Amsterdam Nikhef May 13 2022

String loop corrections and de Sitter vacua

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GREECE

based on works with:
I. Antoniadis, Y. Chen 1803.08941, 1909.10525
I. Antoniadis, O. Lacombe 2007.10362
Pramod Shukla 2203.03362

On the occasion of the retirement of

Prof. Bert Schellekens

Reminiscences

▲ *Late 1980's at CERN:* acquaintance with Bert's work

▲ c. 2005 influenced by his work with Elias, Dijkstra and Pascal

▲ c.2010 Collaboration with Bert, Pascal and Richter

SU(5) D-brane realizations, and proton stability-2010

▲ 2018– Ioannina-Nikhef

I had the chance to know him better:

a friendly person, a nice character and a brilliant physicist.

Why dS vacua?

A few facts about Cosmology

▲ *Major Observational Discovery* ~ 2 *decades ago:*

Accelarating Expansion of the Universe

⇒ *explained with* **Dark Energy** *permeating all of space*

▲ In General Relativity Equations ▲

⇒ *expressed in equations with a positive cosmological constant:*

$\Lambda \approx 10^{-120}$ (*in 4-d* M_P^4 *Planck units*)

or equivalently:

Positive Vacuum Energy

▲ Simple Effective Field Theory description▲

Potential Energy $V(\phi)$ of a scalar field, ϕ

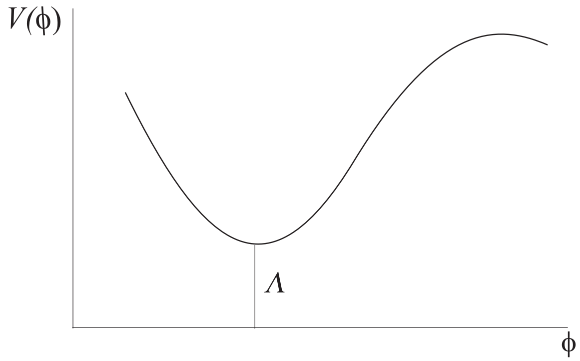
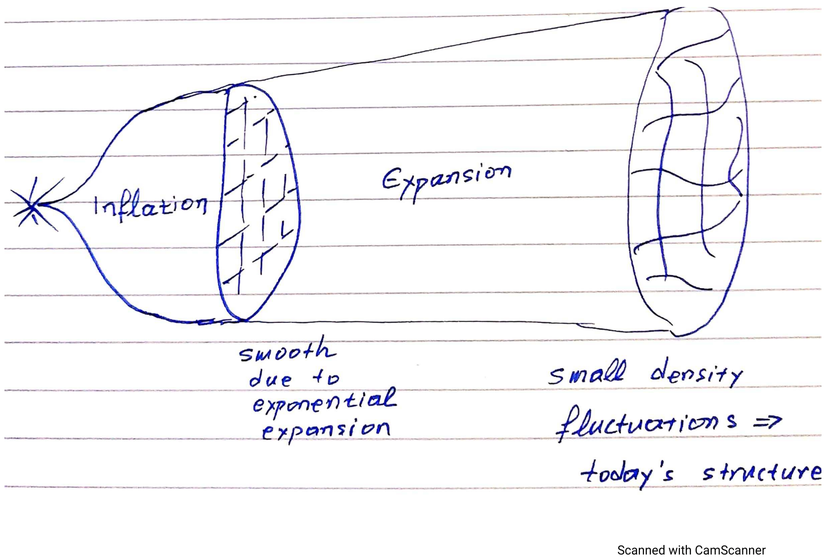


Figure 1: Potential with local minimum and positive Λ

▲ de Sitter vacua ▲

With a few additional requirements:

$V(\phi)$ appropriate for cosmological inflation



▲ **Challenges :**

Finding dS and embedding inflationary scenario in String Theory

but

▲ ∃ related issues in String derived EFT ▲

Compactifications characterised by **large numbers** of moduli:

... *in general*

▲ Deformations of **Compactifications** correspond to **massless scalar fields** in four dimensions

$$\Downarrow$$

problems with fifth forces and other cosmological issues...

▲ Tasks ▲

▲ CY compactification inducing an EFT with $V_{\min} > 0$ and assure positive masses-squared for all moduli fields ⇒

