V^{-1} H\right)^{-1} \times H^T V^{-1} \times Y} \qquad V = J \times \begin{pmatrix} \sigma_1^2 & . & . & . & 0 \\ . & \sigma_2^2 & . & . & . \\ . & . & \sigma_3^2 & . & \\ . & . & . & . & . \\ 0 & . & . & . & \sigma_n^2 \end{pmatrix} \times J^T \,. \qquad J_{ij} = \delta Y_i \;/\; \delta t_j \,.$$

with V for the correlation matrix

Vertex prefit (fitting algorithm)

- Secondary grid scan of vertex hypotheses around the initial reconstructed vertex from step 1. (outcome of step 1. still very sensitive to inclusion of background hits)
  - From –20 m to +20 m in 3 steps for (x,y,z)
  - From –50 ns to +50 ns in 4 steps for (t)
- For each of these, make an L0-only hit cluster, with D < 80 m and Δt < 100 ns
- The resulting hit clusters are fitted using an analytical PDF (see Janník's thesis) which penalizes PMT hits not oriented towards the assumed vertex position:

$$P(t_{res}, \psi) = \frac{1}{\sqrt{a^2 + (t_{res}/ns)^2 + p(\psi)}}$$

$$p(\psi) = \begin{cases} 0 & \text{if } \cos\psi < 0 \\ p_{max} & \text{if } \cos\psi > \cos\psi_{th} \\ p_{max} \times \cos\psi / \cos\psi_{th} & \text{else.} \end{cases}$$

$$\cos\psi_{th} = \frac{d_{ref}}{\sqrt{d_{ref}^2 + d^2}}$$

Maximal penalty angle

Prevent singularity

Hit time residual

Penalty function