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String Theory, Particle Physics and Emergent Gravity

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Prelude

• In 1984 I was a last-year undergraduate at the University of Athens.

• I went for three months at CERN, to finish my undergraduate thesis, on perturbative QCD Predictions for the E-537 experiment at Fermilab, studying lepton pair production in hadron collisions.

• My advisor (an experimentalist who knew good theory) brought me a small book and asked me to read it carefully.

• It was Bert's PhD thesis:


PERTURBATIVE QCD AND LEPTON PAIR PRODUCTION

A.N. Schellekens (Nijmegen U.) (Jun, 1981)

cite

String Theory: the original hype

- String Theory has been introduced, indirectly, to realize an example of crossing (dual) symmetry for hadron amplitudes.

- It has been abandoned after the SLAC deep inelastic data have been linked to QCD, coupled with the problems that string theory exhibited: tachyons, 26 dimensions for Lorentz invariance etc.

- A few persistent souls, have changed its target in 1976: they realized that string theory is an example of a theory containing a theory of quantum gravity. But nobody else noticed.

- In 1984, the theory come back in fashion for a reason that in retrospect looks bizzare: rarity, or uniqueness of anomaly cancellation.

- I was starting my PhD in 1984, and was a very early witness to this hype that surrounded anomaly cancellation in string theory.

• This same hype, "uniqueness", was for several years the main "credo" of the leaders of the field.

• Suddenly, many major problems seemed being solved by a unique theory:

♠ The quantization of gravity, a notoriously difficult problem till then (and some would say up to today)

♠ The unification of gravity with gauge theories.

♠ The appearance of chirality from a higher-dimensional theory.

♠ A unique theory for all of the above.

String Theory: doubts on uniqueness

• Narain's paper on the modular invariant torus partition functions (1985) came as a shock!

• And then Bert with Wolfgang and Dieter, gave a much larger list of "string theory vacua" using a general lattice construction in 1986.

CHIRAL FOUR-DIMENSIONAL HETEROTIC STRINGS FROM SELF-DUAL LATTICES

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It is shown how our previous work on lattice constructions of ten-dimensional heterotic strings can be applied to four dimensions. The construction is based on an extension of Narain's lattices by including the bosonized world-sheet fermions and ghosts, and uses conformal field theory as its starting point. A natural embedding of all these theories in the bosonic string is automatically provided. Large numbers of chiral string theories with and without $N = 1$ supersymmetry can be constructed. Many features of their spectra have a simple interpretation in terms of properties of even self-dual lattices. In particular we find an intriguing relation between extended supersymmetry and exceptional groups.

• Slightly later, another paper appeared that constructed many 4d string vacua using free fermions only on the internal part of the world sheet.

• Needless to say, these papers were met by the incredulity of experts.

• It took a lot of effort and time for them to be accepted as bona-fide vacua of string theory.

• Moreover, the same belief was rampant: all of these, except the unique physical vacuum realizing the SM and its extension, must be non-perturbatively inconsistent. And that included also the maximally supersymmetric ten-dimensional vacua.

• Bert and others had already been suspecting that uniqueness was down the drain.

• But it was not easy to prove it: it was geometrically known that the vacua studied, were parts of moduli spaces and it was not easy to count them as distinct, let allow claim they were non-perturbatively unstable.

String Theory: the second string revolution

• But in 1995, the tide turned:

♠ Witten, with a slight of hand, argued that all five, maximally supersymmetric string theories in 10 (and even 11) dimensions are equivalent (dual).

• Suddenly, uniqueness seemed restored, albeit in a somewhat different spirit.

• But as time went on, and physicists were analyzing the non-trivial lower-dimensional dualities, it started downing on them that the culprit responsible for this magnificent uniqueness was maximal supersymmetry.

• And then flux compactifications have started entering the game.

String Theory: the king is naked

• One of the most important problems in modern theoretical physics is the cosmological constant problem:

♠ Why the observed cosmological constant today is many-many orders of magnitude smaller than that predicted by Quantum Field Theory and (to a certain extend) String Theory.

• In string theory, as gravity is always part of the game, a non-zero cosmological constant at tree level, meant the "death of the Minkowski vacuum".

♠ At loop order, the equations of motion are not satisfied and the vacuum needs to be corrected: a computational catastrophy.

• Therefore, only supersymmetric vacua, with zero cosmological constant have been mostly been discussed in string theory for over 40 years.

• But it was known that supersymmetry had to break, and the breaking scale cannot be too low otherwise we would have seen it.

• Therefore, the clash with the cosmological constant, persists in string theory.

• In 1998 Bert, in his inaugural address to his professorship position at his alma matter, Nijmegen University, has dared to voice his opinion on what he thought was string theory realizing.

• He argued for a large "landscape" of string theory vacua, that may allow the anthropic resolution of some physics questions (like the cosmological constant problem).

• This speech was not published, until the flux compactifications and most notably, the KKLT construction of flux vacua became an accepted extension of the landscape of string vacua, and where it was giving a discretum of vacua, rather than continuous moduli spaces as before.

• Sussking called this whole set, the "anthropic landscape of string theory", and Bert published his essay in the ArXiv:

arXiv > physics > arXiv:physics/0604134

Physics > History and Philosophy of Physics

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The Landscape "avant la lettre"

A.N. Schellekens

This is a translation of an inaugural speech given originally in Dutch in 1998. The topic of that speech, intended for a general audience, was what is now called "The Anthropic Landscape of String Theory".

Comments:	28 pages (10 in Dutch). Misprints corrected, references added
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String Theory: The landscape

• String theory regions around vacua look either like moduli spaces (large flat areas), or a discretum of minima separated by "mountains".

• The landscape is very complex: at each "transition boundary" it generically changes dimension, and is therefore not a manifold.

• But from this point of view, it looks more like what you would consider as the space of QFTs: around each "vacuum" (=CFT) the dimension of the space is given by the dimension of the space of relevant operators.

♠ This changes around different CFTs.

♠ This makes the overall space not a manifold.

♠ But it would have been a sin to claim that there was a similarity between QFT and string theory. String theory was considered far superior!

Searching the String Landscape

• Once the landscape has been accepted by some, searching it for desirable features (the (chiral) SM spectrum, or simpler features) has become a noble enterprize.

• Bert, in a long sequence of works, has systematically developed (and algorithmized) the CFT techniques for building CFTs as parts of the closed strings and BCFTs as parts of orientifolds of string theory (realizing D-branes).

• Being also a computer wizard, he built and refined computer programs that automatized such constructions of vacua, and therefore made possible the large scale searches that followed.