$$\Rightarrow \mathcal{M}oduli\ \mathcal{S}tabilisation \Leftarrow$$

▲ Look for possible **Inflaton** candidates among *moduli*

▲ at **String Theory** level: ▲

▲ # CY of **Compactifications** and # fluxes ⇒
enormous number of String Vacua

⇓

String Landscape

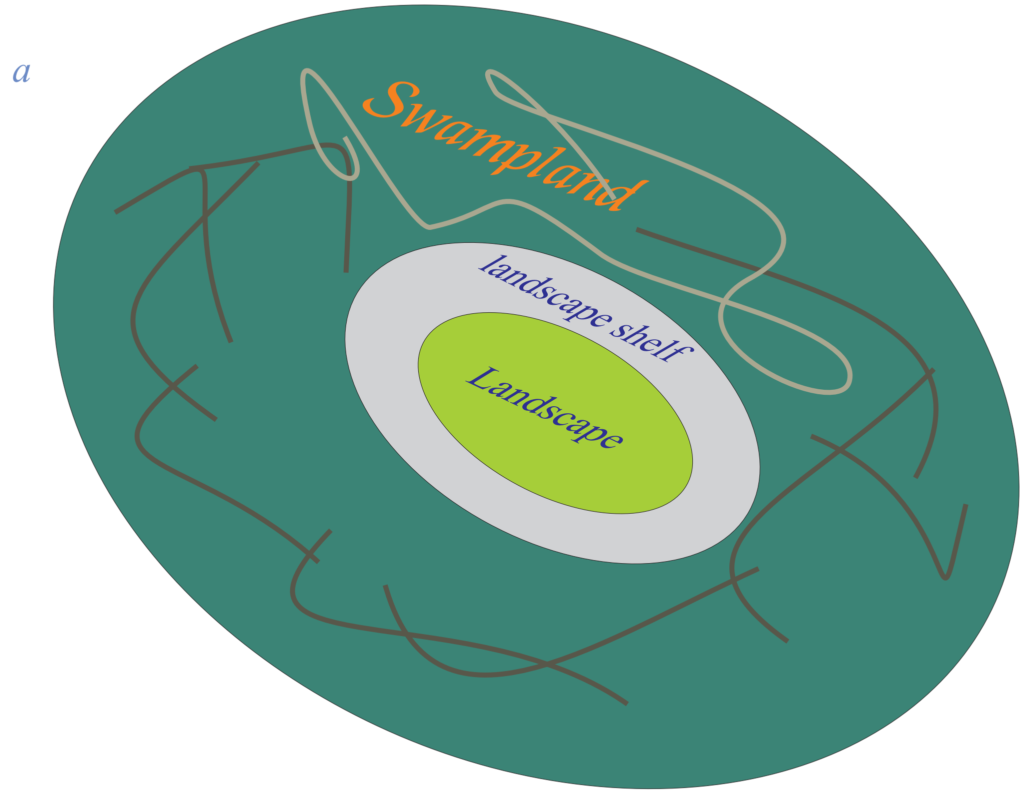
▲ **Long standing Question** ▲

▲ Are there any de Sitter vacua in the **Landscape**?
... If the answer is Yes...they are...

⇒ *Certainly Scarce* ⇐

However, ⇒**...according to recent claims...** ⇒

... all dS vacua fall into the *SWAMPLAND*!



String landscape is surrounded by a vast swampland of inconsistent field theories of dS vacua. Grayzone (inspired by sea-borders) includes proposed 'stringy' dS vacua but not unanimously adopted

$\mathcal{F}$ocus of present $\mathcal{T}$alk

Context: type $\mathcal{IIB}$ theory:

- A solution to the Moduli Stabilisation problem

- Find a *de Sitter* vacuum in String Theory

… based only on **perturbative** corrections [a]

- If yes:

- Examine cosmological implications such as inflation.

[a] We stick to this case because we have perturbative control, as opposed to models with NP-corrections.

Outline of the present Talk

- *Effective Supergravity from type II-B String Theory*
- R^4-terms and localised gravity
- D7 branes and logarithmic corrections
- F-term and D-term potential
- on de Sitter vacua
- Concluding Remarks

★ Type II-B/F-theory

★ **Moduli** Space (*notation*)

▲ Graviton, dilaton and Kalb-Ramond (KR)-field

$$g_{\mu\nu},\ \phi,\ B_{\mu\nu} \to B_2$$

▲ Scalar, 2- and 4-index fields (*p-form potentials*)

$$\mathbf{C_0}, C_{\mu\nu}, C_{\kappa\lambda\mu\nu} \to C_p,\ p = 0, 2, 4$$

1. ▲ $C_0,\ \phi$ $\to$ *combined to* **axion-dilaton** *modulus:*

$$S = C_0 + i\, e^{-\phi} \equiv C_0 + \frac{i}{g_s}$$

2. z_a : *Complex Structure (CS)* moduli (*shape*)

3. T_i : Kähler moduli (*size*) $\leftrightarrow$ $J = g_{ij} dz^i \wedge dz^j$

Type II-B effective Supergravity

Basic 'ingredients':

Superpotential $\mathcal{W}$ *and Kähler potential* $\mathcal{K}$

▲ The Superpotential $\mathcal{W}$ ▲

▲ *Field strengths:*

$$F_p := d\,C_{p-1},\ H_3 := d\,B_2,\ \Rightarrow \mathbf{G_3} := F_3 - S H_3$$

▲ *Holomorphic* $(3,0)$*-form :* $\Omega(z_a)$

Flux-induced superpotential (*G.V.W. hep-th/9906070*):

$$\mathcal{W}_0 = \int \mathbf{G_3} \wedge \Omega(z_a)$$

▲ $\mathcal{W}$-Flatness conditions:

$$\mathcal{D}_{z_a}\mathcal{W} = 0, \quad \mathcal{D}_S\mathcal{W} = 0\ :$$

$\Rightarrow$ z_a *and* S **stabilised** $\Leftarrow$

but!

