Quaternions in Jpp

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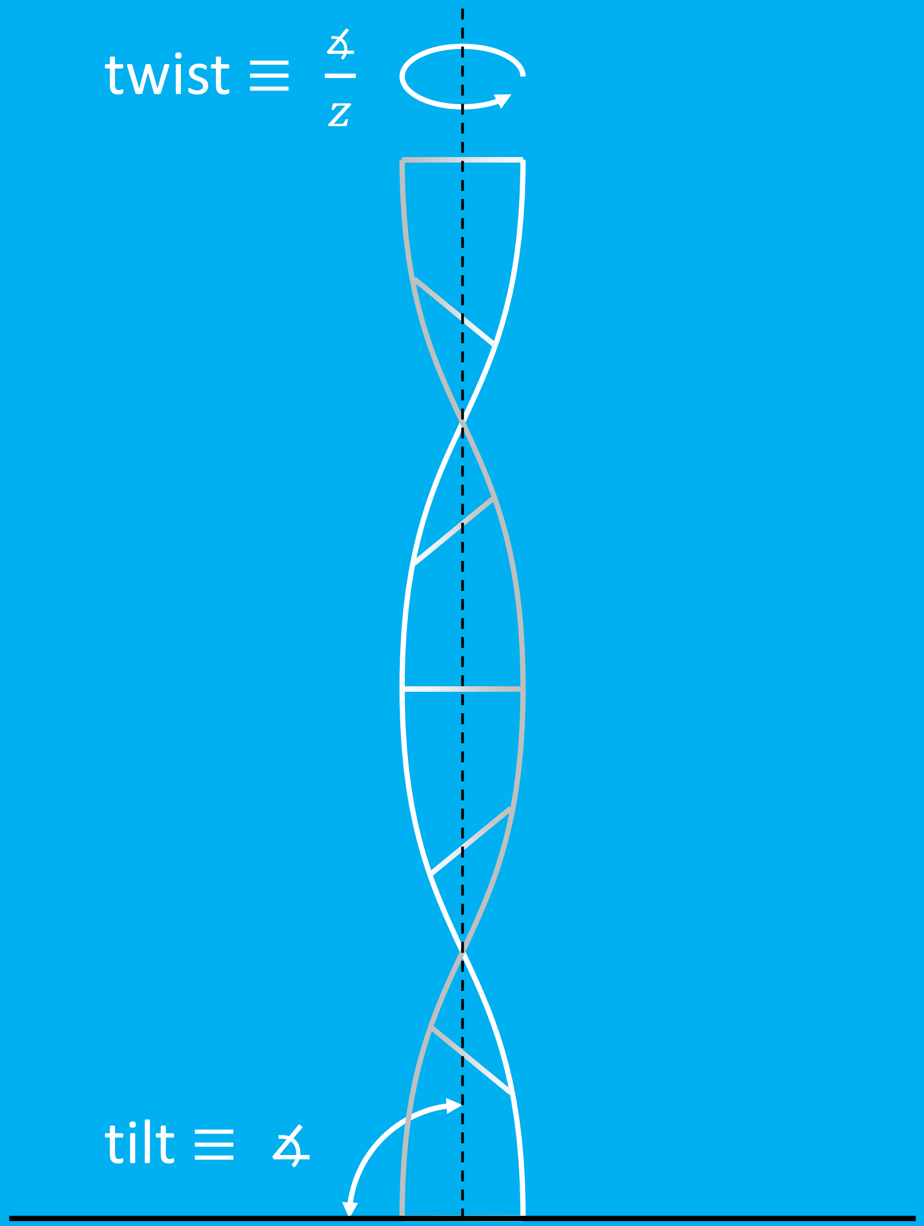
Introduction

- JQuaternion3D
  - defined as a rotation with angle θ around axis $\hat{u}$
    - $Q \equiv \left(\cos\frac{\theta}{2}, \sin\frac{\theta}{2}\hat{u}\right)$
  - basic mathematical operations
  - geodesic distance
  - conversion to and from rotation matrix
  - interpolation
  - averaging

Here as he walked by
on the 16th of October 1843
Sir William Rowan Hamilton
in a flash of genius discovered
the fundamental formula for
quaternion multiplication
$\mathbf{i}^2 = \mathbf{j}^2 = \mathbf{k}^2 = \mathbf{i}\,\mathbf{j}\,\mathbf{k} = -1$
& cut it on a stone of this bridge.