Supersymmetric standard model spectra from RCFT orientifolds

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Abstract

We present supersymmetric, tadpole-free $d = 4$, $N = 1$ orientifold vacua with a three family chiral fermion spectrum that is identical to that of the standard model. Starting with all simple current orientifolds of all Gepner models we perform a systematic search for such spectra. We consider several variations of the standard four-stack intersecting brane realization of the standard model, with all quarks and leptons realized as bifundamentals and perturbatively exact baryon and lepton number symmetries, and with a $U(1)_Y$ vector boson that does not acquire a mass from Green–Schwarz terms. The number of supersymmetric Higgs pairs $H_1 + H_2$ is left free. In order to cancel all tadpoles, we allow a "hidden" gauge group, which must be chirally decoupled from the standard model. We also allow for non-chiral mirror-pairs of quarks and leptons, non-chiral exotics and (possibly chiral) hidden, standard model singlet matter, as well as a massless B–L vector boson. All of these less desirable features are absent in some cases, although not simultaneously. In particular, we found cases with massless Chan–Paton gauge bosons generating nothing more than $SU(3) \times SU(2) \times U(1)$. We obtain almost 180 000 rationally distinct solutions (not counting hidden sector degrees of freedom), and present distributions of various quantities. We analyse the tree level gauge couplings, and find a large range of values, remarkably centered around the unification point.

Orientifolds, hypercharge embeddings and the Standard Model

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Abstract

The embedding of the SM hypercharge into an orientifold gauge group is studied. Possible embeddings are classified, and a systematic construction of bottom–up configurations and top–down orientifold vacua is achieved, solving the tadpole conditions in the context of Gepner orientifolds. Some hypercharge embeddings are strongly preferred compared to others. Configurations with chiral anti-symmetric tensors are suppressed. We find among others, genuine examples of supersymmetric $SU(5)$, flipped $SU(5)$, Pati–Salam and trinification vacua with no chiral exotics.

AdS/CFT

• The most surprising development of the last 25 years, came with the AdS/CFT correspondence:

• It postulates the equivalence of a 4d gauge theory: N=4 supersymetric YM and a +10d string theory, type IIB string theory on a very symmetric but curved space, $AdS_5 \times S^5$.

• This is a strong-weak coupling duality: when strings are weakly stretched , gluons are strongly coupled and vice versa.

• It has not been proven to date, but we have sufficient evidence for its validity that we are exploring its consequences.

• It has been generalized in many directions. In its most general accepted form:

♠ Quantum Field Theory ⇔ string theory/membrane theory

• This class of dualities reinstatedthe symmetry between QFT and string theory:

When string theory is asympotically AdS, then its dynamics can be mapped to QFT and vice versa.

• In this case, the string landscape is none else than the QFT Landscape.

• This observation turns upside down our view of string vacua and expectations of uniqueness. This is not how we think about QFT models of particle physics.

• It should be stressed though that an analogous statement for asymptotically Minkowski, string theory vacua and their holographic duals, is still obscure.

Some lessons from AdS/CFT

• The AdS/CFT correspondence (known also as the holographic correspondence or holography) has tought us some radical lessons:

♠ The number (and nature) of the dimensions of space-time are not an invariant property of a theory, but depend on the description.

♠ The physics of any theory containing (weakly-coupled) gravity is holographic: it is (uniquely) encoded in the boundary S-matrix.

♠ The quantum theory of gravity, as realized by string theory can be thought of as the effective theory of gluon composites:The graviton, and the other string states are composites of (super-gluons).

♠ Black holes are dual to the canonical ensembles of QFTs.

• Information is not lost in black holes. The information paradox is a subtler problem that is still being investigated.

String Theory before and after AdS/CFT

- Before AdS/CFT string theory is only defined in perturbation theory.

- It provides a quantum theory of gravity which is reliable and computable up to scales that are parametrically smaller than the Planck scale.

- It cannot answer questions for energy transfers at or above the Planck scale.

- The "fundamental" degrees of freedom are strings.

♠ After AdS/CFT we can use, dual QFTs that can be non-perturbatively defined to give a non-perturbative definition of the dual string theory.

♠ Gravity, and space-time are emergent concepts from the QFT data.

♠ The colorless string states are composites of the QFT degrees of freedom.

♠ The graviton, in particular is a QFT composite, associated to the QFT energy momentum tensor.

♠ We can therefore say that the short-distance gravitational degrees of freedom are QFT "partons".

♠ For gravity to be weak, we must have a large number N of partons (or colors)

How quantized (composite) gravity might work

• A good example for us will be the low-energy theory of the strong interactions: It is the IR-free (but non-renormalizable) theory of pions, that reminds us quite well the problems with quantizing gravity.

• In that theory, it was eventually understood, that one can quantize the low energy degrees of freedom (pions) in the chiral Lagrangian, but this description has a cutoff, $\Lambda \sim GeV$ and a large number of counterterms are needed.

• Instead, the high-energy degrees of freedom (quarks+gluons) are different and the QFT associated to them is UV complete (and effectively strongly-coupled in the IR)

• Taking this as clue, it would suggest that the non-renormalizability of the graviton appears because of its compositeness: the graviton is a low-energy bound-state.

• This idea is VERY old: Many attempts were made in the past to construct gravity theories where the graviton is a composite field, made out of more elementary fields, of all types: scalars, fermions, vectors etc.

• All such attempts failed to go beyond the classical and provide a dynamical explanation of why the bound state appears "feature-less" at low energies.

The energy momentum tensor of the QFT

• The state generated out of the vacuum by the (conserved) energy-momentum tensor has the quantum numbers of the graviton

$$T_{\mu\nu}(p)|0\rangle \equiv |\epsilon_{\mu\nu}, p\rangle$$

• It is transverse because of energy conservation.

• In weakly-coupled theories, this is a spin-2 multi-particle state and therefore its interactions are expected to be non-local.

• If however, the interactions are strong and make this state a true tightly-bound state with a "size" L, then maybe we can reproduce gravity at scales $\gg L$.

• In particular, in the limit of infinitely-strong interactions we would expect to obtain a good point-like interaction theory for this bound-state graviton.

• We must remember however that, in the presence of strong attractive interactions in the spin-two channel, there may be, generically speaking, a tower of states generated from the $T_{\mu\nu}$ acting on $|0\rangle$.

• If the theory is not conformal, such a spectrum will be discrete, and would be associated with the (generically complex) poles of the two-point function of the stress tensor.

• Again, generically speaking, many such states will be unstable.

• If the theory is conformal, such states will form a continuum.

• This is the case in the original example of AdS/CFT.

We have learned from the holograhic correspondence that:

♠ Strong coupling in QFT makes gravitons tightly bound states.

♠ Large N makes gravitons weakly interacting.

and both of the above give an effective semiclassical theory of (composite) quantum gravity.

Gravitons from hidden sectors

To have a (composite or emergent) graviton couple to the SM:

♠ It can emerge from a "hidden sector".

♠ The hidden sector will be coupled to the SM at some high scale.

♠ Only a few interactions will survive in the IR between the two theories as all interactions will be IR-irrelevant.

♠ This will match with the IR-freedom and non-renormalizability of gravity.