▲ Kähler moduli $\notin \mathcal{W}_0 \Rightarrow$ remain unfixed! ▲

▲ The Kähler potential ▲

$$\mathcal{K}_0 = -\sum_{i=1}^{3} \ln(-i(T_i - \bar{T}_i)) - \ln(-i(S - \bar{S})) - \ln(i \int \Omega \wedge \bar{\Omega}) \cdot$$

▲ The scalar potential ▲

$$V = e^{\mathcal{K}} (\sum_{I,J} \mathcal{D}_I \mathcal{W}_0 \mathcal{K}^{I\bar{J}} \mathcal{D}_{\bar{J}} \mathcal{W}_0 - 3|\mathcal{W}_0|^2)$$

$$= e^{\mathcal{K}} \sum_{I,J=z_a,\neq T_i} D_I \mathcal{W}_0 \mathcal{K}^{-1}_{I\bar{J}} D_{\bar{J}} \mathcal{W}_0 \quad (D_I \mathcal{W}_0 = 0, \text{ flatness})$$

$$+ e^{\mathcal{K}} \left(\sum_{I,J=T_i} \mathcal{K}_0^{I\bar{J}} D_I \mathcal{W}_0 D_{\bar{J}} \mathcal{W}_0 - 3|\mathcal{W}_0|^2 \right) \quad (= 0, \text{ no scale})$$

Kähler moduli completely **undetermined**!

Kähler moduli not fixed due to no-scale structure of classical theory

Engineering the appropriate geometric set up and compute:

T_i-dependent **QUANTUM corrections**

to obtain the sought-after dS vacuum!

What kind of corrections are appropriate?

... this question motivates the following analysis

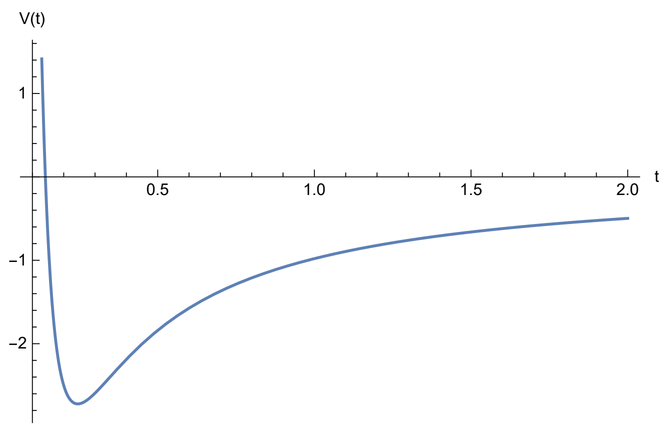
As we 've seen, in type II-B, at the classical level $V_{\rm eff}$ *vanishes.*

Hence, any τ-**dependent perturbative quantum corrections must** *vanish for* $\tau \to \infty$ (τ =*Im*T)

▲ *If* $V(\tau \to \infty)$ *vanishes from below:*

$$\lim_{\tau\to\infty} V_{eff} \to 0^-$$

the expected shape of the potential is of the form

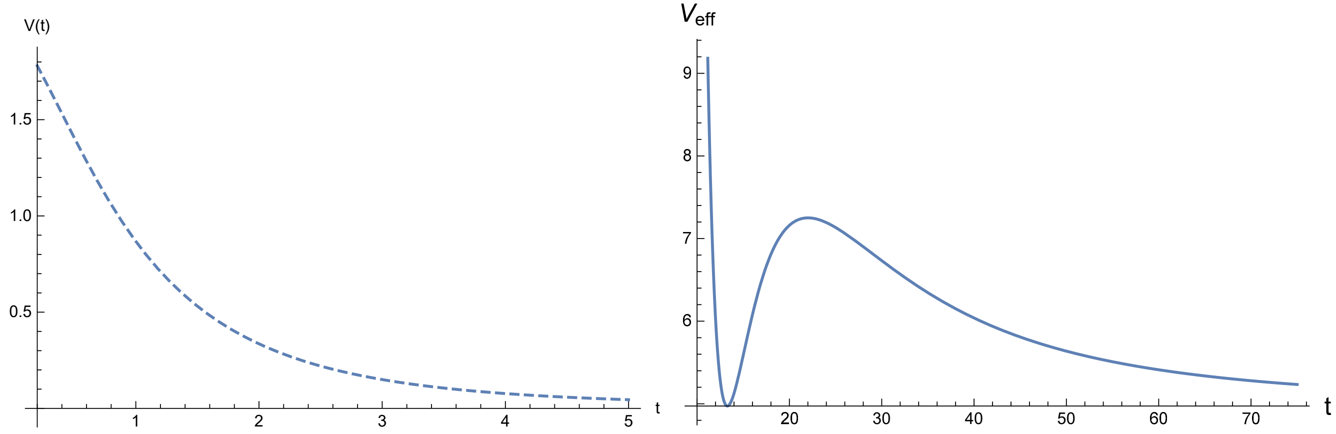


This is AdS minimum! **not acceptable!**

▲ Hence, vanishing at infinity must occur from positive values

$$\lim_{\tau \to \infty} V_{eff} \to 0^{+}$$

Expected shape of the potentials:



