Fourier noise reduction methods

Antares data-set

Looking for periodic sources in Antares data

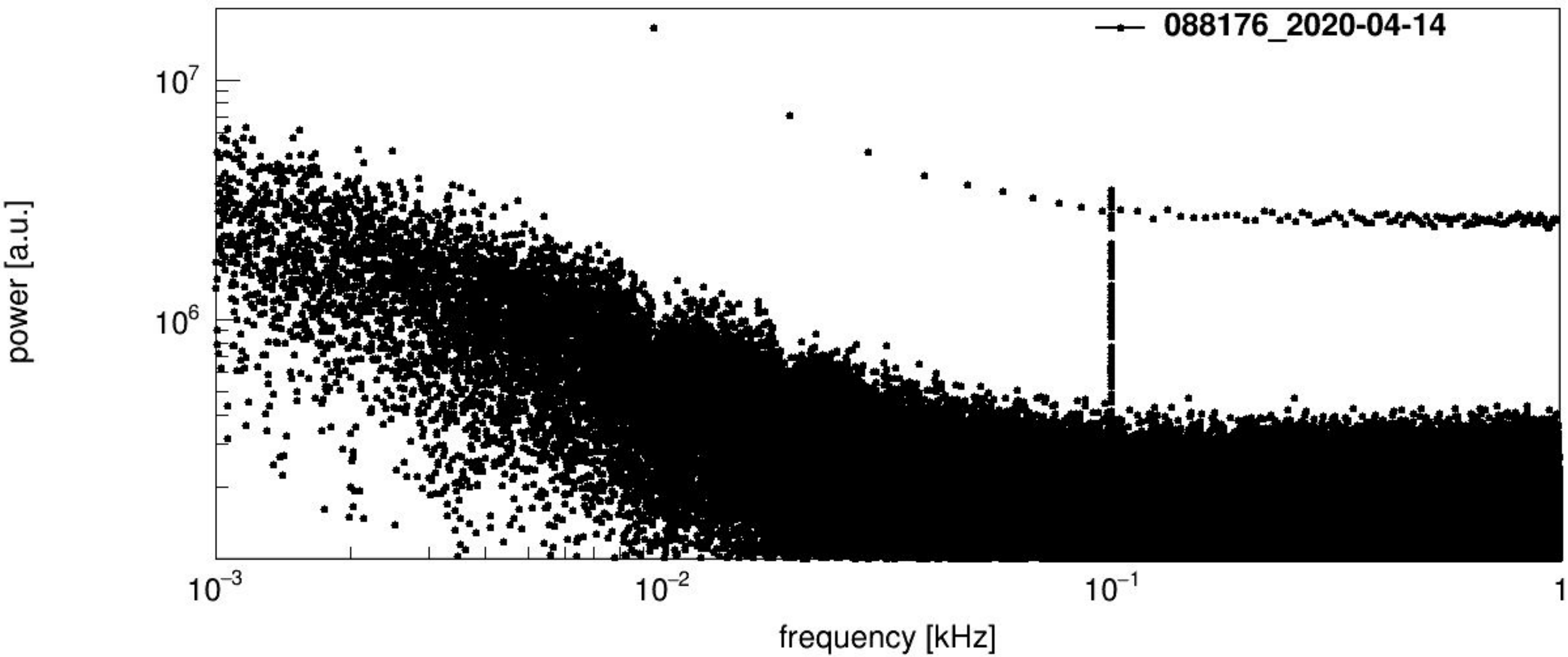
All data is used

Energy ~ MeV

Histogram into (standard) fast fourier transform (FFT)

Noise increases near lower frequencies

Applied transform



Methods of noise reduction

Spectrum Subtraction (SS)

- Subtracting estimate of background from signal

Short time fourier transform (STFT)

- Cuts histogram into smaller pieces
- Difficult to achieve compatibility between t and f

Non Harmonic Analysis

Fourier coefficient is estimated by least squares method

Repeatedly applies steepest descent and Newton's method

Non Harmonic Analysis

Accurate estimation of the frequency f, amplitude A, and initial phase ϕ, avoiding the dominance of the analysis window length