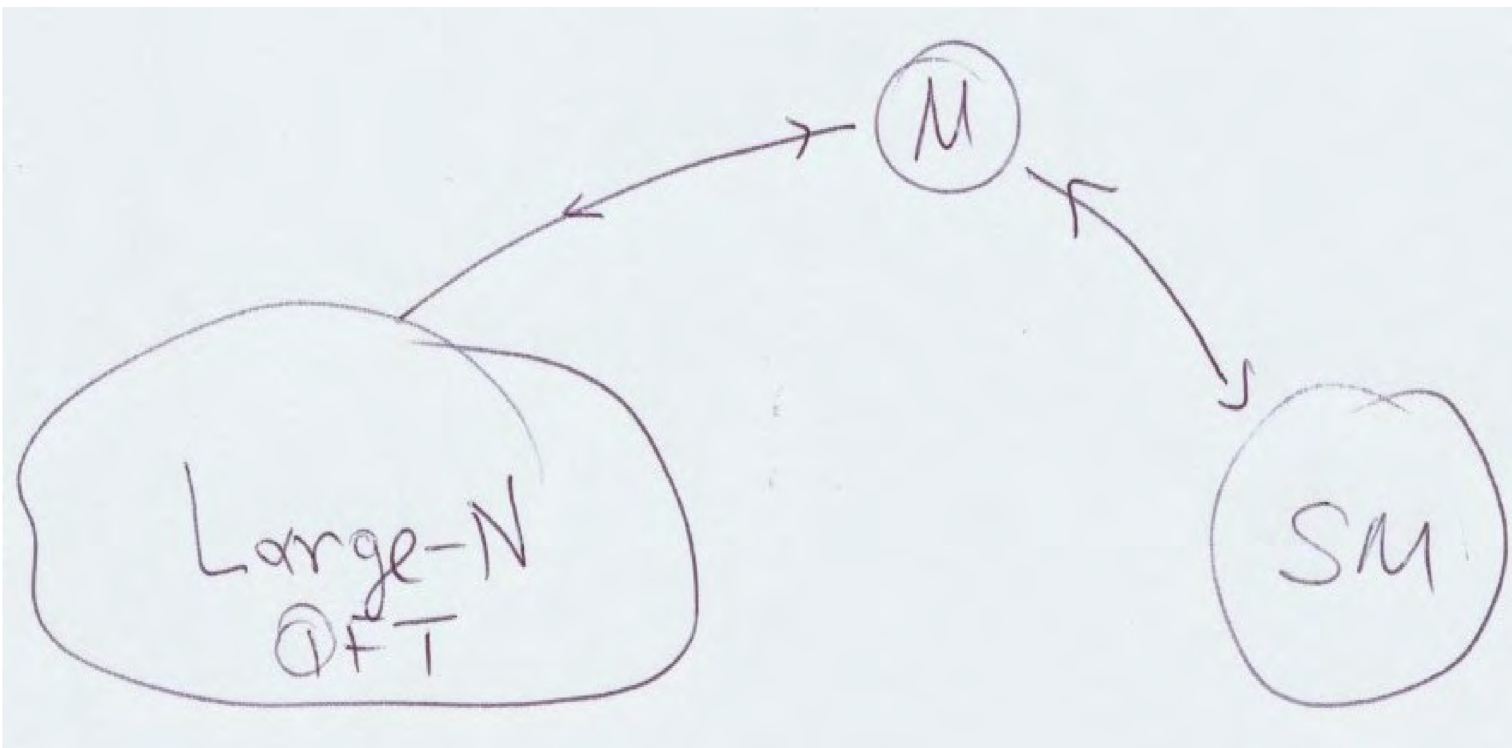
• If we want this graviton to be tightly bound and weakly coupled, then this hidden sector theory must be a large-N, strongly coupled (ie holographic) QFT.

• We are led therefore to couple a large-N theory to the SM in a UV complete fashion.

• One therefore should postulate a massive bifundamental messenger sector to couple the two theories together.

$$QFT_N \quad \times \quad Bifundamentals \quad \times \quad StandardModel$$

• If the bifundamentals have mass M, we can integrate them out to obtain a theory below M with contact interactions between QFT_N and the SM.



• Therefore, the largest piece of the UV theory to which the SM is part of, is associated with gravity, and its degrees of freedom are vastly larger than those of the SM.

• The gravity theory of emergent gravity is a bi-gravity theory:

♠ There is a fixed metric $h_{\mu\nu}$ associated to the metric of the QFT from which dynamical gravity emerges.

♠ and there is the dynamical metric $g_{\mu\nu} = \langle T_{\mu\nu} \rangle$.

• In holography, the fixed metric is implemented by a boundary condition at the AdS boundary.

• A remarkable property holds: If $h_{\mu\nu} = \eta_{\mu\nu}$ then $g_{\mu\nu} = \eta_{\mu\nu}$ is always a solution to the equations of dynamical gravity, *independent of quantum corrections*.

• Therefore: there is no traditional cosmological constant problem.

Comments and Open ends

- Other protected fields can appear together with the graviton: the universal axion and global conserved currents (graviphotons).

Anastasopoulos+Betzios+Bianchi+Consoli+Kiritsis

- There may be interesting particle physics (and cosmological physics) associated with such particles from the "gravitational sector",

- It is not yet clear how to bridge the ultimate IR non-linear theory with one of the massive graviton theories.

- There is a source for "dark energy" and "dark matter" in the hidden theory.

- When the hidden theory is a holographic QFT, then this description transforms intothe brane-in-bulk (or brane-world) description (with Neumann boundary conditions at the AdS boundary).

• In this case, one can self-tune the cosmological constant and always obtain a massive graviton on the brane from the DGP pole with mass $\sim N^{-1}$.

Charmousis+Kiritsis+Nitti

• Adding also the natural (bulk) axion one can, in principle, correlate the self-tuning of the brane CC to a solution of the hierarchy problem.

Hamada+Kiritsis+Nitti+Witkowski

• There is always a scalar mode (the "dilaton") beyond a massive graviton that is always positive in a unitary QFT.

• This is also a natural dark matter or dark energy candidate. Can one make it comply with the principle of equivalence?

• The signature of the metric in emergent gravity can change. If a stress tensor is close to that of a cosmological constant then the signature is Minkowski. If it is of the photonic type, it has Euclidean signature.

• There are many answered questions in order to tie this to observable gravity and asymptotically flat string theory.

Many wishes to Bert (and Beatriz) for life and physics goes on

The Weinberg-Witten Theorem

• The WW theorem assumes Lorentz invariance and a conserved Lorentz-covariant Energy-Momentum tensor.

• It proceeds to prove that no massless particle with spin $S > 1$ can couple to the stress tensor and no massless particles with $S > 1/2$ to a global conserved current.

• This does not rule out a theory that contains a "fundamental" massless graviton, as there exists a loop-hole: The stress tensor is not conserved in the presence of a metric, and projecting on helicity-2 is also non-covariant in a general metric.

• There are also other ways of avoiding the theorem:

• In the case of massless vectors the statement says that no massless (non-abelian) vectors can couple to a conserved Lorentz-covariant global current. It seems that Yang-Mills theory is excluded.

• This is avoided in standard non-abelian gauge theories as the conserved current is not Lorentz-covariant (only up to a gauge transformation).

• A final caveat: Lorentz invariance is crucial: otherwise the notion of masslessness is not well-defined. (even in dS or AdS the notion changes)

• In conclusion: WW can be evaded but it is a serious litmus test for all emergent graviton theories.

• We shall find that although the essence of the WW theorem remains true, the effective theories for composite gravitons are very rich.

The Weinberg-Witten loop-hole

- In GR the stress tensor is not conserved but covariantly conserved.

- One can add corrections to the tress tensor (involving also the flat metric) to make is strictly conserved and Lorentz covariant. This is however NOT a tensor under general coordinate transformations (but this is OK with WW).