The potential on the RHS exhibits local minimum and maximum

This shape suggests that there should be **two** *competing terms*

Let's introduce **Quantum corrections with** $f(\tau)$
generic function **"breaking"** no-scale structure:

$$\mathcal{K} \;=\; -2\log\left(\mathcal{V} + \xi + \eta f(\tau)\right) + \cdots$$

where ξ, η constants and $\mathcal{V} = \tau^{\frac{3}{2}}$.

Some possible $f(\tau)$ functions:

▲▲α) power-law corrections

$$f(\tau) \propto \tau^n \;\Rightarrow$$

$$\boxed{V_F \propto \tau^{n-\frac{9}{2}}}$$

$\Rightarrow V_F$ monotonic function of $\tau \;\Rightarrow\; \nexists\, (V_F)_{min}$

▲▲β) **logarithmic**: $f(\tau) \propto \log \tau$:
(*reminiscent of the Coleman-Weinberg mechanism*)

$$V_F \quad \propto \quad \frac{1}{\mathcal{V}^3} \left(\alpha\xi + \beta + \eta \log(\mathcal{V})\right)$$

$$\Rightarrow\ \exists\ (V_F)_{min} \quad \leftrightarrow \quad \forall\, \eta < 0\, ;\ \xi > 0 \Leftarrow \qquad (1)$$

At $(V_F)_{min}$ *volume size controlled by parameter* ξ*:*

$$\mathcal{V}_{min} = e^{\frac{13}{3} - \frac{\xi}{\eta}}$$

$\Rightarrow$ **large volume expansion** for $\xi \gg |\eta|$

The Kähler potential $\mathcal{K}$

Origin of ξ and $f(\tau)$

from

$\mathcal{PERTURBATIVE}$

String Loop Corrections

▲ α' *corrections* ▲

▲ α'^3 Corrections ▲

Imply redefinition of 4-d dilaton (*Becker et al, hep-th:0204254*)

$$e^{-2\phi_4} = e^{-2\phi_{10}}(\mathcal{V}+\xi)$$
$$= e^{-\frac{1}{2}\phi_{10}}(\hat{\mathcal{V}}+\hat{\xi}) \quad \text{(Einstein frame)}$$

$\mathcal{V}(t^k)$ *6d-volume*, with t^k =ImT^k:

$$\mathcal{V} = \frac{1}{3!}\kappa_{ijk}t^i t^j t^k$$
$$t^k = -\text{Im}(T^k) \equiv \hat{t}^k\, g_s^{1/2}$$
$$\xi = -\frac{\zeta(3)}{4(2\pi)^3}\chi \equiv \hat{\xi}\, g_s^{3/2}$$

Hence, $\hat{\xi}$ incorporated into Kähler potential through the **shift** $(\hat{\mathcal{V}}+\hat{\xi})$.

▲▲

String Coupling Loop Corrections and D7-branes

Two ingredients needed for log-corrections

▲▲

$\mathcal{A}$) Intersecting **D7-brane configuration**:

D7s	Minkowski				compact dimensions					
	0	1	2	3	4	5	6	7	8	9
$D7_a$		*	*	*	*	*	*	*		
$D7_b$		*	*	*	*	*			*	*
$D7_c$		*	*	*			*	*	*	*

$\mathcal{B}$) **Higher derivative couplings in curvature**

(generated by multigraviton scattering)

(see hep-th/9704145; 9707013; 9707018)

Leading correction term in type II-B action:

proportional to the fourth power of curvature:

$$\boxed{\propto R^4}$$

After reduction on $\mathcal{M}_4 \times \mathcal{X}_6$, (with $\mathcal{M}_4$ 4-d Minkowski) R^4 induces a **novel** Einstein-Hilbert term $\mathcal{R}_{(4)} \propto$ by the Euler characteristic χ:

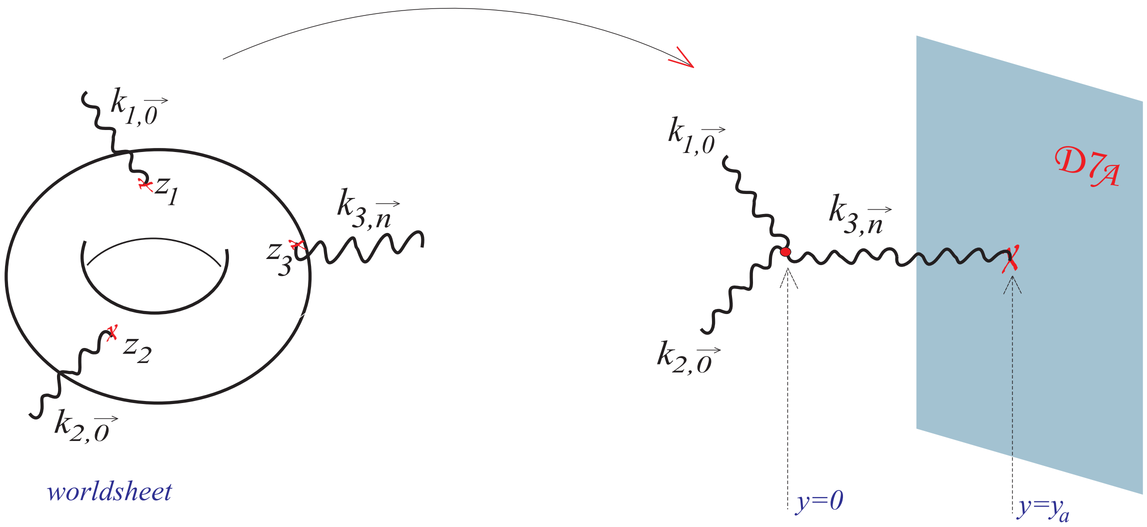
$$\propto \quad \underbrace{\chi \int_{M_4} (\zeta(2) - \zeta(3)e^{-2\phi})\mathcal{R}_{(4)}}_{induced\ \mathcal{EH}\ term},$$

▲▲ *this $\mathcal{EH}$ term possible in 4-dimensions only*!

▲▲ New $\mathcal{EH}$-term localised at points with $\chi \neq 0$▲▲

Localised vertices can emit gravitons and KK-excitations in 6d

$\Rightarrow$ KK-exchange between graviton vertices and $D7$-branes



Corrections

(*assuming 3 intersecting D7 branes*)

$$\propto \zeta(2)\chi \int_{M_4} (1 + \sum_{i=1,2,3} e^{2\phi} T_i \log(R^i_\perp) \mathcal{R}_{(4)} \ ,$$

- ▲ T_i : D7-brane tension
- ▲ $R^i_\perp$: D7-transverse 2-dimension

Extracting the coefficients of the Kähler potential

$$\eta = -\frac{1}{2} g_s T_0 \xi \ \ ; \ \ \xi = -\frac{\chi}{4} \times \begin{cases} \frac{\pi^2}{3} g_s^2 & \text{for orbifolds} \\ \zeta(3) & \text{for smooth CY} \end{cases} \tag{2}$$

Kähler moduli

STABILISATION

A concrete Global Model:

(*GKL & Pramod Shukla* **2203.03362**)

Explicit CY_3 Manifold

from Kreuzer-Skarke dataset with

$h^{1,1} = 3,\ h^{2,1} = 115,\ \chi = -224$

6d-volume given by:

$$\mathcal{V} = 2\, t^1 t^2 t^3 = \frac{1}{\sqrt{2}}\sqrt{\tau_1 \tau_2 \tau_3}$$

($t^i \rightarrow$ 2-cycle, $\tau_i \rightarrow$ 4-cycle moduli, subject to $\tau_i = 2\, t^j t^k$)

▲ Kähler potential including α' and loop corrections:

$$\mathcal{K} = -\log\{-i(S-\bar{S})\} - 2\ln\mathcal{U} + K_{cs} \tag{3}$$

with

$$\mathcal{U} = \mathcal{V} + \frac{\hat{\xi}}{2} + \mathcal{U}_1 \tag{4}$$

Master formula for F-term potential ($generic\ \mathcal{U}_1$)

⇓

$$V_{\alpha'+\log} = e^{\mathcal{K}}\left(\frac{3\mathcal{V}}{2\mathcal{U}^2}\left(1+\frac{\partial\mathcal{U}_1}{\partial\mathcal{V}}\right)^2\frac{4\mathcal{V}^2+\mathcal{V}\hat{\xi}+4\hat{\xi}^2}{\mathcal{V}-\hat{\xi}} - 3\right)|W|^2$$

For α' and log corrections $\mathcal{U}_1$ is:

$$\mathcal{U}_1 = -\hat{\eta} + \hat{\eta}\log\mathcal{V} \tag{5}$$

Large Volume Limit

$$V_F \approx C\frac{\hat{\xi} - 4\hat{\eta} + 2\hat{\eta}\log(\mathcal{V})}{\mathcal{V}^3}$$