Higher accuracy than theoretical upper limit of DFT

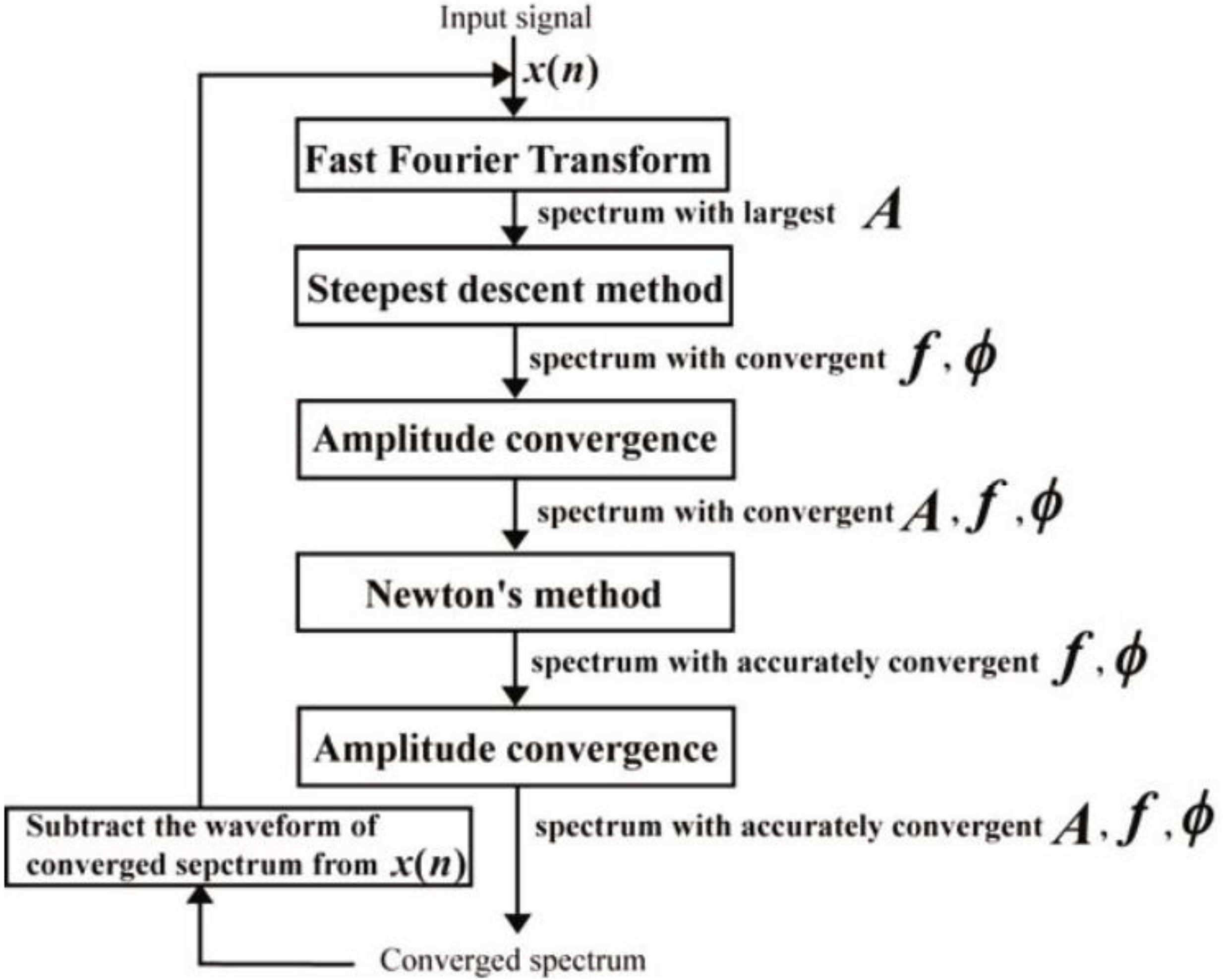


Figure 2 NHA algorithm.

$$F(A, f, \varphi) = \frac{1}{N} \sum_{n=0}^{N-1} \{x(t) - A\cos(2\pi f t + \varphi)\}^2,$$

Steepest descent

Sum over histogram

Guess a wave, any wave!

f and ϕ first, A after

$$\hat{f}_{m+1} = \hat{f}_m - \mu_m \frac{\partial F}{\partial f},$$

$$\hat{\varphi}_{m+1} = \hat{\varphi}_m - \mu_m \frac{\partial F}{\partial \varphi}.$$