String

- orientation of optical module
  - measured by AHRS or LSM device
    - yaw, pitch and roll
  - related through string mechanics
    - tilt
    - twist



Model

- Orientation of floor i at height z_i is defined as
  - $Q_i = Q_0 Q_1^{z_i}$
    - $Q_0 \Rightarrow$ tilt of the string
    - $Q_1 \Rightarrow$ twist of string

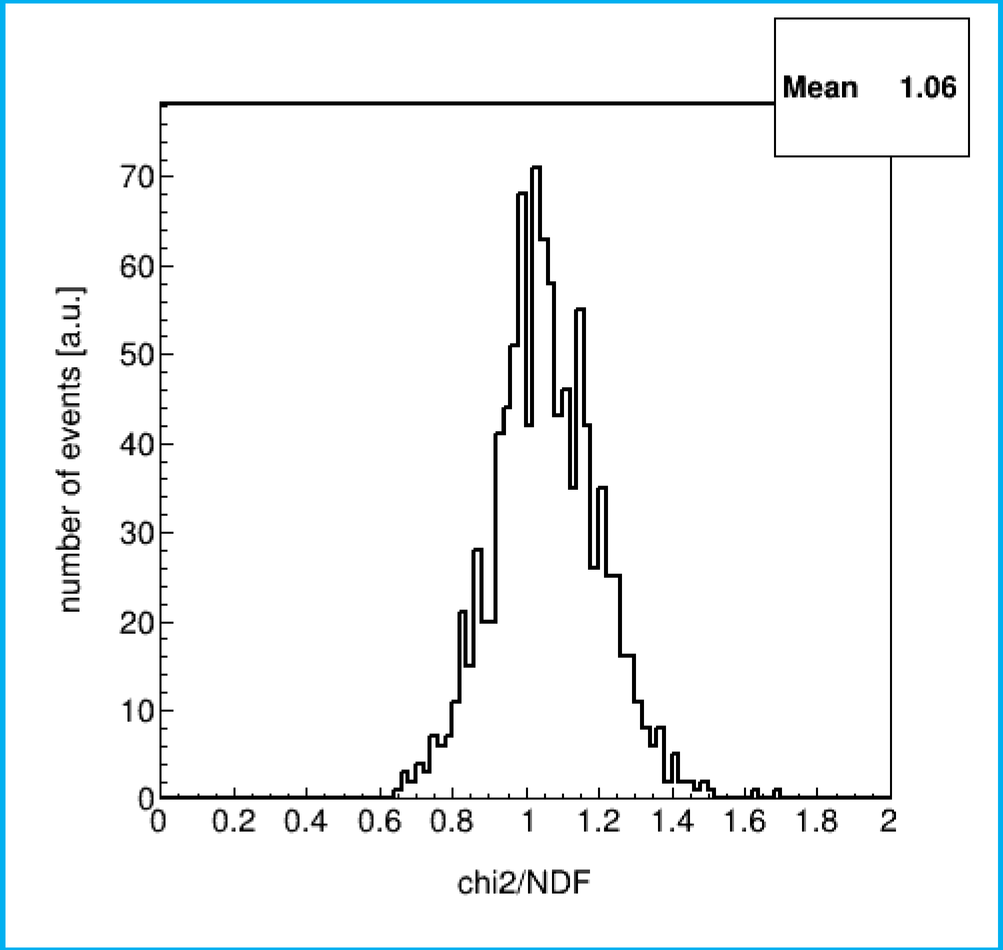
Simulation

- Gaussian smearing (arbitrary)
  - $\sigma_x = 1°$
    $\sigma_y = 1°$
    $\sigma_z = 3°$