Properties

- Minimum exists for $\hat{\eta} < 0$.
- Stabilisation at **large** volume:

$$\mathcal{V}_{\min} = e^{\frac{7}{3} + \frac{\hat{\xi}}{2|\hat{\eta}|}}$$

- For F-term potential, AdS-minimum

$$(V_F)_{\min} \propto \frac{\eta}{\mathcal{V}^3} < 0$$

▲ Uplift to dS occurs through $\mathcal{D}$**-terms** associated with universal $U(1)$'s of D7-stacks:

$$V_{\mathcal{D}} = \frac{g^2_{D7_i}}{2}\left(Q_i \partial_{T_i} K + \sum_j q_j |\Phi_j|^2\right)^2 , \quad \frac{1}{{g_{D7_i}}^2} = \mathrm{Re} T_i + \cdots$$

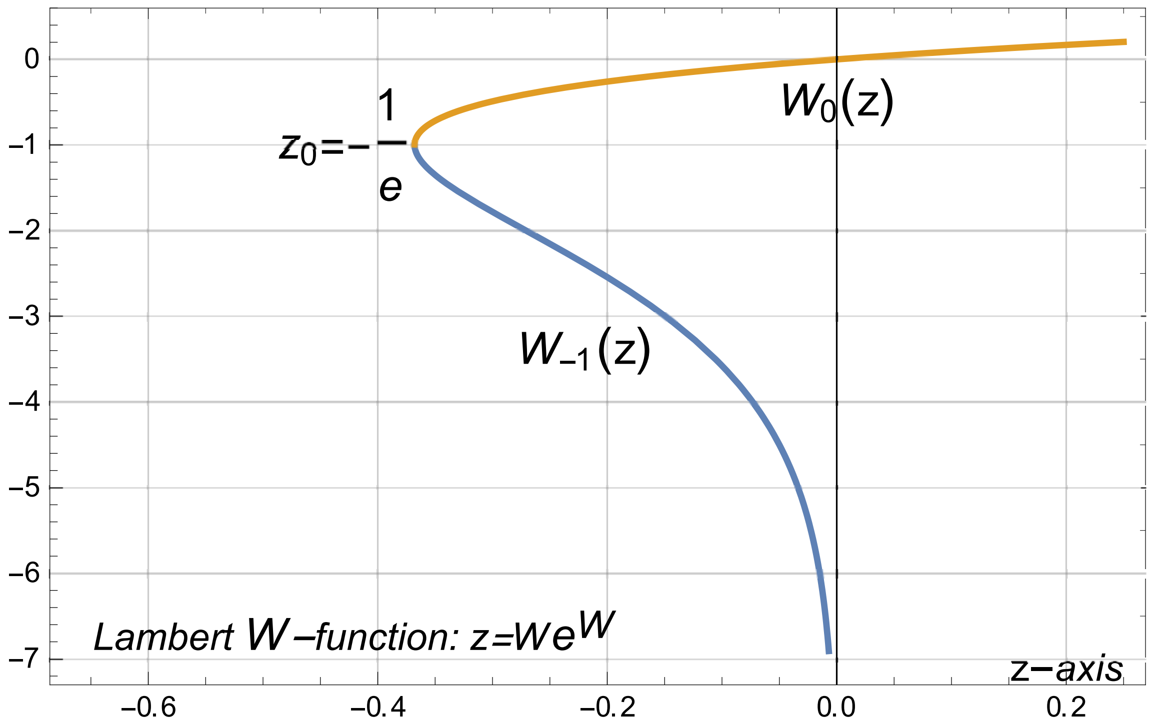
Minimising the total potential:

$$V_{\rm eff} = V_F + V_{\mathcal{D}}$$

$\Rightarrow$ a minimum and a maximum defined by the double-valued **Lambert** W-function:

$$\boxed{\mathcal{V}_{\rm min} = \frac{\hat{n}}{d} \mathbf{W_{0/-1}}\left(\frac{d}{\hat{n}} e^{\frac{7}{3} - \frac{\hat{\xi}}{2\hat{n}}}\right)}$$