- To make a pure helicity-two state, we must project out the (unphysical) helicity 1 and 0 states. This projection is NOT Lorentz covariant (but only up to a gauge transformation).

- We may appeal to diff-invariance to decouple the helicity 0 and 1 states but then we are stuck: $T_{\mu\nu}$ is now NOT fully covariant.

- Therefore GR and many other theories with an explicit dynamical graviton avoid the WW theorem.

Translation Ward identity

• We consider a theory with Lagrangian $\mathcal{L}$. For concreteness, we focus on four-dimensional QFTs.

• Under an infinitesimal diffeomorphism generated by a vector ξ_μ

$$\delta_\xi \mathcal{L} = \frac{1}{2} \left(\partial_\mu \xi_\nu + \partial_\nu \xi_\mu\right) T^{\mu\nu}$$

$$\delta_\xi T^{\mu\nu} = \xi^\sigma \partial_\sigma T^{\mu\nu} + T^{\sigma\nu} \partial^\mu \xi_\sigma + T^{\mu\sigma} \partial^\nu \xi_\sigma$$

• The invariance of the partition function $Z = e^{i \int d^4x \, \mathcal{L}}$ under the infinitesimal translation implies the conservation equation

$$\partial_\mu \langle T^{\mu\nu} \rangle = 0$$

• Similarly, the invariance of the one-point function of the energy-momentum tensor

$$\langle T^{\rho\sigma}(y)\rangle = \frac{\int D\Phi\, e^{i\int d^4x\,\mathcal{L}}\, T^{\rho\sigma}(y)}{\int D\Phi\, e^{i\int d^4x\,\mathcal{L}}}$$

under the infinitesimal translations implies the Ward identity

$$-i\langle\partial_\mu T^{\mu\nu}(x)T^{\rho\sigma}(y)\rangle + \delta(x-y)\langle\partial^\nu T^{\rho\sigma}(x)\rangle + \partial^\nu\delta(x-y)\langle T^{\rho\sigma}(x)\rangle$$

$$-\partial^\rho\left(\delta(x-y)\langle T^{\nu\sigma}(x)\rangle\right) - \partial^\sigma\left(\delta(x-y)\langle T^{\rho\nu}(x)\rangle\right) = 0$$

• In addition, Lorentz invariance implies that the one-point function of the energy-momentum tensor is

$$\langle T^{\mu\nu}(x)\rangle = a\eta^{\mu\nu}$$

where a is a dimensionfull constant.

Consequently, we set

$$\langle\partial^\nu T^{\rho\sigma}(x)\rangle = 0$$

and use it to simplify the Ward identity

$$i\langle\partial_\mu T^{\mu\nu}(x)T^{\rho\sigma}(y)\rangle - \partial^\nu\delta(x-y)\langle T^{\rho\sigma}(x)\rangle$$

$$+\partial^\rho\left(\delta(x-y)\langle T^{\nu\sigma}(x)\rangle\right) + \partial^\sigma\left(\delta(x-y)\langle T^{\rho\nu}(x)\rangle\right) = 0$$

- In momentum space we obtain instead:

$$k_\mu\langle T^{\mu\nu}(k)T^{\rho\sigma}(-k)\rangle = ia\left(-k^\nu\eta^{\rho\sigma} + k^\rho\eta^{\nu\sigma} + k^\sigma\eta^{\rho\nu}\right)$$

- This allows us to deduce the 2-point function as **??**

$$\langle T^{\mu\nu}(k)T^{\rho\sigma}(-k)\rangle$$

$$= ia\left(-\eta^{\mu\nu}\eta^{\rho\sigma} + \eta^{\mu\rho}\eta^{\nu\sigma} + \eta^{\mu\sigma}\eta^{\rho\nu}\right) + b(k^2)\,\Pi^{\mu\nu\rho\sigma}(k) + c(k^2)\,\pi^{\mu\nu}(k)\pi^{\rho\sigma}(k)$$

with

$$\Pi^{\mu\nu\rho\sigma}(k) = \pi^{\mu\rho}(k)\pi^{\nu\sigma}(k) + \pi^{\mu\sigma}(k)\pi^{\nu\rho}(k) \quad , \quad \pi^{\mu\nu}(k) = \eta^{\mu\nu} - \frac{k^\mu k^\nu}{k^2}$$

Aside: String theory vs the swampland

• Conjectures talk about "quantum gravity" but everyone means "string theory"

• The (plausible) assumption that string theory is the space of large-N strongly coupled QFTs, has an automatic avatar:

• The "swampland" corresponds to QFTs that are either weakly-coupled, or are not at large N.

• This explains for example, the generic towers of states that appear at the boundaries of moduli spaces.

• It also suggests why there might be no de Sitter solution in "string theory".

• The notion of string theory used above is certainly more general that the conventional one based on 2d CFTs

• It involves also 3, 4, 5 and 6-dimensional CFTs.

• It might be illuminating to try to see the swampland conjectures via this point of view.

Higher spin

• It is one of the obvious next questions to ask: what about doing this for other operators of your QFT:

• For fields up to $S = 1/2$ this is a standard procedure, and has been done in many contexts.

• The case of $S = 1$ is interesting as it would describe emergent gauge theory. It is qualitatively different than the gravity case.

• When $S > 2$ one can again do the same procedure as here.

• In that case however for interacting theories, higher spin fields are not conserved. The effective theory one obtains will be massive, with characteristic mass the overall cutoff (in string theory this is the string scale).

• They are therefore less interesting for low-energy physics.

• In a free QFT however they are conserved and then one can construct massless actions (of an infinite number of them)

Douglas+Razamat, Leigh

WW versus AdS/CFT

• Is AdS/CFT compatible with the WW theorem?

• The WW theorem involves a subtle limit to define the helicity amplitudes that determine the couplings of massless states to the stress tensor or a local current.

• This limiting procedure is not valid in theories where the states form a continuum.

• This is the case in AdS/CFT.

• From the point of view of the QFT, the effective gravitational coupling is non-local.

• Therefore the WW-theorem does not apply to this case.

• What about non-CFTs?

WW versus nAdS/nCFT

- Consider a familiar example: four-dimensional, large-N YM theory.

- Its string-theory dual is stringy (and nearly tensionless) near the AdS-boundary (weak QFT coupling).

- We expect a gravitational description at low energies (strong QFT coupling).

- The theory has a gap and a discrete spectrum and therefore the emergent gravitational interactions must be local.

- Also gravity must be weakly coupled (and it is, due to large N limit).

• The low energy spectrum contains two stable (lightest) massive scalars (0^{++}, 0^{-+}), and a stable massive graviton (2^{++}). All other glueballs are resonances, and are not asymptotic states.

• The higher cousins of the graviton are unstable.

• A massive graviton is compatible with WW.

• It is also compatible with a fully diff invariant theory of a massless graviton in 5 dimensions.

• The 4d graviton mass is due to the non-trivial 5d background, hence a gravitational "Higgs effect".

• The above gives some credence to the idea that heavy-ion collisions form (unstable) black holes of a massive gravity theory that quickly Hawking evaporate.

Nastase, Kiritsis+Taliotis

The stress tensor vev as a (classical) dynamical metric

- We would like to implement directly the idea of an emergent graviton as the state generated by the energy-momentum tensor.