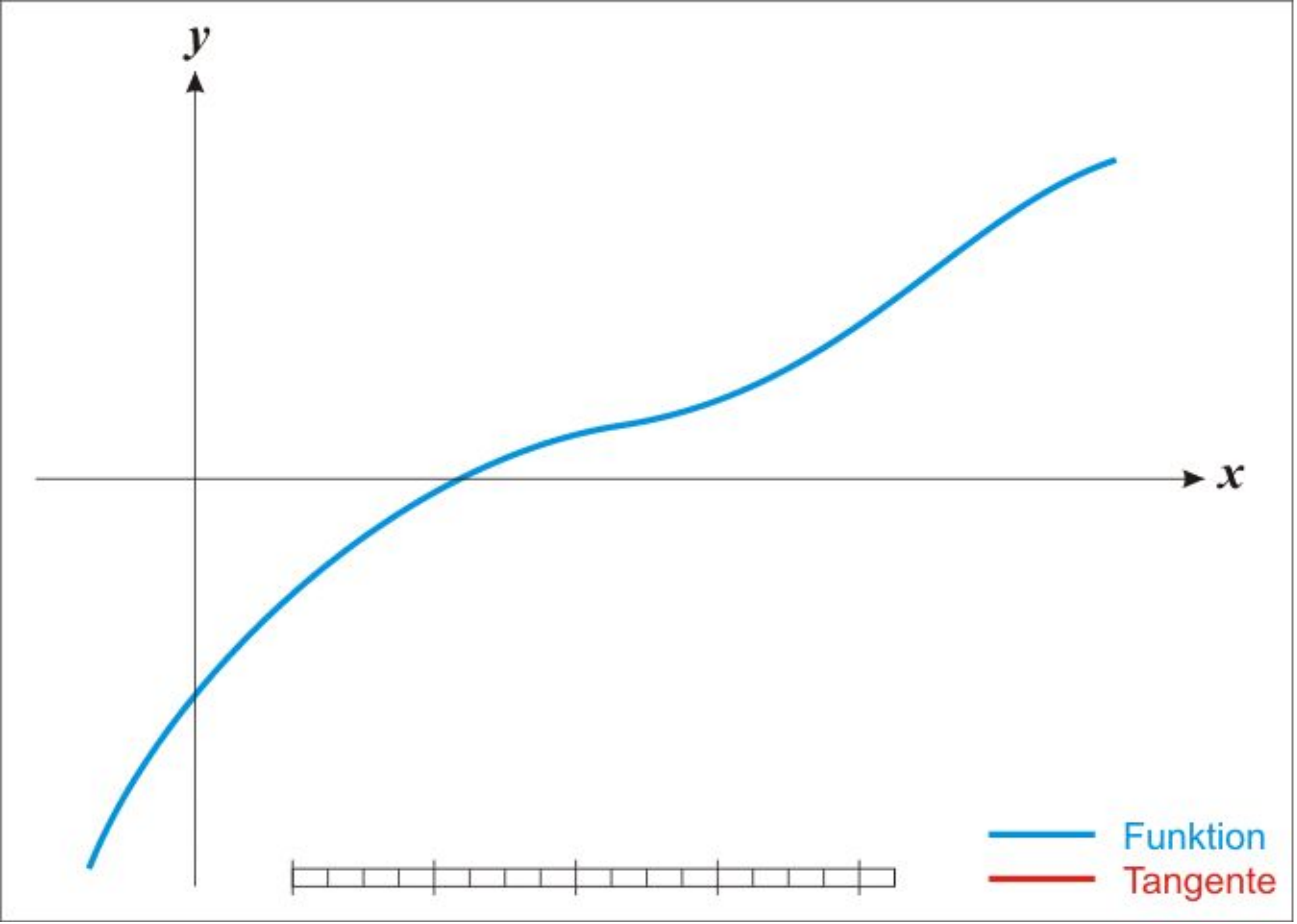
$$\hat{A}_{m+1} = \hat{A}_m - \mu_m \frac{\partial F}{\partial A},$$

Newton's method

Find root for cost function

Since if F =0 we have found a perfect fit for our wave

second order


y
x
Funktion
Tangente

Second order

$$f(x+\Delta x, p+\Delta p) = f(x,p) + f_x \Delta x + \tfrac{1}{2} f_{xx} \Delta^2 x + f_p \Delta p + \frac{1}{2} f_{pp} \Delta^2 p + f_{xp} \Delta p \Delta x \quad (8)$$

$$\Delta x = -\frac{f_p \Delta p + \frac{1}{2} f_{pp} \Delta^2 p}{f_x + f_{xp} \Delta p} \left[1 + \frac{f_p \Delta p + \frac{1}{2} f_{pp} \Delta^2 p}{(f_x + f_{xp} \Delta p)^2} f_{xx} \right] \quad (9)$$