The two branches of the Lambert function $W_0(z)$ and $W_{-1}(z)$



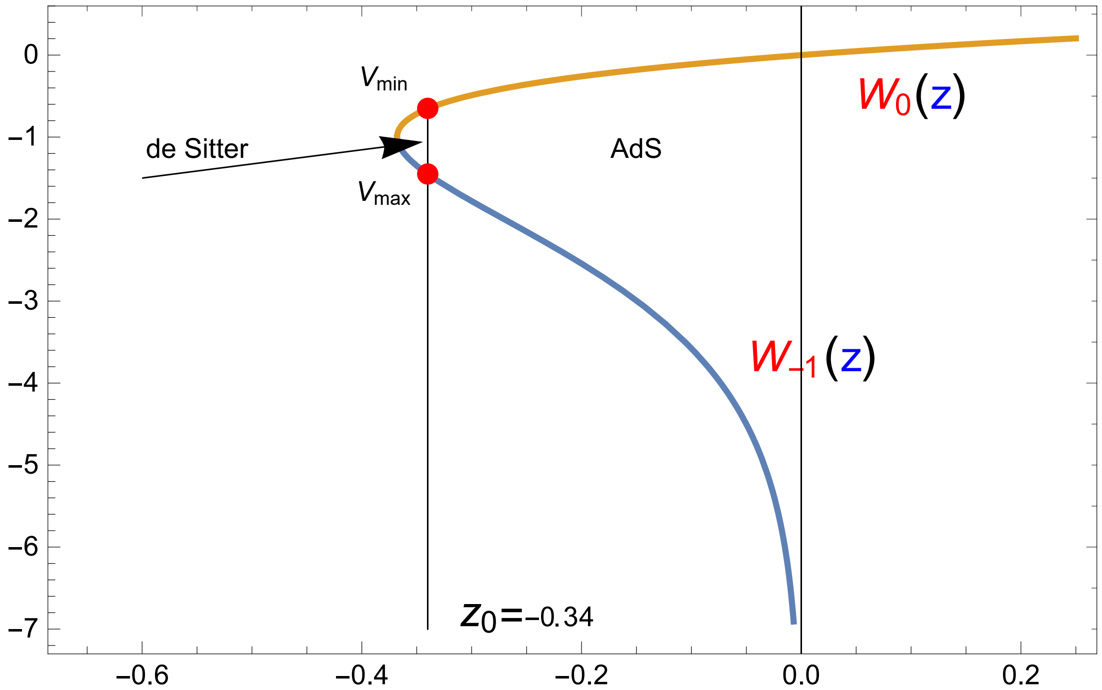
- Real W_0, W_{-1} values $\forall z \geq z_0 = -e^{-1}$.
- Double values for $z \leq 0$.
- We need two extrema (max and min), hence

$$-e^{-1} < z < 0$$

▲ de Sitter vacua ▲

minimum $V_{\rm eff} = V_F + V_{\mathcal{D}}$ *at* $\mathcal{V}_0$ *must be positive:*

$$V_{\rm eff}^{min} = \frac{c}{\mathcal{V}_0^3} + \frac{d}{\mathcal{V}_0^2} > 0]$$

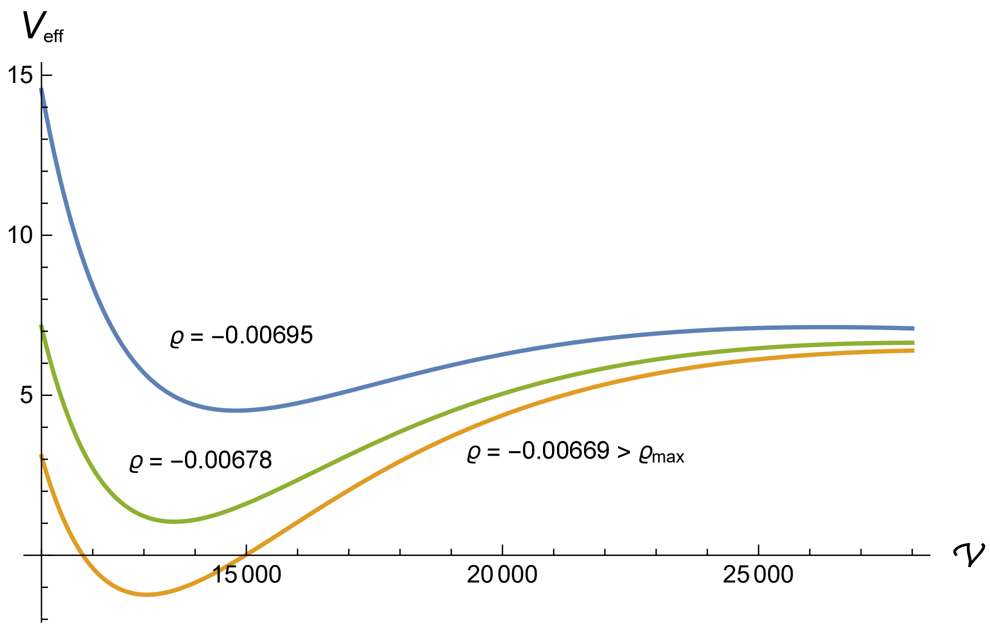


This dramatically constrains acceptable string vacua (fluxes etc)

Plot of $V_{\rm eff}$ vs $\mathcal{V}$ for fixed $\varrho \propto \frac{d}{\hat{\eta}}$.

The lower curve corresponds to AdS vacuum.

At large volume, the potential vanishes asymptotically



★ Conclusions ★

★ *IIB/F-theory:*

• Stabilisation of Kähler Moduli possible with Perturbative Corrections only:

$$\mathcal{K} = -2\ln\left(\mathcal{V} + \hat{\xi}/2 + \hat{\eta}\ln\mathcal{V}\right) + \cdots$$

Origin of log-corrections:

Induced Einstein-Hilbert terms from R^4-couplings in 10-d theory.

This $\mathcal{EH}$-term $\exists$ in $4d$ only!