- We will construct the theory that describes the dynamics of such a graviton in any QFT.

- As a warm-up, we consider a translationally invariant QFT at a fixed background metric $\mathbf{g}_{\mu\nu}$ and a scalar source J coupled to a scalar operator O (for purposes of illustration).

- The presence of an arbitrary background metric $\mathbf{g}_{\mu\nu}(x)$ breaks translation invariance.

• A redefinition of the derivative→ covariant derivative "restores" energy-momentum conservation (in the absence of other non-constant sources):

$$T_{\mu\nu} \equiv \frac{1}{\sqrt{\mathbf{g}}}\frac{\delta S(\mathbf{g}, J)}{\delta \mathbf{g}^{\mu\nu}} \quad , \quad \nabla^{\mu}_{\mathbf{g}}\langle T_{\mu\nu}\rangle \sim \partial_\nu J$$

where $S(\mathbf{g}, J)$ is the action of the theory coupled to the fixed metric $\mathbf{g}$ and to the scalar source J.

• Consider the Schwinger functional

$$e^{-W(g_{\mu\nu},J)} = \int \mathcal{D}\phi \; e^{-S(\phi,g_{\mu\nu},J)}$$

• $g_{\mu\nu}$ is an arbitrary background metric, ϕ are the "quantum fields".

• We assume the presence of a cutoff that preserves diff invariance so that the quantities above are finite.

- This is tricky business but for the moment we can have dim-reg in mind.

- $W(g, J)$ is now diff-invariant:

$$W(g'_{\mu\nu}(x'), J(x')) = W(g_{\mu\nu}(x), J(x)) \quad , \quad g'_{\mu\nu} = g_{\rho\sigma}\frac{\partial x^\rho}{\partial x'^\mu}\frac{\partial x^\sigma}{\partial x'^\nu}$$

- $W(h)$ encodes the interaction energy between energy-momentum sources $h_{\mu\nu}$ with
$g_{\mu\nu} = \mathbf{g}_{\mu\nu} + h_{\mu\nu}$.

- The (quantum) vev of the stress tensor is:

$$h_{\mu\nu} \equiv \frac{1}{\sqrt{\det g}}\frac{\delta W(g, J)}{\delta g^{\mu\nu}}$$

and we will use it to define the associated effective action:

$$\Gamma(h, J, \mathbf{g}) \equiv -W(g, J) + \int d^4x\sqrt{g}\ h_{\mu\nu}\ (g^{\mu\nu} - \mathbf{g}^{\mu\nu})$$

via a modified Legendre transform.

• Γ is the generating functional of 1-PI energy-momentum tensor correlators and is extremal,

$$\left.\frac{\delta\Gamma(h_{\mu\nu},J)}{\delta h_{\mu\nu}}\right|_{g=\mathbf{g}} = 0 \quad , \quad \left.\Gamma(h^*_{\mu\nu},J)\right|_{g=\mathbf{g}} = W(\mathbf{g},J)$$

• The description above in terms of the energy-momentum tensor "effective action" is a theory of (classical) dynamical gravity.

• The dynamical metric is (almost) the energy-momentum tensor vev, $h_{\mu\nu}$.

• Other sources like J represent energy-momentum carrying sources.

• This description is diff-invariant by construction. The related theory is a bi-gravity as it involves a dynamical metric $h_{\mu\nu}$ and a fixed fiducial metric, $\mathbf{g}_{\mu\nu}$.

• The interactions mediated by this graviton are essentially summarizing exchanges of the energy-momentum tensor as we had postulated.

• The emergent graviton propagator (by construction) is generated by the poles of the energy-momentum tensor two-point function in the original theory.

We obtain at quadratic order, around flat space, by definition

$$S_{int} = \int \frac{d^4k}{(2\pi)^4}\ h_{\mu\nu}(k)\ \langle T^{\mu\nu} T^{\rho\sigma}\rangle\ h_{\rho\sigma}(-k) \tag{1}$$

where the general form of the TT two-point function in momentum space is

$$\langle T_{\mu\nu} T_{\rho\sigma}\rangle(k) = -\frac{V}{2}\left(\eta_{\mu\nu}\eta_{\rho\sigma} + \eta_{\mu\rho}\eta_{\nu\sigma} + \eta_{\mu\sigma}\eta_{\nu\rho}\right)$$

$$+B_2(k)\ \left[\pi_{\mu\rho}\pi_{\nu\sigma} + \pi_{\mu\sigma}\pi_{\mu\rho} - \frac{2}{3}\pi_{\mu\nu}\pi_{\rho\sigma}\right] + \frac{B_0(k)}{3}\ \pi_{\mu\nu}\pi_{\rho\sigma}$$

$$B_0 = \frac{\pi^2}{40}k^4\ \int_0^\infty d\mu^2 \frac{\rho_0(\mu^2)}{k^2+\mu^2} \quad , \quad B_2 = \frac{3\pi^2}{80}k^4\ \int_0^\infty d\mu^2 \frac{\rho_2(\mu^2)}{k^2+\mu^2}$$

where

$$\langle T_{\mu\nu}\rangle \equiv V\eta_{\mu\nu} \quad , \quad \pi_{\mu\nu} = \eta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\ , \tag{2}$$

• There three types of contact terms in $\langle TT\rangle$. The $O(k^0)$ are fixed by the translational Ward identity.

• There are $O(k^2)$ terms

$$\delta\langle T_{\mu\nu}T_{\rho\sigma}\rangle(k) = \frac{3A_4}{4}\ k^2\ \left[\pi_{\mu\rho}\pi_{\nu\sigma} + \pi_{\mu\sigma}\pi_{\mu\rho} - \frac{2}{3}\pi_{\mu\nu}\pi_{\rho\sigma}\right]\delta_2 + \frac{A_4}{6}k^2\ \pi_{\mu\nu}\pi_{\rho\sigma}\ \delta_0$$

For IR regularity:

$$6\delta_2 + \delta_0 = 0$$

• There are $O(k^4)$ terms (scheme dependent)

$$\delta\langle T_{\mu\nu}T_{\rho\sigma}\rangle(k) = \left[\pi_{\mu\rho}\pi_{\nu\sigma} + \pi_{\mu\sigma}\pi_{\mu\rho} - \frac{2}{3}\pi_{\mu\nu}\pi_{\rho\sigma}\right]k^4 A_2 + \frac{B_0(k)}{3}\pi_{\mu\nu}\pi_{\rho\sigma}k^4 A_0$$

• Ignoring the contact terms, the interaction mediated by $T_{\mu\nu}$ is given at the quadratic level by

$$W_2^{nl} = \frac{1}{2}\int \frac{d^4p}{(2\pi)^4}\left[2B_2^{nl}(k)\left(h^{\mu\nu}(k)h_{\mu\nu}(-k) - \frac{1}{3}h(k)h(-k)\right) + \frac{B_0^{nl}(k)}{3}h(k)h(-k)\right]$$

The tensor structure is that of a massive spin-2 exchange. For the non-contact contributions at small k

$$B_2(k) = \mathcal{O}(k^6) \quad , \quad B_0 \simeq c_{IR}k^4 \log\frac{k^2}{M^2} + \mathcal{O}(k^6)$$

• The interaction depends crucially on the structure of $B^{nl}_{2,0}$. If there is a mass gap and discrete states then near a pole we can approximate

$$B_{2,0} \simeq \frac{R_{2,0}}{k^2 + m^2_{2,0}}$$

where the residue $R_{2,0}$ has mass dimension six as B has mass dimension four.