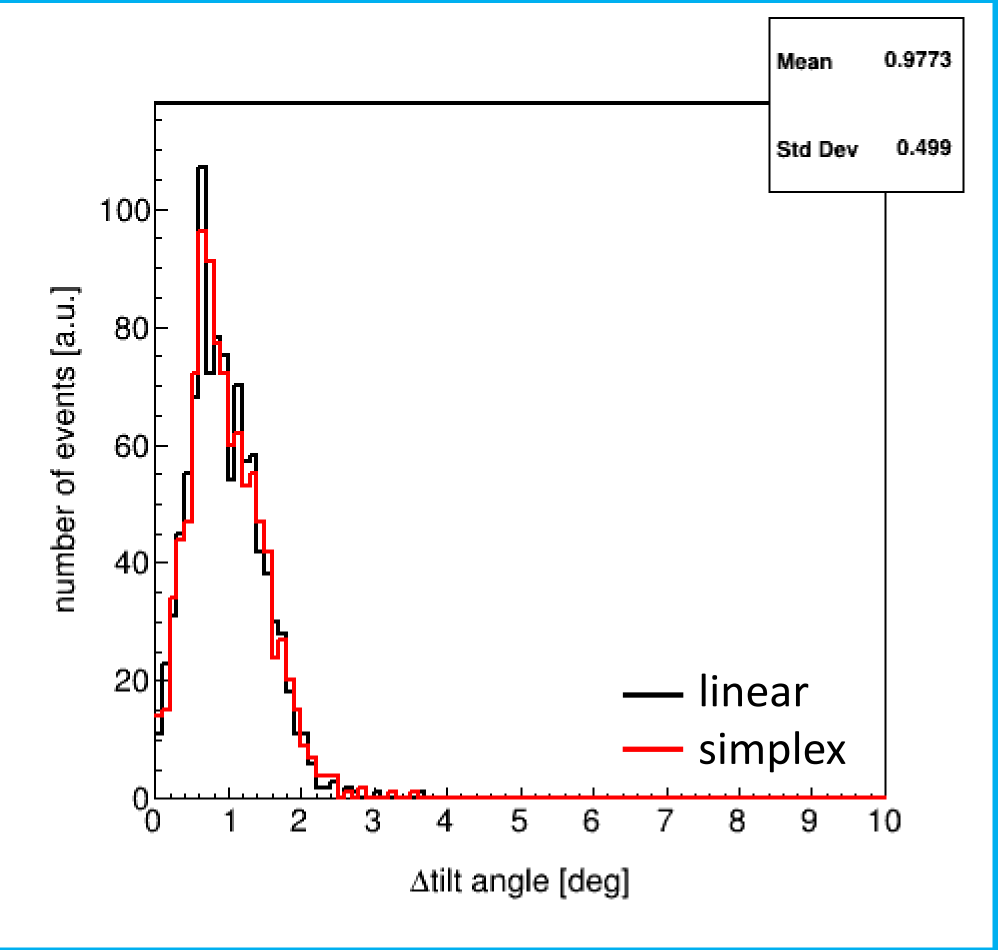
- Applied as random rotations to each floor
  - $Q \equiv Q_z Q_y Q_x$