$\Downarrow$

★ *induced* $\mathcal{EH}$–term ... indispensable element for a:

$4d$ de Sitter Universe

★ **Thank you for your attention** ★

APPENDIX

In String Theory:

multigraviton scattering generates higher derivative couplings in curvature

(Green et al, hep-th/9704145; Antoniadis, et al hep-th/9707013, Kiritsis, et al hep-th/9707018)

Leading correction term in type II-B action:

proportional to the fourth power of curvature:

$$\boxed{\propto R^4}$$

Reduction on $\mathcal{M}_4 \times \mathcal{X}_6$, (with $\mathcal{M}_4$ 4-d Minkowski) induces:

$$\Rightarrow \quad \underbrace{\frac{\alpha}{l_s^8} \int_{M_4 \times \mathcal{X}_6} e^{-2\phi} \mathcal{R}_{(10)}}_{standard\ \mathcal{EH}\ term} + \underbrace{\frac{\beta}{l_s^2} \chi \int_{M_4} (\zeta(2) - \zeta(3) e^{-2\phi}) \mathcal{R}_{(4)}}_{induced\ \mathcal{EH}\ term},$$

Induced Einstein Hilbert ($\mathcal{EH}$) term $\propto$ Euler characteristic:

$$\chi \propto \int R \wedge R \wedge R$$

▲▲ *this $\mathcal{EH}$ term possible in 4-dimensions only*!

▲▲ Introducing 7-branes ▲▲

Localised vertices can emit gravitons and KK-excitations in 6d

⇒ KK-exchange between graviton vertices and $D7$-branes

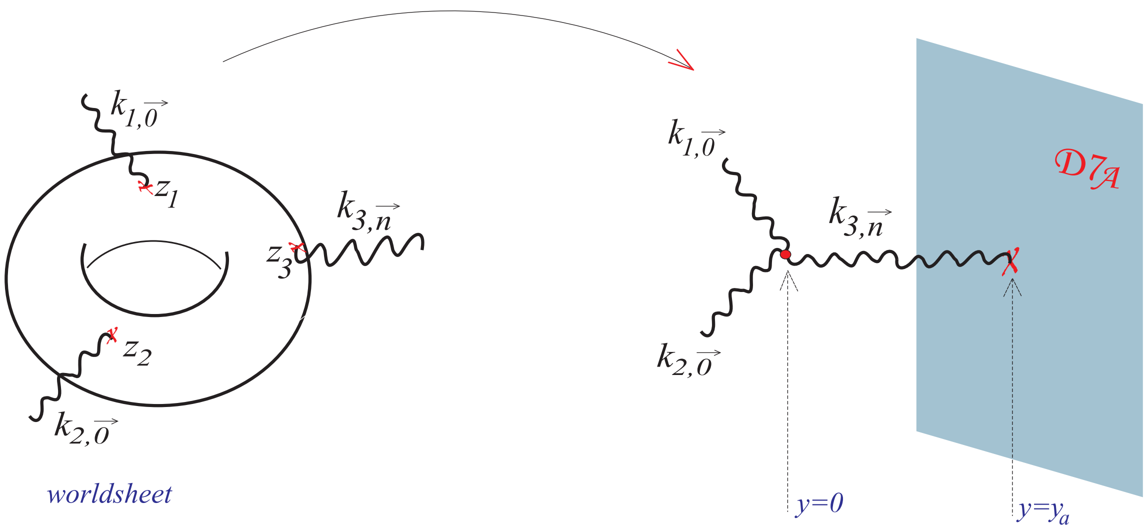


Figure: 3-graviton scattering (2 massless 1 KK) KK-propagating in 2-d towards $D7$

Corrections

(assuming 3 intersecting D7 branes)

$$\frac{1}{(2\pi)^3}\int_{M_4\times\mathcal{X}_6} e^{-2\phi}\mathcal{R}_{(10)} + \frac{4\zeta(2)\chi}{(2\pi)^3}\int_{M_4}(1+\sum_{i=1,2,3} e^{2\phi}T_i \log(R^i_\perp)\mathcal{R}_{(4)} \ ,$$

- T_i : D7-brane tension
- $R^i_\perp$: D7-transverse 2-dimension

Extracting the coefficients of the Kähler potential

$$\eta = -\frac{1}{2}g_s T_0 \xi \ \ ; \ \ \xi = -\frac{\chi}{4}\times \begin{cases} \frac{\pi^2}{3}g_s^2 & \text{for orbifolds} \\ \zeta(3) & \text{for smooth CY} \end{cases} \tag{6}$$

The analysis of the divisor topologies using *cohomCalg* shows that they can be represented by the following Hodge diamonds:

$$K3 \equiv \begin{matrix} & & 1 & & \\ & 0 & & 0 & \\ 1 & & 20 & & 1 \\ & 0 & & 0 & \\ & & 1 & & \end{matrix}\ , \qquad \mathrm{SD} \equiv \begin{matrix} & & 1 & & \\ & 0 & & 0 & \\ 27 & & 184 & & 27 \\ & 0 & & 0 & \\ & & 1 & & \end{matrix}\ .$$