• The resulting interaction involves a massive spin-2 particle of mass m_2 and a massive spin-0 particle with mass m_0.

• In a unitary theory all residues are positive and the exchanges are never ghostlike.

• By an appropriate rescaling of the interacting densities, we find the associated "Planck scales" to be given by

$$M^2_{0,2} \sim \frac{V^2}{R_{2,0}}$$

• The associated field theory is a bi-gravity theory.

An explicit IR parametrization

• We assume that the theory has a uniform mass gap for simplicity.

• We will now parametrize the Schwinger functional W in an IR expansion below the mass gap as

$$W(g,J)=\int\sqrt{g}\left[-V(J)+M^2(J)R(g)-\frac{Z(J)}{2}(\partial J)^2+\mathcal{O}(\partial^4)\right]$$

• We calculate

$$h_{\mu\nu}=\frac{V}{2}g_{\mu\nu}+M^2G_{\mu\nu}-(\nabla_\mu\nabla_\nu-g_{\mu\nu}\Box)M^2-\frac{1}{2}T_{\mu\nu}+\cdots$$

$$G_{\mu\nu}=R_{\mu\nu}-\frac{1}{2}R\ g_{\mu\nu}\quad ,\quad T_{\mu\nu}=Z(J)\left(\partial_\mu J\partial_\nu J-\frac{1}{2}g_{\mu\nu}(\partial J)^2\right)$$

• The $h_{\mu\nu}$ appears uniquely determined, but there is a initial+boundary condition dependence in this formula.

• Note that for arbitrary external source J, this energy-momentum tensor vev is not conserved.

$$\nabla_g{}^\mu h_{\mu\nu}=\frac{1}{2}\left[V(J)'-Z(J)\Box_g J-\frac{1}{2}Z'(J)(\partial J)^2-(M(J)^2)'\ R\right]\partial_\nu J$$

• We may now solve $g_{\mu\nu}$ as a function of $h_{\mu\nu}$:

$$g_{\mu\nu} = \tilde{h}_{\mu\nu} - \delta\tilde{h}_{\mu\nu} \quad , \quad \tilde{h}_{\mu\nu} = \frac{2}{V} h_{\mu\nu}$$

$$\delta\tilde{h}_{\mu\nu} = \frac{2}{V}\left[M^2 \tilde{G}_{\mu\nu} - (\tilde{\nabla}_\mu \tilde{\nabla}_\nu - \tilde{h}_{\mu\nu}\tilde{\Box})M^2\right] - \frac{1}{V}\tilde{T}_{\mu\nu} + \cdots$$

• All the tensors above are written in terms of $\tilde{h}_{\mu\nu}$.

• $\tilde{h}_{\mu\nu}$ is dimensionless and plays the role of the emergent dynamical metric.

• We may rewrite it as an Einstein equation coupled to "matter"

$$M^2 \; \tilde{G}_{\mu\nu} = \frac{V(J)}{2}\left(\tilde{h}_{\mu\nu} - \mathbf{g}_{\mu\nu}\right) + \frac{1}{2}\tilde{T}_{\mu\nu}(J) + (\tilde{\nabla}_\mu \tilde{\nabla}_\nu - \tilde{h}_{\mu\nu}\tilde{\Box})M(J)^2 + \cdots$$

• The effective gravitational equation above is equivalent to $\frac{\delta\Gamma}{\delta h_{\mu\nu}} = 0$.

• The background metric $\mathbf{g}_{\mu\nu}$ appears as an external source and contributes like a cosmological constant.

• This is an "unusual" bigravity theory.

• Other sources act as sources of energy and momentum.

• This description is non-singular only if $V \neq 0$.

• If $V = 0$, then the gravitational theory is non-local but can be constructed.

• Note that when $J(x) \neq 0$ the original QFT is not translationally invariant and its energy-momentum tensor is not conserved.

• The emergent gravity theory is however still diff. invariant, and the diff. invariance is broken "spontaneously" because of the presence of the scalar source $J(x)$ and the fixed (fiducial) metric of the original QFT.

BACK

The linearized coupling

• We therefore consider a coupling between the "hidden theory" and the "visible theory" of the form

$$S_{int} = \int d^4x \left(\lambda \mathsf{T}_{\mu\nu}(x)\,\widehat{\mathsf{T}}^{\mu\nu}(x) + \lambda' \mathsf{T}(x)\widehat{\mathsf{T}}(x)\right)$$

at a high scale M. This is an irrelevant coupling with $\lambda \sim M^{-4}$.

• $\mathsf{T}_{\mu\nu}$ is the SM energy-momentum tensor, $\widehat{\mathsf{T}}_{\mu\nu}$ is the hidden one.

• We also define

$$\mathfrak{c} \equiv \frac{\lambda'}{\lambda} \quad , \quad \mathbf{T}_{\mu\nu} \equiv \mathsf{T}_{\mu\nu} + \mathfrak{c}\,\mathsf{T}\,\eta_{\mu\nu}$$

so that

$$S_{int} = \lambda \int d^4x\, \mathbf{T}_{\mu\nu}(x)\,\widehat{\mathsf{T}}^{\mu\nu}(x)$$

• Note that the expectation value of the hidden energy momentum tensor, acts as an external metric for the SM.

$$\lambda \int d^4x \;\; \mathbf{T}_{\mu\nu}(x)\,\widehat{\mathsf{T}}^{\mu\nu}(x) \quad \to \quad \int d^4x \;\; \mathbf{T}_{\mu\nu}(x) h^{\mu\nu}$$

• The coupling has introduced the following effective interactions in the visible theory:

$$\delta S_{vis} = \lambda\hat{\Lambda}\int d^4x\,\mathbf{T}(x) - \frac{1}{2}\lambda^2\int d^4x_1 d^4x_2\,\mathbf{T}_{\mu\nu}(x_1)\,\mathbf{T}_{\rho\sigma}(x_2)\,\hat{\mathsf{G}}^{\mu\nu,\rho\sigma}(x_1-x_2)$$

• The second term can be written in momentum space as

$$\delta S^{TT}_{vis} \equiv -\frac{1}{2}\frac{\lambda^2}{(2\pi)^4}\int d^4k\,\mathbf{T}_{\mu\nu}(-k)\,\mathbf{T}_{\rho\sigma}(k)\,\hat{\mathsf{G}}^{\mu\nu,\rho\sigma}(k)$$

and is an induced quadratic energy-momentum interaction in the visible theory.

• This interaction can be reformulated in terms of a classical spin-2 field $h_{\mu\nu}$

$$\delta S^{TT}_{eff} = \int d^4k\left[-h_{\mu\nu}(-k)\mathbf{T}^{\mu\nu}(k) + \frac{(2\pi)^4}{2\lambda^2}h_{\mu\nu}(-k)\quad \mathcal{P}^{\mu\nu,\rho\sigma}(k)\quad h_{\rho\sigma}(k)\right]$$

• The inverse propagator $\mathcal{P}^{\mu\nu,\rho\sigma}(k)$ of the emerging spin-2 field is the inverse of the hidden sector 2-point function $\hat{\mathsf{G}}^{\mu\nu,\rho\sigma}(k)$.

