BULK FIELDS IN CFT

( & beyond)

Bert fest

13 May 2022

Context

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- **chiral CFT** :

  vertex operator algebra $\mathcal{V}$ and its representations

- **full local CFT** :

  combine left and right chiral parts

  - **bulk fields** carry a representation F of $\mathcal{V}_{\mathrm{l}} \otimes_{\mathbb{C}} \mathcal{V}_{\mathrm{r}}$
  - boundary conditions & defect lines
  - boundary fields & defect fields

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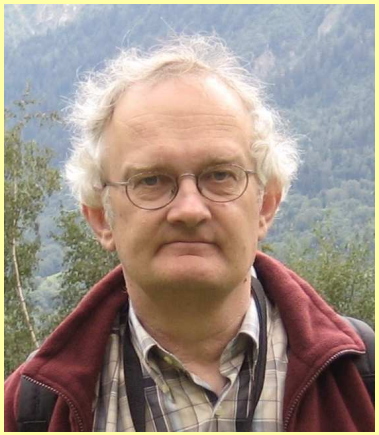
full local *rational* CFT : $F = \bigoplus_{i,j} \mathbb{C}^{Z_{ij}} \otimes_{\mathbb{C}} (R_i \boxtimes R_j)$

- modular invariance : $[Z, T] = 0 = [Z, S]$



pre-Bert

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find full local CFTs for a given chiral rational CFT by finding modular invariants

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e.g.

- 1984 C-diagonal modular invariant $Z_{ij} = C_{ij} = \delta_{i,\overline{j}}$ [B-P-Z]
- 1986 modular invariance of the torus partition function [Ca]
- 1986 D_n-type invariant for minimal models [I-Z]
- 1987 D_n, E_6 [Ge]
- 1987 E_7, E_8 [Ka C-I-Z]

- 1989 selection rules for extensions and coset constructions [M-S]

pre- and not-so-pre- Bert

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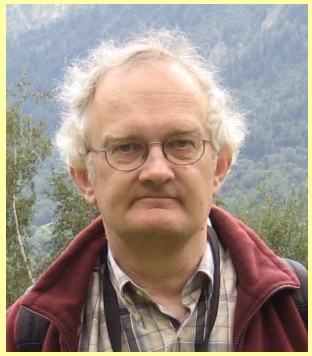
- 1986 conformal embeddings
- 1987 [B-B]



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Bert & Shimon

much better idea :

model-independent systematic construction of modular invariants :

via **simple currents** = invertible objects

- 1989 modular invariants from simple currents
- 1990 application to tensor products of N=2 models and string compactifications
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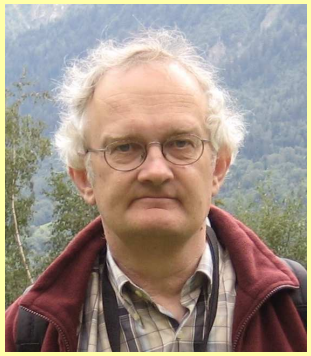
- provides *almost all* physical modular invariants
- terminology : modular invariant *not* coming from simple currents = *exceptional*

- another application of simple currents : twists of Grothendieck-Verdier structures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Bert & Beatriz & Christoph & me

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model-independent systematic construction of modular invariants :

via *Galois symmetries* of the modular S-matrix

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one lesson : many *unphysical* modular invariants

- 2003 example with unphysical true diagonal modular invariant
- 2016 criterion for the true diagonal modular invariant being unphysical [Da]

Bert & Christoph & me

another idea : simple current constructions via Lie theory

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- quantum subgroups e.g. 2021 [Ed]
- anomalies in gauge theories e.g. 2022 [H-L-M-M]
- missed opportunity : anyon condensation e.g. 2014 [E-R-B]

"[in l.c. ...] shown how KAC computes the S matrix for simple current extensions"

Other developments

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a few other developments:

more exceptional invariants – e.g.