Applied to the cost function

$$\hat{f}_{m+1} = \hat{f}_m - \frac{\upsilon_m}{J} \begin{vmatrix} \frac{\partial F}{\partial f} & \frac{\partial^2 F}{\partial f \partial \varphi} \\ \frac{\partial F}{\partial \varphi} & \frac{\partial^2 F}{\partial \varphi^2} \end{vmatrix},$$

$$\hat{\varphi}_{m+1} = \hat{\varphi}_m - \frac{\upsilon_m}{J} \begin{vmatrix} \frac{\partial^2 F}{\partial f^2} & \frac{\partial F}{\partial f} \\ \frac{\partial^2 F}{\partial f \partial \varphi} & \frac{\partial F}{\partial \varphi} \end{vmatrix},$$

where

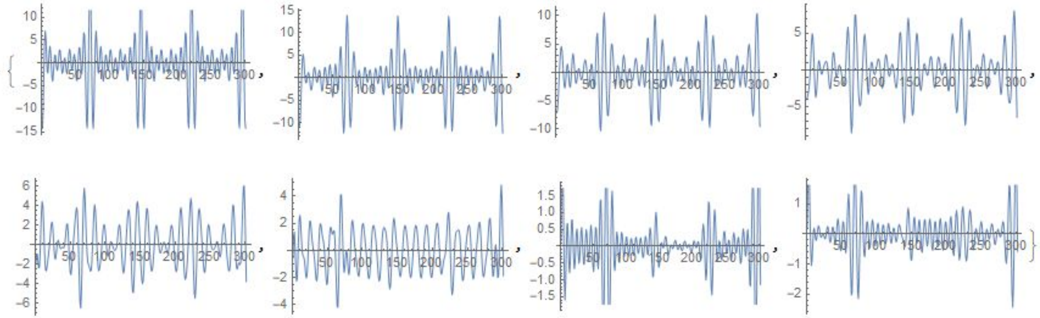
$$J = \begin{vmatrix} \frac{\partial^2 F}{\partial f^2} & \frac{\partial^2 F}{\partial f \partial \varphi} \\ \frac{\partial^2 F}{\partial f \partial \varphi} & \frac{\partial^2 F}{\partial \varphi^2} \end{vmatrix}.$$

Idealized data

Searching iteratively

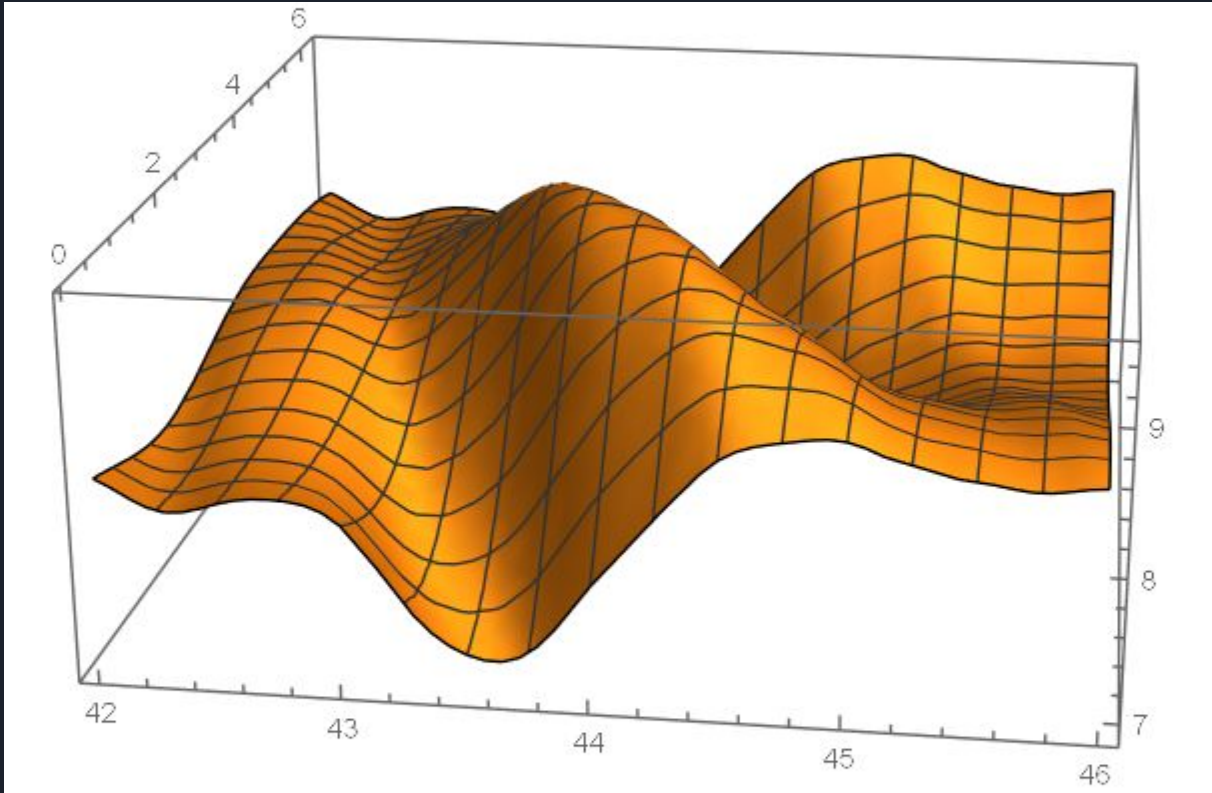
```
In[128]:= SpecifiedTestFunction = Sum[.4 k Cos[4 k 2 Pi var + .4 k], {k, 5, 10}];
Plotlist = {};
Guesslist = {};
For[s = 0, s < 8, s ++,
 ourtestdata = testdata[300, 1 / 300, SpecifiedTestFunction];
 ourplot = ListLinePlot[ourtestdata];
 Print[s];
 Plotlist = Append[Plotlist, ourplot];
 {FOut, PhiOut, AOut} = CompleteFunction[200, 1 / 200, 300, 0.05, SpecifiedTestFunction, 10^(-6)];
 Guesslist = Append[Guesslist, {FOut, PhiOut, AOut}];
 guess = AOut Cos[FOut 2 Pi var + PhiOut];
 SpecifiedTestFunction = SpecifiedTestFunction - guess
]
Plotlist
Guesslist

0
1
2
3
4
5
6
7
```