• It remains to examine under what circumstances $\mathcal{P}^{\mu\nu,\rho\sigma}(k)$ is well-defined and what tensor structures it involves.

• We assume that the hidden theory is a Lorentz-invariant QFT.

$$\widehat{G}^{\mu\nu,\rho\sigma}(k) = \widehat{\Lambda}\left(\eta^{\mu\nu}\eta^{\rho\sigma}+\eta^{\mu\rho}\eta^{\nu\sigma}+\eta^{\mu\sigma}\eta^{\rho\nu}\right)+\widehat{b}(k^2)\Pi^{\mu\nu\rho\sigma}(k)+\widehat{c}(k^2)\pi^{\mu\nu}(k)\pi^{\rho\sigma}(k)$$

with

$$\pi^{\mu\nu} = \eta^{\mu\nu} - \frac{k^\mu k^\nu}{k^2} \quad , \quad \Pi^{\mu\nu,\rho\sigma}(k) = \pi^{\mu\rho}(k)\pi^{\nu\sigma}(k) + \pi^{\mu\sigma}(k)\pi^{\nu\rho}(k)$$

• The only combination of tensor structures which is analytic at quadratic order in momentum, in the long-wavelength limit $k^2 \to 0$, is the one that has

$$\widehat{b}(k^2) = \widehat{b}_0\, k^2 + \mathcal{O}(k^4)\ , \quad \widehat{c}(k^2) = -2\widehat{b}_0\, k^2 + \mathcal{O}(k^4)$$

• If $\widehat{\Lambda} = 0$, the two-point function has zero modes which are proportional to k^μ and is therefore not invertible.

• In this case, one must invert in the space orthogonal to the zero modes. This gives rise to a non-local effective theory for the graviton.

- Up to quadratic order in the momentum expansion

$$\mathcal{P}^{\mu\nu\rho\sigma}(k) = -\frac{1}{4\widehat{\Lambda}}(\eta^{\mu\nu}\eta^{\rho\sigma} - \eta^{\mu\rho}\eta^{\nu\sigma} - \eta^{\mu\sigma}\eta^{\nu\rho})$$

$$+2\widehat{b}_0\widehat{\Lambda}^{-2}\Bigg[\frac{k^2}{8}(\eta^{\mu\nu}\eta^{\rho\sigma} - \eta^{\mu\rho}\eta^{\nu\sigma} - \eta^{\mu\sigma}\eta^{\nu\rho})$$

$$+\frac{1}{8}(\eta^{\nu\sigma}k^\mu k^\rho + \eta^{\nu\rho}k^\mu k^\sigma + \eta^{\mu\sigma}k^\nu k^\rho + \eta^{\mu\rho}k^\nu k^\sigma)\Bigg] + \mathcal{O}(k^4)$$

Emergent quadratic gravity

- We now re-define:

$$h_{\mu\nu} = -\mathfrak{h}_{\mu\nu} + \frac{1}{2}\mathfrak{h}\,\eta_{\mu\nu} + \lambda\hat{\Lambda}\,\eta_{\mu\nu}\;, \quad \mathfrak{h} = \mathfrak{h}^{\rho\sigma}\eta_{\rho\sigma}$$

$$\mathfrak{T}^{\mu\nu} \equiv \mathbf{T}^{\mu\nu} - \frac{1}{\lambda}\left(1 + \frac{1}{2\lambda\,\hat{\Lambda}}\right)\eta^{\mu\nu}\;, \quad \mathfrak{T} = \mathfrak{T}^{\mu\nu}\eta_{\mu\nu}$$

- The full effective action of the visible QFT at this order in the λ-expansion and at the two-derivative level is

$$S_{eff} = S_{vis} + \int d^4x\,\left(\mathfrak{h}_{\mu\nu}\mathfrak{T}^{\mu\nu} - \frac{1}{2}\mathfrak{h}\,\mathfrak{T}\right) + \frac{1}{16\pi G}\int d^4x\left[\sqrt{g}\,(R + \Lambda)\right]^{(2)}_{g_{\mu\nu}=\eta_{\mu\nu}+\mathfrak{h}_{\mu\nu}}$$

with the identification of parameters

$$\Lambda = \frac{\hat{\Lambda}}{\hat{b}_0}\;, \quad \frac{1}{16\pi G} \equiv M_P^2 = -\frac{(2\pi)^8\,\hat{b}_0}{\lambda^2\hat{\Lambda}^2}$$

- The sign of Newton's constant is positive when $\hat{b}_0$ is negative

• This seems to be the case with simple QFTs but we have no general proof.

• The second term, which describes the coupling of the visible QFT to the emergent graviton, can be expressed in terms of the original energy-momentum tensor of the visible QFT

$$\int d^4x \left(\mathfrak{h}_{\mu\nu}\mathfrak{T}^{\mu\nu} - \frac{1}{2}\mathfrak{h}\,\mathfrak{T}\right) = \int d^4x\sqrt{g}\; g^{\mu\nu}\;\mathfrak{T}_{\mu\nu}\Big|_{g_{\mu\nu}=\eta_{\mu\nu}+\mathfrak{h}_{\mu\nu}}$$

• There is a non-trivial shift of the energy due to the coupling of the two theories.

• Because of the presence of "dark energy" the flat (fiducial) metric is always a solution to the equations of emergent gravity.

We can compute the (non-contact part of the) induced interaction between SM sources without expanding in momenta

$$L_{int} = -\frac{\lambda^2}{2}\left[2B_2(k)\left(T_{\mu\nu}(-k)T^{\mu\nu}(k) - \frac{1}{3}T(-k)T(k)\right) + \frac{(1+3\mathfrak{c})^2}{3}B_0(k)\right] + \cdots$$

where $\mathfrak{c}$ is defined by

$$S_{int} = \lambda\int d^4x\left[\hat{T}_{\mu\nu}T^{\mu\nu} + \mathfrak{c}\ \hat{T}T\right]$$

- The tensor structure is that of massive gravity.

- At the special (integrable) value $\mathfrak{c} = -\frac{1}{3}$ the scalar dilaton decouples. *Taylor*

- The interaction is always attractive and stable.

- Around a massive pole, of mass m_2 we have

$$M_P^2 \sim \frac{M^8}{R_2} \sim M^2\ \left(\frac{M}{m_2}\right)^6$$

- A generalization of the formalism of the effective action allows us to (formally) construct the full non-linear theory.

Emergent quadratic gravity:Comments

• A coupling of stress tensors between two theories induces gravity at the quadratic level.

• This is true in the generic case: $\widehat{\Lambda} \neq 0$.

• Otherwise the graviton theory is non-local.

• There is always an effective cosmological constant for the emerging gravity in the local case.

• There is also a shift of the stress tensor giving a “dark” energy. It is a reflection of the coupling to the hidden theory.

• We parametrize $\lambda = \frac{1}{NM^4}$ where M a large scale controlling the coupling of the two theories and N the number of colors of the hidden theory.