- 1992 tensor products of SU(2) WZW models [F-K-S-V]
- 1994 SU(3) ("ADE ade") [Ga]
- 2000 SO(n) at level 2 and 3 [Ga]
- 2000 twisted doubles of finite groups [C-G-R]

Other developments

a few other developments:

- 2002 very many unphysical invariants [Ga]

  not a surprise – modular invariance is just one out of infinitely many constraints

  physical invariant $\equiv$ torus partition function of a consistent RCFT

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"if you cannot prove s.th.
then sooner or later you will find a counter example"

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- 2008 *physical invariants* $\overset{=}{\longleftrightarrow}$ *Morita classes of simple special symmetric Frobenius algebras internal to* $\mathcal{C}$ [F-F-R-S]

  $\overset{=}{\longleftrightarrow}$ *indecomposable pivotal* $\mathcal{C}$*-module categories*

$$\bigoplus_{i,j} \mathbf{Hom}_{AA}(R_i \otimes^{+} A \otimes^{-} R_j, A) \otimes_{\mathbb{C}} R_i \boxtimes R_j$$

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e.g.

simple current invariants

Schellekens algebra:

$$A = \Big(\bigoplus_{g \in H} R_g \; ; \; \omega \Big) \qquad \mathrm{d}\omega = \psi|_H \qquad H \leq \mathbf{Pic}^{\circ}(\mathcal{C})$$

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e.g.

C-diagonal invariant
simple current invariants
permutation invariants
of tensor product theories
} are all physical

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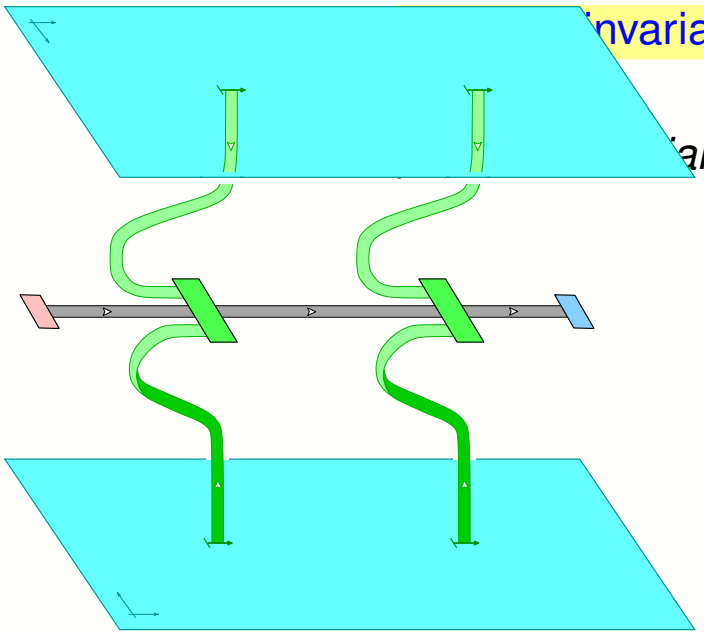

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- 2000 – 2006 given $\mathcal{C}$ and the Frobenius algebra A in $\mathcal{C}$
  can construct consistently all correlators [F^{π}-R-S]
  on world sheets of arbitrary topology

Other developments

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- 2021 bulk fields directly in terms of the module category $\mathcal{M}$:

  $=$ internal natural transformations [F-S]

$$F = \underline{\mathbf{Nat}}(\mathbf{Id}_{\mathcal{M}}, \mathbf{Id}_{\mathcal{M}}) \in \mathcal{Z}(\mathcal{C})$$

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  - braided equivalence $\mathcal{Z}(\mathcal{C}) \xrightarrow{\simeq} \mathcal{Z}(\mathcal{C}^*_{\mathcal{M}})$

    maps $\underline{\mathbf{Nat}}(\mathbf{Id}_{\mathcal{M}}, \mathbf{Id}_{\mathcal{M}}) \longmapsto \int_{G \in \mathcal{C}^\star_{\mathcal{M}}} G^{\mathrm{r.a.}} \circ G \overset{\mathrm{ssi}}{=} \bigoplus_m G_m^{\mathrm{r.a.}} \circ G_m$

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    i.e. every bulk algebra is C-diagonal when regarded as an object in $\mathcal{Z}(\mathcal{C}^*_{\mathcal{M}})$

    i.e. **"every physical partition function is C-diagonal"**

Outlook