Out[132]=



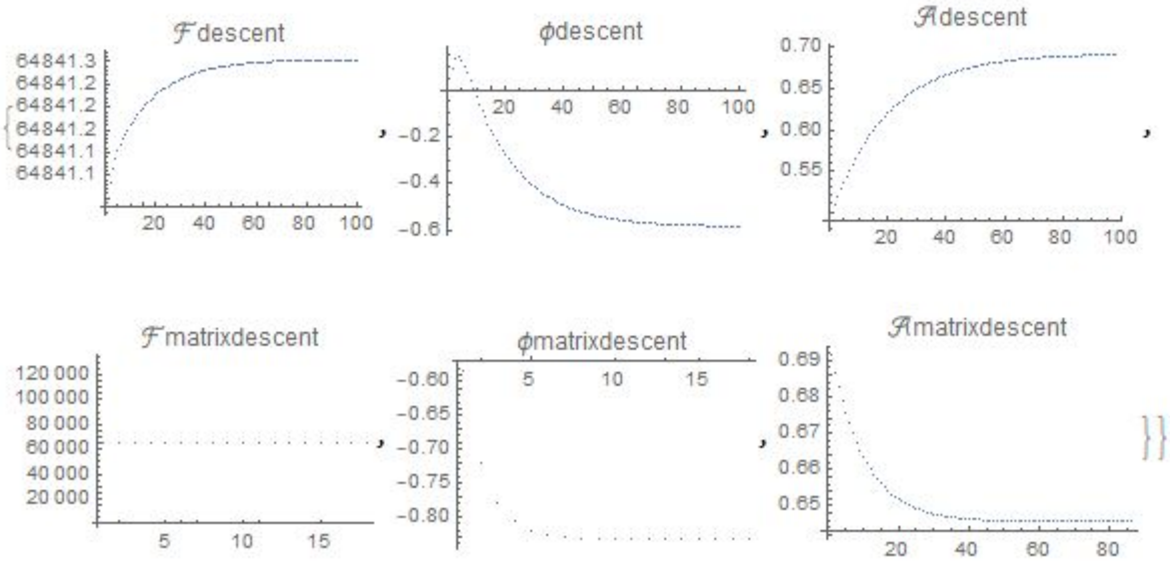
```
Out[133]= {{40.0849, -2.59294, 4.12941}, {36.0899, -3.00513, 3.64459}, {32.0901, -3.4029, 3.18481}, {28.0819, 2.50439, 2.74158},
 {24.0595, 2.16869, 2.32578}, {20.0019, 1.94435, 1.94196}, {40.7223, -1.63267, 0.579088}, {37.521, -0.0320461, -0.0134654}}
```

Optimization space



Applied to real Antares data

```
In[]:= ourproducts = Timing[CompleteFunction[10^4, 10^(-4), 100, 0.05, 10^(-4)]]

Out[]= {38.1406, {{64841.3, -0.834105, 0.64555},
```




```
}}}
```