• Also from calculations

$$\hat{b}_0 = -\kappa N^2 m^2 \quad , \quad \kappa \sim O(1) \quad , \quad \hat{\Lambda} = \epsilon \; N^2 \; m^4 \quad , \quad \epsilon = \pm 1 \qquad (3)$$

We may now calculate the relevant ratios of scales

$$\frac{\Lambda}{M_P^2} = -\frac{\epsilon}{\kappa^2 x^2} \quad , \quad \frac{\Lambda_{dark}}{M_P^2} = -\frac{\frac{N}{x} + \frac{\epsilon}{2(2\pi)^4}}{(1+4\mathfrak{c})\kappa^2 \; x^2} \quad , \quad \frac{m}{M_P} = \frac{1}{\sqrt{\kappa} \; x} \qquad (4)$$

$$\frac{\Lambda_{dark}}{\Lambda} = \frac{\epsilon\frac{N}{x} + \frac{1}{2(2\pi)^4}}{(1+4\mathfrak{c})} \quad , \quad \frac{M^4}{M_P^4} = \frac{1}{\kappa^2 x^3} \quad , \quad x \equiv \frac{M^4}{m^4} \gg 1 \qquad (5)$$

• We always have semiclassical gravity, $\Lambda \ll M_P^2$.

• If $N \lesssim x$ then

$$\Lambda \sim \Lambda_{\rm dark} \sim \mathcal{O}(m^2) \ll M^2 \ll M_P^2$$

• If $x \ll N \ll x^{\frac{3}{2}}$ then

$$\Lambda \ll \Lambda_{\rm dark} \ll M^2 \ll M_P^2$$

• If $x^{\frac{3}{2}} \ll N \ll x^3$ then

$$\Lambda \ll M^2 \ll \Lambda_{\rm dark} \ll M_P^2$$

• If $N \gg x^3$ then

$$\Lambda \ll M^2 \ll M_P^2 \ll \Lambda_{\rm dark}$$

• For phenomenological purposes $x \lesssim 10^{20}$ so that the messenger scale is above experimental thresholds.

• Note that so far the SM quantum effects are not included.

The non-linear analysis

- We start again from the Schwinger functional of the coupled QFTs

$$e^{-W(\mathcal{J},\hat{\mathcal{J}},\mathbf{g})} = \int [D\Phi]\,[D\widehat{\Phi}]\,e^{-S_{visible}(\Phi,\mathcal{J},\mathbf{g})-S_{hidden}\left(\widehat{\Phi},\mathbf{g},\hat{\mathcal{J}}\right)-S_{int}\left(\mathcal{O}^i,\widehat{\mathcal{O}}^i,\mathbf{g}\right)}$$

- Φ^i and $\widehat{\Phi}^i$ are respectively the (quantum) fields of the visible QFT and the hidden $\widehat{\text{QFT}}$.

- $\mathcal{J}$ and $\hat{\mathcal{J}}$ are (scalar) sources in the visible and hidden theories respectively.

- For energies $E \ll M$, we can integrate out the hidden theory and obtain

$$e^{-W(\mathcal{J},\hat{\mathcal{J}},\mathbf{g})} = \int [D\Phi][D\widehat{\Phi}]\,e^{-S_{visible}(\Phi,\mathcal{J},\mathbf{g})-S_{hidden}\left(\widehat{\Phi},\hat{\mathcal{J}},\mathbf{g}\right)-S_{int}}$$

$$= \int [D\Phi]\,e^{-S_{visible}(\Phi,\mathcal{J},\mathbf{g})-\mathcal{W}(\mathcal{O}^i+\hat{\mathcal{J}}^i,\mathbf{g},)}$$

- The interaction part is defined as:

$$S_{int} = \int d^4x\sqrt{\mathbf{g}}\sum_i \lambda_i\,\mathcal{O}_i(x)\,\widehat{\mathcal{O}}_i(x)$$

• We now put the full theory on a curved manifold with metric $g_{\mu\nu}$ and define again the generating functional in the presence of the background metric as

$$e^{-W(\mathcal{J},g,\hat{\mathcal{J}})} = \int [D\Phi]\, e^{-S_{visible}(\Phi,\mathcal{J},g) - \mathcal{W}(\mathcal{O}^i+\hat{\mathcal{J}}^i,g)}$$

• We define

$$h_{\mu\nu} \equiv \frac{1}{\sqrt{g}} \frac{\delta \mathcal{W}\left(\mathcal{O}^i, g, \hat{\mathcal{J}}\right)}{\delta g^{\mu\nu}}\Big|_{g_{\mu\nu}=\mathbf{g}_{\mu\nu}} = \langle \widehat{\mathsf{T}}_{\mu\nu} \rangle$$

• This will eventually play the role of an emergent metric for the visible theory.

• The diffeomorphism invariance of the functional $W(\mathcal{J}, g, \hat{\mathcal{J}})$ is reflecting (as usual) the translational invariance of the underlying QFT.

• We define the Legendre-transformed functional

$$S_{eff}(h,\Phi,\mathcal{J},\hat{\mathcal{J}},\mathbf{g}) = S_{vis}(\mathbf{g},\Phi,\mathcal{J}) - \int d^4x\,\sqrt{g(\mathcal{O}^i+\hat{\mathcal{J}}^i,h)}\,h_{\mu\nu}\times$$

$$\times\left[g^{\mu\nu}(\mathcal{O}^i+\hat{\mathcal{J}}^i,h)-\mathbf{g}^{\mu\nu}\right]+\mathcal{W}\left(\mathcal{O}^i+\hat{\mathcal{J}}^i,g(\mathcal{O}^i+\hat{\mathcal{J}}^i,h)\right)$$

We can show that:

♠ This functional satisfies

$$\left.\frac{\delta S_{eff}}{\delta h_{\mu\nu}}\right|_{g_{\mu\nu}=\mathbf{g}_{\mu\nu}} = 0$$

♠ These are the emerging non-linear gravitational equations.

♠ When evaluated in the solution of the above equation gives the original action.

$$,\quad \left.S_{eff}\right|_{g_{\mu\nu}=\mathbf{g}_{\mu\nu}} = S_{visible}+\mathcal{W}\left(\mathcal{O}^i+\hat{\mathcal{J}}^i,\mathbf{g}\right)$$

• Therefore, $S_{eff}(h,\Phi,\mathcal{J},\hat{\mathcal{J}},\mathbf{g})$ is the emergent gravity action that generalizes the linearized computation.

Detailed plan of the presentation

- Title page 0 minutes
- Prelude 1 minutes
- String Theory: the original hype 3 minutes
- String Theory: doubts on uniqueness 5 minutes
- String Theory: the second string revolution 6 minutes
- String Theory: the king is naked 8 minutes
- String Theory: The landscape 9 minutes
- Searching the String Landscape 11 minutes
- AdS/CFT 13 minutes
- Lessons from AdS/CFT 14 minutes
- String Theory before and after AdS/CFT 16 minutes
- How quantized (composite) gravity might work 17 minutes
- The energy-momentum tensor 19 minutes
- Gravitons from hidden sectors 22 minutes
- Comments and open ends 24 minutes