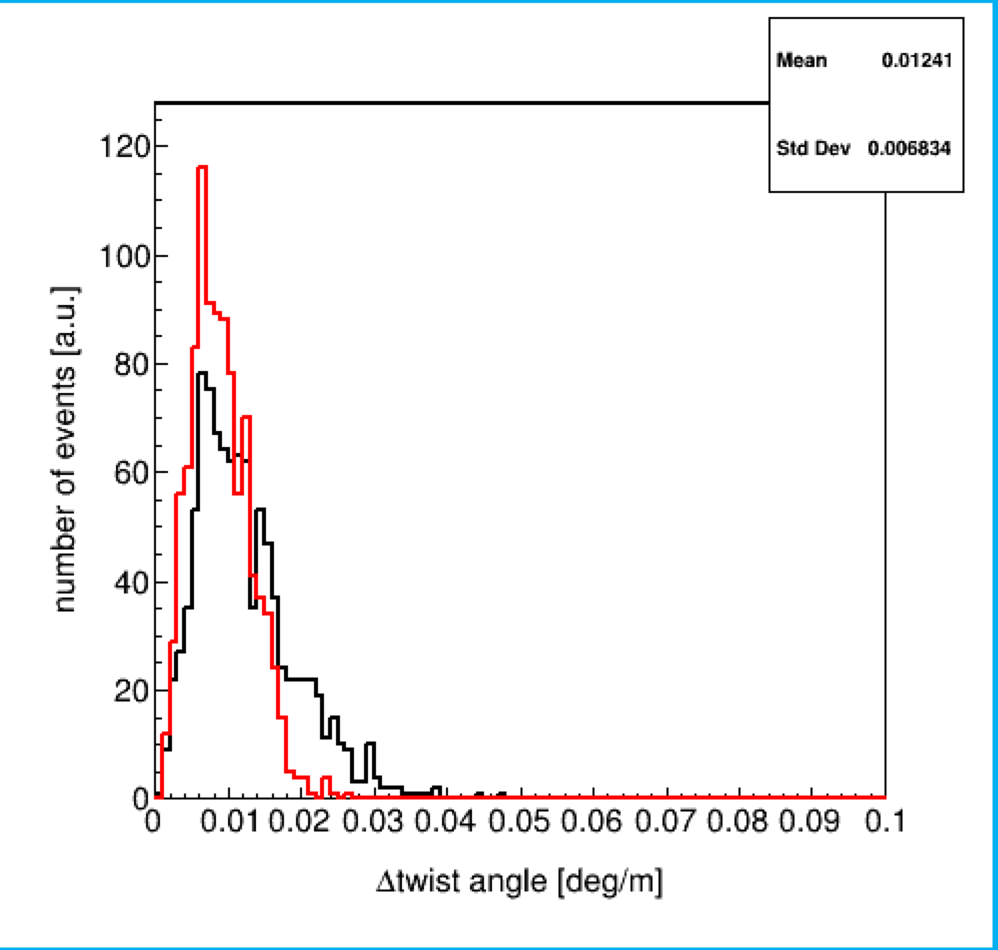
Fit

- Resolution (for χ^2 evaluation)
  - $\sigma \equiv \frac{1}{2}\sqrt{\sigma_x^2 + \sigma_y^2 + \sigma_z^2}$
- Metric
  - geodesic distance between two quaternions
- Minimiser
  - JSimplex works out-of-the-box
    - step corresponds to multiplication $\quad \mathrm{Q} \pm \Delta Q \quad \Rightarrow \quad Q\,\Delta Q^{\pm 1}$
    - scaling corresponds to power $\quad \mathrm{Q} \times y \quad \Rightarrow \quad Q^{y}$



Mean 1.06
number of events [a.u.]
chi2/NDF
0 10 20 30 40 50 60 70
0 0.2 0.4 0.6 0.8 1 1.2 1.4 1.6 1.8 2


Mean 0.9773
Std Dev 0.499
number of events [a.u.]
Δtilt angle [deg]
linear
simplex


Mean 0.01241
Std Dev 0.006834
number of events [a.u.]
Δtwist angle [deg/m]

Conclusions & Outlook

- Quaternions are part of Jpp
  - can be used to apply rotations
  - allow for (basic) mathematical operations
  - allow for fits